7.1

For the given state of stress, calculate principal normal stresses. Then use the maximum normal stress fracture criterion to obtain the effective stress and safety factor.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\tau 3} \pm \tau_3, \qquad \sigma_3 = \sigma_z$$

$$\bar{\sigma}_{NT} = \mathrm{MAX}(\sigma_1, \sigma_2, \sigma_3), \qquad (\bar{\sigma}_{NT} > 0,\ |\sigma_{max}| > |\sigma_{min}|)$$

$$X = \sigma_{ut} / \bar{\sigma}_{NT}$$

Stresses in MPa $\qquad$ Si_3N_4 (Table 3.10)

σ_x	σ_y	σ_z	τ_{xy}	$\sigma_{\tau 3}$	τ_3
125.00	15.00	0.00	-25.00	70.00	60.42

σ_1	σ_2	σ_3	σ_{ut}	$\bar{\sigma}_{NT}$	X
130.42	9.58	0.00	450	130.42	**3.45**

7.2

For the given state of stress, calculate principal normal stresses. Then use the maximum normal stress fracture criterion to obtain the effective stress and safety factor.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\tau 3} \pm \tau_3 , \qquad \sigma_3 = \sigma_z$$

$$\bar{\sigma}_{NT} = \text{MAX}(\sigma_1, \sigma_2, \sigma_3), \qquad (\bar{\sigma}_{NT} > 0,\ |\sigma_{max}| > |\sigma_{min}|)$$

$$X = \sigma_{ut} / \bar{\sigma}_{NT}$$

Stresses in MPa Gray cast iron

σ_x	σ_y	σ_z	τ_{xy}	$\sigma_{\tau 3}$	τ_3
50.00	80.00	0.00	20.00	65.00	25.00

σ_1	σ_2	σ_3	σ_{ut}	$\bar{\sigma}_{NT}$	X
90.00	40.00	0.00	214	90.00	**2.38**

7.3

For the given state of stress, calculate principal normal stresses. Then use the maximum normal stress fracture criterion to obtain the effective stress and safety factor.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\tau 3} \pm \tau_3 , \qquad \sigma_3 = \sigma_z$$

$$\bar{\sigma}_{NT} = \mathrm{MAX}(\sigma_1, \sigma_2, \sigma_3), \qquad (\bar{\sigma}_{NT} > 0,\ |\sigma_{max}| > |\sigma_{min}|)$$

$$X = \sigma_{ut} / \bar{\sigma}_{NT}$$

Stresses in MPa SiC (Table 3.10)

σ_x	σ_y	σ_z	τ_{xy}	$\sigma_{\tau 3}$	τ_3
50.00	10.00	-20.00	-15.00	30.00	25.00

σ_1	σ_2	σ_3	σ_{ut}	$\bar{\sigma}_{NT}$	X
55.00	5.00	-20.00	307	55.00	**5.58**

7.4 Spherical pressure vessel, $t = 2.5$,
$d_2 = 150\ mm$, $p = 0.5\ MPa$, $k_t = 1.2$
Glass, similar to soda-lime glass
$\sigma_{ut} = 50\ MPa$ (Table 3.10)

$$\sigma_x = \sigma_y = \frac{p r_1}{2t} \times k_t = \frac{(0.5\ MPa)(72.5\ mm)(1.2)}{2(2.5\ mm)}$$

$$r_1 = d_2/2 - t = 75 - 2.5 = 72.5\ mm$$

$$\sigma_x = \sigma_y = 8.7\ MPa, \quad \sigma_z = -p = -0.5\ MPa$$

$$\sigma_1, \sigma_2, \sigma_3 = \sigma_x, \sigma_y, \sigma_z = 8.7, 8.7, -0.5\ MPa$$

$$\bar{\sigma}_{NT} = MAX(\sigma_1, \sigma_2, \sigma_3) = 8.7\ MPa$$

$$X_u = \frac{\sigma_{ut}}{\bar{\sigma}_{NT}} = \frac{50}{8.7} = 5.75$$ ◀

7.5 Pipe with closed ends, $t = 5\ mm$, $d_1 = 3\ m$, $p = 2\ MPa$, 18 Ni maraging steel. Find safety factor against yielding. Use Fig. A.7(a).

$$\sigma_t = \frac{pr_1}{t} = \frac{(2\ MPa)(1500\ mm)}{5\ mm} = 600\ MPa$$

$$\sigma_x = \frac{pr_1}{2t} = 300\ MPa$$

$\sigma_r = -p = -2\ MPa$ (inside), $\sigma_r = 0$ (outside)

$$\sigma_1, \sigma_2, \sigma_3 = \sigma_t, \sigma_x, \sigma_r \qquad (\text{since } \tau_{tx}, \tau_{xr}, \tau_{rt} = 0)$$

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right), \qquad X_S = \sigma_o / \bar{\sigma}_S$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}, \qquad X_H = \sigma_o / \bar{\sigma}_S$$

Stresses in MPa — **18 Ni maraging steel (Table 4.2)**

Location	σ_t	σ_x	σ_r	σ_1	σ_2	σ_3
inside	600.00	300.00	-2.00	600.00	300.00	-2.00
outside	600.00	300.00	0.00	600.00	300.00	0.00

		Max Shear		Oct Shear	
Location	σ_o	$\bar{\sigma}_S$	X_S	$\bar{\sigma}_H$	X_H
inside	1791	602.00	**2.98**	521.35	**3.44**
outside	1791	600.00	**2.99**	519.62	**3.45**

Inside has a slightly lower safety factor.
$X_S = 2.98$ (max shear), $X_H = 3.44$ (oct shear) ◀

7.6

For the given state of stress, determine the safety factor against yielding by (a) the maximum shear stress yield criterion, and (b) the octahedral shear stress yield criterion. First calculate principal normal stresses, and then use these to obtain the effective stress and safety factor for each method. Or for (b), the alternate form can be applied without first calculating principal stresses.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\tau 3} \pm \tau_3, \qquad \sigma_3 = \sigma_z$$

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right), \qquad X_S = \sigma_o / \bar{\sigma}_S$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}, \qquad X_H = \sigma_o / \bar{\sigma}_H$$

Or use alternate form for $\bar{\sigma}_H$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

Stresses in MPa AISI 1020 steel (Table 4.2)

σ_x	σ_y	σ_z	τ_{xy}	$\sigma_{\tau 3}$	τ_3	σ_1	σ_2	σ_3
-100.00	40.00	0.00	-50.00	-30.00	86.02	56.02	-116.02	0.00

	Max Shear		Oct Shear	
σ_o	$\bar{\sigma}_S$	X_S	$\bar{\sigma}_H$	X_H
260	172.05	**1.51**	151.99	**1.71**

7.7

For the given state of stress, determine the safety factor against yielding by (a) the maximum shear stress yield criterion, and (b) the octahedral shear stress yield criterion. First calculate principal normal stresses, and then use these to obtain the effective stress and safety factor for each method. Or for (b), the alternate form can be applied without first calculating principal stresses.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\tau 3} \pm \tau_3\,, \qquad \sigma_3 = \sigma_z$$

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right), \qquad X_S = \sigma_o / \bar{\sigma}_S$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}\,, \qquad X_H = \sigma_o / \bar{\sigma}_H$$

Or use alternate form for $\bar{\sigma}_H$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

Stresses in MPa 7075-T6 aluminum (Table 4.2)

σ_x	σ_y	σ_z	τ_{xy}	$\sigma_{\tau 3}$	τ_3	σ_1	σ_2	σ_3
100.00	140.00	-60.00	80.00	120.00	82.46	202.46	37.54	-60.00

	Max Shear		Oct Shear	
σ_o	$\bar{\sigma}_S$	X_S	$\bar{\sigma}_H$	X_H
469	262.46	**1.79**	229.78	**2.04**

7.8

For the given state of stress, determine the yield strength required for a safety factor against yielding of 3.0 by (a) the maximum shear stress yield criterion, and (b) the octahedral shear stress yield criterion. First calculate principal normal stresses, and then use these to obtain the effective stress and required yield strength for each method. Or for (b), the alternate form can be applied without first calculating principal stresses.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\tau 3} \pm \tau_3\,, \qquad \sigma_3 = \sigma_z$$

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right), \qquad \sigma_o = X_S \bar{\sigma}_S$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}\,, \qquad \sigma_o = X_H \bar{\sigma}_H$$

Or use alternate form for $\bar{\sigma}_H$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

Stresses in MPa

σ_x	σ_y	σ_z	τ_{xy}	$\sigma_{\tau 3}$	τ_3	σ_1	σ_2	σ_3
150.00	30.00	0.00	-45.00	90.00	75.00	165.00	15.00	0.00

Max Shear			Oct Shear		
$\bar{\sigma}_S$	X_S	σ_o	$\bar{\sigma}_H$	X_H	σ_o
165.00	3.00	**495**	158.03	3.00	**474**

7.9

For the given state of stress, determine the yield strength required for a safety factor against yielding of 2.0 by (a) the maximum shear stress yield criterion, and (b) the octahedral shear stress yield criterion. First calculate principal normal stresses, and then use these to obtain the effective stress and required yield strength for each method. Or for (b), the alternate form can be applied without first calculating principal stresses.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\tau 3} \pm \tau_3, \qquad \sigma_3 = \sigma_z$$

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right), \qquad \sigma_o = X_S \bar{\sigma}_S$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}, \qquad \sigma_o = X_H \bar{\sigma}_H$$

Or use alternate form for $\bar{\sigma}_H$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

Stresses in MPa

σ_x	σ_y	σ_z	τ_{xy}	$\sigma_{\tau 3}$	τ_3	σ_1	σ_2	σ_3
120.00	-50.00	200.00	60.00	35.00	104.04	139.04	-69.04	200.00

Max Shear			Oct Shear		
$\bar{\sigma}_S$	X_S	σ_o	$\bar{\sigma}_H$	X_H	σ_o
269.04	2.00	**538**	244.34	2.00	**489**

◀

7.10 AISI 1020 steel, $\sigma_o = 260$ MPa,
$E = 203$ GPa, $\nu = 0.293$ (Tables 4.2, 5.2)
$\varepsilon_x = 190 \times 10^{-6}$, $\varepsilon_y = -760 \times 10^{-6}$, $\gamma_{xy} = 300 \times 10^{-6}$
Plane stress, $X = ?$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu\sigma_y]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu\sigma_x]$$

$$\gamma_{xy} = \tau_{xy}/G, \quad G = \frac{E}{2(1+\nu)} \quad 78.5 \text{ GPa}$$

$$(190 \times 10^{-6})(203{,}000) = \sigma_x - 0.293\,\sigma_y \text{ MPa}$$
$$(-760 \times 10^{-6})(203{,}000) = -0.293\,\sigma_x + \sigma_y \text{ MPa}$$

Solve; $\sigma_x = -7.26$, $\sigma_y = -156.4$ MPa

$\tau_{xy} = G\gamma_{xy} = 23.55$ MPa

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

$\bar{\sigma}_H = 158.3$

$X = \sigma_o/\bar{\sigma}_H = 1.64$ ◀

7.11 Rosette strain gage measurements on 7075-T6 Al surface: $\varepsilon_x = 1200\times10^{-6}$, $\varepsilon_y = -650\times10^{-6}$, $\varepsilon_{45} = 1900\times10^{-6}$. Find safety factor against yielding.

