

Filtragem no domínio de frequência

Filtragem no domínio de frequência

Modificar a transformada de Fourier de uma imagem e computar a inversa para obter o resultado.

Dada uma imagem $f(x,y)$, $M \times N$, a equação básica de filtragem tem a forma

$$g(x, y) = \mathfrak{F}^{-1}[H(u, v)F(u, v)]$$

onde $H(u,v)$ é a função de filtro, $F(u,v)$ é a transformada da imagem original e $g(x,y)$ é a imagem filtrada.

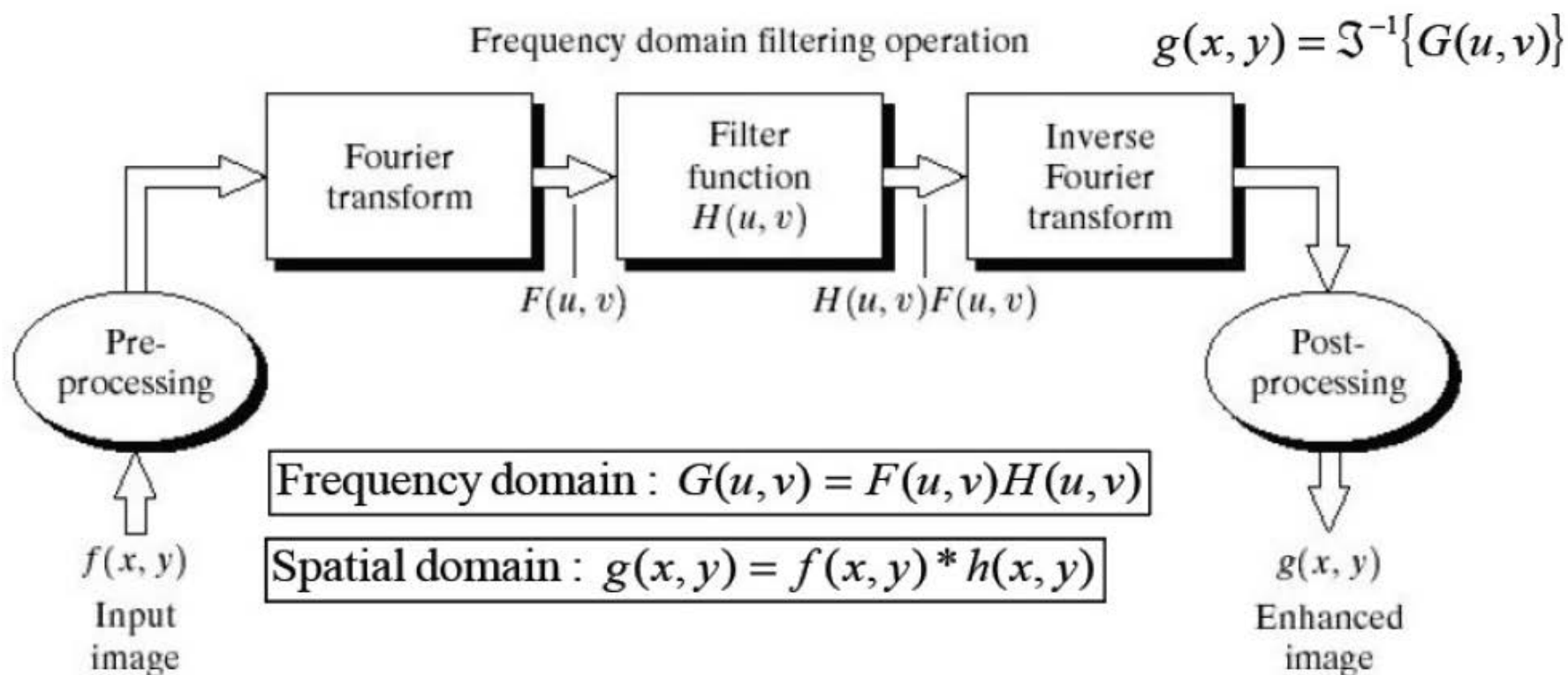
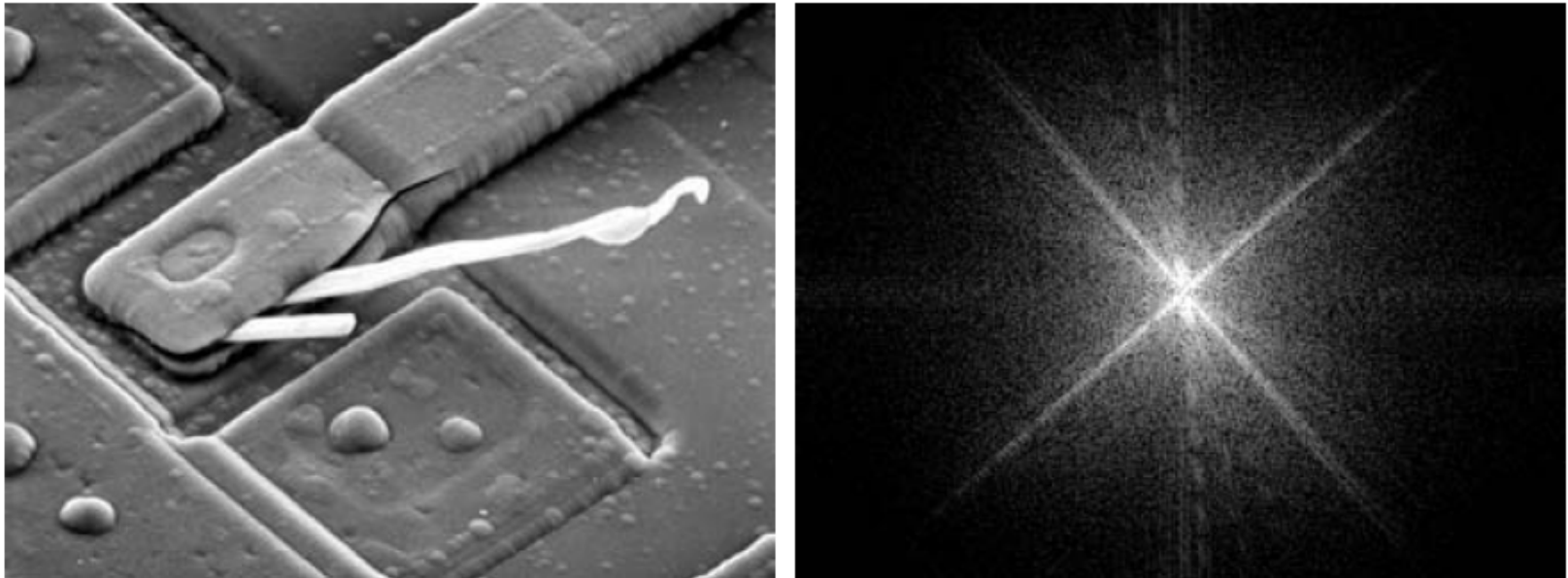


FIGURE 4.5 Basic steps for filtering in the frequency domain.



a b

FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

Ex. filtro simples

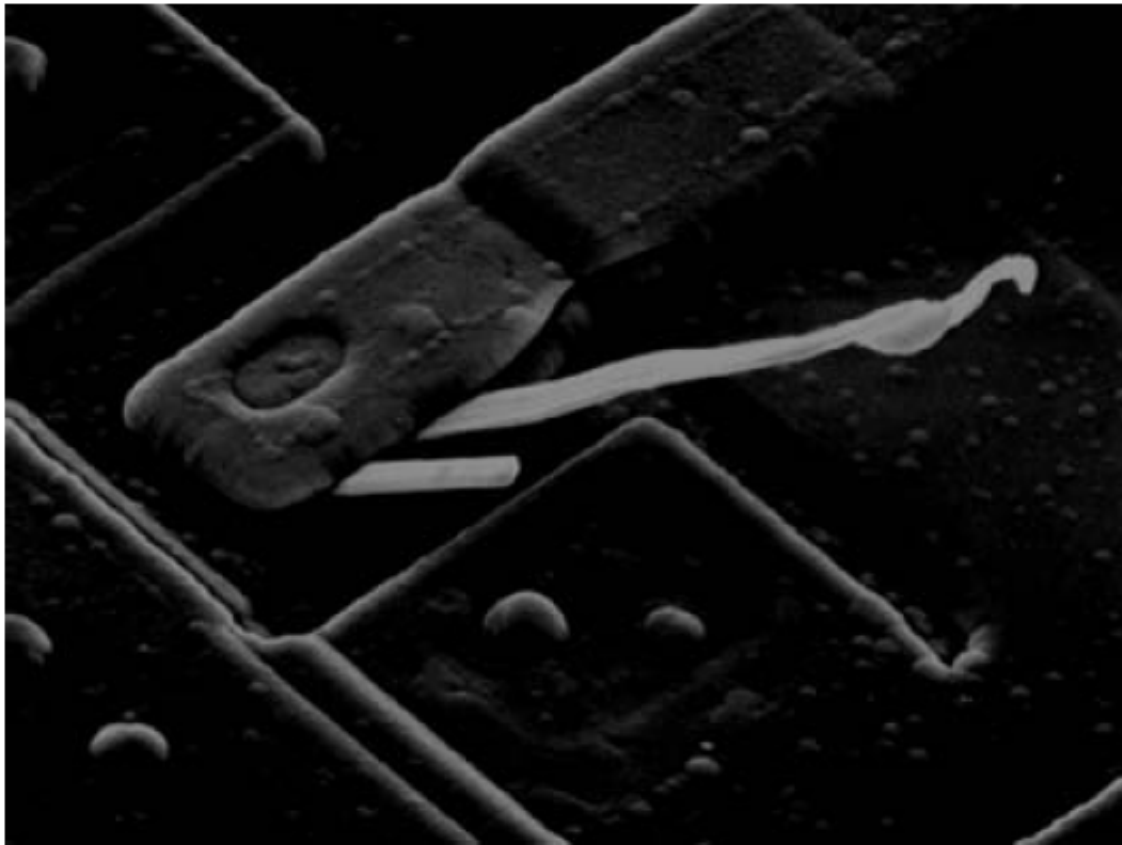
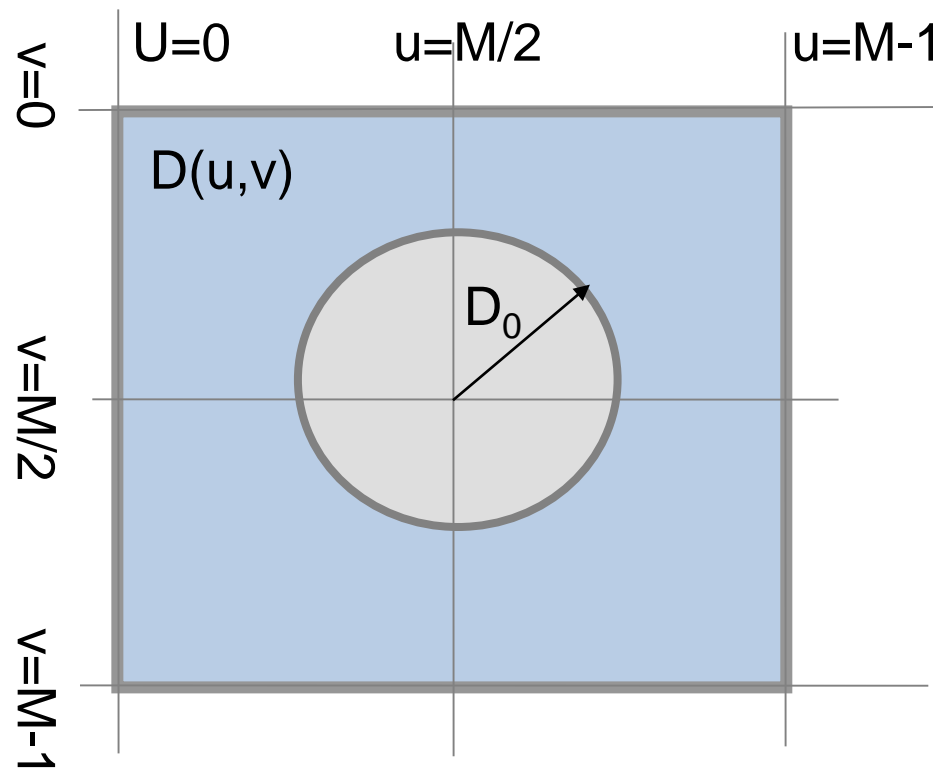


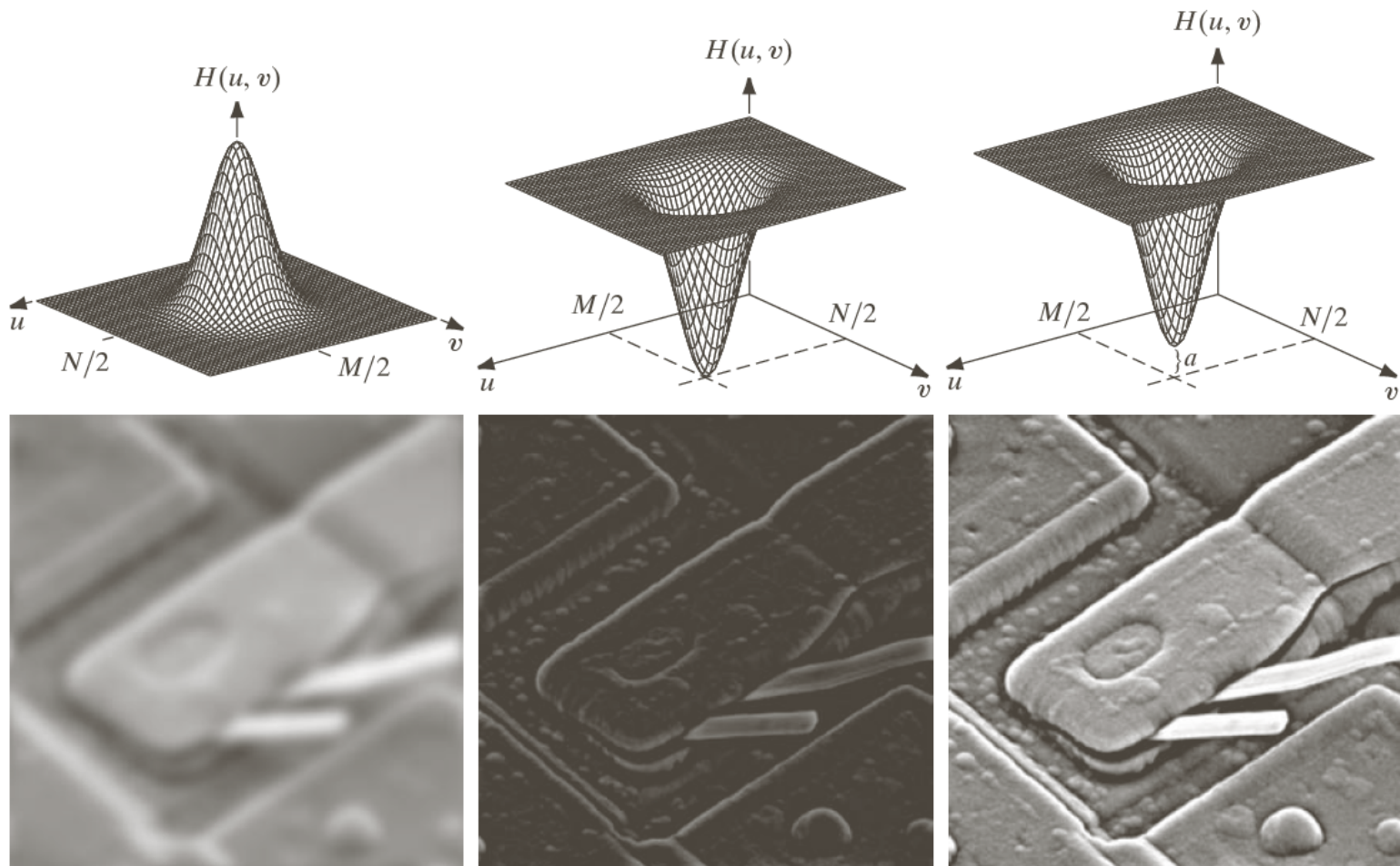
FIGURE 4.30

Result of filtering the image in Fig. 4.29(a) by setting to 0 the term $F(M/2, N/2)$ in the Fourier transform.

