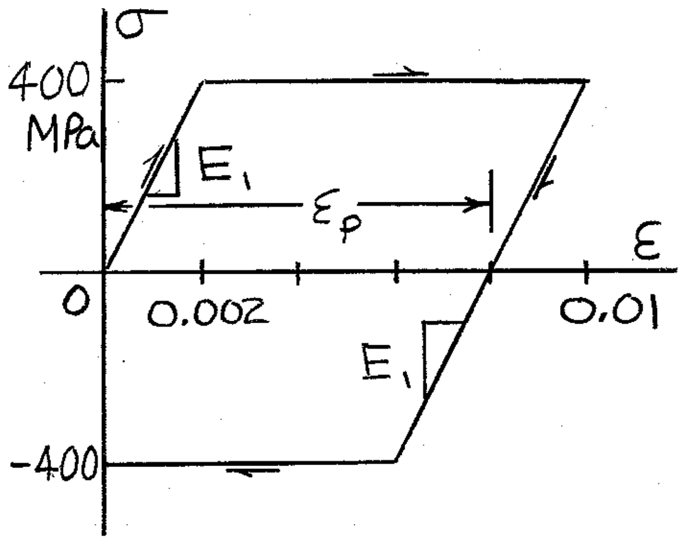
5.2 Elastic, perfectly plastic model:

$E_1 = 200$ GPa, $\sigma_o = 400$ MPa

(a) ε to 0.01, then (b) back to zero.



$$\varepsilon = \frac{\sigma}{E_1} + \varepsilon_p$$

$$\varepsilon_p = 0.01 - \frac{400 \text{ MPa}}{200{,}000 \text{ MPa}}$$

$$\varepsilon_p = 0.008$$

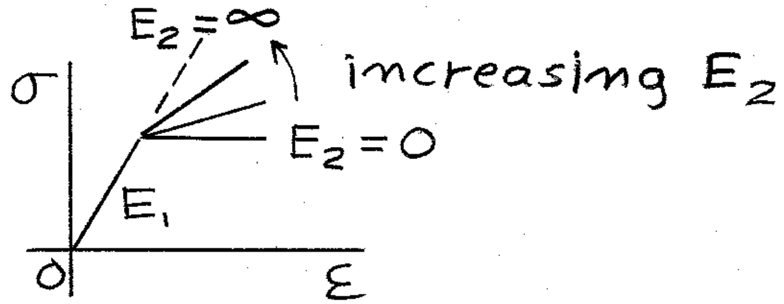
$$\varepsilon_e = 0.002$$

5.3 Elastic, linear-hardening model.

$$\varepsilon = \frac{\sigma}{E_1} + \frac{\sigma - \sigma_0}{E_2} \qquad (\sigma > \sigma_0)$$

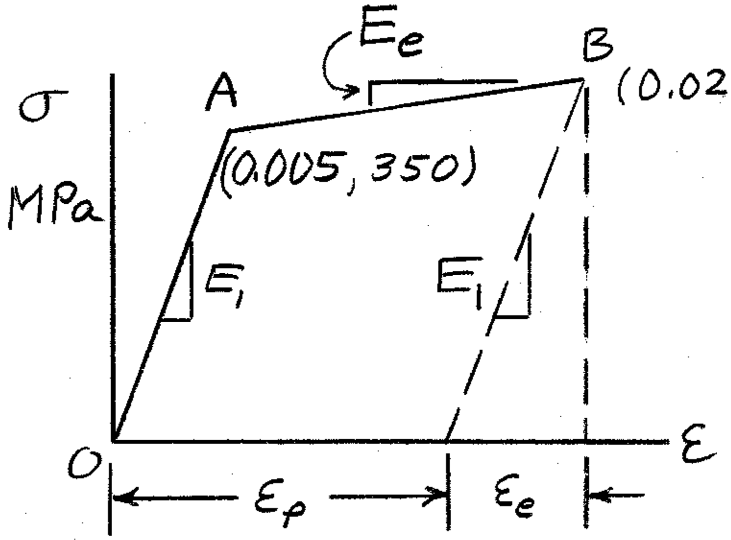
$$\frac{d\varepsilon}{d\sigma} = \frac{1}{E_1} + \frac{1}{E_2}, \quad \frac{d\sigma}{d\varepsilon} = \frac{1}{\frac{1}{E_1} + \frac{1}{E_2}}$$ ◀

This is the stiffness of E_1 and E_2 in series.



◀

5.4 Elastic, linear hardening model.



$E_1 = 70{,}000$ MPa

$E_2 = 2000$ MPa

$\sigma_o = 350$ MPa

Point A: Begin yielding

$$\varepsilon = \frac{\sigma}{E_1} = \frac{\sigma_o}{E_1} = 0.005$$

Point B: $\varepsilon = 0.02$

$$E_e = \frac{1}{1/E_1 + 1/E_2} = 1944 \text{ MPa} \qquad \text{(Eq. 5.10)}$$

$$\frac{\Delta\sigma}{\Delta\varepsilon} = E_e, \quad \Delta\sigma = 1944 \text{ MPa}(0.02 - 0.005)$$

$$\Delta\sigma = 29 \text{ MPa}$$

$$\sigma_B = \sigma_A + \Delta\sigma = 350 + 29 = 379 \text{ MPa}$$

$$\varepsilon_e = \frac{\sigma_B}{E_1} = \frac{379}{70{,}000} = 0.0054 \quad ◀$$

$$\varepsilon_p = \varepsilon_A - \varepsilon_e = 0.02 - 0.0054 = 0.0146 \quad ◀$$

5.5 Elastic, steady-state creep model:

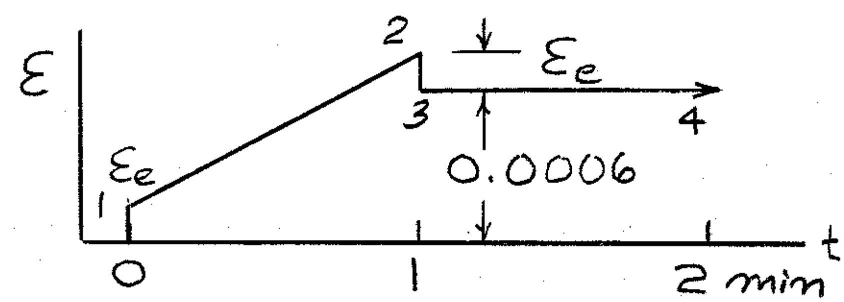
$E_1 = 60$ GPa, $\eta_1 = 1000$ GPa·s

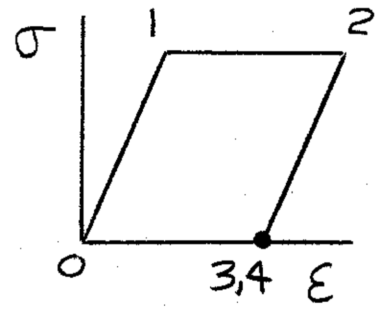
$\sigma = 10$ MPa for 60 s, then removed.