From Ex. 6.9: $\gamma_{xy} = 2\varepsilon_{45} - \varepsilon_x - \varepsilon_y = 3250\times10^{-6}$

Substituting ε_x, ε_y, and γ_{xy} into Hooke's Law gives the nonzero stresses σ_x, σ_y, τ_{xy}. (See Prob. 6.52 solution.) Then calculate $\bar{\sigma}_S$ or $\bar{\sigma}_H$ and safety factor.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{\tau 3} \pm \tau_3, \qquad \sigma_3 = \sigma_z$$

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right), \qquad X_S = \sigma_o / \bar{\sigma}_S$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}, \qquad X_H = \sigma_o / \bar{\sigma}_H$$

Or use alternate form for $\bar{\sigma}_H$

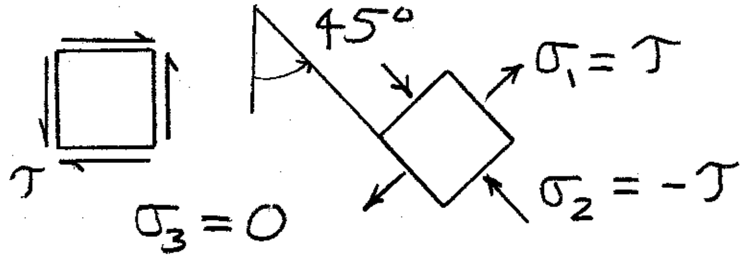
$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

Stresses in MPa 7075-T6 aluminum (Table 4.2)

σ_x	σ_y	σ_z	τ_{xy}	$\sigma_{\tau 3}$	τ_3	σ_1	σ_2	σ_3
77.86	-18.83	0.00	84.93	29.52	97.73	127.25	-68.22	0.00

	Max Shear		Oct Shear	
σ_o	$\bar{\sigma}_S$	X_S	$\bar{\sigma}_H$	X_H
469	195.46	**2.40**	171.83	**2.73**

7.12 Solid circular shaft in torsion



$2r = d = \text{diam.}$

$$\tau = \frac{Tr}{J}, \quad J = \frac{\pi r^4}{2}$$

$$\tau = \frac{2T}{\pi r^3} = \frac{16T}{\pi d^3}$$

(a) $d = ?$ for τ_{max} criterion.

Assume safety factor $X = \frac{\sigma_o}{\bar{\sigma}_S}$ also given.

$$\frac{\sigma_o}{X} = \bar{\sigma}_S = MAX(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) = 2\tau$$

$$\frac{\sigma_o}{X} = \frac{32T}{\pi d^3}, \quad d_S = \left(\frac{32TX}{\pi \sigma_o}\right)^{1/3}$$ ◀

(b) $d = ?$ for τ_h criterion, and compare.

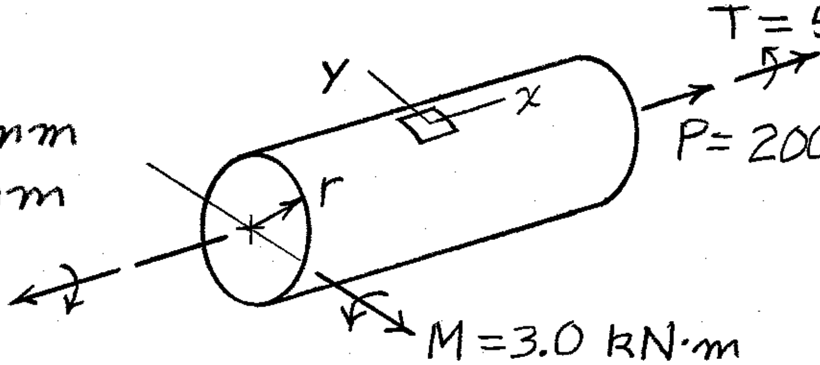
$$\frac{\sigma_o}{X} = \bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\frac{\sigma_o}{X} = \sqrt{3}\,\tau = \frac{16\sqrt{3}\,T}{\pi d^3}, \quad d_H = \left(\frac{16\sqrt{3}\,TX}{\pi \sigma_o}\right)^{1/3}$$ ◀

$$\frac{d_S}{d_H} = \left(\frac{32}{16\sqrt{3}}\right)^{1/3} = 1.049$$

(a) requires d to be about 5% larger than does (b)

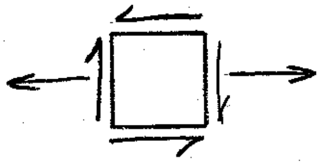
7.13 Solid circular shaft, $X_0 = 2$, $\sigma_0 = ?$



$d = 60$ mm

$r = 30$ mm

$\sigma_y = 0$



$$\sigma_x = \frac{P}{A} + \frac{Mr}{I} \qquad I = \frac{\pi r^4}{4}$$

$$\tau_{xy} = \frac{Tr}{J} \qquad J = \frac{\pi r^4}{2}$$

$$\sigma_x = \frac{200{,}000\ \text{N}}{\pi (30)^2\ \text{mm}^2} + \frac{(3.0 \times 10^6\ \text{N}\cdot\text{mm})(30\ \text{mm})}{\pi (30)^4/4\ \text{mm}^4}$$

$$\sigma_x = 212.2\ \text{MPa}$$

$$\tau_{xy} = \frac{(5 \times 10^6)(30)}{\pi (30)^4/2} = 117.9\ \text{MPa}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1, \sigma_2 = 106.1 \pm 158.6 = 264.7, -52.5\ \text{MPa}$$

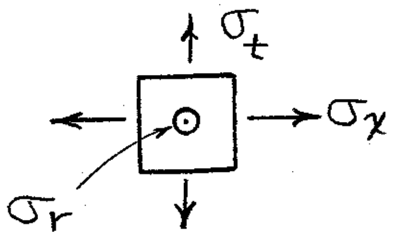
$$\sigma_3 = \sigma_z = 0$$

$$\bar{\sigma}_S = \text{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right)$$

$$\bar{\sigma}_S = 317.2\ \text{MPa} = \sigma_0 / X_0$$

$$\sigma_0 = 634\ \text{MPa}$$

7.14 Pipe with closed ends: $d_2 = 80$, $t = 3.0$ mm, $p = 20$ MPa, $M = 2.0$ kN·m. 7075-T6 AL.
Find safety factor by (a) $\bar{\sigma}_S$, (b) $\bar{\sigma}_H$.



$r_2 = 40$, $r_1 = 37$ mm

$r_{avg} = 38.5$ mm

Use Figs. A.7 (a), A.8

$$\sigma_t = \frac{p r_1}{t} = \frac{(20\text{ MPa})(37\text{ mm})}{3.0\text{ mm}} = 246.7\text{ MPa}$$

$$\sigma_x = \frac{p r_1}{2t} \pm \frac{M}{\pi r_{avg}^2 t} = 123.3 \pm \frac{2.0\times10^6\text{ N·mm}}{\pi(38.5)^2(3.0)\text{ mm}^3}$$

$\sigma_x = 266.5, -19.8$ MPa (T or C side of bending)

$\sigma_r = 0$ (outside), $\sigma_r = -p = -20$ MPa (inside)

$$\sigma_1, \sigma_2, \sigma_3 = \sigma_t, \sigma_x, \sigma_r \qquad (\text{since } \tau_{tx}, \tau_{xr}, \tau_{rt} = 0)$$

$$\bar{\sigma}_S = \text{MAX}\left(|\sigma_1-\sigma_2|, |\sigma_2-\sigma_3|, |\sigma_3-\sigma_1|\right), \qquad X_S = \sigma_o / \bar{\sigma}_S$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}, \qquad X_H = \sigma_o / \bar{\sigma}_S$$

7075-T6 aluminum (Table 4.2): σ_o, MPa = 469

Stresses in MPa

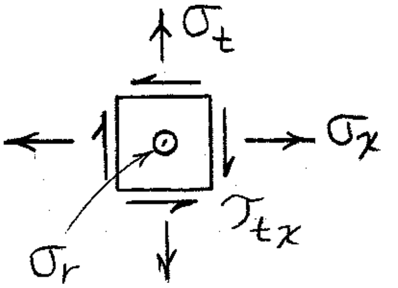
Location	σ_t	σ_x	σ_r	τ_{tx}	σ_1	σ_2	σ_3
outside, T	246.7	266.5	0.0	0.0	246.7	266.5	0.0
inside, T	246.7	266.5	-20.0	0.0	246.7	266.5	-20.0
outside, C	246.7	-19.8	0.0	0.0	246.7	-19.8	0.0
inside, C	246.7	-19.8	-20.0	0.0	246.7	-19.8	-20.0

(7.14, p.2)

	Max Shear		Oct Shear	
Location	$\bar{\sigma}_S$	X_S	$\bar{\sigma}_H$	X_H
outside, T	266.5	1.76	257.2	1.82
inside, T	286.5	**1.64**	277.1	**1.69**
outside, C	266.5	1.76	257.2	1.82
inside, C	266.7	1.76	266.6	1.76

Inside on T-side of bending has lowest safety factor, $X_S = 1.64$ (max shear), or $X_H = 1.69$ (oct. shear)

7.15 Pipe with closed ends: $d_2 = 80, t = 3.0$ mm, $p = 20$ MPa, $M = 2.0$, $T = 3.0$ kN·m, 7075-T6 Al. Find safety factor by (a) $\bar{\sigma}_S$, (b) $\bar{\sigma}_H$.