Filtrando em 2D

Distribuição de frequências em 2D





a	b	c
d	e	f

FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

Sobre *padding*...

- Eq. da filtragem: produto de duas funções => convolução no domínio espacial, pelo teorema da convolução.
 - Vimos que se as funções não forem adequadamente preenchidas (*padding*), teremos erro de *wraparound*.
- Imagem é, em geral, dada no domínio espacial, mas filtro pode ser especificado no domínio da frequência...
 - uma ideia seria construir um filtro do tamanho da imagem, computar o IDFT para obter o filtro espacial correspondente, fazer o *padding* do filtro no domínio espacial e então computar o DFT para retornar ao domínio da frequência.
 - O exemplo 1-D a seguir ilustra o perigo dessa abordagem...

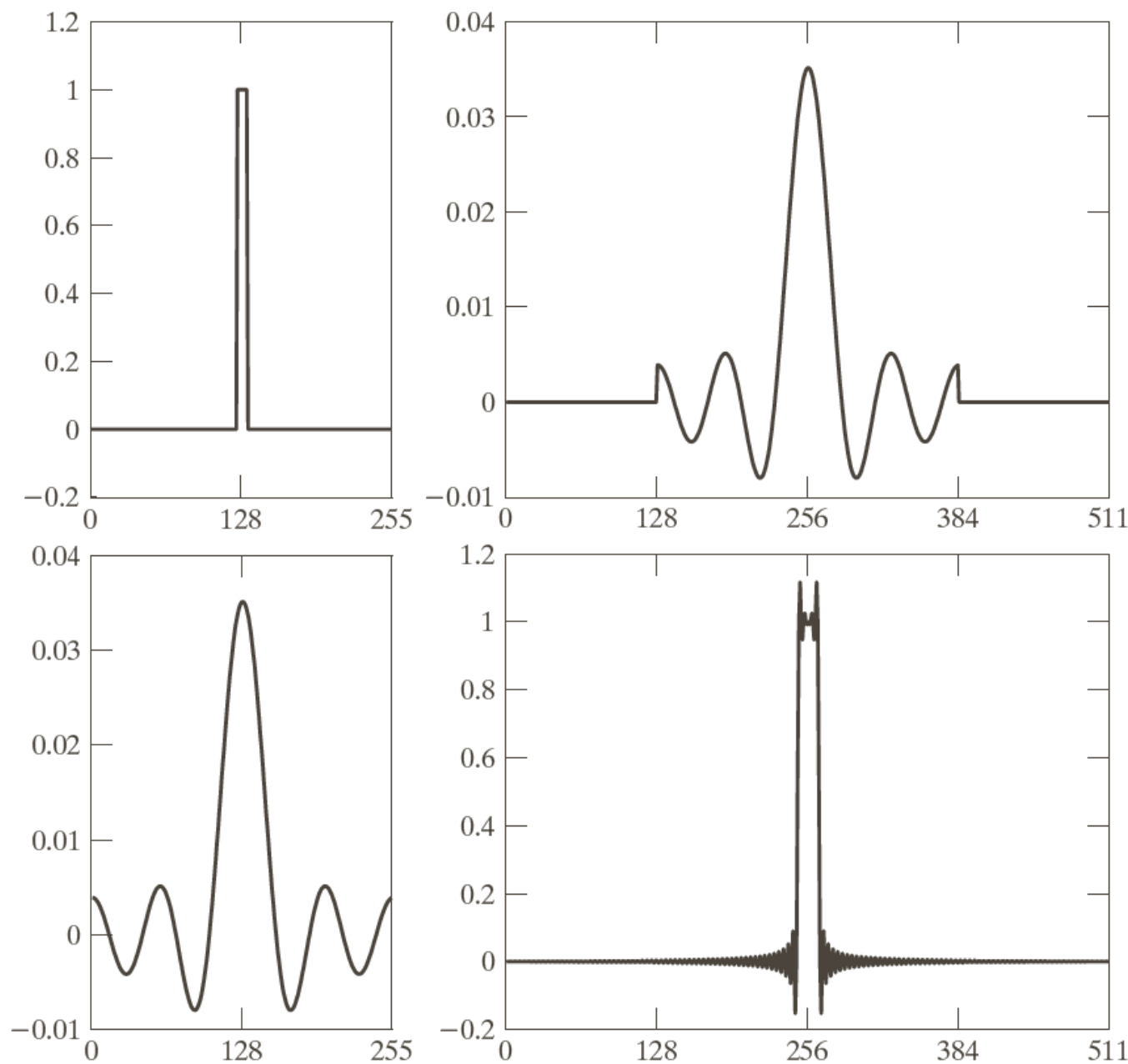


FIGURE 4.34
 (a) Original filter specified in the (centered) frequency domain.
 (b) Spatial representation obtained by computing the IDFT of (a).
 (c) Result of padding (b) to twice its length (note the discontinuities).
 (d) Corresponding filter in the frequency domain obtained by computing the DFT of (c). Note the ringing caused by the discontinuities in (c). (The curves appear continuous because the points were joined to simplify visual analysis.)

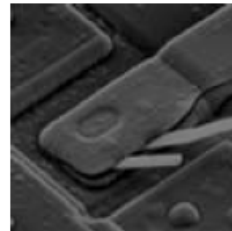
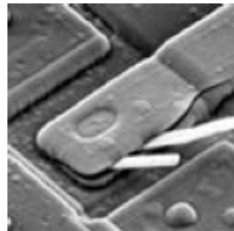
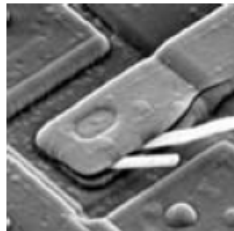
Passos p/ filtragem no domínio de frequência

1. dada uma imagem de entrada $f(x,y)$, $M \times N$, obter $P=2M$ e $Q=2N$ (valores para o *padding*).
2. formar a imagem com *padding*, $f_p(x,y)$, de tamanho $P \times Q$, colocando o número necessário de zeros em $f(x,y)$
3. multiplicar $f_p(x,y)$ por $(-1)^{x+y}$ para centralizar a transformada.
4. computar a DFT, $F(u,v)$, da imagem do passo 3.
5. gerar uma função de filtro real, simétrica, $H(u,v)$, de tamanho $P \times Q$ com centro em $P/2$ e $Q/2$. Formar o produto $G(u,v) = H(u,v)F(u,v)$ usando a multiplicação matricial.
6. obter a imagem processada:

$$g_p(x, y) = \left\{ r \left[\mathcal{F}^{-1}[G(u, v)] \right] \right\} (-1)^{x+y}$$

onde a parte real é selecionada para ignorar os componentes complexos espúrios.

7. obter o resultado final, $g(x,y)$, recortando a região $M \times N$ na área superior esquerda de $g_p(x,y)$.



a	b	c
d	e	f
g	h	

FIGURE 4.36

- (a) An $M \times N$ image, f .
 (b) Padded image, f_p of size $P \times Q$.
 (c) Result of multiplying f_p by $(-1)^{x+y}$.
 (d) Spectrum of F_p . (e) Centered Gaussian lowpass filter, H , of size $P \times Q$.
 (f) Spectrum of the product HF_p .
 (g) g_p , the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p .
 (h) Final result, g , obtained by cropping the first M rows and N columns of g_p .

Filtros *zero-phase shift*

Analisemos o ângulo de fase da transformada filtrada.

Devido ao DFT ser uma matriz complexa, pode-se expressar os termos em partes reais e imaginárias:

$$F(u,v) = R(u,v) + jI(u,v)$$

A Eq. de filtragem fica

$$g(x, y) = \mathfrak{F}^{-1} \left[H(u, v) R(u, v) + j (-H^*(u, v)) I(u, v) \right]$$

O ângulo de fase não é alterado pois $H(u,v)$ cancela quando é calculada a fase:

$$\Phi(u, v) = \arctan \left[\frac{I(u, v)}{R(u, v)} \right]$$

Esse tipo de filtro é chamado de *zero-phase-shift*. É o tipo de filtro que consideramos...

Filtros: correspondência espaço e frequência

Filtros Gaussianos são interessantes porque ambas as transformadas de uma função Gaussiana são funções Gaussianas reais.

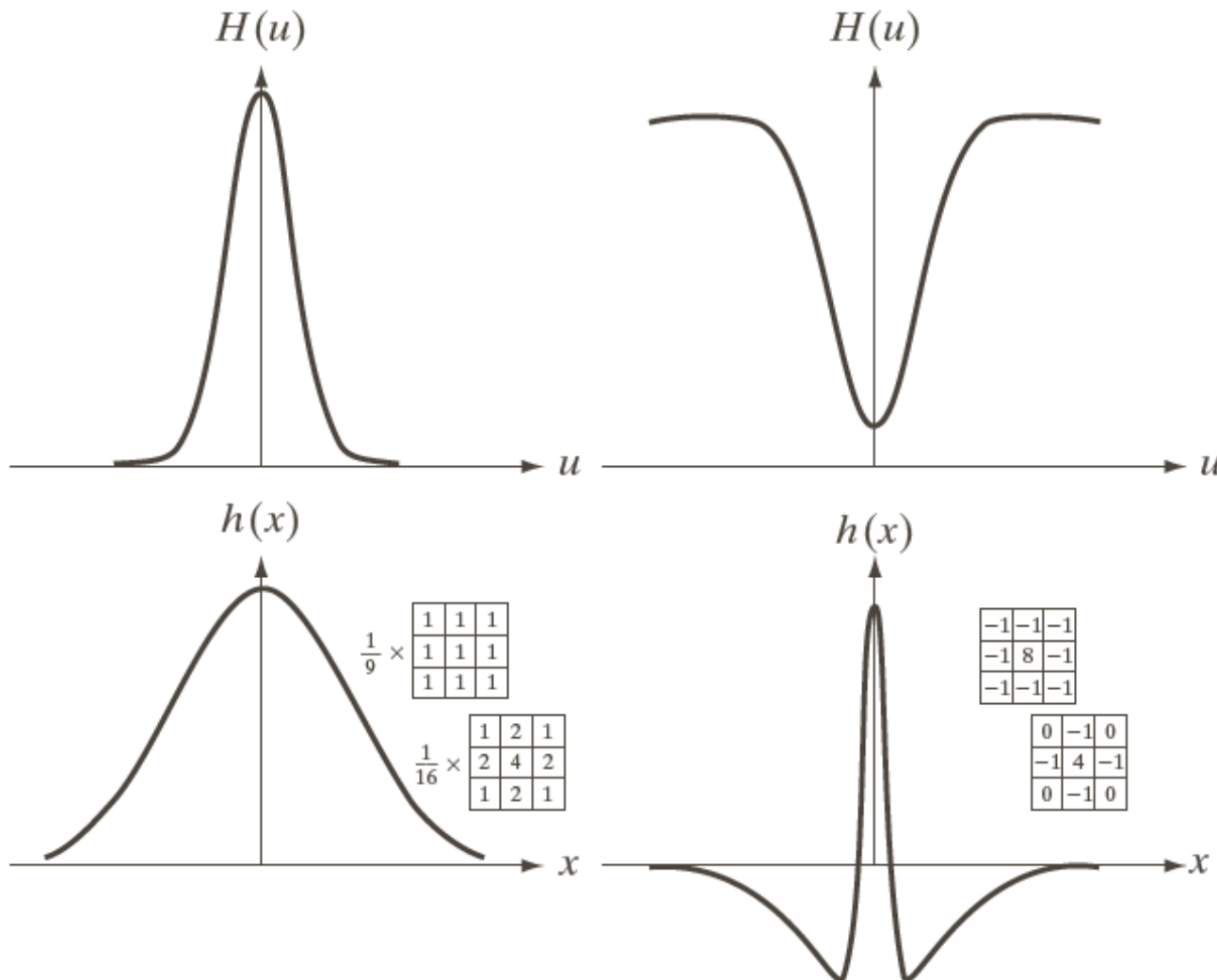
Seja $H(u)$ um filtro Gaussiano no domínio da frequência 1-D

$$H(u) = A e^{-u^2/2\sigma^2}$$

onde σ é o desvio padrão da curva Gaussiana. O filtro correspondente no domínio espacial (inversa de $H(u)$) é

$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2 x^2}$$

Filtros Gaussianos

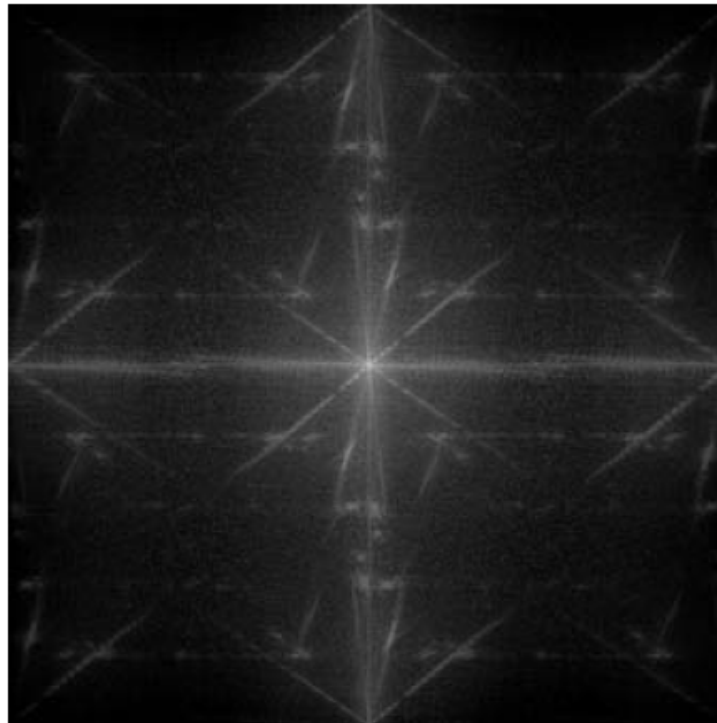


a	c
b	d

FIGURE 4.37

(a) A 1-D Gaussian lowpass filter in the frequency domain. (b) Spatial lowpass filter corresponding to (a). (c) Gaussian highpass filter in the frequency domain. (d) Spatial highpass filter corresponding to (c). The small 2-D masks shown are spatial filters we used in Chapter 3.

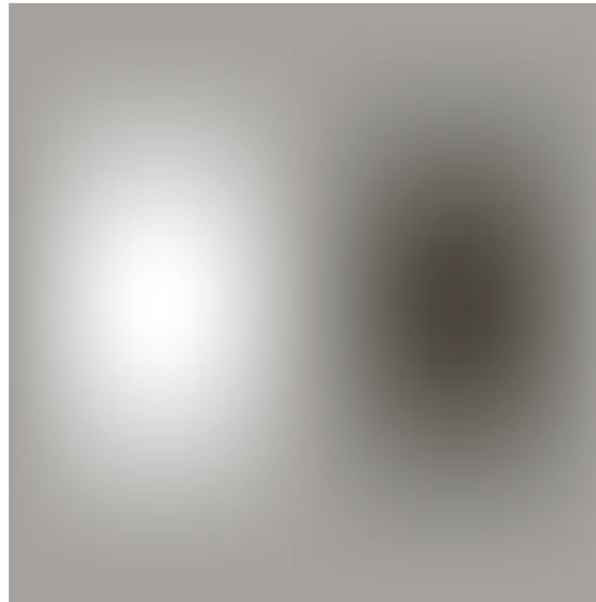
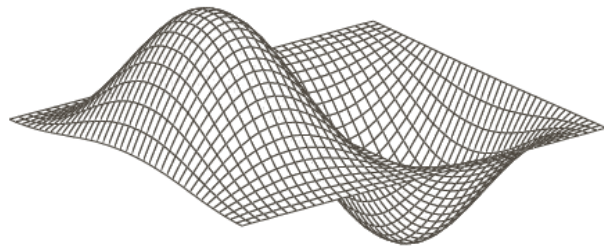
Ex. 4.15 (Gonzalez)



a b

FIGURE 4.38
(a) Image of a building, and
(b) its spectrum.