$$\varepsilon_e = \frac{\sigma}{E_1} = \frac{10 \text{ MPa}}{60{,}000 \text{ MPa}} = 1.67 \times 10^{-4}$$

$$\varepsilon = \varepsilon_e + \frac{\sigma t}{\eta_1} = 1.67 \times 10^{-4} + \frac{(10 \text{ MPa})(60 \text{ s})}{10^6 \text{ MPa·s}}$$

$\varepsilon = 7.67 \times 10^{-4}$ at $t = 60$ s





5.6 $\frac{d\varepsilon_c}{dt} = \frac{\sigma - E_2\varepsilon_c}{\eta_2}$, $\sigma = \text{constant}$

$$\frac{d\varepsilon_c}{\sigma - E_2\varepsilon_c} = \frac{dt}{\eta_2}, \quad d(\sigma - E_2\varepsilon_c) = -E_2\, d\varepsilon_c$$

$$\frac{d(\sigma - E_2\varepsilon_c)}{\sigma - E_2\varepsilon_c} = -\frac{E_2}{\eta_2}dt$$

Integrate both sides. $\sigma = \sigma' = \text{constant}$

$$\ln(\sigma' - E_2\varepsilon_c) = \frac{-E_2 t}{\eta_2} + C$$

When $t = 0$, $\varepsilon_c = 0$, so $C = \ln\sigma'$

$$\ln\left(1 - \frac{E_2\varepsilon_c}{\sigma'}\right) = -\frac{E_2 t}{\eta_2}$$

$$e^{-\frac{E_2 t}{\eta_2}} = 1 - \frac{E_2\varepsilon_c}{\sigma'}, \quad \varepsilon_c = \frac{\sigma'}{E_2}\left(1 - e^{\frac{-E_2 t}{\eta_2}}\right)$$

Finally, add elastic strain.

$$\varepsilon = \varepsilon_e + \varepsilon_c = \frac{\sigma'}{E_1} + \frac{\sigma'}{E_2}\left(1 - e^{-E_2 t/\eta_2}\right)$$

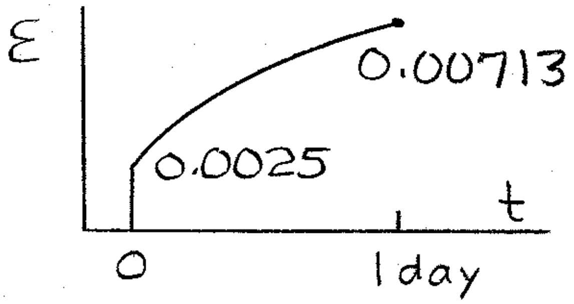
5.7 Elastic, transient creep model.

$E_1 = 6$, $E_2 = 3$ GPa, $\eta_2 = 10^5$ GPa• s

$\sigma = 15$ MPa applied for 1 day = 86,400 s.

$$\varepsilon = \frac{\sigma}{E_1} + \frac{\sigma}{E_2}\left(1 - e^{-\frac{E_2 t}{\eta_2}}\right)$$

Substitute various values of t in seconds and calculate correspon ing ε



Then plot these ε versus t.

5.8

$\dot{\varepsilon} = B\sigma^m$

$m = 3 \text{ to } 7$

Hold at constant ε'

Find $\sigma = f(\varepsilon', t, B, m, E_1)$

$\sigma = E_1 \varepsilon'$ at $t = 0$, when ε' applied

$$\varepsilon' = \varepsilon_e + \varepsilon_c, \quad 0 = \dot{\varepsilon}_e + \dot{\varepsilon}_c = \frac{\dot{\sigma}}{E_1} + B\sigma^m$$

$$\frac{d\sigma}{dt} = -BE_1\sigma^m$$

$$\int_0^t dt = -\frac{1}{BE_1}\int_{E_1\varepsilon'}^{\sigma} \frac{d\sigma}{\sigma^m}, \quad t = -\frac{1}{BE_1}\left.\frac{\sigma^{1-m}}{1-m}\right|_{E_1\varepsilon'}^{\sigma}$$

$$-tBE_1(1-m) = \sigma^{1-m} - (E_1\varepsilon')^{1-m}$$

$$\sigma^{1-m} = (E_1\varepsilon')^{1-m} - tBE_1(1-m)$$

$$\sigma = \left[(E_1\varepsilon')^{1-m} - tBE_1(1-m)\right]^{\frac{1}{1-m}}$$ ◀

Further manipulation gives the form of Eq. 15.46.

$$\sigma = \frac{\sigma_i}{\left[tBE_1(m-1)\sigma_i^{m-1} + 1\right]^{1/(m-1)}}$$

where $\sigma_i = E_1\varepsilon'$ (stress at $t = 0$)

5.9 *Elastic, steady state creep model.*

$\sigma_i = E_1 \varepsilon'$ initially

$\sigma = E_1 \varepsilon' e^{-E_1 t/\eta_1} = \sigma_i e^{-E_1 t/\eta_1}$

$\frac{\sigma}{\sigma_i} = e^{-E_1 t/\eta_1}$ Assume initial stress σ_i and E_1, η_1 the same for all bands.

$\sigma/\sigma_i = 0.9$ for $t = 3$ months

$0.9 = e^{(-E_1/\eta_1)3}$

$\ln 0.9 = \left(-\frac{E_1}{\eta_1}\right)3$, $\frac{E_1}{\eta_1} = 0.0351$ 1/mos

For $\sigma/\sigma_i = 0.5$, $t = t_A$, mos

$0.5 = e^{-0.0351\, t_A}$

$\ln 0.5 = -0.0351\, t_A$, $t_A = 19.7$ mos ◀

5.10 PC plastic, $L_i = 250$, retangular section, $w = 30$, $t = 2.50$ mm.
$P = 2000$ N, $\Delta L = 2.7$, $\Delta w = -0.11$ mm

(a) $\sigma_x = \dfrac{2000 \text{ N}}{30 \times 2.5 \text{ mm}^2} = 26.67 \text{ MPa}$ ◀

(b) $\varepsilon_x = \dfrac{\Delta L}{L_i} = \dfrac{2.7 \text{ mm}}{250 \text{ mm}} = 0.0108$ ◀

(c) $\varepsilon_y = \dfrac{\Delta W}{W} = \dfrac{-0.11 \text{ mm}}{30 \text{ mm}} = -0.00367$ ◀

(d) $E = \dfrac{\sigma_x}{\varepsilon_x} = \dfrac{26.67 \text{ MPa}}{0.0108} = 2470 \text{ MPa} = 2.47 \text{ GPa}$ ◀

(e) $\nu = -\dfrac{\varepsilon_y}{\varepsilon_x} = -\dfrac{-0.00367}{0.0108} = 0.340$ ◀

(f) $G = \dfrac{E}{2(1+\nu)} = \dfrac{2.47 \text{ GPa}}{2(1+0.34)} = 0.922 \text{ GPa}$ ◀

(g) In Table 5.2, $E = 2.4$ GPa, $\nu = 0.38$, which is reasonable agreement. ◀

5.11 Bar of Al alloy; $L_i = 150$, $d_i = 40$ mm, $P = 250$ kN (below yield)

(a) $\sigma_x = \frac{P}{A} = \frac{250{,}000\ \text{N}}{\pi(40\ \text{mm})^2/4} = 198.9\ \text{MPa}$ ◀