$r_2 = 40,\ r_1 = 37$ mm

$r_{avg} = 38.5$ mm

Use Figs. A.7(a) and A.8.

$$\sigma_t = \frac{p r_1}{t} = \frac{(20\text{ MPa})(37\text{ mm})}{3.0\text{ mm}} = 246.7\text{ MPa}$$

$$\sigma_x = \frac{p r_1}{2t} \pm \frac{M}{\pi r_{avg}^2 t} = 123.3 \pm \frac{2.0\times10^6\text{ N·mm}}{\pi(38.5)^2(3.0)\text{ mm}^4}$$

$\sigma_x = 266.5, -19.8$ MPa (T or C side)

$\sigma_r = 0$ (outside), $\sigma_r = -p = -20$ MPa (inside)

$$\tau_{tx} = \frac{T}{2\pi r_{avg}^2 t} = \frac{3.0\times10^6\text{ N·mm}}{2\pi(38.5)^2(3.0)\text{ mm}^4} = 107.4\text{ MPa}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_t + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_t - \sigma_x}{2}\right)^2 + \tau_{tx}^2} = \sigma_{\tau 3} \pm \tau_3, \qquad \sigma_3 = \sigma_r$$

$$\bar{\sigma}_S = \text{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right), \qquad X_S = \sigma_o / \bar{\sigma}_S$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}, \qquad X_H = \sigma_o / \bar{\sigma}_H$$

Or use alternate form for $\bar{\sigma}_H$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_t - \sigma_x)^2 + (\sigma_x - \sigma_r)^2 + (\sigma_r - \sigma_t)^2 + 6(\tau_{tx}^2 + \tau_{xr}^2 + \tau_{rt}^2)}$$

(7.15, p.2)

7075-T6 aluminum (Table 4.2): σ_o, MPa = 469

Stresses in MPa

Location	σ_t	σ_x	σ_r	τ_{tx}	$\sigma_{\tau 3}$	τ_3
outside, T	246.7	266.5	0.0	107.4	256.6	107.8
inside, T	246.7	266.5	-20.0	107.4	256.6	107.8
outside, C	246.7	-19.8	0.0	107.4	113.4	171.1
inside, C	246.7	-19.8	-20.0	107.4	113.4	171.1

			Max Shear		Oct Shear	
σ_1	σ_2	σ_3	$\bar{\sigma}_S$	X_S	$\bar{\sigma}_H$	X_H
364.4	148.8	0.0	364.4	1.29	317.4	1.48
364.4	148.8	-20.0	384.4	**1.22**	333.7	**1.41**
284.5	-57.7	0.0	342.3	1.37	317.4	1.48
284.5	-57.7	-20.0	342.3	1.37	325.0	1.44

Inside on T-side of bending has lowest safety factor, $X_S = 1.22$ (max shear), or $X_H = 1.41$ (oct. shear). ◀

7.16 Solid shaft, diam. d, $P = 200$ kN, $T = 1.50$ kN·m. (a) $d = 50$ mm, $X = ?$
(b) $d = ?$, $X = 2.0$, AISI 1020 steel.

(a) $\sigma_x = \frac{P}{A} = \frac{P}{\pi d^2/4}$, $\tau_{xy} = \frac{Tc}{J} = \frac{T(d/2)}{\pi d^4/32}$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

$\sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$, $\bar{\sigma}_H = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$, $X = \sigma_o / \bar{\sigma}_H$

P	T	σ_o
N	N-mm	MPa
2.00E+05	1.50E+06	260

AISI 1020 Steel

					Oct Shear		
d	J	A	σ_x	τ_{xy}	$\bar{\sigma}_H$	X	
mm	mm^4	mm^2	MPa	MPa	MPa		
50.00	6.136E+05	1.963E+03	101.86	61.12	146.90	**1.77**	(a) ◀
52.50	7.457E+05	2.165E+03	92.40	52.80	130.00	**2.00**	(b)

(b) $\bar{\sigma}_H = \frac{\sigma_o}{X} = \sqrt{\left(\frac{4P}{\pi d^2}\right)^2 + 3\left(\frac{16T}{\pi d^3}\right)^2}$, $X = 2.0$

$$\frac{\sigma_o}{X} = \frac{4}{\pi d^2}\sqrt{P^2 + 48\left(\frac{T}{d}\right)^2}$$

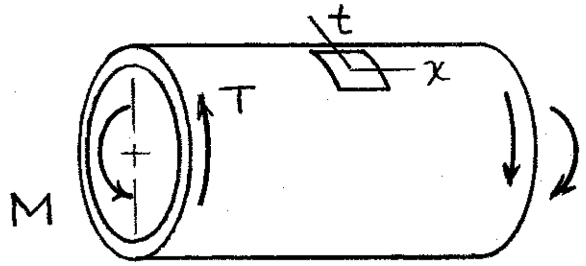
$$130 \text{ MPa} = \frac{4}{\pi d^2}\sqrt{(200{,}000\text{ N})^2 + 48\left(\frac{1.50\times10^6 \text{ N}\cdot\text{mm}}{d\text{, mm}}\right)^2}$$

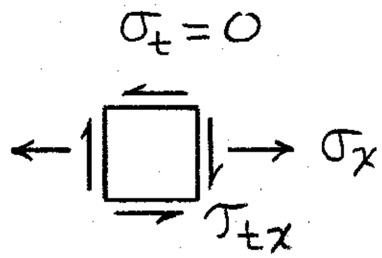
Solve iteratively, $d = 52.50$ mm ◀

7.18 A circular tube must support $M = 4.5$, $T = 7.0$ kN·m, ASTM A514 steel, $t = 3.0$ mm.

(a) $X_o = ?$ for $d_2 = 80$ mm.

(b) $d_2 = ?$ for $X_o = 1.5$



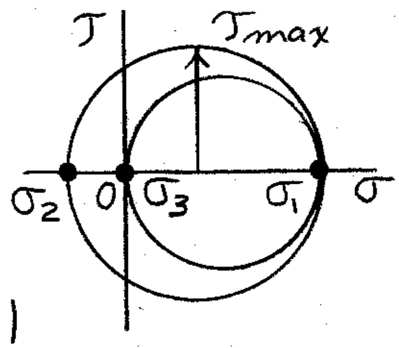


Use Fig. A.8

$$\sigma_x = \frac{M}{\pi r_{avg}^2 t}, \quad \tau_{tx} = \frac{T}{2\pi r_{avg}^2 t}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_t + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_t - \sigma_x}{2}\right)^2 + \tau_{tx}^2}, \quad \sigma_3 = 0$$

$$\sigma_1, \sigma_2 = \frac{1}{2\pi r_{avg}^2 t}\left[M \pm \sqrt{M^2 + T^2}\right]$$



$\sigma_1 > 0$, $\sigma_2 < 0$, $\sigma_3 = 0$

So σ_1, σ_2 determine τ_{max}

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right)$$

$$\bar{\sigma}_S = \sigma_1 - \sigma_2 = \frac{\sqrt{M^2 + T^2}}{\pi r_{avg}^2 t} = \frac{\sigma_o}{X_o}$$

$\sigma_o = 724$ MPa (Table 4.2)

(a) $d_2 = 80$, $r_2 = 40$, $r_1 = 37$, $r_{avg} = 38.5$ mm

$$X_o = \frac{\pi \sigma_o r_{avg}^2 t}{\sqrt{M^2 + T^2}} = \frac{\pi (724\ \text{MPa})(38.5)^2 (3.0)\ \text{mm}^3}{\sqrt{(4.5 \times 10^6)^2 + (7.0 \times 10^6)^2}\ \text{N·mm}}$$

(7.18, p. 2)

$X_o = 1.215$

(b) $r_{avg}^2 = \frac{X_o \sqrt{M^2 + T^2}}{\pi \sigma_o t}$, $r_{avg} = \sqrt{\frac{X_o}{\pi \sigma_o t} \sqrt{M^2 + T^2}}$

$$r_{avg} = \sqrt{\frac{1.5}{\pi (724\ \text{MPa})(3.0\ \text{mm})} \sqrt{4.5^2 + 7.0^2}\,(10^6)\ \text{N}\cdot\text{mm}}$$

$r_{avg} = 42.77$ mm

$r_2 = r_{avg} + t/2 = 44.27$ mm, $d_2 = 88.54$ mm

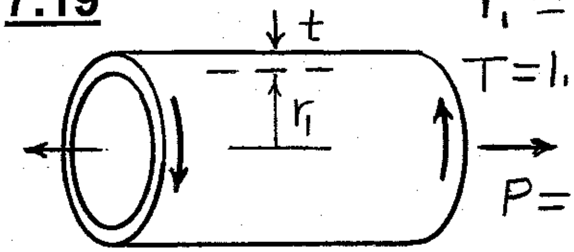
Note: If $\bar{\sigma}_H$ is used: (a) $X_o = 1.34$, (b) $d_2 =$ 84.48 mm.

M	T	t	σ_o
N-mm	N-mm	mm	MPa
4.5E+06	7.0E+06	3.00	724

Case	d_2, mm	r_2, mm	r_{avg}, mm	I, mm^4	J, mm^4	σ_x	σ_y	σ_z	τ_{xy}
(a)	80.00	40.00	38.50	5.38E+05	1.08E+06	322.1	0.0	0.0	250.5
(b)	**88.54**	44.27	42.77	7.37E+05	1.47E+06	261.0	0.0	0.0	203.0
(b)	**84.48**	42.24	40.74	6.37E+05	1.27E+06	287.7	0.0	0.0	223.8

Stresses in MPa						Max Shear		Oct Shear	
Case	$\sigma_{\tau 3}$	τ_3	σ_1	σ_2	σ_3	$\bar{\sigma}_S$	X_S	$\bar{\sigma}_H$	X_H
(a)	161.1	297.8	458.9	-136.8	0.0	595.7	**1.22**	540.4	**1.34**
(b)	130.5	241.3	371.8	-110.8	0.0	482.7	**1.50**	---	---
(b)	143.8	266.0	409.8	-122.2	0.0	---	---	482.7	**1.50**

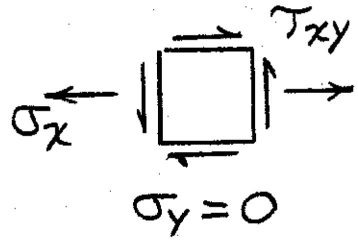
7.19



$r_1 = 23$ mm

$T = 1.0$ kN·m

$P = 60$ kN



7075-T6 AL, $\sigma_o = 469$ MPa (Table 4.2)

(a) $X_o = ?$ if $t = 2.5$ mm, $r_{avg} = 24.25$

$$\sigma_x = \frac{P}{A} = \frac{P}{2\pi r_{avg} t} = \frac{60{,}000\ \text{N}}{2\pi(24.25)(2.5)\ \text{mm mm}} = 157.5\ \text{MPa}$$

$$\tau_{xy} = \frac{TR}{J} \approx \frac{T\, r_{avg}}{2\pi r_{avg}^3 t} = \frac{(10^6\ \text{N}\cdot\text{mm})(24.25\ \text{mm})}{2\pi(24.25)^3(2.5)\ \text{mm}^4}$$