-1	0	1
-2	0	2
-1	0	1



a	b
c	d

FIGURE 4.39

(a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.

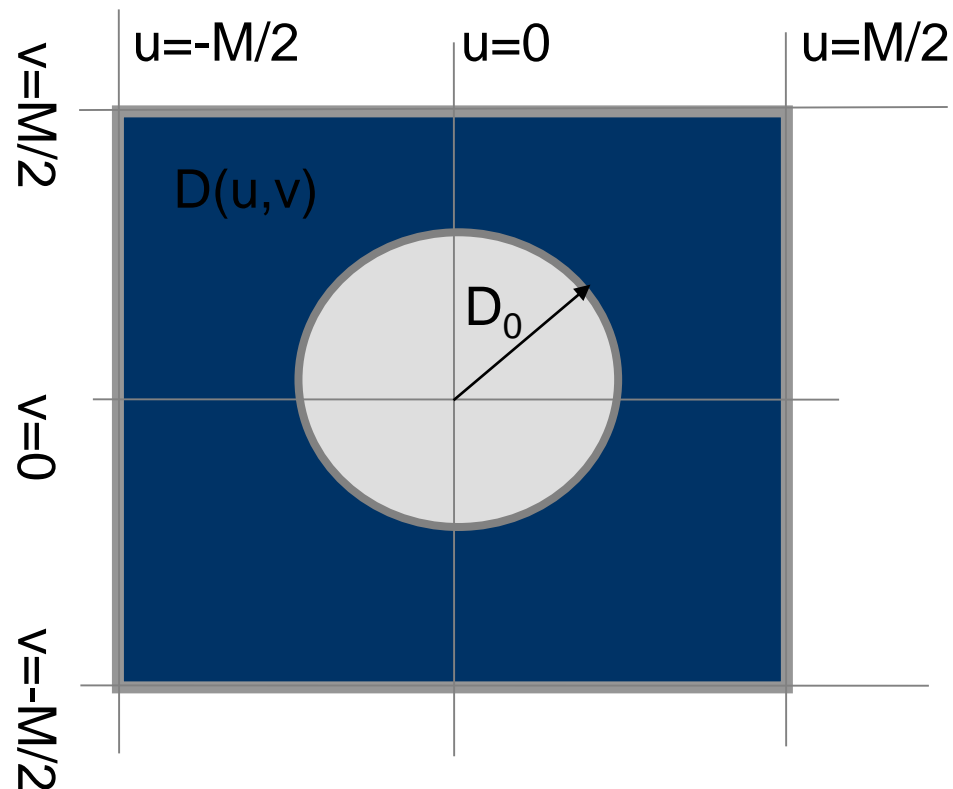


Suavização de imagens com filtros no domínio da frequência

- Três tipos de filtros de suavização (*low-pass filter*): Ideal, *Butterworth* e Gaussiano.
- O filtro *Butterworth* tem um parâmetro chamado *ordem* do filtro. Valores altos de *ordem* aproximam o filtro do Ideal, valores baixos o tornam mais parecido com o Gaussiano.
 - Todas as funções $H(u,v)$, são entendidas como funções discretas de tamanho $P \times Q$.

Filtrando em 2D

Distribuição de frequências em 2D



Filtros passa-baixa

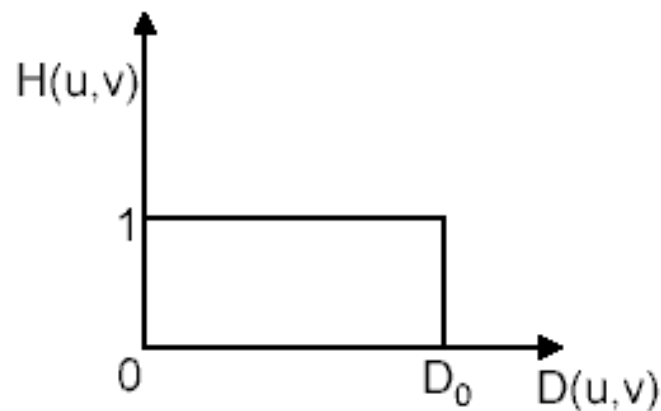
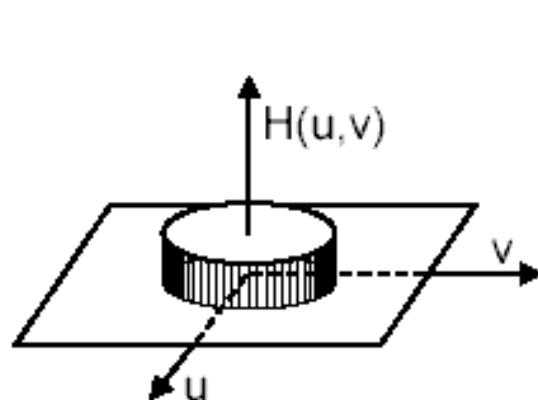
- Preservam apenas as baixas frequências
 - Filtro Ideal (ILPF)
 - Filtro Butterworth (BLPF)
 - Filtro Gaussiano (GLPF)
- Qual o efeito visual?

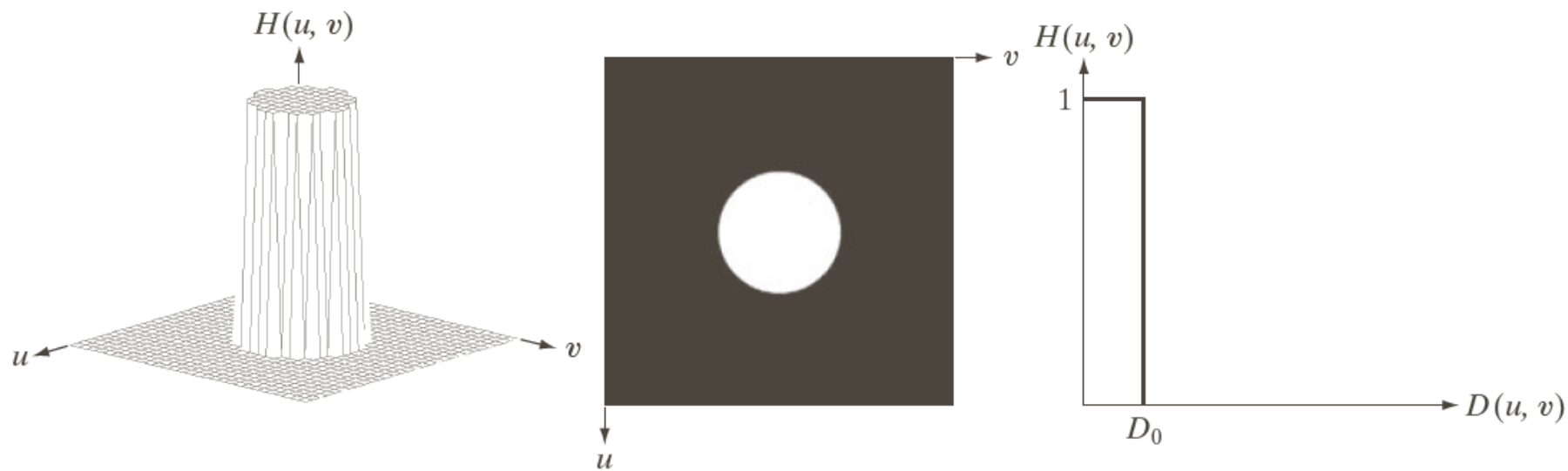
$H(u,v)$ - Ideal Low Pass Filter

$$H(u,v) = \begin{cases} 1 & D(u,v) \leq D_0 \\ 0 & D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \sqrt{u^2 + v^2}$$

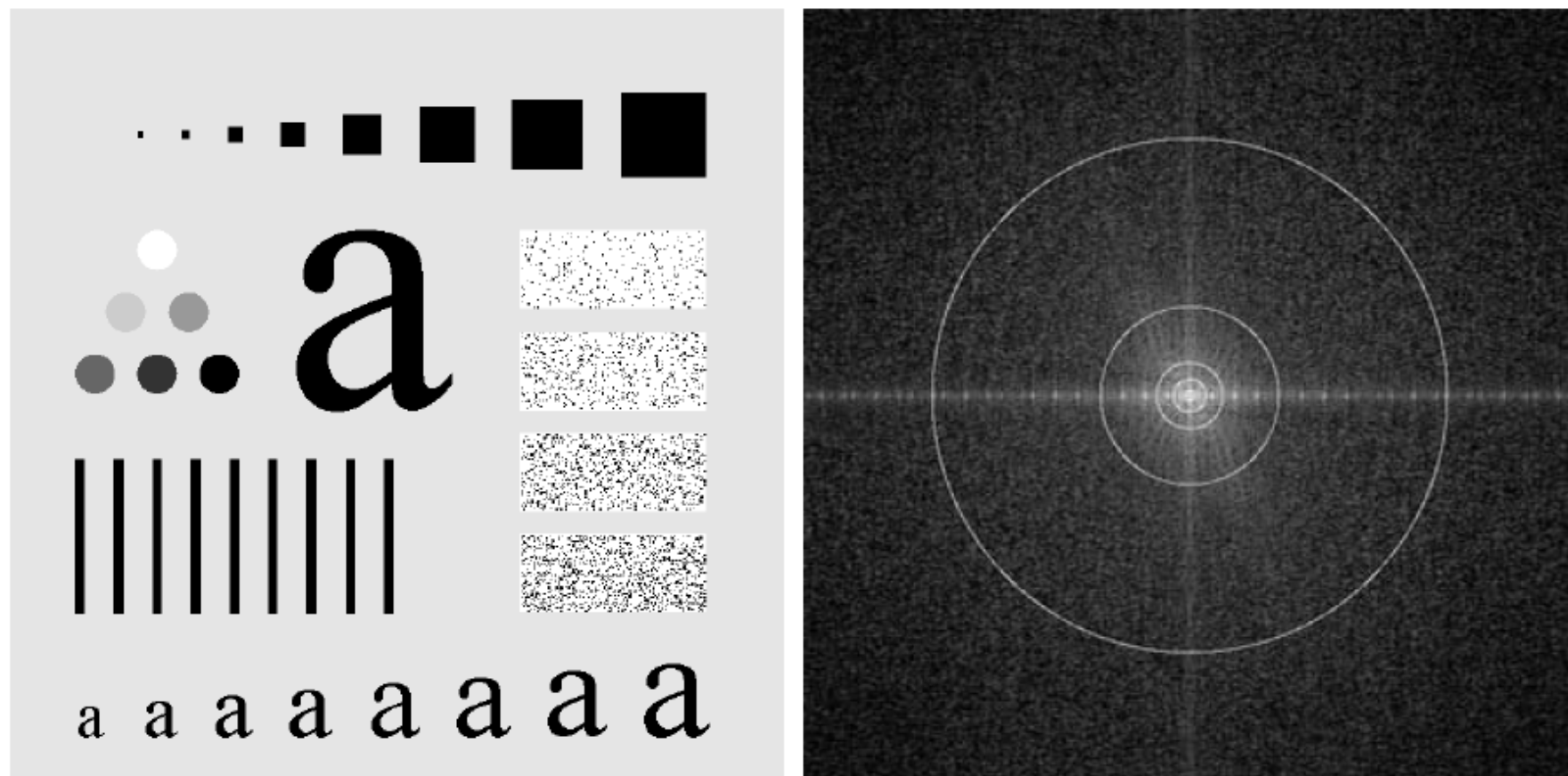
D_0 = cut off frequency





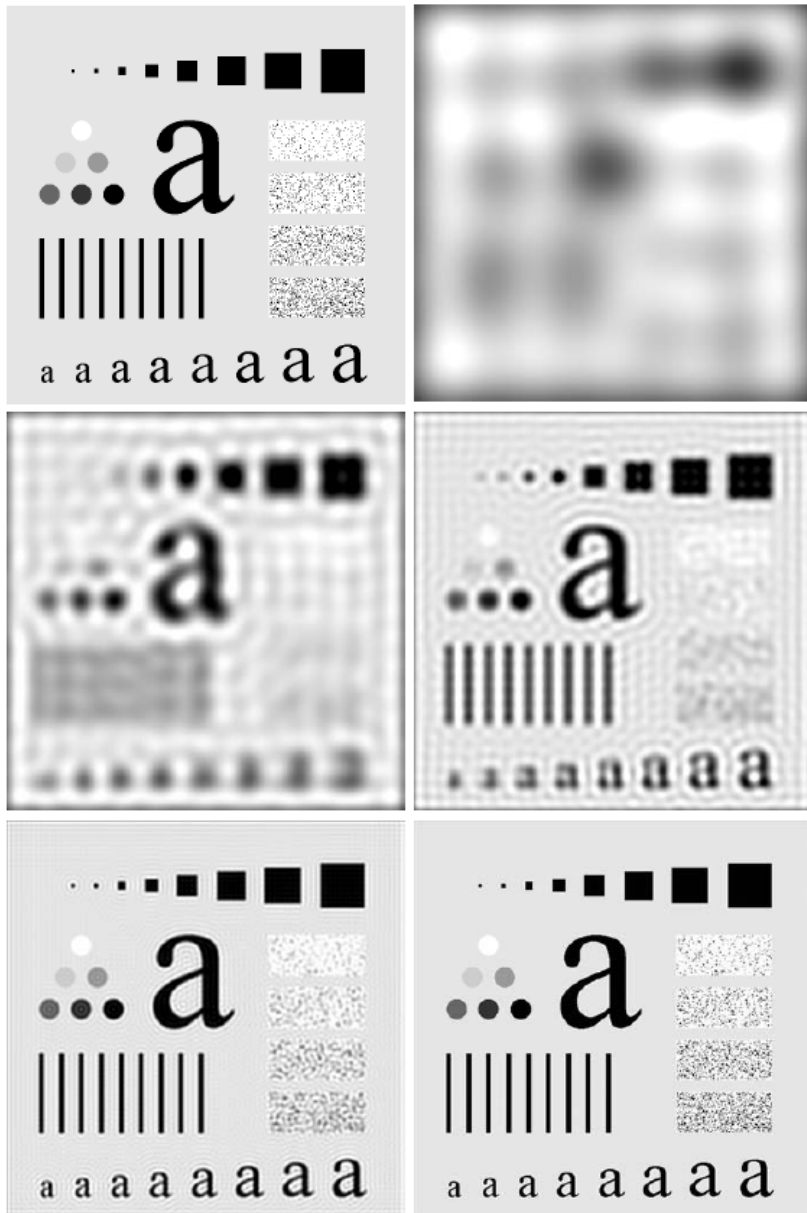
a b c

FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.



a b

FIGURE 4.41 (a) Test pattern of size 688×688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

(a) Imagem original
(b) - (f) resultados da filtragem usando ILPF com frequências de corte nos valores de raio 10, 20, 60, 160 e 460. A potência removida por esses filtros foi de 13, 6.9, 4.3, 2.2 e 0.8% do total respectivamente