(b) $\varepsilon_x = \frac{\sigma_x}{E}$, $E = 70{,}300$ MPa (Table 5.2)

$\varepsilon_x = \frac{198.9\ \text{MPa}}{70{,}300\ \text{MPa}} = 0.00283$ ◀

(c) $\varepsilon_y = -\nu \varepsilon_x$, $\nu = 0.345$ (Table 5.2)

$\varepsilon_y = -0.345\,(0.00283) = -0.000976$ ◀

(d) $L = L_i + \Delta L$, $\Delta L = \varepsilon_x L_i = 0.00283\,(150\ \text{mm})$

$L = 150 + 0.424 = 152.42$ mm ◀

(e) $d = d_i + \Delta d$, $\Delta d = \varepsilon_y d_i = -0.000976\,(40\ \text{mm})$

$d = 40 - 0.0391 = 39.96$ mm ◀

5.12 Simplify Hooke's Law for special cases.

(a) Plane stress: $\sigma_z = \tau_{yz} = \tau_{zx} = 0$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y), \quad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y), \quad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

(b) Uniaxial: only σ_x nonzero

$$\varepsilon_x = \frac{\sigma_x}{E}, \quad \varepsilon_y = \frac{-\nu\sigma_x}{E}, \quad \varepsilon_z = \frac{-\nu\sigma_x}{E}$$

(c) Plane strain: $\varepsilon_z = \gamma_{yz} = \gamma_{zx} = 0$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$0 = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right], \quad \sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

5.13 For plane stress, $\sigma_z = \tau_{yz} = \tau_{zx} = 0$.
(a) Simplify Hooke's Law. (b) Express (a) in terms of $\sigma_x, \sigma_y = f$(strains). (c) Find equation for ε_z.

(a) Let $\sigma_z = \tau_{yz} = \tau_{zx}$ in Eq. 5.26 and 5.27.

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y), \quad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \quad ◀$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y), \quad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

(b) $$\varepsilon_x + \nu\varepsilon_y = \frac{1}{E}\left[(\sigma_x - \nu\sigma_y) + \nu(\sigma_y - \nu\sigma_x)\right]$$

$$\varepsilon_x + \nu\varepsilon_y = \frac{1}{E}\left[\sigma_x - \nu^2\sigma_x\right]$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \quad ◀$$

From similar derivation, or by symmetry

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) \quad ◀$$

(c) $$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{\nu}{1-\nu^2}(1+\nu)(\varepsilon_x + \varepsilon_y)$$

$$\varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) \quad ◀$$

5.14 Solve Hooke's Law, Eq. 5.26, for stresses, and generalize to include ΔT. Combine Eqs. 5.26(a), (b), (c) as follows:

$$E(\varepsilon_x + \nu\varepsilon_y + \nu\varepsilon_z) = \sigma_x - \nu(\sigma_y + \sigma_z) + \nu\sigma_y - \nu^2(\sigma_x + \sigma_z) + \nu\sigma_z - \nu^2(\sigma_x + \sigma_y)$$

$$= \sigma_x(1 - 2\nu^2) - \nu^2(\sigma_y + \sigma_z)$$

$$= \sigma_x(1 - 2\nu^2) + \nu(E\varepsilon_x - \sigma_x)$$

where the last step arises from Eq. 5.26(a). Solving for σ_x, and writing the other two equations by analogy:

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu(\varepsilon_x + \varepsilon_z)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y)\right]$$

For thermal strain case, Eq. 5.40 suggests applying the transformation $\varepsilon_x \rightarrow \varepsilon_x - \alpha(\Delta T)$, $\varepsilon_y \rightarrow \varepsilon_y - \alpha(\Delta T)$, $\varepsilon_z \rightarrow \varepsilon_z - \alpha(\Delta T)$, to above equations, which gives

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)\right] - \frac{E\alpha(\Delta T)}{1-2\nu}$$

and similar equations for σ_y and σ_z.

5.15 On the surface of a steel part ($\sigma_z = 0$), measured $\varepsilon_x = -0.002$, $\varepsilon_y = 0.003$. Find σ_x, σ_y. Complete state of stress?
$E = 212{,}000$ MPa, $\nu = 0.293$ (Table 5.2)

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$\left.\begin{aligned} -0.002(212{,}000) &= \sigma_x - 0.293\,\sigma_y \\ 0.003(212{,}000) &= \sigma_y - 0.293\,\sigma_x \end{aligned}\right\} \text{Solve}$$

$\sigma_x = -260$, $\sigma_y = 560$ MPa ◀

If (x, y) are principal (1,2) axes, then $\tau_{xy} = 0$, and the state of stress is completely determined. Otherwise, another measurement is needed, such as the strain 45° to the x-y axes.

5.16 Al alloy, plane stress

$\varepsilon_x = 0.0004, \ \varepsilon_y = 0.0010, \ \gamma_{xy} = 0.0008$

$\sigma_x, \sigma_y, \tau_{xy}, \varepsilon_z$?

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$E = 70,300 \text{ MPa}, \ \nu = 0.345$ (Tab. 5.2)

$$0.0004\,(70,300) = \sigma_x - 0.345\,\sigma_y$$

$$0.0010\,(70,300) = \sigma_y - 0.345\,\sigma_x$$

Solve simultaneously:

$\sigma_x = 59.45, \ \sigma_y = 90.81 \text{ MPa}$ ◀

$$\tau_{xy} = G\gamma_{xy}, \quad G = \frac{E}{2(1+\nu)}$$

$$\tau_{xy} = \frac{70,300 \text{ MPa}}{2(1+0.345)}\,0.0008 = 20.91 \text{ MPa}$$ ◀

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\varepsilon_z = \frac{1}{70,300}\left[0 - 0.345(59.45 + 90.81)\right]$$

$\varepsilon_z = -0.000737$ ◀

5.17 Strains measured on mild steel part surface: $\varepsilon_x = 190 \times 10^{-6}$; $\varepsilon_y = -760 \times 10^{-6}$; $\gamma_{xy} = 300 \times 10^{-6}$; $\sigma_z = \tau_{yz} = \tau_{zx} = 0$. Find: σ_x, σ_y, τ_{xy}, ε_z

$$E\varepsilon_x = \sigma_x - \nu(\sigma_y + \sigma_z)$$
$$E\varepsilon_y = \sigma_y - \nu(\sigma_x + \sigma_z)$$
$$E = 212{,}000 \text{ MPa}, \ \nu = 0.293 \quad \text{(Table 5.2)}$$

$$\left.\begin{aligned} 212{,}000(1.9 \times 10^{-4}) &= \sigma_x - 0.293\sigma_y \\ 212{,}000(-7.6 \times 10^{-4}) &= \sigma_y - 0.293\sigma_x \end{aligned}\right\} \text{Solve}$$

$$11.802 = 0.293\sigma_x - (0.293)^2\sigma_y$$
$$-149.32 = 0.9142\sigma_y$$
$$\sigma_y = -163.34 \text{ MPa} \quad \blacktriangleleft$$
$$\sigma_x = -7.58 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{xy} = G\gamma_{xy}, \ G = \frac{E}{2(1+\nu)} = 81{,}980 \text{ MPa}$$
$$\tau_{xy} = 24.59 \text{ MPa} \quad \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = 2.36 \times 10^{-4} \quad \blacktriangleleft$$