$\tau_{xy} = 108.3$ MPa

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1, \sigma_2 = 212.6, -55.1$ MPa, $\sigma_3 = \sigma_z = 0$

$\bar{\sigma}_S = \text{MAX}(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) = 267.7$ MPa

$X_o = \sigma_o / \bar{\sigma}_S = 1.75$ ◀

(b) $t = ?$ for $X_o = 2.0$

$$\sigma_x = \frac{P}{2\pi(r_1 + t/2)t}, \quad \tau_{xy} = \frac{T(r_1 + t/2)}{2\pi(r_1 + t/2)^3 t}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}, \quad \sigma_3 = 0$$

$\sigma_1 > 0$, $\sigma_2 < 0$, so $\bar{\sigma}_S = \sigma_1 - \sigma_2 = \sigma_o / X_o$

Vary t until $X_o = 2.0$; $t = 2.82$ mm ◀

(7.19, p.2)   2nd solution; use $\bar{\sigma}_H$

(a) $\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$

$\bar{\sigma}_H = 244.9$ MPa

$X_o = \sigma_o / \bar{\sigma}_H = 1.92$ ◀

(b) Vary t, but with $\bar{\sigma}_H$ as above, to get $X_o = 2.0$; $t = 2.60$ mm ◀

T N-mm	P N	r_1 mm	σ_o MPa
1.00E+06	60,000	23.00	469

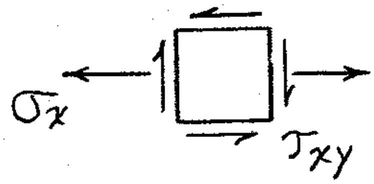
Stresses in MPa

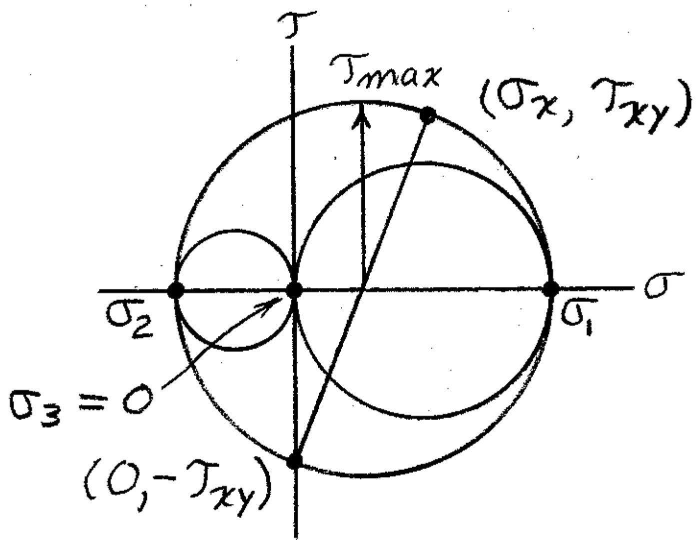
Case	t mm	r_2 mm	r_{avg} mm	J mm^4	A mm^2	σ_x MPa	σ_y MPa	τ_{xy} MPa
(a)	2.50	25.50	24.25	2.240E+05	380.9	157.5	0.0	108.3
(b)	**2.82**	25.82	24.41	2.581E+05	433.0	138.6	0.0	94.6
(b)	**2.60**	25.60	24.30	2.346E+05	397.3	151.0	0.0	103.6

Case	$\sigma_{\tau 3}$	τ_3	σ_1	σ_2	σ_3	$\bar{\sigma}_S$	X_S	$\bar{\sigma}_H$	X_H
(a)	78.8	133.9	212.6	-55.1	0.0	267.7	**1.75**	244.9	**1.92**
(b)	69.3	117.3	186.5	-48.0	0.0	234.5	**2.00**	---	---
(b)	75.5	128.2	203.7	-52.7	0.0	---	---	234.5	**2.00**

7.20 Solid circular shaft under bending and torsion. Find $d = f(\sigma_o, X_o, M, T)$ using (a) $\bar{\sigma}_S$, and (b) $\bar{\sigma}_H$.

$\sigma_y = \sigma_z = 0$





$\sigma_1 > 0, \ \sigma_2 < 0$

$\sigma_3 = \sigma_z = 0$

σ_1, σ_2 give τ_{max}

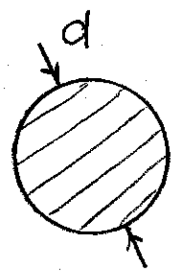
(a)

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1, \sigma_2 = \sigma_x/2 \pm \sqrt{(\sigma_x/2)^2 + \tau_{xy}^2}$$

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right) = \sigma_1 - \sigma_2$$

$$\bar{\sigma}_S = 2\sqrt{(\sigma_x/2)^2 + \tau_{xy}^2} = \sigma_o / X_o$$



$$\sigma_x = \frac{M(d/2)}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3}$$

$$\tau_{xy} = \frac{T(d/2)}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3}$$

$$\bar{\sigma}_S = 2\sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{32}{\pi d^3}\sqrt{M^2 + T^2} = \frac{\sigma_o}{X_o}$$

(7.20, p.2)

$$d = \left(\frac{32 X_o}{\pi \sigma_o} \sqrt{M^2 + T^2} \right)^{1/3} \blacktriangleleft$$

(b)

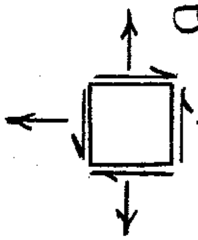
$$\bar{\sigma}_H = \frac{1}{\sqrt{2}} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}} \sqrt{2\sigma_x^2 + 6\tau_{xy}^2} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\bar{\sigma}_H = \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 3\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\sqrt{4M^2 + 3T^2} = \frac{\sigma_o}{X_o}$$

$$d = \left(\frac{16 X_o}{\pi \sigma_o} \sqrt{4M^2 + 3T^2} \right)^{1/3} \blacktriangleleft$$

7.21 Thin-walled tube, closed ends,
$r_1 = 50$, $t = 2$ mm, $p = 24$ MPa, $T = 8.0$ kN·m,
$X = 2.2$. Select a material from Table 4.2



$$\sigma_y = \frac{p r_1}{t} = \frac{24\text{ MPa}(50\text{ mm})}{2\text{ mm}} = 600\text{ MPa}$$

$$\sigma_x = \frac{p r_1}{2t} = 300\text{ MPa}$$

$$\tau_{xy} = \frac{T r_{avg}}{J} = \frac{8\times10^6\text{ N·mm}(51\text{ mm})}{2\pi(51\text{ mm})^3(2\text{ mm})} = 244.8\text{ MPa}$$

$$J = 2\pi r_{avg}^3 t$$

$\sigma_z = 0$ (outside)

Outside:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1, \sigma_2 = 450 \pm 287.1 = 737.1, 162.9\text{ MPa}$$

$$\sigma_3 = \sigma_z = 0$$

$$\bar{\sigma}_S = \text{MAX}(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) = \frac{\sigma_o}{X}$$

$$\bar{\sigma}_S = 737.1 = \sigma_o / 2.2$$

$\sigma_o = 1622$ MPa required ◁

Inside: $\sigma_3 = \sigma_z = -p = -24$ MPa

$$\tau_{xy} = \frac{T r_{avg}}{J} = 244.8\text{ MPa}$$

$$\sigma_x, \sigma_y = 300, 600\text{ MPa}$$

$$\sigma_1, \sigma_2 = 450 \pm 287.1 = 737.1, 162.9\text{ MPa}$$

(7.21, p, 2)

$$\bar{\sigma}_S = MAX\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right) = \frac{\sigma_o}{X}$$

$\bar{\sigma}_S = 761.1$ MPa $= \sigma_o / 2.2$

$\sigma_o = 1674$ MPa required, controls ◁

Possibilities:

AISI 4142 (205 °C) steel, $\sigma_o = 1688$ MPa

18 Ni maraging steel (250), $\sigma_o = 1791$ MPa ◀

Second Solution:

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \frac{\sigma_o}{X}$$

Outside: $\bar{\sigma}_H = 670.6$ MPa $= \sigma_o / 2.2$

$\sigma_o = 1475$ MPa required ◁

Inside: $\bar{\sigma}_H = 687.0 = \sigma_o / 2.2$

$\sigma_o = 1511$ MPa required, controls

Possibilities: 4142 (205 °C) steel, 18 Ni (250) steel, as above, or AISI 4142 (370 °C) steel with $\sigma_o = 1584$ MPa. Choose the latter as most ductile. ◀

- Do not use 4142 (as quenched) — too brittle.

7.22 Block confined in y dir., free in x.

$\varepsilon_y = 0$, $\sigma_x = 0$, $\sigma_x, \sigma_y, \sigma_z = \sigma_1, \sigma_2, \sigma_3$

$\sigma_z = ?$ at yieding. Poisson's ratio effect?

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z), \quad 0 = \sigma_y - \nu\sigma_z, \quad \sigma_y = \nu\sigma_z$$

$$\bar{\sigma}_S = MAX(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) = \sigma_o \quad (a)$$

$$MAX(|0 - \nu\sigma_z|, |\nu\sigma_z - \sigma_z|, |\sigma_z - 0|) = \sigma_o$$

$\sigma_z = \sigma_o$, No effect of confinement or ν. ◀

(b) Proceed similarly for octahedral shear.

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma_o$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(0 - \nu\sigma_z)^2 + (\nu\sigma_z - \sigma_z)^2 + (\sigma_z - 0)^2} = \sigma_o$$

$$\sigma_z = \frac{\sigma_o}{\sqrt{1 - \nu + \nu^2}}$$

ν	0	0,3	0,5
σ_z/σ_o	1	1.125	1.155

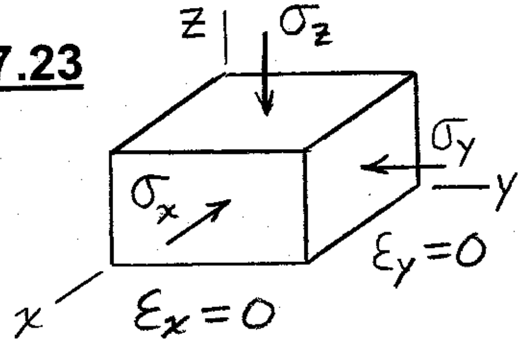
In this case there is a modest effect of ν. ◀

(c) AISI 1020 steel: $\sigma_o = 260$ MPa, $\nu = 0.293$ (Tables 4.2, 5.2). Assume $\sigma_o = -260$ MPa in C.