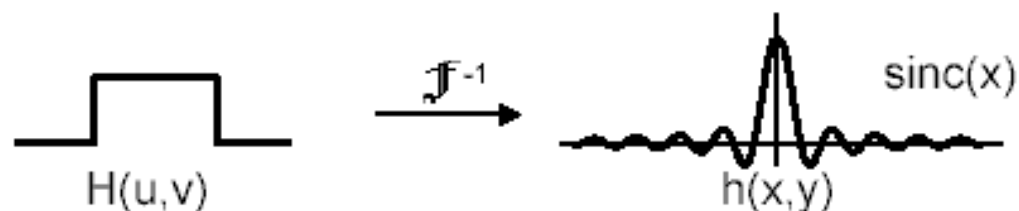
The Ringing Problem

$$G(u,v) = F(u,v) \cdot H(u,v)$$



Convolution Theorem

$$g(x,y) = f(x,y) * h(x,y)$$



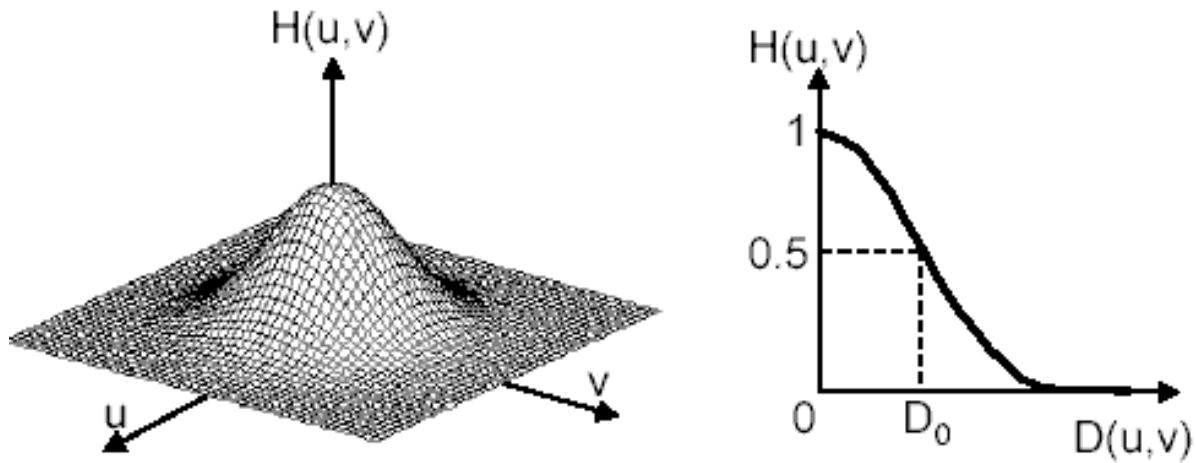
$\uparrow D_0 \longrightarrow \downarrow$ Ringing radius + blur

a b

FIGURE 4.43

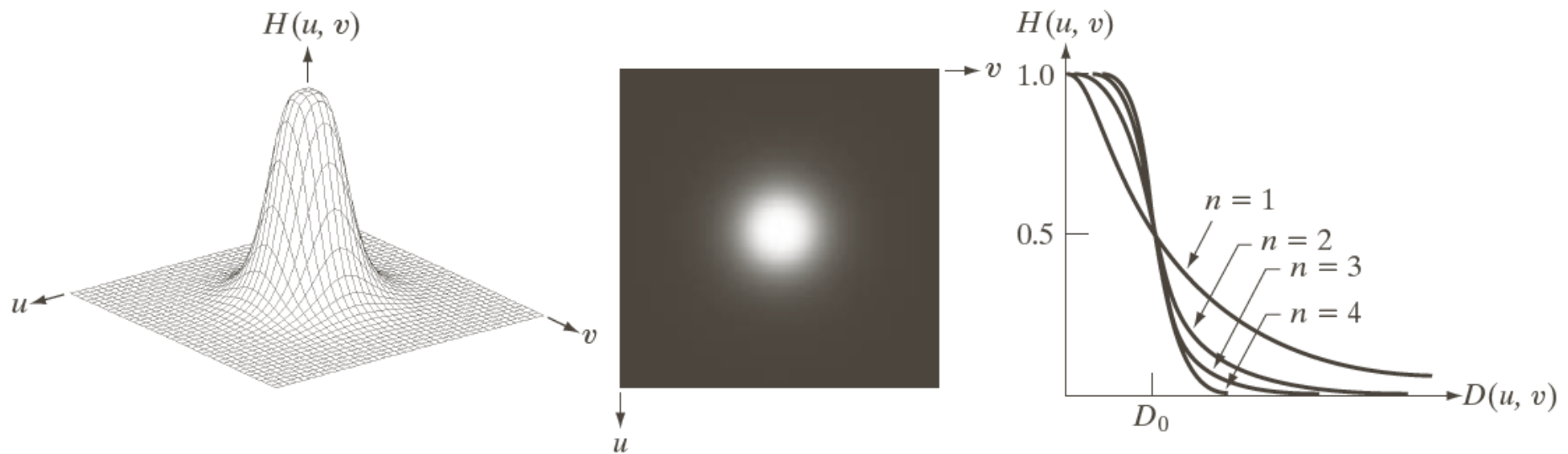
(a) Representation in the spatial domain of an ILPF of radius 5 and size 1000×1000 .
(b) Intensity profile of a horizontal line passing through the center of the image.

$H(u,v)$ - Butterworth Filter



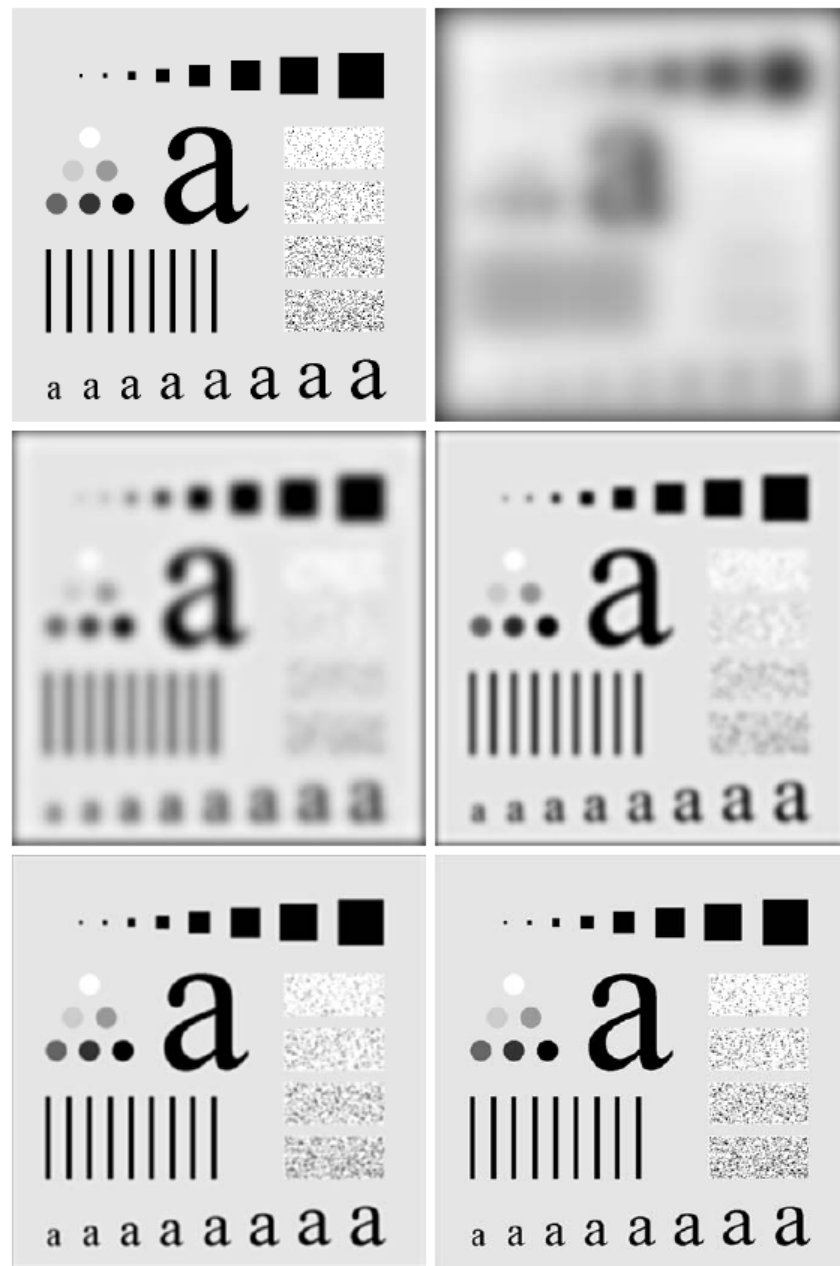
$$H(u,v) = \frac{1}{1 + (D(u,v)/D_0)^{2n}}$$

$$D(u,v) = \sqrt{u^2 + v^2}$$



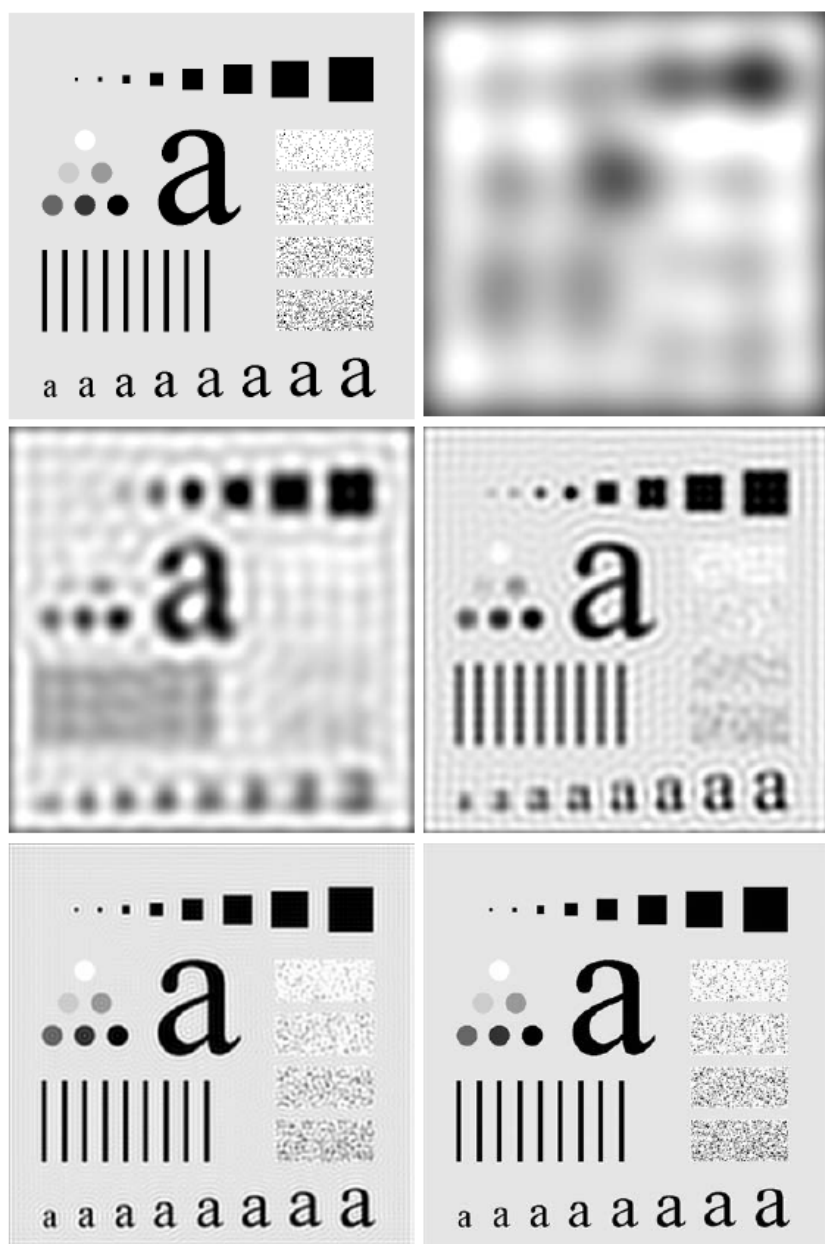
a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



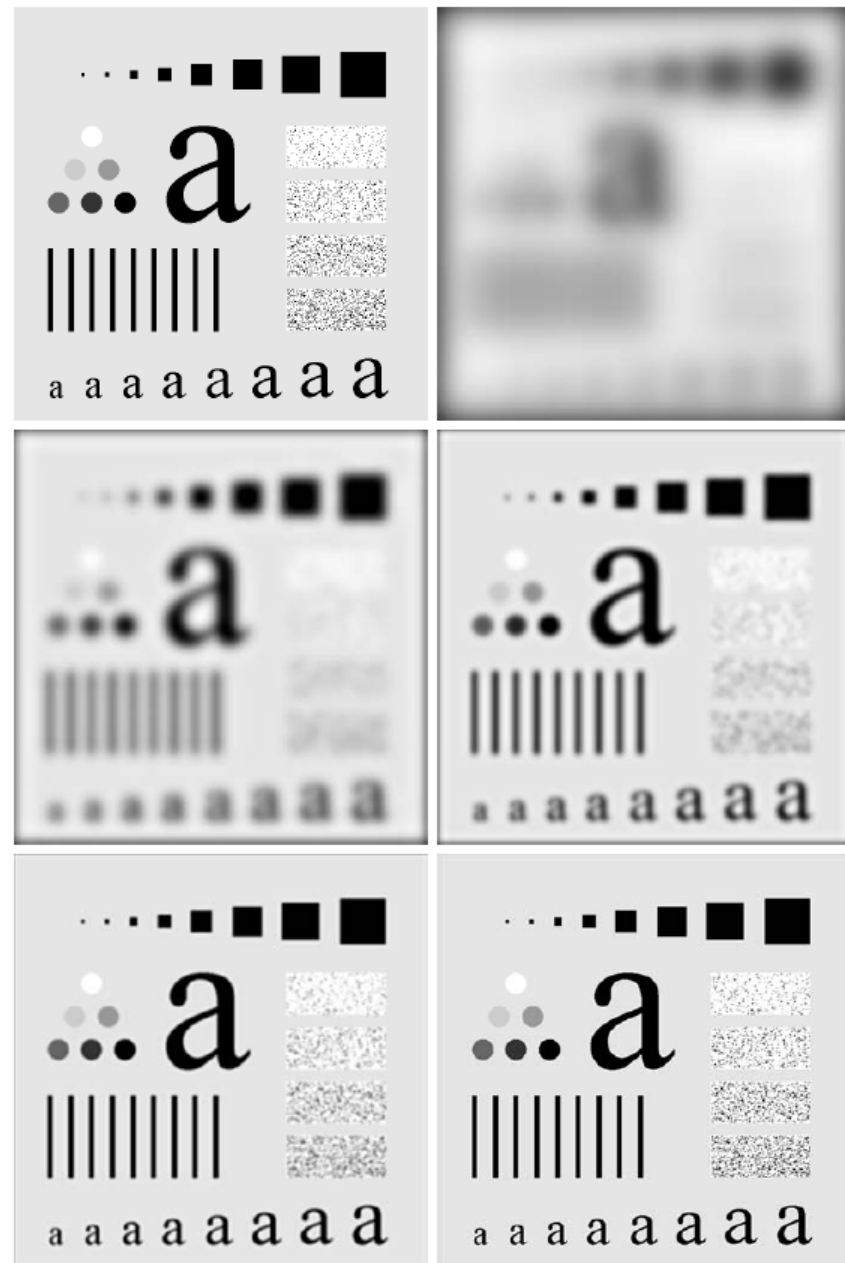
a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