5.18 On surface of a Ti alloy part:

$\varepsilon_x = 38.0 \times 10^{-4}$, $\varepsilon_y = 1.6 \times 10^{-4}$, $\gamma_{xy} = 7.2 \times 10^{-4}$

$\sigma_x, \sigma_y, \tau_{xy}, \varepsilon_z = ?$ $\quad \sigma_z = \tau_{yz} = \tau_{zx} = 0$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$E = 120$ GPa, $\nu = 0.361$ (Table 5.2)

$$38 \times 10^{-4} = \frac{1}{120{,}000 \text{ MPa}}(\sigma_x - 0.361\,\sigma_y)$$

$$1.6 \times 10^{-4} = \frac{1}{120{,}000 \text{ MPa}}(\sigma_y - 0.361\,\sigma_x)$$

Solving simultaneously:

$\sigma_x = 532.3$, $\sigma_y = 211.4$ MPa ◀

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \quad G = \frac{E}{2(1+\nu)}$$

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} = \frac{120{,}000 \text{ MPa}}{2(1+0.361)} 7.2 \times 10^{-4}$$

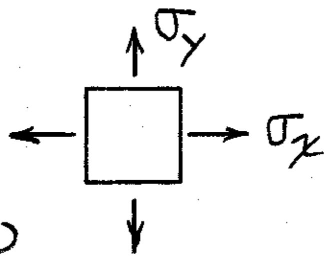
$\tau_{xy} = 31.74$ MPa ◀

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\varepsilon_z = \frac{1}{120{,}000 \text{ MPa}}\left[0 - 0.361\,(532.3 + 211.4)\right] \text{MPa}$$

$\varepsilon_z = -22.4 \times 10^{-4}$ ◀

5.19 For a spherical pressure vessel, find (a) $\Delta r = f(p, r, t, \text{mat'l.})$; (b) $\Delta t = g(p, r, \text{etc.})$



$\sigma_z \approx 0$

$r \approx r_1$, $r \approx r_{avg}$

$$\sigma_x = \sigma_y = \frac{pr}{2t} \quad \text{(Fig. A.7)}$$

$$\varepsilon_x = \varepsilon_y = \frac{\Delta(2\pi r)}{2\pi r} = \frac{\Delta r}{r}$$

$$\varepsilon_z = \frac{\Delta t}{t}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) = \frac{1}{E}\left(\frac{pr}{2t} - \nu\frac{pr}{2t}\right)$$

$$\varepsilon_x = \frac{pr(1-\nu)}{2tE} = \frac{\Delta r}{r}$$

$$\Delta r = \frac{pr^2(1-\nu)}{2tE} \quad ◀$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = \frac{-\nu pr}{tE} = \frac{\Delta t}{t}$$

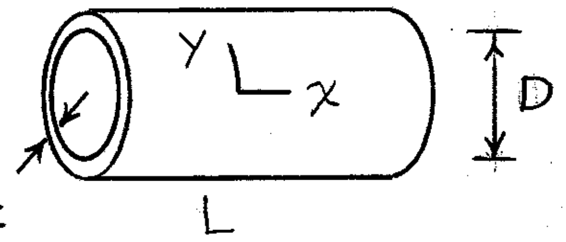
$$\Delta t = -\frac{\nu pr}{E} \quad ◀$$

Comment: The simplifying assumption $\sigma_z \approx 0$, and thus the above result, is accurate only if $r >> t$, say $r > 10t$, which is true for thin-walled vessels.

5.20 Thin walled tube, closed ends. Find $dV_e/V_e = f(p, D, t, \text{mat'l.})$. Use Fig. A.7(a).

$\sigma_z \approx 0$

$V_e = \dfrac{\pi D^2 L}{4}$



$\sigma_x = \dfrac{pD}{4t}$

$\sigma_y = \dfrac{pD}{2t}$

$$dV_e = \frac{\partial V_e}{\partial D}\,dD + \frac{\partial V_e}{\partial L}\,dL = \frac{\pi}{4}\left[2\,dL\,dD + D^2\,dL\right]$$

$$\frac{dV_e}{V_e} = 2\frac{dD}{D} + \frac{dL}{L} = 2\,\varepsilon_y + \varepsilon_x$$

$$\varepsilon_x = \frac{dL}{L}, \quad \varepsilon_y = \frac{d(\pi D)}{\pi D} = \frac{dD}{D}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = \frac{pD}{4tE}(1 - 2\nu)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] = \frac{pD}{4tE}(2 - \nu)$$

$$\frac{dV_e}{V_e} = \frac{pD}{4tE}(5 - 2\nu)$$

5.21 Thin walled spherical pressure vessel. Find $dV_e / V_e = f(p, D, t, \text{mat'l.})$. Use Fig. A.7 (b).

$$V_e = \frac{\pi D^3}{6}, \quad dV_e = \frac{\pi D^2}{2} dD$$

$$\sigma_x = \sigma_y = \frac{pD}{4t}, \quad \varepsilon_x = \varepsilon_y = \frac{d(\pi D)}{\pi D} = \frac{dD}{D}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = \frac{pD}{4tE}(1-\nu)$$

$$\frac{dV_e}{V_e} = 3\frac{dD}{D} = \frac{3pD}{4tE}(1-\nu) \quad ◀$$

5.22 Block of material with $\sigma_y = \lambda \sigma_x$, $\sigma_z = 0$. Find as $f(\sigma_x, \lambda, \text{mat'l.})$: (a) ε_z, (b) $E' = \sigma_x / \varepsilon_x$. Also, (c) compare E' with E.

(a) $$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{\nu \sigma_x}{E}(1+\lambda)$$ ◀

(b) $$E' = \frac{\sigma_x}{\varepsilon_x}, \quad \varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\varepsilon_x = \frac{\sigma_x}{E}(1 - \nu\lambda), \quad E' = \frac{E}{1 - \nu\lambda}$$ ◀

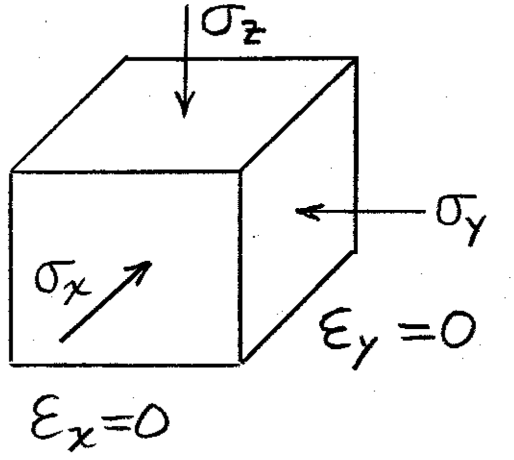
(c) Assume $\nu = 0.3$, a typical value.