Max shear: $\sigma_z = \sigma_o = -260$ MPa ◀

Oct. shear: $\sigma_z = \dfrac{-260\text{ MPa}}{\sqrt{1 - 0.293 + 0.293^2}} = -292$ MPa ◀

7.23



From Prob. 5.23:

$$\sigma_x = \sigma_y = \frac{\nu \sigma_z}{1-\nu}$$

$$\sigma_1 = \sigma_z$$

$$\sigma_2 = \sigma_3 = \frac{\nu \sigma_z}{1-\nu}$$

(a) Max. shear

$$\bar{\sigma}_S = \text{MAX}(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) = \sigma_o \quad \text{(at yld.)}$$

$$\text{MAX}\left(\left|\sigma_z - \frac{\nu\sigma_z}{1-\nu}\right|, \left|\frac{\nu\sigma_z}{1-\nu} - \frac{\nu\sigma_z}{1-\nu}\right|, \left|\frac{\nu\sigma_z}{1-\nu} - \sigma_z\right|\right) = \sigma_o$$

$$\left|\sigma_z - \frac{\nu\sigma_z}{1-\nu}\right| = \left|\frac{1-2\nu}{1-\nu}\sigma_z\right| = \sigma_o$$

$$\sigma_z = \sigma_o\left(\frac{1-\nu}{1-2\nu}\right) \qquad (\sigma_z, \sigma_o \text{ both negative}) \blacktriangleleft$$

	$\nu = 0$	$\nu = 0.1$	$\nu = 0.3$	$\nu = 0.5$
$\frac{\sigma_z}{\sigma_o} =$	1.0	1.125	1.75	∞

Yes, big effect; no yielding if const. volume because then $\sigma_1 = \sigma_2 = \sigma_3$, which is simple hydrostatic compression. ◀

(b) Octahedral shear

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma_o$$

(7.23, p.2)

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{2\left(\sigma_z - \frac{\nu\sigma_z}{1-\nu}\right)^2} = \frac{1-2\nu}{1-\nu}\sigma_z = \sigma_o$$

$$\sigma_z = \sigma_o\left(\frac{1-\nu}{1-2\nu}\right) \qquad \text{(same as (a))} \quad \blacktriangleleft$$

(c) AISI 1020 steel: $\sigma_o = 260$ MPa, $\nu = 0.293$ (Tables 4.2, 5.2). Assume $\sigma_o = -260$ MPa in C.

$$\sigma_z = (-260\text{ MPa})\left(\frac{1-0.293}{1-2(0.293)}\right) = -444\text{ MPa} \quad \blacktriangleleft$$

This substantially exceeds σ_o.

7.24 $\sigma_x = \lambda \sigma_y$, $\varepsilon_z = 0$, λ varies $+1$ to -1.

Determine σ_y at yielding. (Fig. 7.27)

Assume elastic, perfectly plastic behavior so that Hooke's law applies up to yielding.

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y), \quad 0 = \frac{\sigma_z}{E} - \frac{\nu}{E}(\lambda\sigma_y + \sigma_y)$$

$$\sigma_z = \nu\sigma_y(1+\lambda)$$

$$\sigma_x, \sigma_y, \sigma_z = \sigma_1, \sigma_2, \sigma_3$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\lambda\sigma_y - \sigma_y)^2 + [\sigma_y - \nu\sigma_y(1+\lambda)]^2 + [\nu\sigma_y(1+\lambda) - \lambda\sigma_y]^2} = \sigma_o \quad \text{(at yld.)}$$

$$\sigma_y = \frac{\sigma_o}{[\lambda^2 - \lambda + 1 + \nu(\nu-1)(1+\lambda)^2]^{1/2}}$$

$\lambda = 1$: $\sigma_y = \sigma_o/(1-2\nu)$

$\lambda = 0$: $\sigma_y = \sigma_o/(1-\nu+\nu^2)^{0.5}$

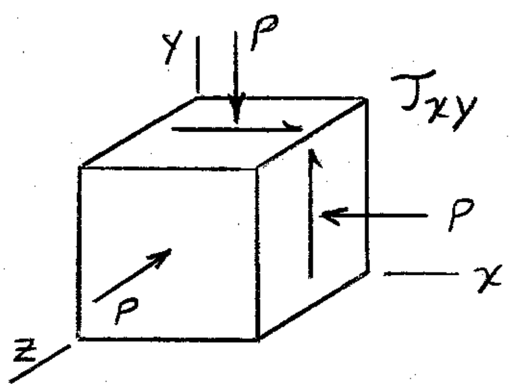
$\lambda = -1$: $\sigma_y = \sigma_o/\sqrt{3}$

For typical $\nu = 0.3$:

$\lambda =$	1	0.5	0	-0.5	-1
$\sigma_y/\sigma_o =$	2.500	1.898	1.125	0.768	0.577

Hence, yielding occurs at $\sigma_y > \sigma_o$ for $\lambda \geq 0$, but the effect reverses for some $\lambda < 0$. ◀

7.25



2024-T4 Al, $\sigma_o = 303$ MPa
(Table 4.2)

(a) $X_o = 2.5$, $\tau_{xy} = ?$

$p = 100$ MPa

$\sigma_x = \sigma_y = \sigma_z = -p$

(b) Discuss effect of p.

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(-p+p)^2+(-p+p)^2+(-p+p)^2+6\tau_{xy}^2} = \frac{\sigma_o}{X_o}$$

$$\sqrt{3}\,\tau_{xy} = \sigma_o/X_o = 303/2.5 \text{ MPa}$$

$$\tau_{xy} = 70.0 \text{ MPa}$$ ◀

(b) The pressure drops out of the $\bar{\sigma}_H$ calculation. The same will happen for $\bar{\sigma}_S$. Pressure has no effect on either yield criterion as it does not affect the value of τ_{max} or τ_h, the quantities that cause yielding in the two criteria.

7.26 $\sigma_z = -120$ MPa, and τ_{xy}, on block.

(a) $\tau_{xy} = ?$ for $X = 2$, (b) Discuss σ_z effect.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm \tau_{xy}$$

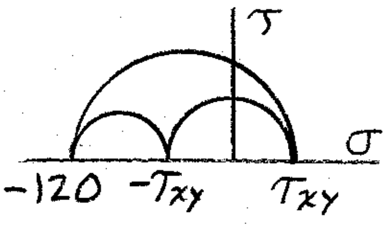
$$\sigma_3 = \sigma_z = -120 \text{ MPa}$$

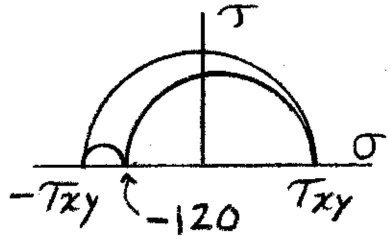
$$\bar{\sigma}_S = \text{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right)$$

$$\bar{\sigma}_S = \tau_{xy} + 120$$

or

$$\bar{\sigma}_S = 2\tau_{xy}$$





Try the first case: AISI 1020 steel

$\bar{\sigma}_S = \sigma_o / X$, $\sigma_o = 260$ MPa (Table 4.2)

$\tau_{xy} + 120 = 130$ MPa, $\tau_{xy} = 10$ MPa ◁

Second case:

$2\tau_{xy} = 130$ MPa, $\tau_{xy} = 65$ MPa ◁

Controlling answer: $\tau_{xy} = 10$ MPa ◀

If σ_z decreases, 2nd case can control, and $\tau_{xy} = 65$ MPa, so the result is very sensitive to σ_z. ◀

Second Solution

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma_o / X$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(2\tau_{xy})^2 + (-\tau_{xy} + 120)^2 + (-120 - \tau_{xy})^2}$$

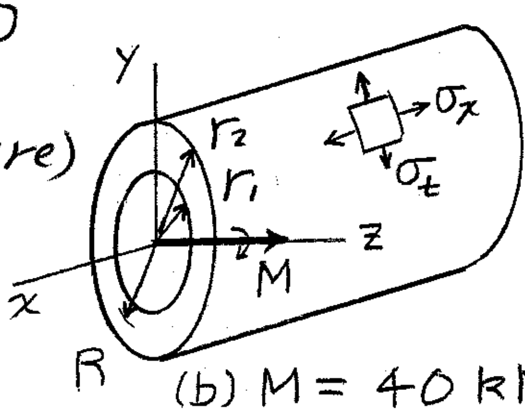
$$\bar{\sigma}_H = \sqrt{3\tau_{xy}^2 + 120^2} = 130 \text{ MPa}, \quad \tau_{xy} = 28.9 \text{ MPa}$$ ◀

7.27 Thick-walled tube, $p = 100$ MPa

$r_1 = 40$, $r_2 = 50$ mm, 18-Ni maraging steel

$\sigma_o = 1791$ MPa (Table 4.2)

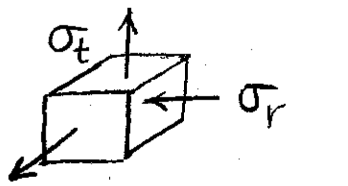
(a) $X_o = ?$ for $M = 0$



$$\sigma_{xp} = \frac{p r_1^2}{r_2^2 - r_1^2} \quad \text{(pressure)}$$

$$\sigma_{xp} = \frac{100(40)^2}{50^2 - 40^2} = 178.8 \text{ MPa}$$

(b) $M = 40$ kN·m



$\sigma_x = \sigma_{xp} + \sigma_{xb}$

$$\sigma_t = \sigma_x\left(\frac{r_2^2}{R^2} + 1\right) = \sigma_x\left(\frac{50^2}{R^2} + 1\right)$$

$$\sigma_r = -\sigma_x\left(\frac{r_2^2}{R^2} - 1\right) = -\sigma_x\left(\frac{50^2}{R^2} - 1\right)$$

$\sigma_x, \sigma_t, \sigma_r = \sigma_1, \sigma_2, \sigma_3$ (no shear)

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$X_o = \sigma_o / \bar{\sigma}_H = 1791 \text{ MPa} / \bar{\sigma}_H$

For various R, calculate σ_t, σ_r, $\bar{\sigma}_H$, and X_o

See Table (a) on next page.