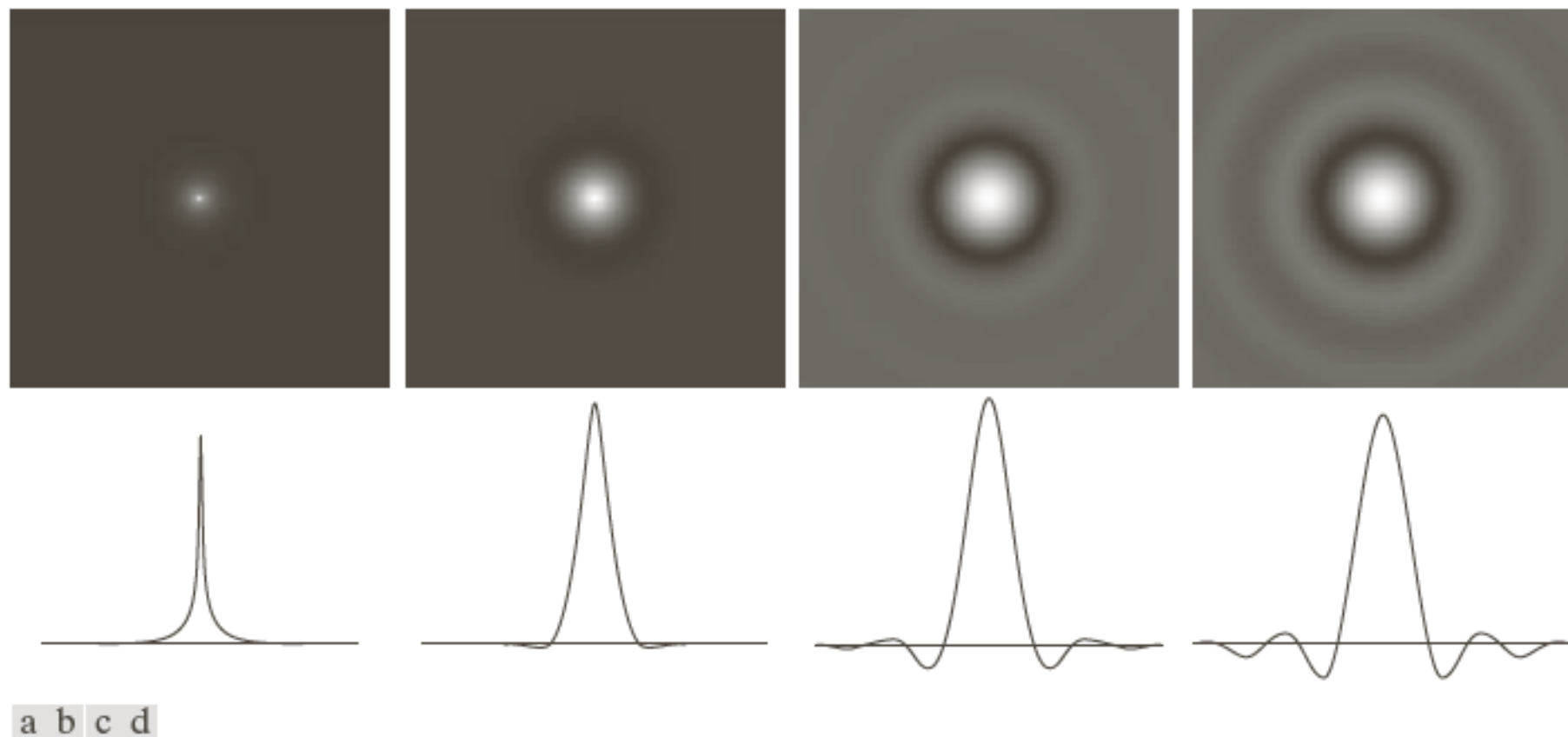


FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000×1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Filtros Gaussianos

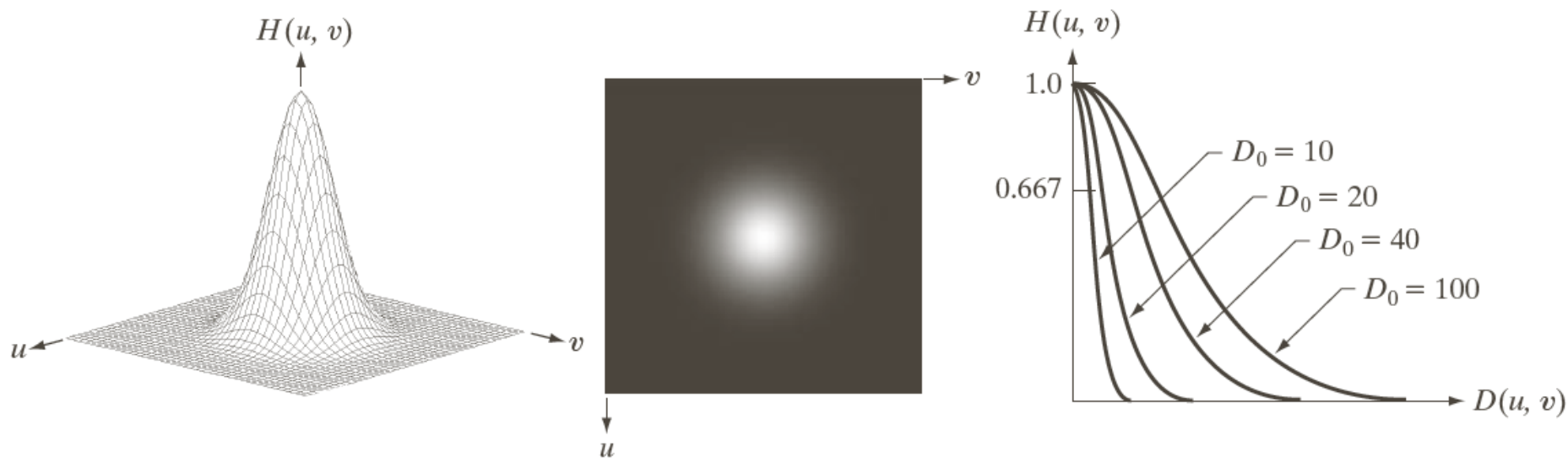
Os filtros de passa baixa Gaussianos (GLPF) são dados por

$$H(u, v) = e^{-D^2(u, v)/2\sigma^2}$$

onde $D(u, v)$ é a distância Euclidiana. Se fizermos $\sigma = D_0$, a frequência de corte, temos uma notação compatível com os demais filtros

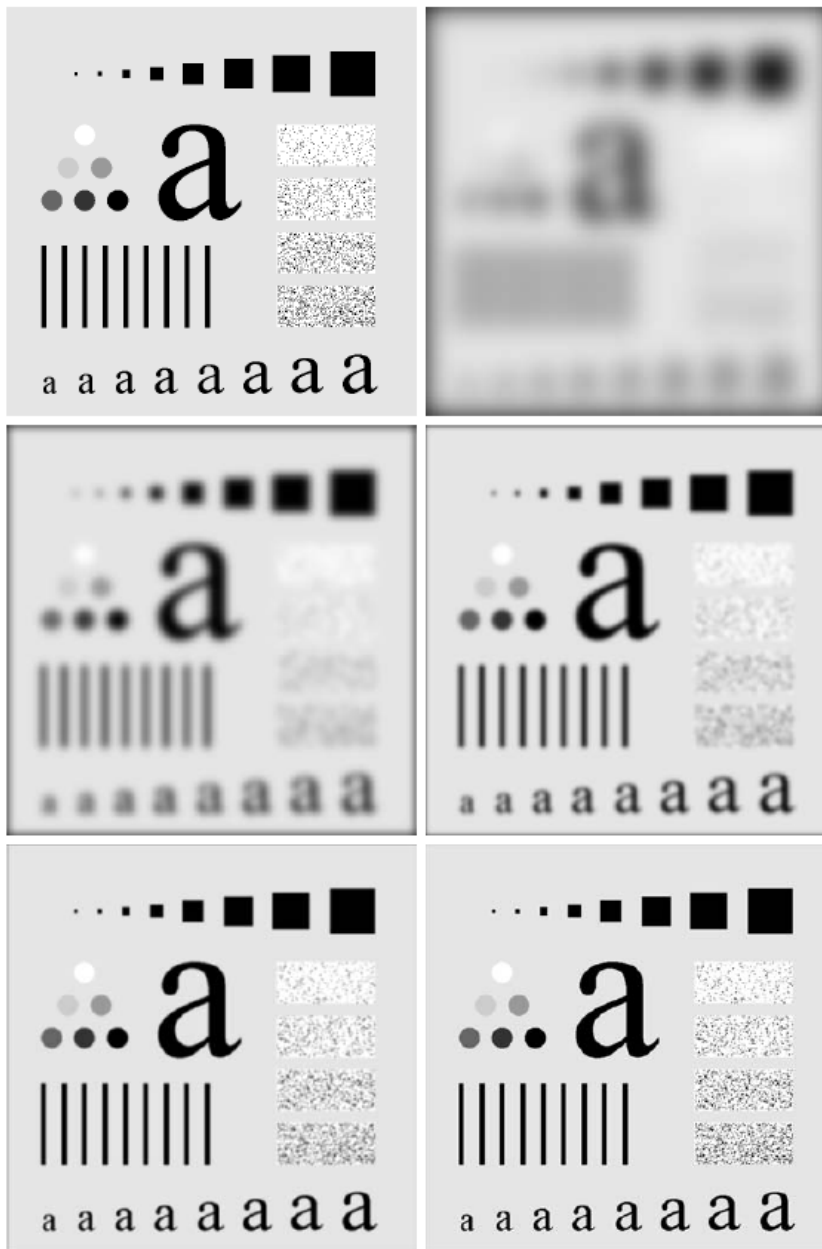
$$H(u, v) = e^{-D^2(u, v)/2D_0^2}$$

A transformada inversa de Fourier de GLPF tb. é Gaussiana => filtro Gaussiano espacial obtido pela IDFT não apresenta efeito oscilatório (*ringing*).



a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0 .



- (a) Imagem original
- (b) - (f) resultado da
filtragem usando GLPF
com frequência de corte
de raios iguais a Fig.4.41b

a b
c d
e f

FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Filtros passa-baixa

TABLE 4.4

Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$	$H(u, v) = \frac{1}{1 + [D(u, v)/D_0]^{2n}}$	$H(u, v) = e^{-D^2(u,v)/2D_0^2}$

Filtros passa-baixa: comparação

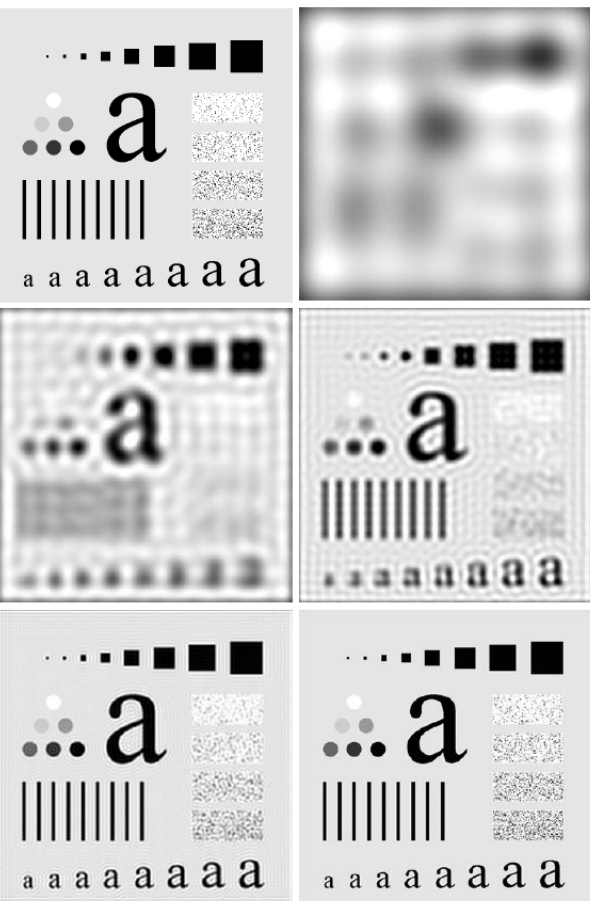


FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

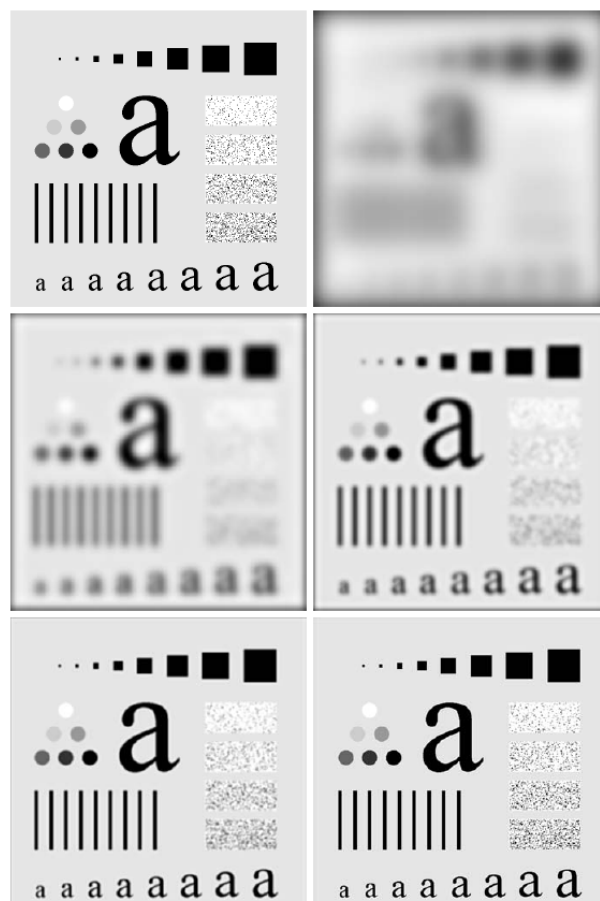


FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

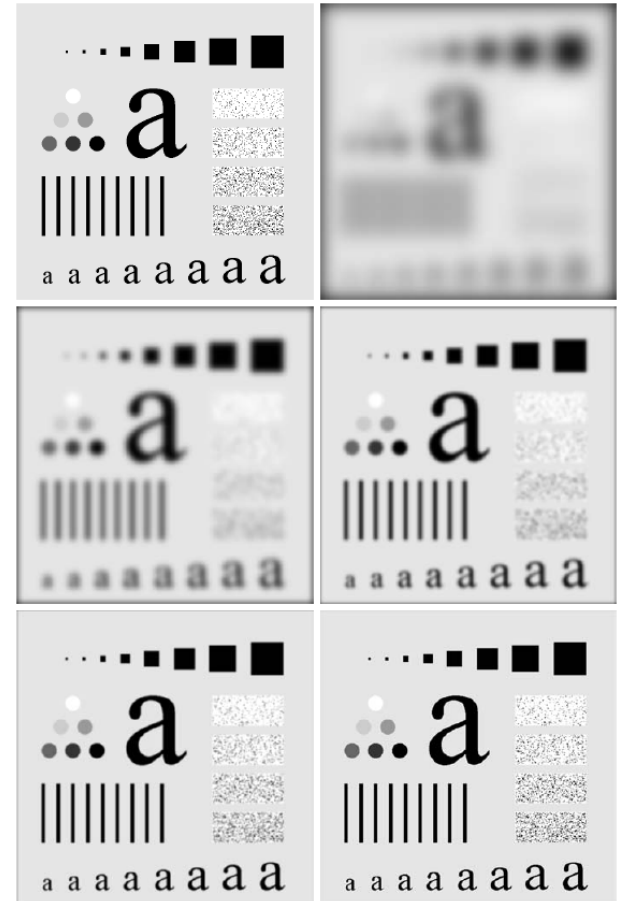


FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs 4.42 and 4.45.