λ	-1	0	$+1$
E'	$0.77E$	E	$1.43E$

◀

The effect of λ on E' is substantial.

5.23 Block of material confined by rigid die in x- and y-directions.



(a) Yes, σ_x and σ_y occur.

(b) $E' = \dfrac{\sigma_z}{\varepsilon_z}$

(c) $\nu = 0.5$

$$\left.\begin{aligned} E\varepsilon_x &= \sigma_x - \nu(\sigma_y + \sigma_z) = 0 \\ E\varepsilon_y &= \sigma_y - \nu(\sigma_x + \sigma_z) = 0 \end{aligned}\right\} \text{ Solve}$$

$$\underline{\nu\sigma_x - \nu^2(\sigma_y + \sigma_z) = 0}$$

$$\sigma_y(1-\nu^2) - \sigma_z(\nu + \nu^2) = 0$$

$$\sigma_y = \frac{\nu\sigma_z}{1-\nu}, \quad \sigma_x = \frac{\nu\sigma_z}{1-\nu} \quad \blacktriangleleft$$

$$E\varepsilon_z = \sigma_z - \nu(\sigma_x + \sigma_y) = \sigma_z - \nu\left(\frac{2\nu\sigma_z}{1-\nu}\right)$$

$$E\varepsilon_z = \sigma_z\left(\frac{1-\nu-2\nu^2}{1-\nu}\right)$$

$$E' = \frac{\sigma_z}{\varepsilon_z} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \quad \blacktriangleleft$$

$$E' \to \infty \text{ as } \nu \to \frac{1}{2} \quad \blacktriangleleft$$

5.24 Block of material confined by rigid walls in z-direction, with $\sigma_y = \lambda\sigma_x$. Find as $f(\sigma_x, \lambda, \text{mat'l.})$: (a) σ_z, (b) $E' = \sigma_x/\varepsilon_x$. Also, compare E' and E.

(a) Yes, a σ_z develops. Substitute $\varepsilon_z = 0$ into Eq. 5.26 (c).

$$0 = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = \frac{\sigma_z}{E} - \frac{\nu\sigma_x}{E}(1+\lambda)$$

$$\sigma_z = \nu\sigma_x(1+\lambda) \qquad \blacktriangleleft$$

(b) $\varepsilon_x E = \sigma_x - \nu(\sigma_y + \sigma_z)$

$$\varepsilon_x E = \sigma_x - \nu\lambda\sigma_x - \nu^2\sigma_x(1+\lambda)$$

$$\frac{\sigma_x}{\varepsilon_x} = E' = \frac{E}{1-\nu\lambda-\nu^2(1+\lambda)} \qquad \blacktriangleleft$$

λ	-1	0	$+1$
E'	0.77E	1.10 E	1.92 E

(c) ◀

The effect of λ on E' for $\nu = 0.3$ is substantial.

5.25 A block of brass (70Cu-30Zn) is confined by a die in the z-direction, $\varepsilon_z = 0$. If $\sigma_x = -60$, $\sigma_y = -100$ MPa, find σ_z, ε_x, and ε_y.

$E = 101{,}000$ MPa, $\nu = 0.350$ (Table 5.2)

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = 0$$

$$\sigma_z = \nu(\sigma_x + \sigma_y) = -56 \text{ MPa} \quad \blacktriangleleft$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) = -53.5 \times 10^{-6} \quad \blacktriangleleft$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) = -588 \times 10^{-6} \quad \blacktriangleleft$$

5.26 Mg alloy confined so that $\varepsilon_x = \varepsilon_y = 0$, $\sigma_z = -50$ MPa, no friction, no yielding.

(a) $\sigma_x, \sigma_y = ?$ $E = 44.7$ GPa, $\nu = 0.291$ (Table 5.2)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = 0, \quad \sigma_y = \frac{\sigma_x - \nu\sigma_z}{\nu}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] = 0$$

$$\frac{\sigma_x - \nu\sigma_z}{\nu} - \nu(\sigma_x + \sigma_z) = 0$$

$$\sigma_x - \nu\sigma_z - \nu^2(\sigma_x + \sigma_z) = 0$$

$$\sigma_x = \frac{\nu + \nu^2}{1 - \nu^2}\sigma_z = \frac{\nu\sigma_z}{1-\nu}$$

Similarly $\sigma_y = \dfrac{\nu\sigma_z}{1-\nu} = \sigma_x$

$$\sigma_y = \sigma_x = \frac{0.291}{1-0.291}(-50\text{ MPa}) = -20.52\text{ MPa} \quad ◀$$

(b) $\varepsilon_z = \dfrac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$, $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$$\varepsilon_z = \frac{1}{44{,}700\text{ MPa}}\left[-50 - 0.291(-20.52 \times 2)\right]\text{MPa}$$

$$\varepsilon_z = -8.51 \times 10^{-4} \quad ◀$$

$$\varepsilon_v = 0 + 0 - 8.51 \times 10^{-4} = -8.51 \times 10^{-4} \quad ◀$$

(c) $E' = \dfrac{\sigma_z}{\varepsilon_z} = 58{,}700\text{ MPa} = 58.7\text{ GPa}$ ◀

$E' > E$ due to Poisson strains from σ_x and σ_y ◀

5.27 Block of Al alloy confined in the z-direction, with $\sigma_x = \sigma_y = -100$ MPa.

$E = 70.3$ GPa, $\nu = 0.345$ (Table 5.2)

(a) $\sigma_z = ?$, (b) $E' = \sigma_x / \varepsilon_x$, (c) $\varepsilon_v = ?$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = 0$$

$$\sigma_z = \nu(\sigma_x + \sigma_y) = 0.345(-100-100)$$

$$\sigma_z = -69 = 69 \text{ MPa compression}$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_x = \frac{1}{70{,}300 \text{ MPa}}\left[\underset{\text{MPa}}{-100} - 0.345(\underset{\text{MPa}}{-100-69})\right]$$

$$\varepsilon_x = -0.0005931$$

$$E' = \frac{\sigma_x}{\varepsilon_x} = \frac{-100 \text{ MPa}}{-5.931 \times 10^{-4}} = 168.6 \text{ GPa}$$

$\varepsilon_y = \varepsilon_x$ by symmetry

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = 2(-5.931 \times 10^{-4}) + 0$$

$$\varepsilon_v = -1.186 \times 10^{-3} \text{ (− means } V \text{ decreases)}$$

5.28

$$\sigma = -\frac{E\alpha\,\Delta T}{1-\nu}, \quad \Delta T = -\frac{\sigma(1-\nu)}{E\alpha}$$

For each material, tabulate α and ν from Table P5.28, and E, σ_{ut}, and σ_{uc} from Table 3.10. Give σ_{uc} a negative sign as it is a compressive stress. Then let $\sigma = \sigma_{ut}$ to calculate ΔT for a down shock.

$$\Delta T = -\frac{140\text{ MPa}\,(1-0.18)}{(280{,}000\text{ MPa})(13.5\times10^{-6}\ 1/°C)}$$

$$\Delta T = -30.4\ °C \quad (\text{MgO; others similarly})$$

Next, let $\sigma = \sigma_{uc}$ and calculate ΔT for an up shock. For MgO, this is

$$\Delta T = -\frac{(-840)(1-0.18)}{(280{,}000)(13.5\times10^{-6})} = 182\ °C$$

Material	α 10^{-6}/deg C	ν	E GPa	σ_{ut} MPa	σ_{uc} MPa	Down deg C	Up deg C
MgO	13.5	0.18	280	140	-840	-30	182
Al_2O_3	8.0	0.22	372	262	-2620	-69	687
ZrO_2	10.2	0.30	210	147	-2100	-48	686
SiC	4.5	0.22	393	307	-2500	-135	1103
Si_3N_4	2.9	0.27	310	450	-3450	-365	2801 ◀

The smaller ΔT's for a down shock indicate that sudden cooling is more

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likely than sudden heating to cause a given material to crack. The ΔT's for either down shock or up shock vary by more than a factor of 10 among the various materials, with Si_3N_4 clearly having the best resistance to sudden temperature changes.