$X_o = 3.72$ at $R = r_1$ (minimum X_o) ◀

(b) $$\sigma_{xb} = \frac{My}{I} = \frac{My}{\pi(r_2^4 - r_1^4)/4} \quad \text{(bending)}$$

$$\sigma_{xb} = \frac{(40 \times 10^6 \text{ N}\cdot\text{mm})(y, \text{mm})}{\pi(50^4 - 40^4)/4 \text{ mm}^4}$$

$\sigma_{xb} = 13.802\, y$ MPa (y in mm)

For various y from +40 to +50 mm, and from −40 to −50 mm, calculate $\sigma_x = \sigma_{xp} + \sigma_{xb}$, σ_t, and σ_r, where $R = |y|$ for the last two. Then calculate $\bar{\sigma}_H$ and X_o as for (a). See Table (b)

Minimum $X_o = 2.37$ at $y = \pm 50$ ◀

(a)

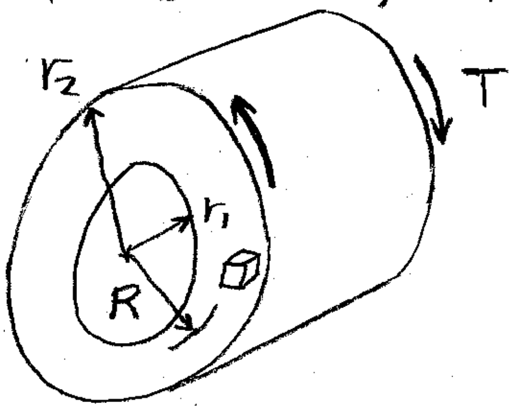
R mm	σ_x MPa	σ_t MPa	σ_r MPa	eff σ_H MPa	X
50	177.8	355.6	0.0	307.9	5.82
48	177.8	370.7	-15.1	334.1	5.36
46	177.8	387.8	-32.3	363.8	4.92
44	177.8	407.3	-51.8	397.6	4.50
42	177.8	429.7	-74.2	436.4	4.10
40	177.8	455.6	-100.0	481.1	3.72 ◁

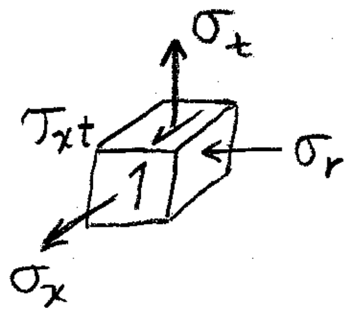
(b)

Y mm	σ_{xp} MPa	σ_t MPa	σ_r MPa	σ_{xb} MPa	total σ_x MPa	eff σ_H MPa	X
50	177.8	355.6	0.0	690.1	867.9	755.7	2.37 ◁
48	177.8	370.7	-15.1	662.5	840.3	742.0	2.41
46	177.8	387.8	-32.3	634.9	812.7	731.7	2.45
44	177.8	407.3	-51.8	607.3	785.1	725.9	2.47
42	177.8	429.7	-74.2	579.7	757.5	725.6	2.47
40	177.8	455.6	-100.0	552.1	729.9	732.3	2.45
-40	177.8	455.6	-100.0	-552.1	-374.3	732.3	2.45
-42	177.8	429.7	-74.2	-579.7	-401.9	725.6	2.47
-44	177.8	407.3	-51.8	-607.3	-429.5	725.9	2.47
-46	177.8	387.8	-32.3	-634.9	-457.1	731.7	2.45
-48	177.8	370.7	-15.1	-662.5	-484.7	742.0	2.41
-50	177.8	355.6	0.0	-690.1	-512.3	755.7	2.37 ◁

7.29 Thick-walled tube, $r_1 = 25$, $r_2 = 50$ mm, $p = 100$ MPa, $T = 75$ kN·m

18 Ni maraging steel, $\sigma_o = 1791$ MPa (Table 4.2). $X_o = ?$





$$\sigma_x = \frac{p r_1^2}{r_2^2 - r_1^2} = C$$

$$\sigma_t = C\left(\frac{r_2^2}{R^2} + 1\right), \quad \sigma_r = -C\left(\frac{r_2^2}{R^2} - 1\right)$$

$$\tau_{xt} = \frac{TR}{J}, \quad J = \frac{\pi}{2}\left(r_2^4 - r_1^4\right)$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_t}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_t}{2}\right)^2 + \tau_{xt}^2}$$

$$\sigma_3 = \sigma_r$$

First Solution: Max. shear yield crit.

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right)$$

$$X_S = \sigma_o / \bar{\sigma}_S$$

For various R from r_1 to r_2, calculate σ_x, σ_t, σ_r, τ_{xt}, then $\sigma_1, \sigma_2, \sigma_3, \bar{\sigma}_S$, X_S.

(7.29, p.2)

p	T	r_1	r_2	σ_o	J
MPa	kN-m	mm	mm	MPa	mm^4
100	75	25	50	1791	9.204E+06

R	Stresses in MPa							
mm	σ_x	σ_t	$\sigma_r = \sigma_3$	τ_{xt}	σ_1	σ_2	$\bar{\sigma}_S$	X_S
25.00	33.3	166.7	-100.0	203.7	314.3	-114.3	428.7	4.18
30.00	33.3	125.9	-59.3	244.5	328.4	-169.2	497.6	3.60
35.00	33.3	101.4	-34.7	285.2	354.6	-219.9	574.5	3.12
40.00	33.3	85.4	-18.8	325.9	386.4	-267.6	654.0	2.74
45.00	33.3	74.5	-7.8	366.7	421.2	-313.4	734.5	2.44
50.00	33.3	66.7	0.0	407.4	457.8	-357.8	815.6	**2.20**

Yielding first at r_2, where $X_S = 2.20$ ◄

Second Solution: Octahedral shear yield criterion.

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$X_H = \sigma_o / \bar{\sigma}_H$$

R		
mm	$\bar{\sigma}_H$	X_H
25.00	421.7	4.25
30.00	452.8	3.96
35.00	507.8	3.53
40.00	571.7	3.13
45.00	639.1	2.80
50.00	708.1	**2.53**

Yielding first at r_2, where $X_H = 2.53$ ◄

7.31 Solid circular shaft, $L = 1.0\ m$, $M = 1.0$, $T = 1.5\ kN \cdot m$, $X_o = 2.0$.

(a) If AISI 1020 steel, find d, m.

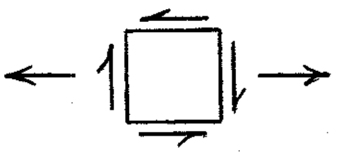
(b) Calculate d, m for other materials named.

(c) $C_m = 1$ for AISI 1020, $C_m = 6$ for Al's, $C_m = 3$ for AISI 4140. Compare relative cost.

(d) Select a material.

(a) AISI 1020: $\sigma_o = 260$ MPa, $\rho = 7.87\ g/cm^3$ (Tables 4.2, 3.1)

$\sigma_y = 0$



$$\sigma_x = \frac{Mr}{I},\quad I = \frac{\pi d^4}{64},\quad r = d/2$$

$$\tau_{xy} = \frac{Tr}{J},\quad J = \frac{\pi d^4}{32}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\,, \qquad \sigma_3 = \sigma_z = 0$$

$$\bar{\sigma}_S = \mathrm{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right), \qquad X_{oS} = \sigma_o / \bar{\sigma}_S$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

$$X_{oH} = \sigma_o / \bar{\sigma}_H$$

Combining the above equations gives expressions for the required d as derived in Prob. 7.20 solution.

(7.31, p.2)

$$d_S = \left(\frac{32 X_o}{\pi \sigma_o} \sqrt{M^2 + T^2} \right)^{1/3} \quad (\bar{\sigma}_S,\ \text{max shear})$$

$$d_H = \left(\frac{16 X_o}{\pi \sigma_o} \sqrt{4M^2 + 3T^2} \right)^{1/3} \quad (\bar{\sigma}_H,\ \text{oct. shear})$$

$$d_S = \left(\frac{32\ (2.0)}{\pi\ (260\ \text{MPa})} \sqrt{(1.0 \times 10^6)^2 + (1.5 \times 10^6)^2}\ \text{N·mm} \right)^{1/3}$$

$$d_S = 52.08\ \text{mm}$$ ◀

$$d_H = \left(\frac{32\ (2.0)}{\pi\ (260)} \sqrt{4\ (1.0 \times 10^6)^2 + 3\ (1.5 \times 10^6)^2} \right)^{1/3}$$

$$d_H = 50.46\ \text{mm}$$ ◀

Mass: Use d_H, note $g/cm^3 = Mg/m^3$

$$m = \rho V = \rho \frac{\pi d^2 L}{4}$$

$$m = \left(7.87 \frac{\text{Mg}}{\text{m}^3} \times 1000 \frac{\text{kg}}{\text{Mg}} \right) \frac{\pi\ (0.05046)^2 (1.0)\ \text{m}^3}{4}$$

$$m = 15.74\ \text{kg}$$ ◀

(b) Similar calculations for the other mat'ls, give values as in table on next page.

(c) Also calculate $C_m m$ values as a measure of relative cost of the material in each shaft.

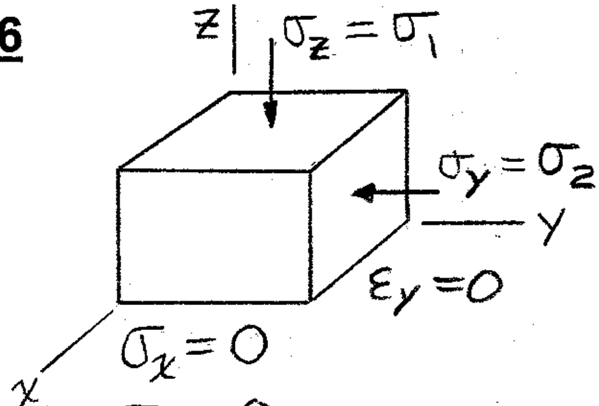
(7.31, p.3)

M, N-mm	T, N-mm	L, m	X_o
1.00E+06	1.50E+06	1.00	2.00

Material	σ_o, MPa	%RA	ρ, g/cm^3	Matl Rel Cost, C_m	d_H, mm	m, kg	Shaft Rel Cost, $C_m m$
1020 steel	260	61	7.87	1	50.46	15.74	15.7
2024-T4 Al	303	35	2.70	6	47.95	4.87	29.2
7075-T6 Al	469	33	2.70	6	41.45	3.64	21.9
4140 (205)	1583	7	7.87	3	27.63	4.72	14.2 ◀
4140 (315)	1560	33	7.87	3	27.77	4.77	14.3
4140 (425)	1399	38	7.87	3	28.79	5.12	15.4
4140 (540)	1158	48	7.87	3	30.67	5.81	17.4
4140 (650)	872	55	7.87	3	33.71	7.02	21.1