Butterworth x Gaussiano

- Butterworth
 - corte mais abrupto
 - ainda apresenta ruído oscilatório
- Gaussiano
 - corte suave - maior *blur*
 - não apresenta ruído oscilatório

Filtro passa-baixa Gaussiano: ex. 1

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



a b

FIGURE 4.49

(a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

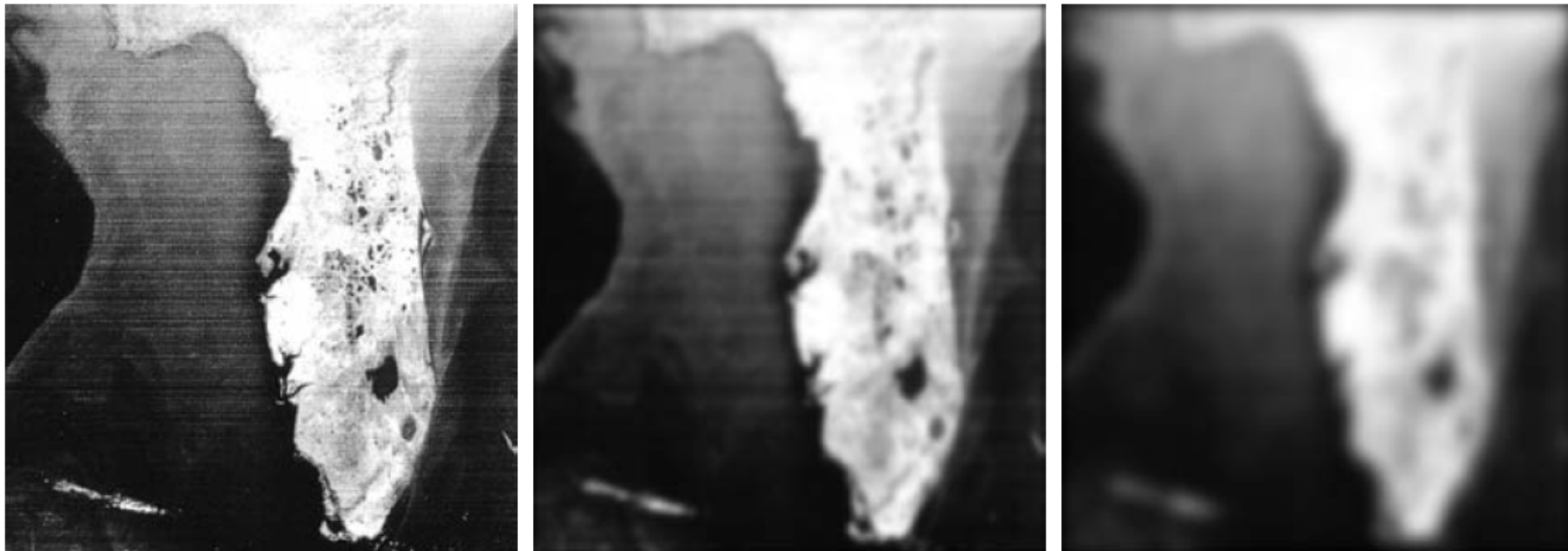
Filtro passa-baixa Gaussiano: ex. 2



a b c

FIGURE 4.50 (a) Original image (784×732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Filtro passa-baixa Gaussiano: ex. 3



a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Filtros de realce de imagens

- Um filtro de passa alta que efetiva o efeito de *sharpening*, contrário à suavização, é obtido de um dado filtro de passa baixa pela equação

$$H_H(\mu, \nu) = 1 - H_L(\mu, \nu)$$

Filtros Passa Alta Ideais

Um filtro 2-D ideal de passa alta (*IHPF, ideal highpass filter*) é definido por

$$H(u, v) = \begin{cases} 0 & \text{se } D(u, v) \leq D_0 \\ 1 & \text{se } D(u, v) > D_0 \end{cases}$$

onde D_0 é a frequência de corte de $D(u, v)$, e $D(u, v) = \sqrt{u^2 + v^2}$

Filtros de Butterworth Passa Alta

A função de transformação de um filtro de passa alta Butterworth (BHPF) de ordem n , e com frequência de corte a uma distância D_0 da origem, é definido por

$$H(u, v) = \frac{1}{1 + [D_0 / D(u, v)]^{2n}}$$

onde $D(u, v) = \sqrt{u^2 + v^2}$

Filtros Passa Alta Gaussianos

A função de transferência do filtro Gaussiano passa alta com frequência de corte e uma distância D_0 do centro do retângulo de frequência é dada por

$$H(u, v) = 1 - e^{-D^2(u, v) / 2D_0^2}$$

$$D(u, v) = \sqrt{u^2 + v^2}$$

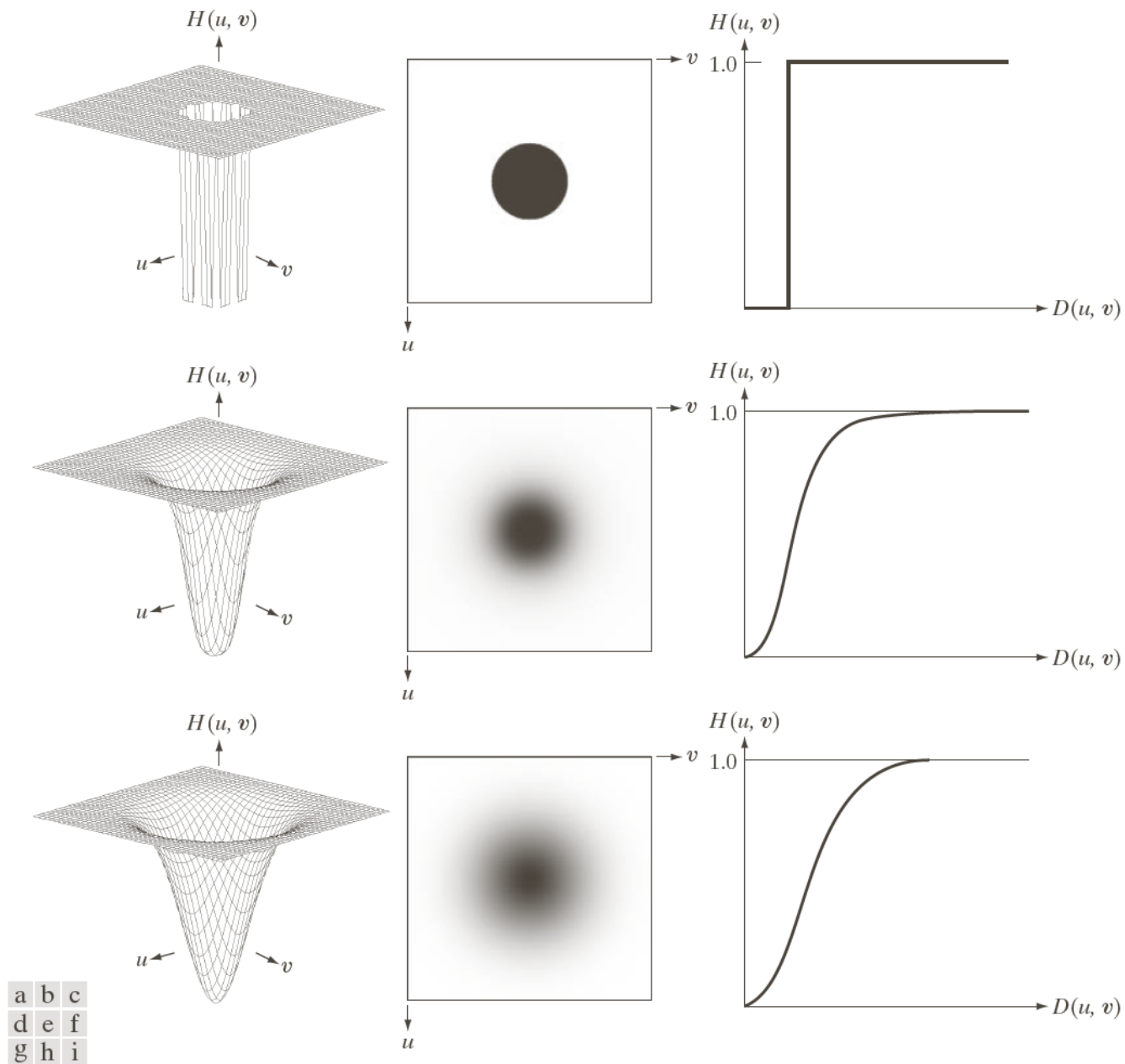


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

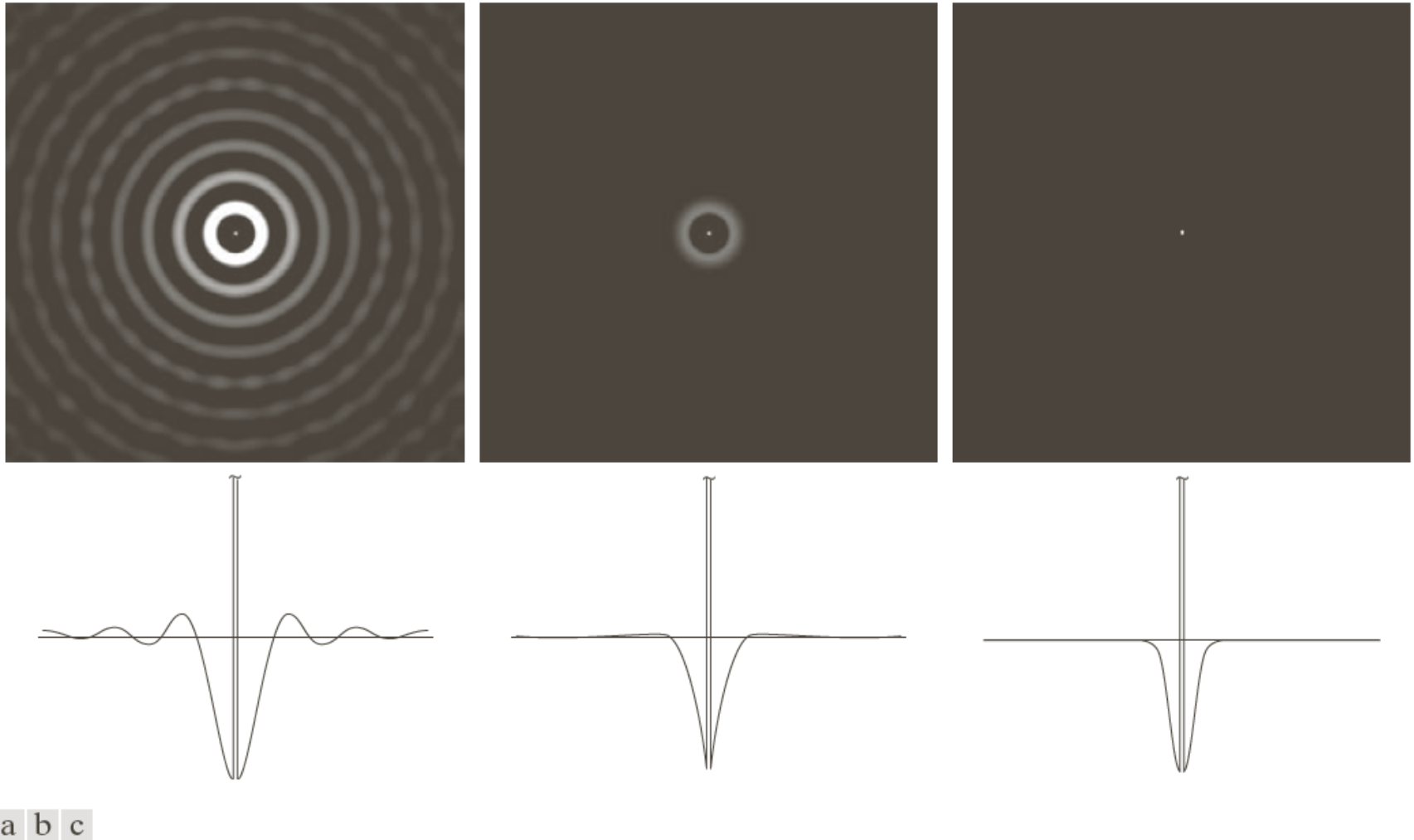


FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

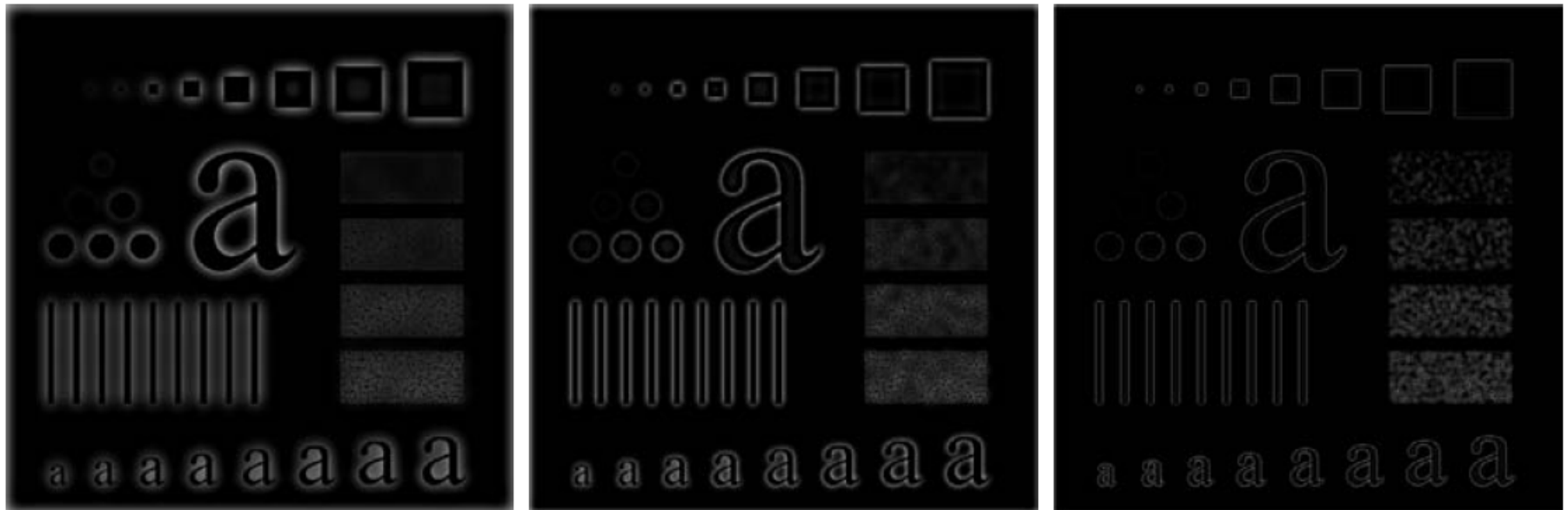
Filtro passa-alta ideal



a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60,$ and 160 .