5.30 Block confined so that $\varepsilon_y = 0$, with $\sigma_x = 0$. $\sigma_z = -75$ MPa, ΔT occurs.

(a) As $T\uparrow$, $|\sigma_y|\uparrow$. Due to the block trying to increase length in the y-direction, σ_y will become more negative to maintain $\varepsilon_y = 0$. For $T\downarrow$, the opposite happens and $|\sigma_y|\downarrow$, but not changing after $\sigma_y = 0$ is reached and contact is lost. ◀

(b) $$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha(\Delta T)$$

$$0 = \frac{1}{E}\left[0 - \nu(0 + \sigma_z)\right] + \alpha(\Delta T)$$

$$\Delta T = \frac{\nu\sigma_z}{E\alpha} = \frac{0.343(-75\text{ MPa})}{130{,}000\text{ MPa}(16.5 \times 10^{-6}\ 1/°\text{C})}$$

$\Delta T = -11.99°C$, that is, $12°C$ decrease ◀

(E and ν for copper are from Table 5.2.)

5.32 Ti alloy with 35% unidir. SiC. Estimate E_X, E_Y, G_{XY}, ν_{XY}, and ν_{YX}

$E_m = 120$ GPa, $\nu_m = 0.361$ (Table 5.2)

$E_r = 396$ GPa, $\nu_r = 0.22$ (")

$$G_m = \frac{E_m}{2(1+\nu_m)} = 44.1 \text{ GPa}, \quad G_r = \frac{E_r}{2(1+\nu_r)} = 162 \text{ GPa}$$

$E_X = V_r E_r + V_m E_m$, $V_r = 0.35$, $V_m = 0.65$

$E_X = 217$ GPa ◀

$$E_Y = \frac{E_r E_m}{V_r E_m + V_m E_r} = 159 \text{ GPa}$$ ◀

$$\nu_{XY} = V_r \nu_r + V_m \nu_m = 0.312$$ ◀

$$\nu_{YX}/E_Y = \nu_{XY}/E_X, \quad \nu_{YX} = 0.228$$ ◀

$$G_{XY} = \frac{G_r G_m}{V_r G_m + V_m G_r} = 59.2 \text{ GPa}$$ ◀

5.33 Epoxy with 60% unidir, E-glass as in Table 5.3. Estimate E_X, E_Y, G_{XY}, ν_{XY}, and ν_{YX}. Compare to table.

$E_r = 72.3$ GPa, $\nu_r = 0.22$, $V_r = 0.60$
$E_m = 3.5$ GPa, $\nu_m = 0.33$, $V_m = 1 - V_r$

$$E_X = V_r E_r + V_m E_m = 44.8 \text{ GPa}$$

$$E_Y = \frac{E_r E_m}{V_r E_m + V_m E_r} = 8.16 \text{ GPa}$$

$$G_r = \frac{E_r}{2(1+\nu_r)} = 29.6 \text{ GPa}, \quad G_m = \frac{E_m}{2(1+\nu_m)} = 1.32 \text{ GPa}$$

$$G_{XY} = \frac{G_r G_m}{V_r G_m + V_m G_r} = 3.08 \text{ GPa}$$

$$\nu_{xy} = V_r \nu_r + V_m \nu_m = 0.264$$
$$\nu_{YX}/E_Y = \nu_{XY}/E_X, \quad \nu_{YX} = 0.048$$

Comparison of these with Table 5.3:

Source	E_X (GPa)	E_Y (GPa)	G_{XY} (GPa)	ν_{XY}	ν_{YX}
Calc.	44.8	8.16	3.08	0.264	0.048
Table	45	12	4.4	0.25	—

Calculated and tabulated values agree roughly. The calculations are approximate, especially for E_Y, which is a lower bound.

5.34 Epoxy with 60% unidir. Kevlar as in Table 5.3. Estimate E_X, E_Y, G_{XY}, ν_{XY} and ν_{YX}. Compare to table.

$E_r = 124$ GPa, $\nu_r = 0.35$, $V_r = 0.60$

$E_m = 3.5$ GPa, $\nu_m = 0.33$, $V_m = 1 - V_r$

$$E_X = V_r E_r + V_m E_m = 75.8 \text{ GPa}$$

$$E_Y = \frac{E_r E_m}{V_r E_m + V_m E_r} = 8.39 \text{ GPa}$$

$$G_r = \frac{E_r}{2(1+\nu_r)} = 45.9 \text{ GPa}, \quad G_m = \frac{E_m}{2(1+\nu_m)} = 1.32 \text{ GPa}$$

$$G_{XY} = \frac{G_r G_m}{V_r G_m + V_m G_r} = 3.15 \text{ GPa}$$

$$\nu_{XY} = V_r \nu_r + V_m \nu_m = 0.342$$

$$\nu_{YX} / E_Y = \nu_{XY} / E_X, \quad \nu_{YX} = 0.038$$

Comparison with Table 5.3:

Source	E_X (GPa)	E_Y (GPa)	G_{XY} (GPa)	ν_{XY}	ν_{YX}
Calc.	75.8	8.39	3.15	0.342	0.038
Table	76	5.5	2.1	0.34	—

E_X and ν_{XY} agree quite well. E_Y and G_{XY} are not very close, but E_Y is only expected to be a lower bound.