(d) If weight is an overring factor, choose 7075-T6 Al. However, this a relatively brittle material (see Section 1.2.3 and Table 8.1). A good combination of ductile behavior (high %RA) and relatively low mass and cost would be AISI 4140 (540 °C temper). ◀

7.36



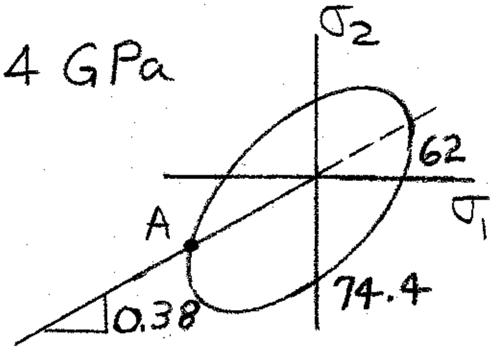
$\sigma_3 = 0$

Polycarbonate

$$\frac{|\sigma_{oc}|}{\sigma_{ot}} = 1.2$$

$\nu = 0.38$ (Table 5.2)

Table 4.3 $\begin{cases} \sigma_{ot} = 62 \text{ MPa} \\ E = 2.4 \text{ GPa} \end{cases}$



$\sigma_z = ?$ at yielding

Show biaxial locus: pt A

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) = 0, \quad \sigma_y = \nu\sigma_z$$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$
$$+ 2(|\sigma_{oc}| - \sigma_{ot})(\sigma_1 + \sigma_2 + \sigma_3) = 2|\sigma_{oc}|\sigma_{ot}$$

$$(\sigma_z - \nu\sigma_z)^2 + (\nu\sigma_z)^2 + (-\sigma_z)^2$$
$$+ 2(1.2 \times 62 - 62)(\sigma_z + \nu\sigma_z) = 2 \times 1.2 \times 62^2$$

$$\sigma_z^2(1 - \nu + \nu^2) + 12.4(1 + \nu)\sigma_z - 4612.8 = 0$$

$$0.7644\,\sigma_z^2 + 17.112\,\sigma_z - 4612.8 = 0$$

Solve quadratic eqn.: $\sigma_z = 67.3, -89.7$ MPa

$\sigma_z = -89.7$ MPa (− root as compression) ◀

7.39 Fit C-M to limestone data. $\mu, \phi, \theta_c, \sigma'_{uc}, \sigma'_{ut} = ?$ Comment.

Based on the relationship

$$|\sigma_1 - \sigma_3| = -m(\sigma_1 + \sigma_3) + 2\tau_i\sqrt{1-m^2}$$

perform a least squares fit, $y = ax + b$, where

$$y = |\sigma_1 - \sigma_3|, \qquad x = \sigma_1 + \sigma_3$$

The resulting coefficients a and b then give

$$m = -a, \qquad \tau_i = \frac{b}{2\sqrt{1-m^2}}, \qquad \phi = \sin^{-1} m, \qquad \mu = \tan\phi$$

$$\theta_c = \frac{90° - \phi}{2}, \qquad \sigma'_{uc} = -2\tau_i\sqrt{\frac{1+m}{1-m}}, \qquad \sigma'_{ut} = 2\tau_i\sqrt{\frac{1-m}{1+m}}$$

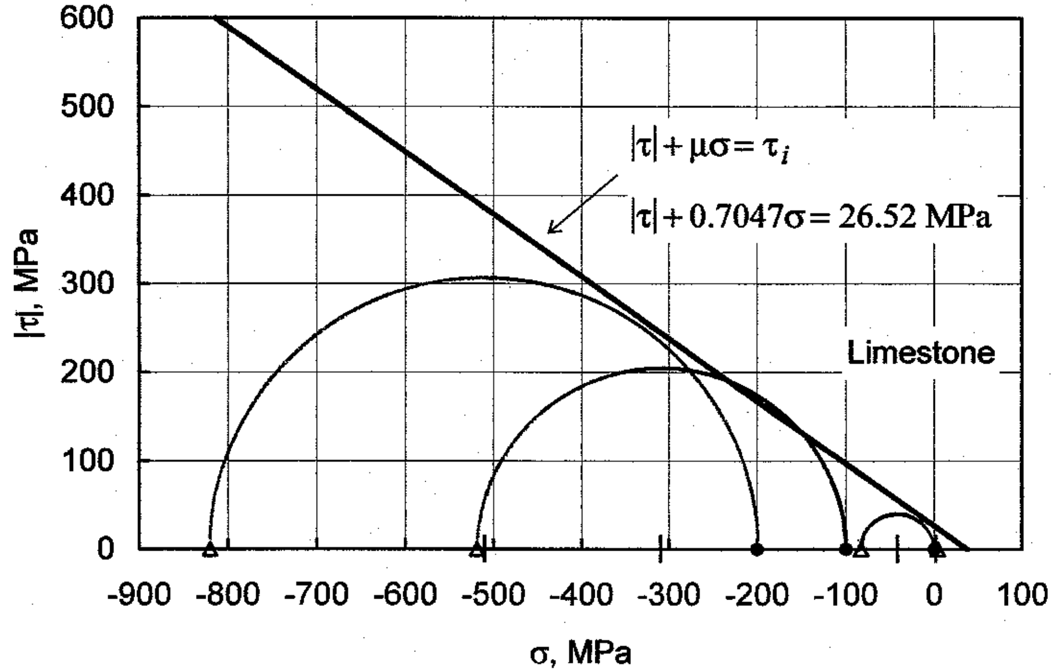
Then plot the envelope line along with the Mohr's circles for each test.

σ_3 MPa	$\sigma_1 = \sigma_2$ MPa	Y $\lvert\sigma_1 - \sigma_3\rvert$	X $\sigma_1 + \sigma_3$	Center $(\sigma_1 + \sigma_3)/2$
4	0	---	---	2.0
-83	0	83	-83	-41.5
-519	-100	419	-619	-309.5
-820	-200	620	-1020	-510.0

a	b, MPa	m	τ_i, MPa	ϕ, rad	μ
-0.5760	43.35	0.5760	26.52	0.6139	0.7047

ϕ, deg	θ_c, deg	σ'_{uc}, MPa	σ'_{ut}, MPa
35.17	27.41	-102.26	27.51

(7.39, p.2)



The fitted line represents the data well. $\sigma'_{uc} = -102$ MPa from the fit is in rough agreement with $\sigma_{uc} = -83$ MPa from the test. But $\sigma'_{ut} = 28$ MPa far exceeds $\sigma_{ut} = 4$ MPa, so the envelope does not fit tension-dominated behavior. ◀

7.40 Fit C-M to siltstone data. μ, ϕ, θ_c, σ'_{uc}, σ'_{ut} = ? Comment.

Based on the relationship

$$|\sigma_1 - \sigma_3| = -m(\sigma_1 + \sigma_3) + 2\tau_i\sqrt{1-m^2}$$

perform a least squares fit, $y = ax + b$, where

$$y = |\sigma_1 - \sigma_3|, \qquad x = \sigma_1 + \sigma_3$$

The resulting coefficients a and b then give

$$m = -a, \qquad \tau_i = \frac{b}{2\sqrt{1-m^2}}, \qquad \phi = \sin^{-1} m, \qquad \mu = \tan\phi$$

$$\theta_c = \frac{90^\circ - \phi}{2}, \qquad \sigma'_{uc} = -2\tau_i\sqrt{\frac{1+m}{1-m}}, \qquad \sigma'_{ut} = 2\tau_i\sqrt{\frac{1-m}{1+m}}$$

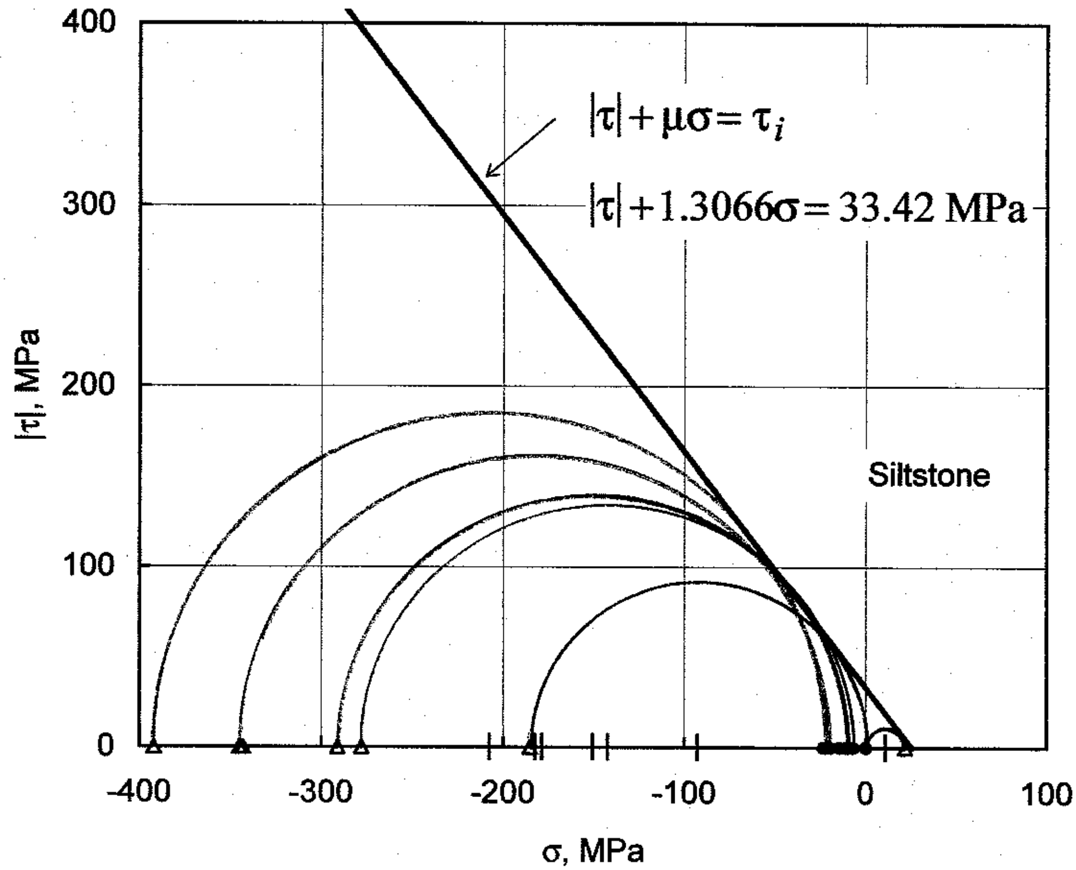
Then plot the envelope line along with the Mohr's circles for each test.