Filtro passa-alta Butterworth



a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Filtro passa-alta Gaussiano

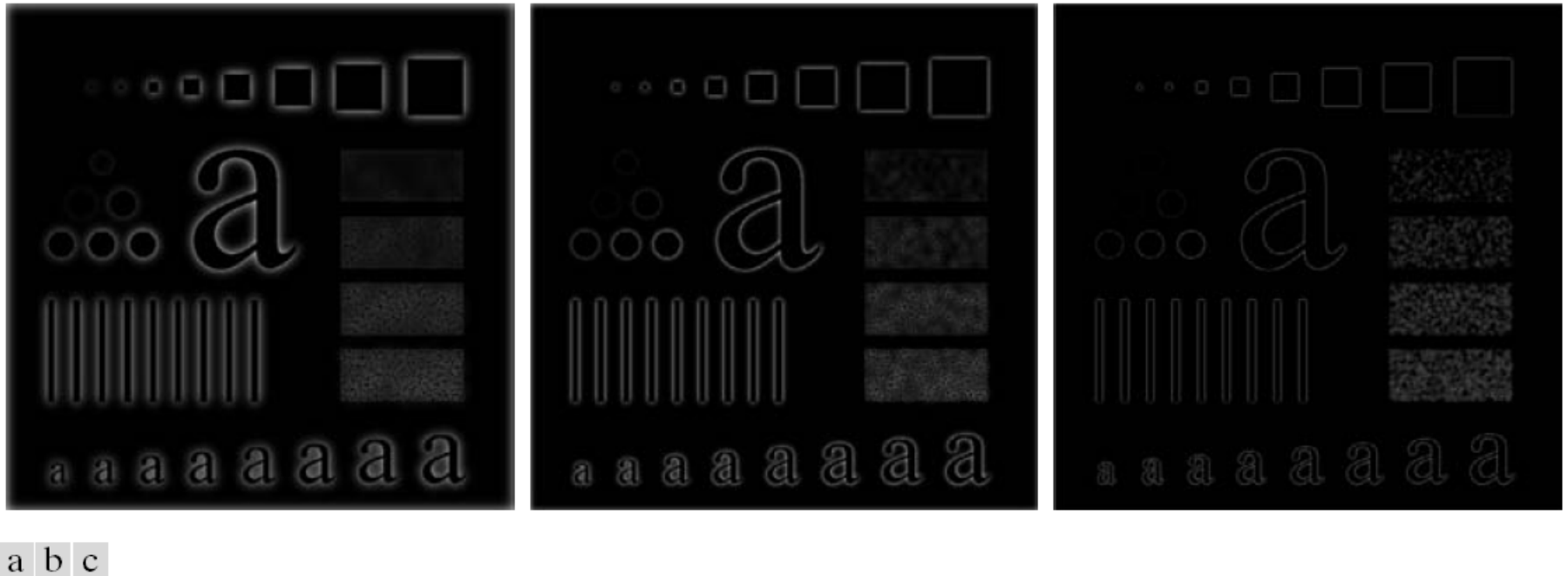
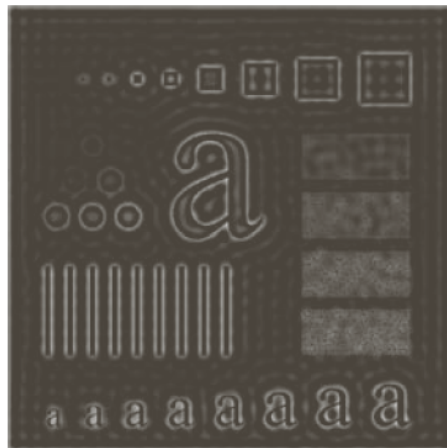
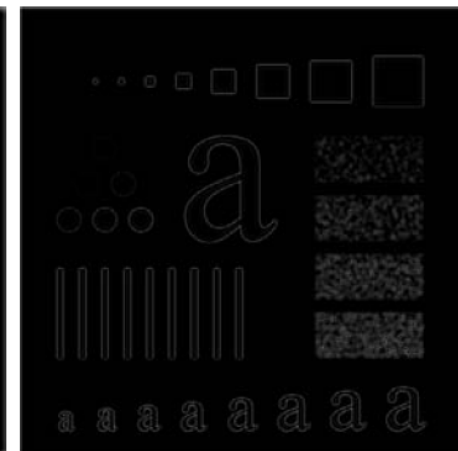


FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60$, and 160 , corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

IHPF



BHPF



GHPF

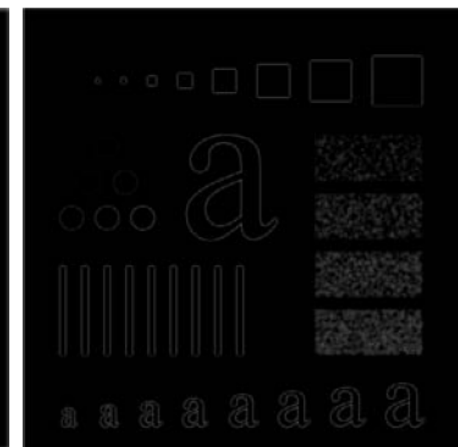
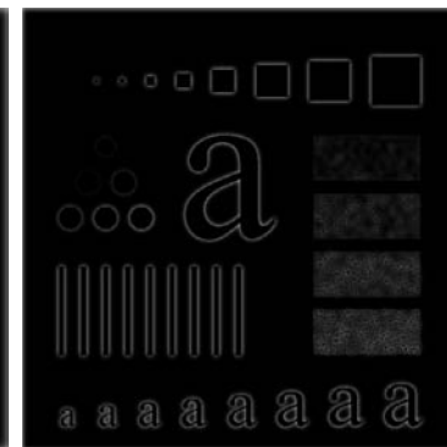
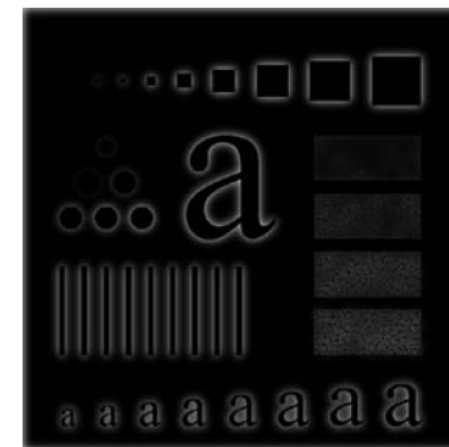


TABLE 4.4

Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$	$H(u, v) = \frac{1}{1 + [D(u, v)/D_0]^{2n}}$	$H(u, v) = e^{-D^2(u,v)/2D_0^2}$

TABLE 4.5

Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \leq D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$	$H(u, v) = \frac{1}{1 + [D_0/D(u, v)]^{2n}}$	$H(u, v) = 1 - e^{-D^2(u,v)/2D_0^2}$

Filtro passa-alta Butterworth: ex.



a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Laplaciano

O Laplaciano foi usado para melhoramento de imagens no domínio espacial. No domínio espacial, ele pode ser implementado usando o filtro:

$$H(u, v) = -4\pi^2(u^2 + v^2)$$

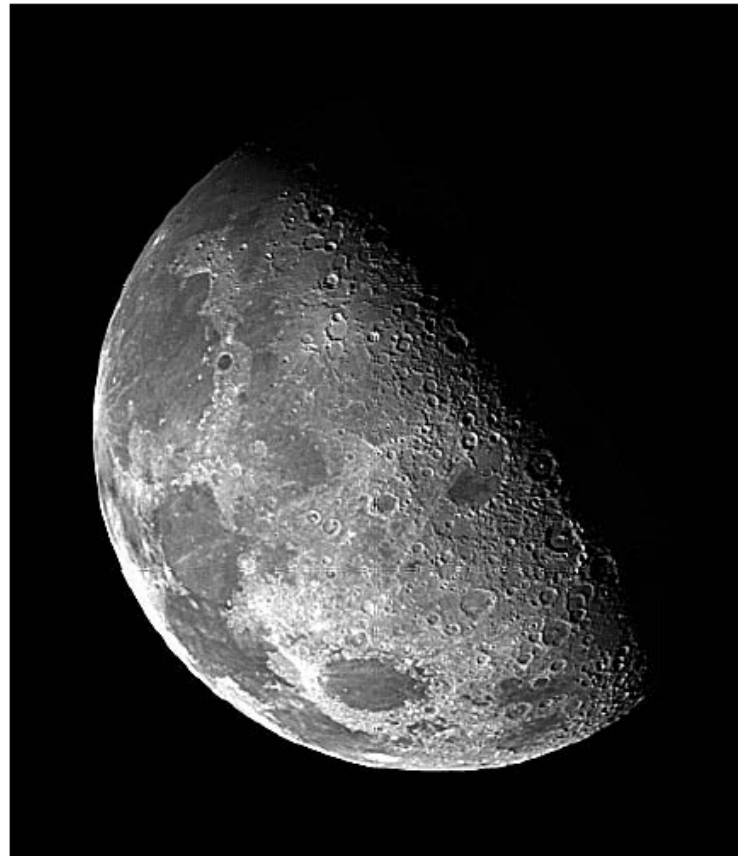
ou, relativamente ao centro do retângulo de frequência:

$$\begin{aligned} H(u, v) &= -4\pi^2[(u - P/2)^2 + (v - Q/2)^2] \\ &= -4\pi^2 D^2(u, v) \end{aligned}$$

então a imagem Laplaciana é obtida por: $\nabla^2 f(x, y) = \mathfrak{F}^{-1}\{H(u, v)F(u, v)\}$

E a imagem melhorada é obtida por $g(x, y) = f(x, y) + c\nabla^2 f(x, y)$

onde $c = -1$, pois $H(u, v)$ é negativa.



a b

FIGURE 4.58

(a) Original, blurry image.

(b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Filtros seletivos

- Os filtros até agora operam sobre o retângulo de frequência como um todo. Existem aplicações em que é de interesse processar bandas específicas de frequência (*bandreject* ou *bandpass filters*) ou pequenas regiões do retângulo de frequência (*notch filters*).

Filtros passa-banda/rejeita-banda

TABLE 4.6

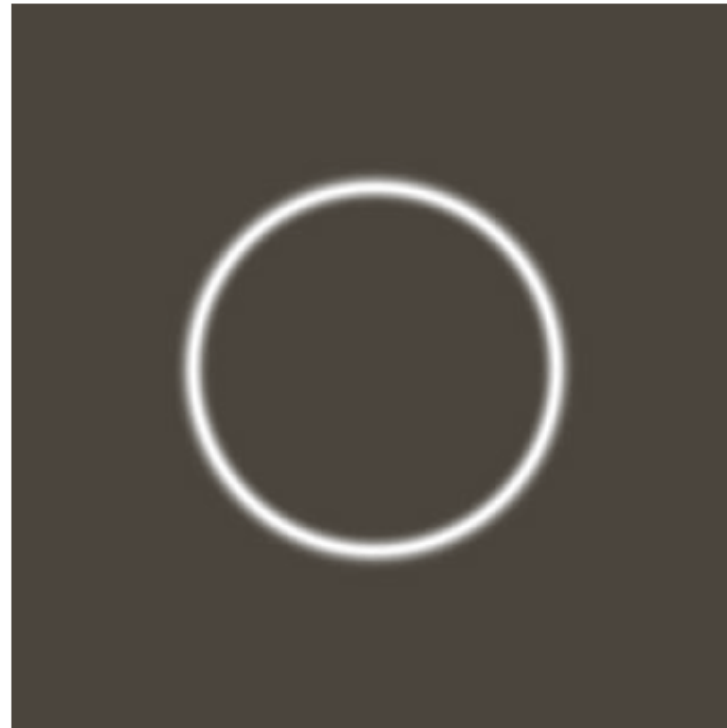
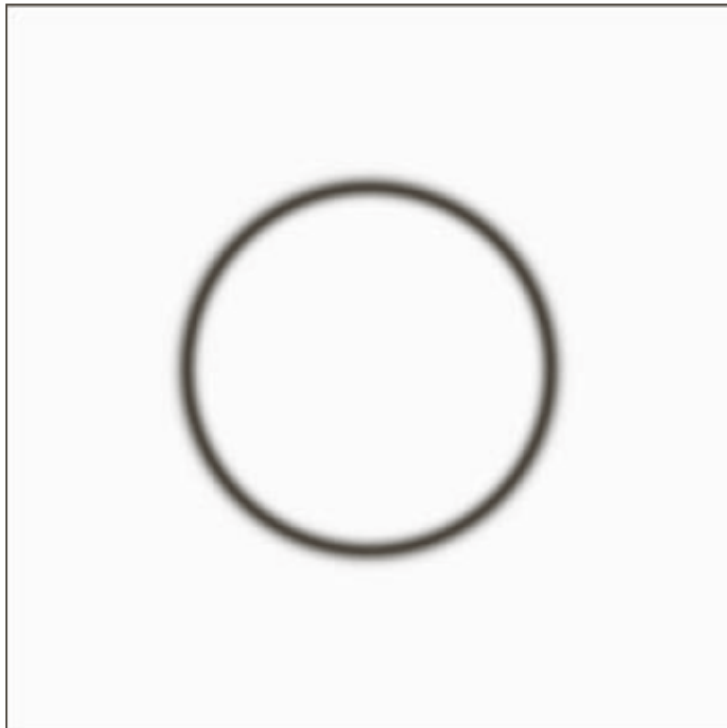
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u, v) = \frac{1}{1 + \left[\frac{DW}{D^2 - D_0^2} \right]^{2n}}$	$H(u, v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW} \right]^2}$

TABLE 4.6

Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u, v) = \frac{1}{1 + \left[\frac{DW}{D^2 - D_0^2} \right]^{2n}}$	$H(u, v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW} \right]^2}$



a b

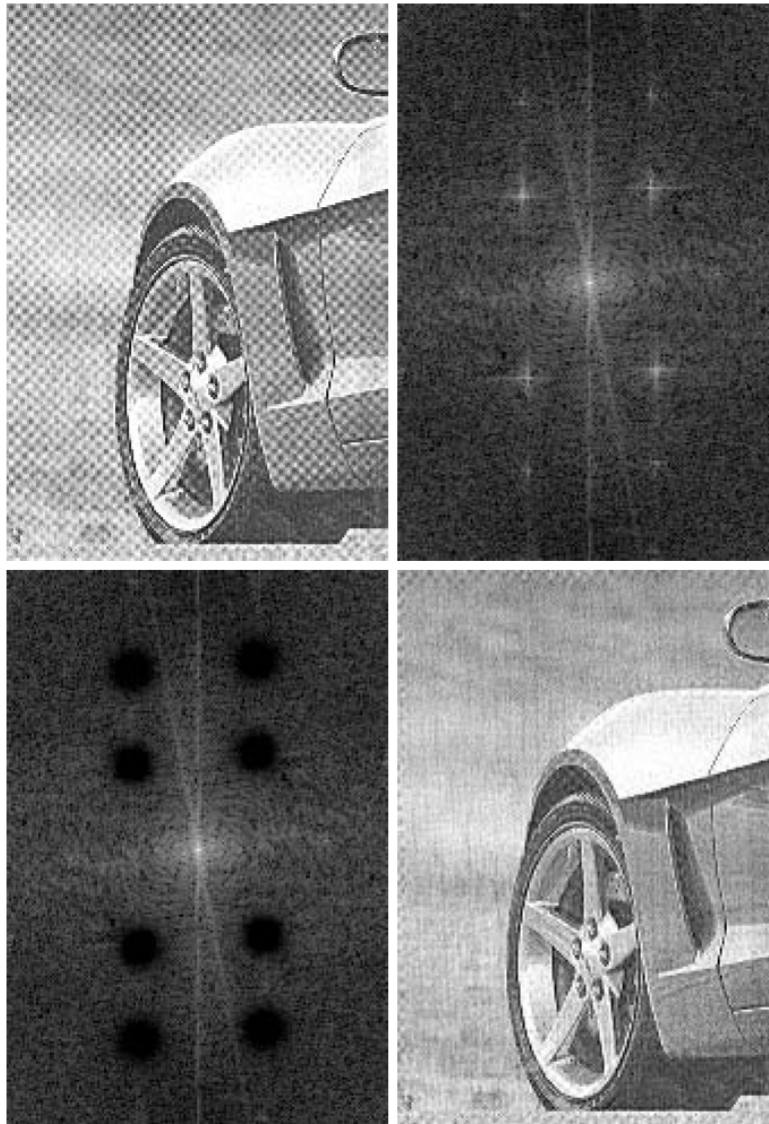
FIGURE 4.63

(a) Bandreject Gaussian filter.
 (b) Corresponding bandpass filter.
 The thin black border in (a) was added for clarity; it is not part of the data.