5.36 Epoxy, with 0 to 100% unidir. E-glass. Calculate E_x, E_y for various V_r, and plot.

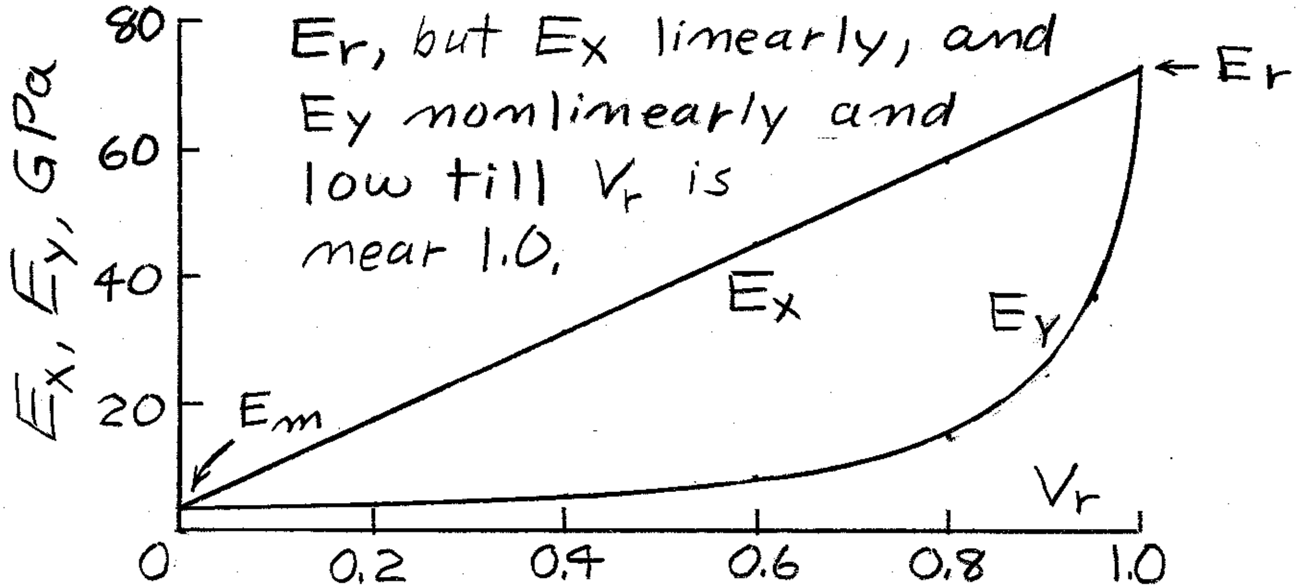
$E_r = 72.3$ GPa, $\nu_r = 0.22$
$E_m = 3.5$ GPa, $\nu_m = 0.33$ (Table 5.3)

$$E_x = V_r E_r + V_m E_m \qquad (\parallel \text{ fibers})$$

$$E_y = \frac{E_r E_m}{V_r E_m + V_m E_r} \qquad (\perp \text{ fibers})$$

V_r	E_x, GPa	E_y, GPa
0	3.5	3.5
0.2	17.3	4.32
0.4	31.0	5.65
0.6	44.8	8.16
0.8	58.5	14.7
1.0	72.3	72.3

Both increase from E_m to E_r, but E_x linearly, and E_y nonlinearly and low till V_r is near 1.0.



5.37 Epoxy with 60% unidir. T-300 graphite, Table 5.3. Find $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ for $\sigma_x = 300, \sigma_y = -100, \tau_{xy} = 15$ MPa.

$E_x = 132, \; E_y = 10.3, \; G_{xy} = 6.5$ GPa

$\nu_{xy} = 0.25$

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}}{E_y}\sigma_y \;, \quad \varepsilon_y = -\frac{\nu_{xy}}{E_x}\sigma_x + \frac{\sigma_y}{E_y}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G_{xy}} \;, \quad \frac{\nu_{yx}}{E_y} = \frac{\nu_{xy}}{E_x} = \frac{0.25}{132{,}000} = 1.894 \times 10^{-6} \; 1/\text{MPa}$$

$$\varepsilon_x = \frac{300}{132{,}000} - (1.894 \times 10^{-6})(-100) = 2.46 \times 10^{-3}$$ ◀

$$\varepsilon_y = -(1.894 \times 10^{-6})(300) + \frac{-100}{10{,}300} = -10.28 \times 10^{-3}$$ ◀

$$\gamma_{xy} = \frac{15}{6{,}500} = 2.31 \times 10^{-3}$$ ◀

5.39 Unidirectional fibers comp. sheet.

$E_m = 3.5$ GPa, $\nu_m = 0.33$, $V_m = 0.30$

$E_r = 531$ GPa, $\nu_r = 0.20$, $V_r = 0.70$

$\varepsilon_X = 0.005$, $\varepsilon_Y = 0.001$; $\sigma_X, \sigma_Y = ?$

First estimate E_X, E_Y, ν_{XY}, then apply anisotropic Hooke's Law, with (X,Y) the in-plane material symmetry directions.

$$E_X = V_r E_r + V_m E_m = 373 \text{ GPa}$$

$$\frac{1}{E_Y} = \frac{V_r}{E_r} + \frac{V_m}{E_m}, \quad E_Y = 11.5 \text{ GPa}$$

$$\nu_{XY} = V_r \nu_r + V_m \nu_m = 0.239$$

$$\nu_{XY}/E_X = \nu_{YX}/E_Y = 0.239/373 \quad 1/\text{GPa}$$

$$\varepsilon_X = \frac{\sigma_X}{E_X} - \frac{\nu_{YX}}{E_Y}\sigma_Y, \quad 0.005 = \frac{\sigma_X}{373} - \frac{0.239}{373}\sigma_Y$$

$$\varepsilon_Y = -\frac{\nu_{XY}}{E_X}\sigma_X + \frac{\sigma_Y}{E_Y}, \quad 0.001 = -\frac{0.239}{373}\sigma_X + \frac{\sigma_Y}{11.5}$$

$$\left.\begin{aligned} \sigma_X - 0.239\,\sigma_Y &= 1.865 \text{ GPa} \\ -\sigma_X + 135.7\,\sigma_Y &= 1.561 \text{ GPa} \end{aligned}\right\} \text{solve}$$

$$135.2\,\sigma_Y = 3.426$$

$$\sigma_Y = 0.0253 \text{ GPa} = 25.3 \text{ MPa}$$

$$\sigma_X = 1.871 \text{ GPa} = 1871 \text{ MPa}$$

5.40 Graphite-epoxy composite, 65% unidirectional fibers. Two experiments:
(1) $\sigma_x = 150$ MPa, $\varepsilon_x = 1138 \times 10^{-6}$, $\varepsilon_y = -372 \times 10^{-6}$
(2) $\sigma_y = 11.2$ MPa, $\varepsilon_y = 1165 \times 10^{-6}$, $\varepsilon_x \approx -22 \times 10^{-6}$
X is // fibers, Y transverse. $E_m = 3.5$ GPa

(a) $E_X, E_Y, \nu_{XY}, \nu_{YX} = ?$ (b) $E_r = ?$

Using (1): $E_X = \dfrac{\sigma_X}{\varepsilon_X} = \dfrac{0.150 \text{ GPa}}{1138 \times 10^{-6}} = 131.8 \text{ GPa}$ ◀

$$\nu_{XY} = -\frac{\varepsilon_Y}{\varepsilon_X} = -\frac{-372 \times 10^{-6}}{1138 \times 10^{-6}} = 0.327$$ ◀

Using (2): $E_Y = \dfrac{\sigma_Y}{\varepsilon_Y} = \dfrac{0.0112 \text{ GPa}}{1165 \times 10^{-6}} = 9.61 \text{ GPa}$ ◀

$$\nu_{YX} = -\frac{\varepsilon_X}{\varepsilon_Y} = -\frac{-22 \times 10^{-6}}{1165 \times 10^{-6}} \approx 0.019$$ ◁

Better value: $\nu_{YX} / E_Y = \nu_{XY} / E_X$

$$\nu_{YX} = 0.327 \, \frac{9.61 \text{ GPa}}{131.8 \text{ GPa}} = 0.024$$ ◀

(b) $E_X = V_r E_r + V_m E_m$, $V_m = 1 - V_r$

$$E_r = \frac{E_X - (1 - V_r) E_m}{V_r} = \frac{131.8 - 0.35(3.5)}{0.65} \text{ GPa}$$

$E_r = 201$ GPa ◀

Note: The equation for E_Y could be used for (b), but this is only a lower bound on E_Y, and subtraction of two nearly equal quantities is encountered. Hence, the $E_r = 162$ GPa obtained is not used.

