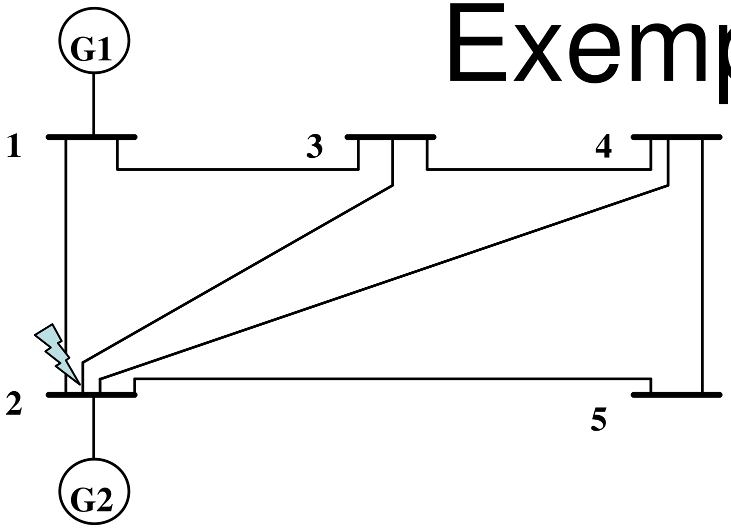
Sincronismo X Estabilidade

Formulação COA e Máquina como Referência

Exemplo Motivacional



Dados de Linha

Barra	Tensão	Ângulo	MW	MVAr
1	1.060	0.0	0	0
2	1.042	-2.7	20	10
3	1.016	-4.9	45	15
4	1.015	-5.2	40	5
5	1.010	-6.1	60	10

Dados de Barra

Barra	Barra	Resist.	Reat	Suscept.
1	2	0.02	0.06	0.03
1	3	0.08	0.24	0.025
2	3	0.06	0.18	0.02
2	4	0.06	0.18	0.02
2	5	0.04	0.12	0.015
3	4	0.01	0.03	0.01
4	5	0.08	0.24	0.025

Dados de Gerador

Barra	Reat. Trans.	M
1	0.25	0.265
2	0.50	0.005

Não existe equilíbrio!

Matriz Sistema Reduzido Pós-Falta

$$Y_{RED} = \begin{bmatrix} 0{,}425 - j1{,}04 & 0{,}205 + j0{,}756 \\ 0{,}205 + j0{,}756 & 0{,}357 - j1{,}132 \end{bmatrix}$$

$$M\dot{\omega}_1 = P_{m1} - E_1^{\,2} G_{11} - E_1 E_2 Y_{12} \cos\left(\phi_{12} - \delta_{12}\right)$$
$$M\dot{\omega}_2 = P_{m2} - E_2^{\,2} G_{22} - E_2 E_1 Y_{21} \cos\left(\phi_{21} - \delta_{21}\right)$$

No equilíbrio:

$$P_{m1} - E_1^{\,2} G_{11} - E_1 E_2 Y_{12} \cos\left(\phi_{12} - \delta_{12}\right) = 0$$
$$P_{m2} - E_2^{\,2} G_{22} - E_2 E_1 Y_{21} \cos\left(\phi_{21} - \delta_{21}\right) = 0$$

$$0{,}754 - 1{,}049\cos(74{,}82^o - \delta_{12}) = 0$$
$$-0{,}092 - 1{,}049\cos(74{,}82^o + \delta_{12}) = 0$$

$$\begin{array}{rl} & 0{,}754 - 1{,}049\cos 74{,}82 \cos\delta_{12} - 1{,}049 \,\mathrm{sen}\, 74{,}82 \,\mathrm{sen}\, \delta_{12} \\ + & \underline{-0{,}092 - 1{,}049\cos 74{,}82 \cos\delta_{12} + 1.049 \,\mathrm{sen}\, 74{,}82 \,\mathrm{sen}\, \delta_{12}} \\ & 0{,}662 - 0{,}549\cos\delta_{12} = 0 \end{array}$$

Formulação Máquina como Referência

$$\dot{\omega}_1 = \frac{P_{m1} - P_{e1}}{M_1}$$

$$\vdots$$

$$\dot{\omega}_n = \frac{P_{mn} - P_{en}}{M_n}$$

$$\dot{\delta}_1 = \omega_1$$

$$\vdots$$

$$\dot{\delta}_n = \omega_n$$

$$\begin{cases} \dot{\omega}_{1n} = \dot{\omega}_1 - \dot{\omega}_n = \dfrac{P_{m1} - P_{e1}}{M_1} - \dfrac{P_{mn} - P_{en}}{M_n} \\ \vdots \qquad \vdots \qquad \qquad \vdots \\ \dot{\omega}_{(n-1)n} = \dot{\omega}_{(n-1)} - \dot{\omega}_n = \dfrac{P_{m(n-1)} - P_{e(n-1)}}{M_{(n-1)}} - \dfrac{P_{mn} - P_{en}}{M_n} \\ \dot{\delta}_{1n} = \dot{\delta}_1 - \dot{\delta}_n = \omega_1 - \omega_n \\ \vdots \qquad \vdots \qquad \vdots \\ \dot{\delta}_{(n-1)n} = \dot{\delta}_{(n-1)} - \dot{\delta}_n = \omega_{(n-1)} - \omega_n \end{cases}$$

$$\begin{cases} \dot{\omega}_n = \dfrac{P_{mn} - P_{en}}{M_n} \\ \dot{\delta}_n = \omega_n \end{cases}$$

Formulação COA – Center of Angle

COA:

$$\delta_o = \frac{1}{M_T}\sum_{i=1}^{n} M_i\delta_i \qquad M_T = \sum_{i=1}^{n} M_i$$

Derivando 2 vezes:

$$\omega_o = \frac{1}{M_T}\sum_{i=1}^{n} M_i\omega_i$$

$$M_T\dot{\omega}_o = \sum_{i=1}^{n}\left(P_{mi} - P_{ei}\right) = P_{COA}$$

Novas variáveis:

$$\theta_i = \delta_i - \delta_o$$

$$\tilde{\omega}_i = \omega_i - \omega_o$$

$$\boxed{\begin{aligned}\dot{\tilde{\omega}}_i &= \frac{P_{mi} - P_{ei}}{M_i} - \frac{1}{M_T}P_{COA}\\ \dot{\theta}_i &= \tilde{\omega}_i\end{aligned}}$$

Exercício: Mostrar para modelo de rede reduzida que

$$P_{COA} = \sum_{i=1}^{n}(P_{mi} - E_i^2 G_{ii}) - 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} D_{ij}\cos\left(\delta_i - \delta_j\right)$$