Filtros Notch

- Usados para filtrar padrões espectrais repetitivos
 - Em geral, percebidos como picos de intensidade em locais que não a origem
 - Para remover o ruído, basta ‘apagar’ esses picos de frequência do espectro

Filtros *notch*



a	b
c	d

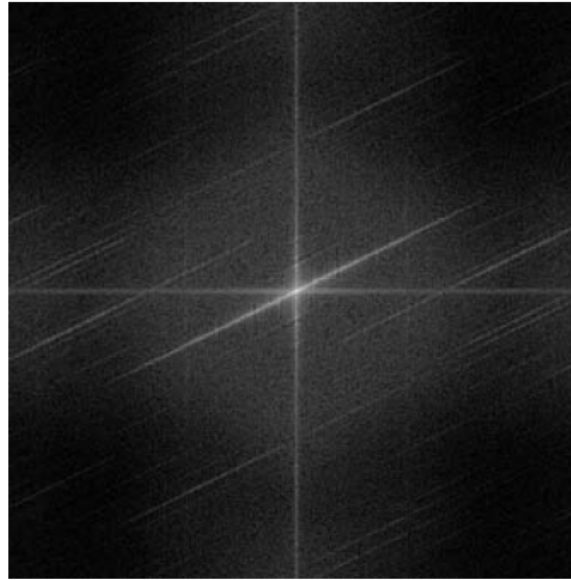
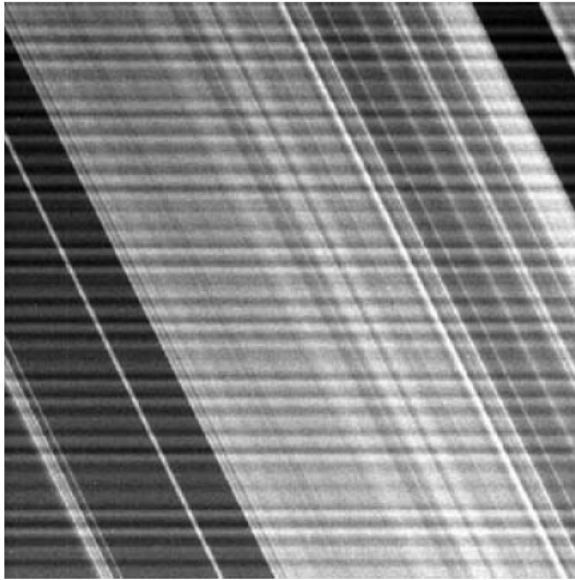
FIGURE 4.64

(a) Sampled newspaper image showing a moiré pattern.

(b) Spectrum.

(c) Butterworth notch reject filter multiplied by the Fourier transform.

(d) Filtered image.



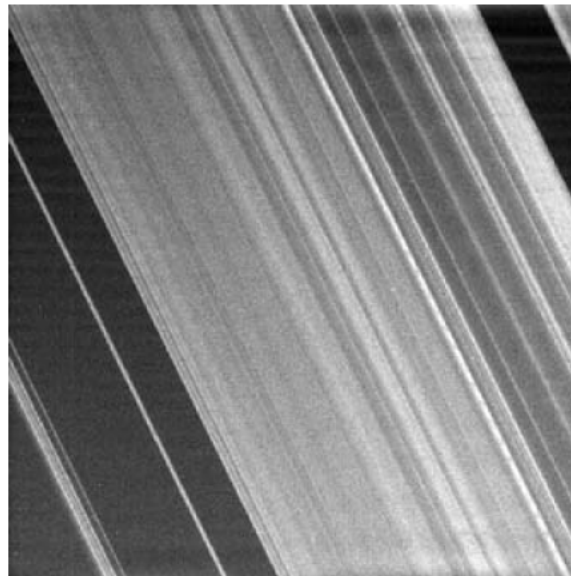
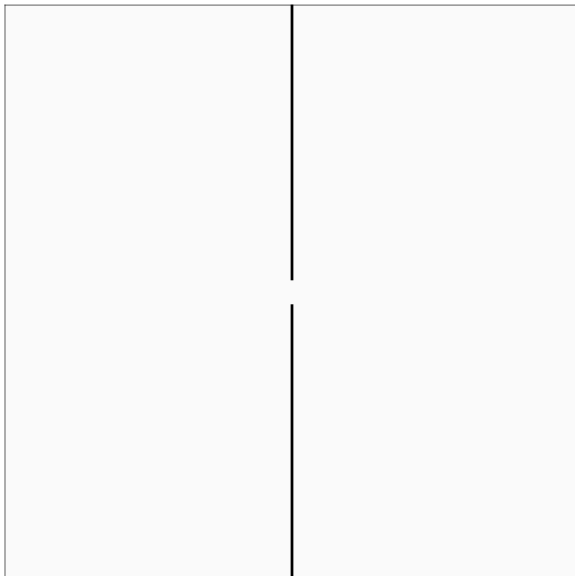
a	b
c	d

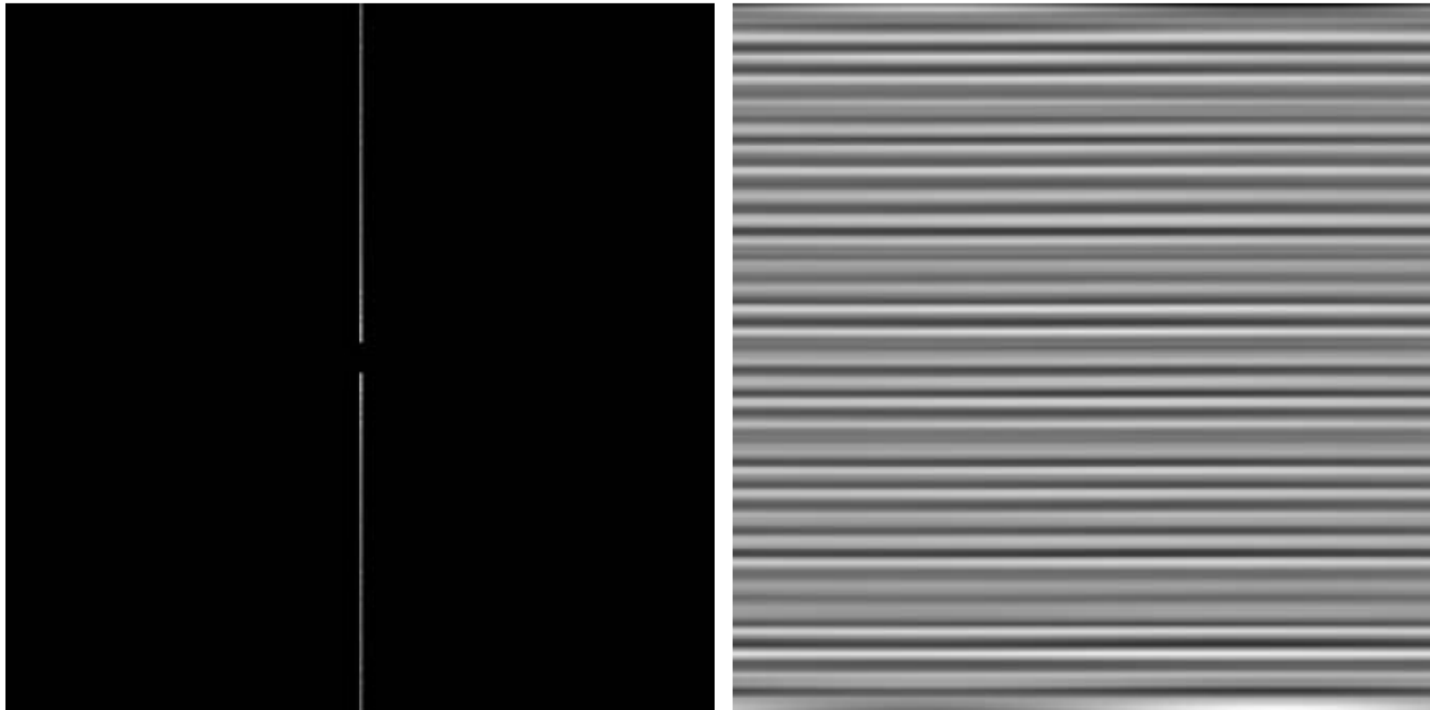
FIGURE 4.65

(a) 674×674 image of the Saturn rings showing nearly periodic interference.

(b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter.

(d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)





a b

FIGURE 4.66

(a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a).
(b) Spatial pattern obtained by computing the IDFT of (a).

Implementação - FFT

- A transformada rápida de Fourier, FFT, é um método de cálculo de DFT

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j 2\pi(ux/M + vy/N)}$$

- O cálculo da DFT requer $(MN)^2$ somatórios e adições. Para imagens de tamanho 1.024x1.024 isso significa da ordem de um trilhão de multiplicações e adições.
- O FFT reduz essa computação para a ordem de $MN \log_2 MN$ multiplicações e adições, o que para uma imagem de 1.024x1.024 seria da ordem de 20 milhões de multiplicações e adições.

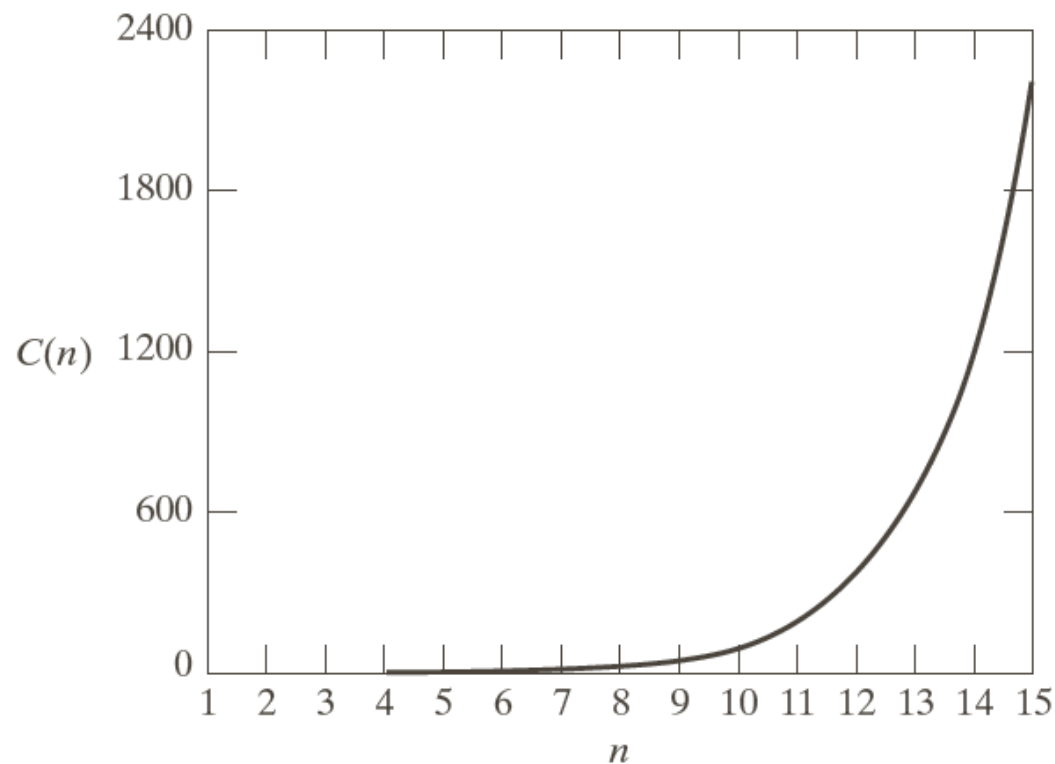


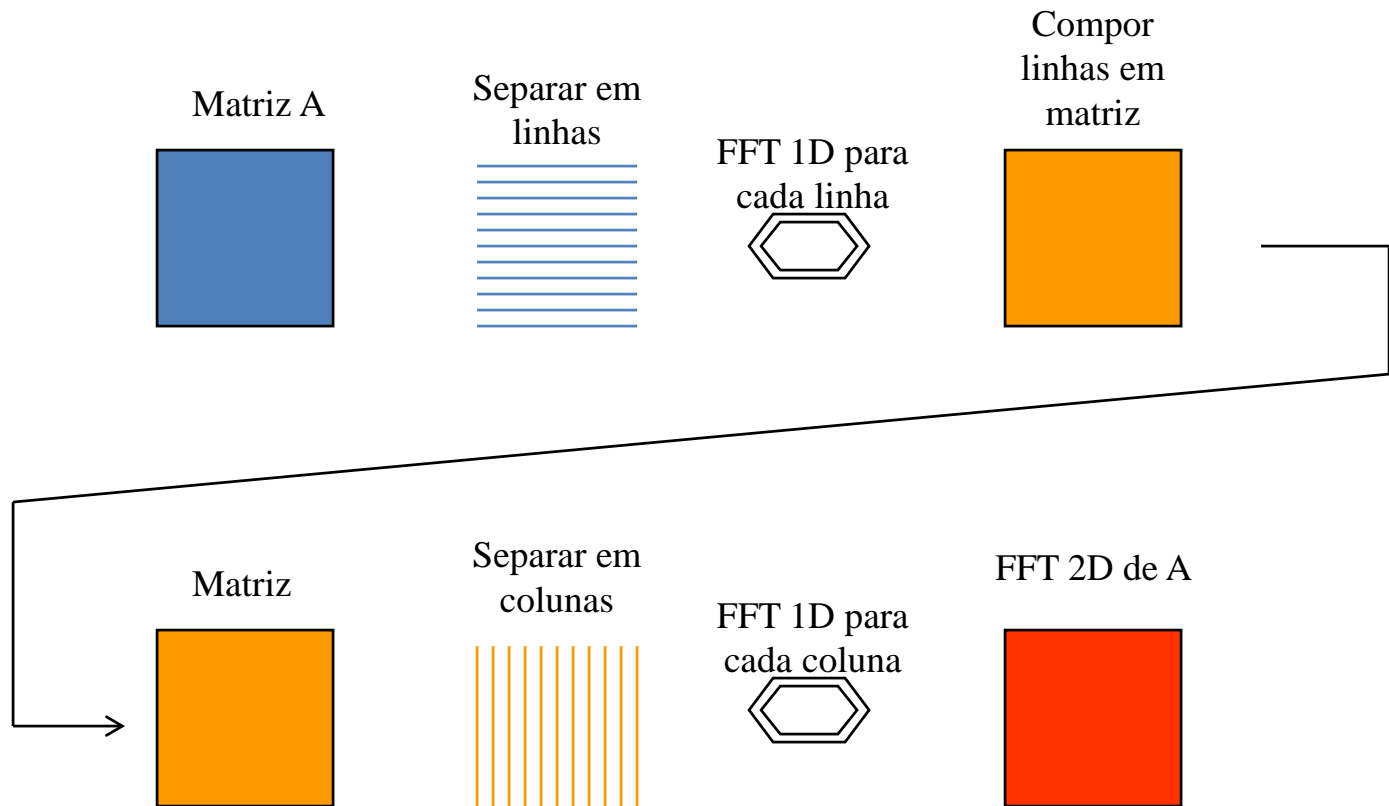
FIGURE 4.67

Computational advantage of the FFT over a direct implementation of the 1-D DFT. Note that the advantage increases rapidly as a function of n .

Implementação

- Separabilidade da DFT 2-D
- Cálculo da IDFT utilizando um algoritmo DFT
- Transformada rápida de Fourier (FFT)

Algoritmo 2D de 1D



Bibliografia

- Fonte: Gonzalez & Wood, Cap. 4