5.41 Composite with unidirectional Kevlar fibers in epoxy matrix. Minimum properties required: $E_x = 70$, $E_y = 10$ GPa. Find (a) V_r, and for resulting composite (b) G_{xy}, and (c) ν_{xy}, ν_{yx}. Use Table 5.3 properties.

First consider E_x requirement.

$$E_X = V_r E_r + V_m E_m$$

$$70 = V_r(124) + (1 - V_r)\,3.5 \quad GPa$$

$$V_r = 0.552 \quad \triangleleft$$

Then consider E_y requirement.

$$\frac{1}{E_Y} = \frac{V_r}{E_r} + \frac{V_m}{E_m}, \quad \frac{1}{10} = \frac{V_r}{124} + \frac{(1-V_r)}{3.5} \quad \frac{1}{GPa}$$

$V_r = 0.669$ Use this as satisfies both requirements ◀

Material	E, GPa	ν	V	G, GPa
Kevlar (r)	124	0.35	0.669	45.92
Epoxy (m)	3.5	0.33	0.331	1.316

$$G_r = \frac{E_r}{2(1+\nu_r)} = 45.92 \ GPa, \quad G_m = \frac{E_m}{2(1+\nu_m)} = 1.316 \ GPa$$

$$\frac{1}{G_{xy}} = \frac{V_r}{G_r} + \frac{V_m}{G_m} = \frac{0.669}{45.92} + \frac{0.331}{1.316}, \quad G_{xy} = 3.76 \ GPa \ \blacktriangleleft$$

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$$\nu_{xy} = V_r \nu_r + V_m \nu_m = 0.669(0.35) + 0.331(0.33)$$

$$\nu_{xy} = 0.343 \blacktriangleleft$$

$$\frac{\nu_{yx}}{E_y} = \frac{\nu_{xy}}{E_x}, \quad E_y = 10 \text{ GPa}$$

$$E_x = V_r E_r + V_m E_m = 0.669(124) + 0.331(3.5)$$

$$= 84.1 \text{ GPa}$$

$$\nu_{yx} = E_y \frac{\nu_{xy}}{E_x} = 10 \frac{0.343}{84.1} = 0.0407 \blacktriangleleft$$

Comment: Note that the $V_r = 0.669$ choice gives $E_y = 10$ GPa, and $E_x = 84.1$ GPa, as calculated just above. However, if the $V_r = 0.552$ choice is made, $E_x = 70$ GPa, but E_y fails its requirement, as below.

$$\frac{1}{E_y} = \frac{V_r}{E_r} + \frac{V_m}{E_m}, \quad \frac{1}{E_y} = \frac{0.552}{124} + \frac{(1-0.552)}{3.5}$$

$$E_y = 7.55 \text{ GPa}, < 10, \text{ no good}$$

5.42 SiC in Ti alloy matrix.

$V_f = ?$ minimum if unidirectional fibers and $E_x = 250$, $G_{xy} = 60$ GPa, minimum

$E_f = 396$ GPa, $\nu_f = 0.22$ (Table 5.2)

$E_m = 120$ GPa, $\nu_m = 0.361$

$$G_f = \frac{E_f}{2(1+\nu_f)} = 162.3 \text{ GPa}$$

$$G_m = \frac{E_m}{2(1+\nu_m)} = 44.1 \text{ GPa}$$

$$E_x = V_f E_f + V_m E_m = V_f E_f + (1-V_f) E_m$$

$$V_f = \frac{E_x - E_m}{E_f - E_m} = \frac{250-120}{396-120} = 0.471$$

$$G_{xy} = \frac{G_f G_m}{V_f G_m + V_m G_f} = \frac{162.3 \times 44.1}{0.471 \times 44.1 + (1-0.471)162.3}$$

$G_{xy} = 67.12$ GPa > 60 GPa, O.K.

$V_f = 0.471$ ◀

5.43 Tungsten wire in aluminum matrix, unidirectional. $E_X = 225$ and $E_Y = 100$ GPa, minimums. (a) $V_r = ?$

$E_r = 411$ GPa, $\nu_r = 0.280$ (Table 5.2)
$E_m = 70.3$ MPa, $\nu_m = 0.345$

$$E_X = V_r E_r + V_m E_m$$

$$225 = V_r(411) + (1 - V_r)70.3 \quad \text{GPa}$$

$$V_r = \frac{225 - 70.3}{411 - 70.3} = 0.454 \quad \triangleleft$$

$$\frac{1}{E_Y} = \frac{V_r}{E_r} + \frac{V_m}{E_m}, \quad \frac{1}{100} = \frac{V_r}{411} + \frac{1 - V_r}{70.3} \quad \frac{1}{\text{GPa}}$$

$$V_r = \frac{1/100 - 1/70.3}{1/411 - 1/70.3} = 0.358 \quad \triangleleft$$

$V_r = 0.454$ controls ◀

(b) $G_{XY} = ?$ for $V_r = 0.454$

$$G_r = \frac{E_r}{2(1+\nu_r)} = \underset{\text{GPa}}{160.5}, \quad G_m = \frac{E_m}{2(1+\nu_m)} = \underset{\text{GPa}}{26.1}$$

$$\frac{1}{G_{XY}} = \frac{V_r}{G_r} + \frac{V_m}{G_m} = \frac{0.454}{160.5} + \frac{(1 - 0.454)}{26.1}$$

$G_{XY} = 42.1$ GPa ◀

5.44 Unidirectional fiber or wire composite

$E_x = 170$, $E_y = 85$ GPa, minimums

(a) Mg matrix, $V_r = 0.60$, $V_m = 1 - V_r = 0.40$

$E_m = 44.7$ GPa, $\nu_m = 0.291$ (Table 5.2)

$E_r = ?$ min required

$E_x = V_r E_r + V_m E_m$

$170 = 0.60 E_r + 0.40(44.7)$ GPa

$E_r = 254$ GPa ◁

$$\frac{1}{E_y} = \frac{V_r}{E_r} + \frac{V_m}{E_m}, \quad \frac{1}{85} = \frac{0.60}{E_r} + \frac{0.40}{44.7} \quad \frac{1}{\text{GPa}}$$

$E_r = 213$ GPa ◁

The larger value controls, $E_r = 254$ GPa ◀

(b) Candidate materials:

SiC, $E_r = 396$ GPa

Tungsten, $E_r = 411$ GPa ◀