σ_3 MPa	$\sigma_1 = \sigma_2$ MPa	Y $\lvert\sigma_1 - \sigma_3\rvert$	X $\sigma_1 + \sigma_3$	Center $(\sigma_1 + \sigma_3)/2$
21.9	0	---	---	11.0
-185.4	0	185.4	-185.4	-92.7
-278	-7.10	270.9	-285.1	-142.6
-291	-10.49	280.5	-301.5	-150.7
-343	-14.34	328.7	-357.3	-178.7
-345	-19.65	325.4	-364.7	-182.3
-392	-23.1	368.9	-415.1	-207.6

a	b, MPa	m	τ_i, MPa	ϕ, rad	μ
-0.7941	40.62	0.7941	33.42	0.9175	1.3066

ϕ, deg	θ_c, deg	σ'_{uc}, MPa	σ'_{ut}, MPa
52.57	18.71	-197.28	22.64

(7.40, p.2)



The fitted line represents the data well. $\sigma'_{uc} = -197$ MPa from the fit is in rough agreement with $\sigma_{uc} = -185$ MPa from the test. Also, $\sigma'_{ut} = 22.6$ MPa is close to $\sigma_{ut} = 21.9$ MPa, but this latter agreement is unusual and is not thought to imply general validity for the envelope in the tension-dominated region.

7.41 Fit C–M to concrete data. μ, ϕ, θ_c, σ'_{uc}, σ'_{ut} = ? Comment.

Based on the relationship

$$|\sigma_1 - \sigma_3| = -m(\sigma_1 + \sigma_3) + 2\tau_i\sqrt{1-m^2}$$

perform a least squares fit, $y = ax + b$, where

$$y = |\sigma_1 - \sigma_3|, \qquad x = \sigma_1 + \sigma_3$$

The resulting coefficients a and b then give

$$m = -a, \qquad \tau_i = \frac{b}{2\sqrt{1-m^2}}, \qquad \phi = \sin^{-1} m, \qquad \mu = \tan\phi$$

$$\theta_c = \frac{90° - \phi}{2}, \qquad \sigma'_{uc} = -2\tau_i\sqrt{\frac{1+m}{1-m}}, \qquad \sigma'_{ut} = 2\tau_i\sqrt{\frac{1-m}{1+m}}$$

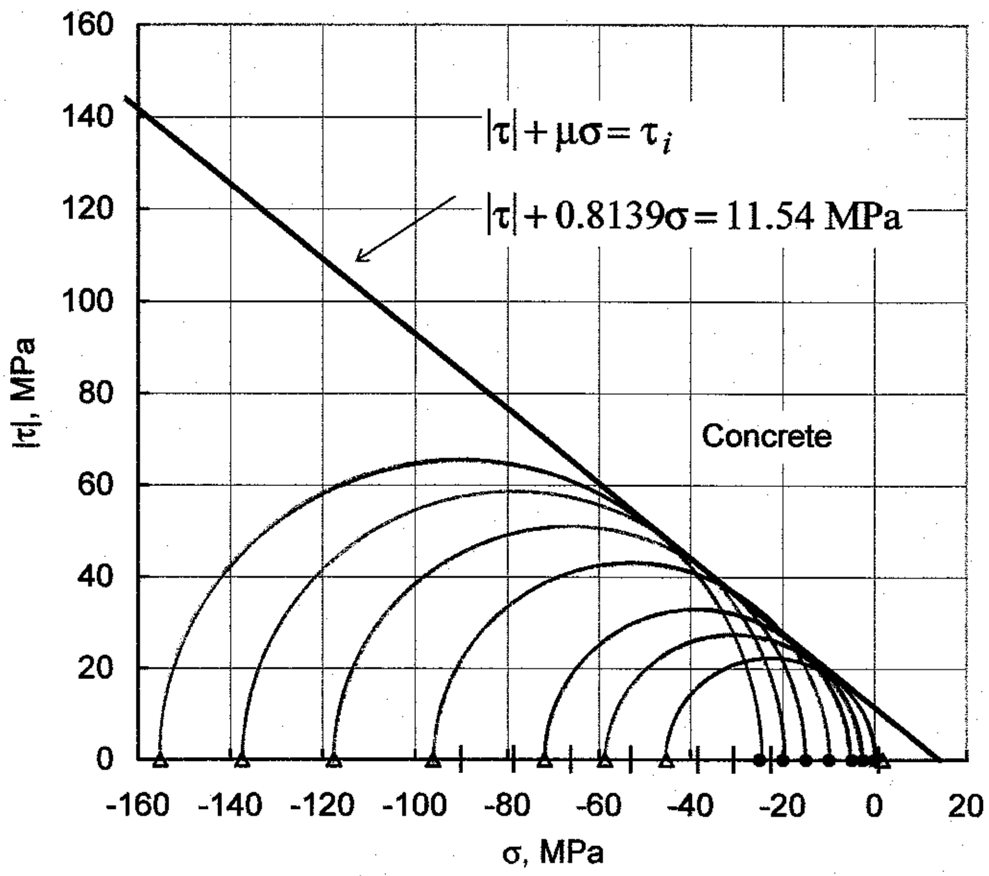
Then plot the envelope line along with the Mohr's circles for each test.

σ_3 MPa	$\sigma_1 = \sigma_2$ MPa	Y $\lvert\sigma_1 - \sigma_3\rvert$	X $\sigma_1 + \sigma_3$	Center $(\sigma_1 + \sigma_3)/2$
1.70	0	---	---	0.85
-45.3	0	45.3	-45.3	-22.7
-58.8	-2.5	56.3	-61.3	-30.7
-72.0	-5.0	67.0	-77.0	-38.5
-96.2	-10.0	86.2	-106.2	-53.1
-117.6	-15.0	102.6	-132.6	-66.3
-137.5	-20.0	117.5	-157.5	-78.8
-155.2	-25.0	130.2	-180.2	-90.1

a	b, MPa	m	τ_i, MPa	ϕ, rad	μ
-0.6312	17.90	0.6312	11.54	0.6831	0.8139

ϕ, deg	θ_c, deg	σ'_{uc}, MPa	σ'_{ut}, MPa
39.14	25.43	-48.54	10.97

(7.41, p.2)



The fitted line represents the data well. $\sigma_{uc}' = -49$ MPa from the fit is in rough agreement with $\sigma_{uc} = -45$ MPa from the test. But $\sigma_{ut}' = 11.0$ MPa far exceeds $\sigma_{ut} = 1.7$ MPa, so the envelope does not fit tension dominated behavior. ◀

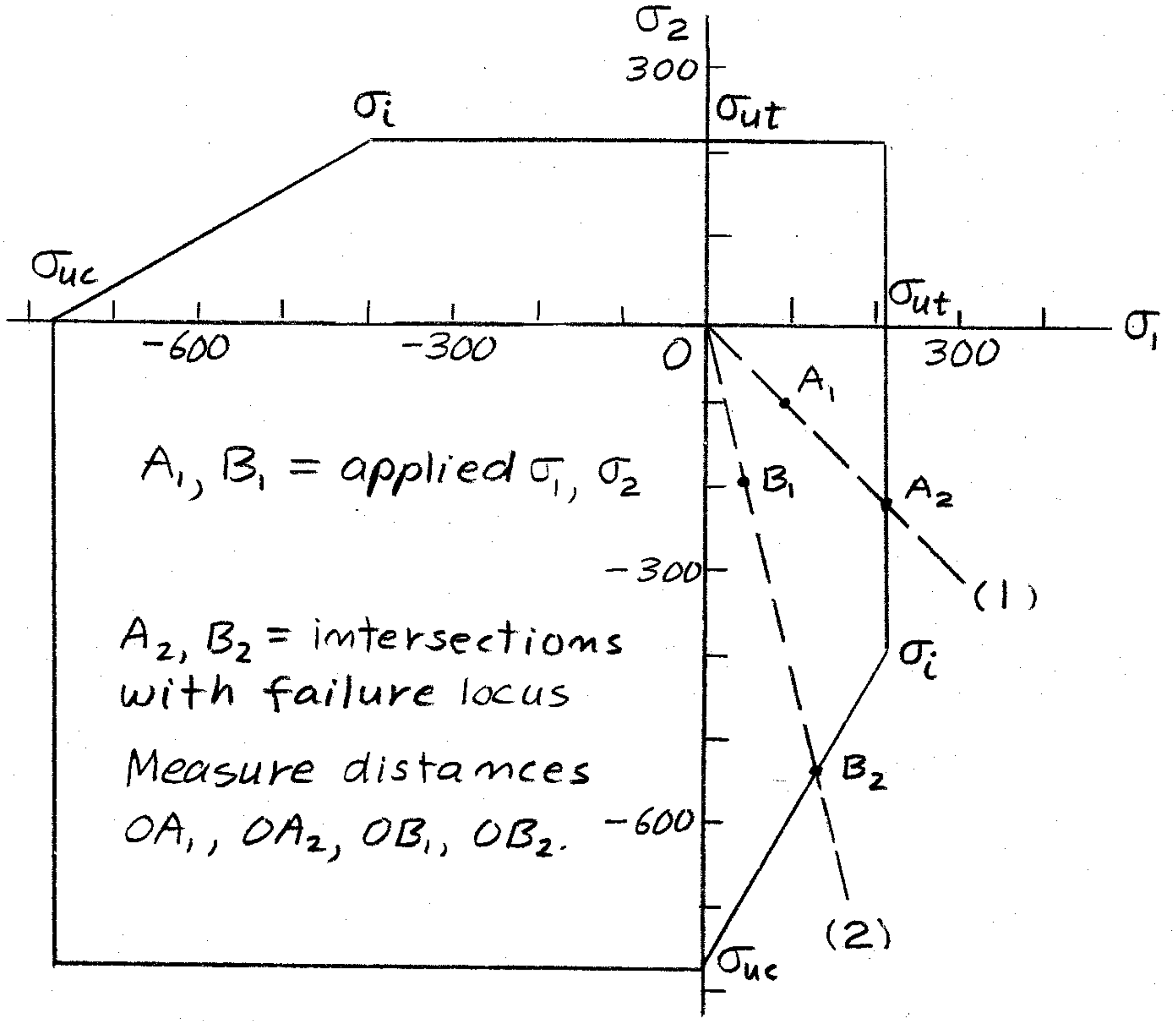
7.43 Plot $\sigma_1 - \sigma_2$ failure locus and graphically verify X values from Ex. 7.8.

(1) $X = 2.27$ for $\sigma_1, \sigma_2 = 94.3, -94.3$ MPa

(2) $X = 2.83$ for $\sigma_1, \sigma_2 = 47.2, -188.6$ MPa

$\sigma_3 = 0$ for both cases

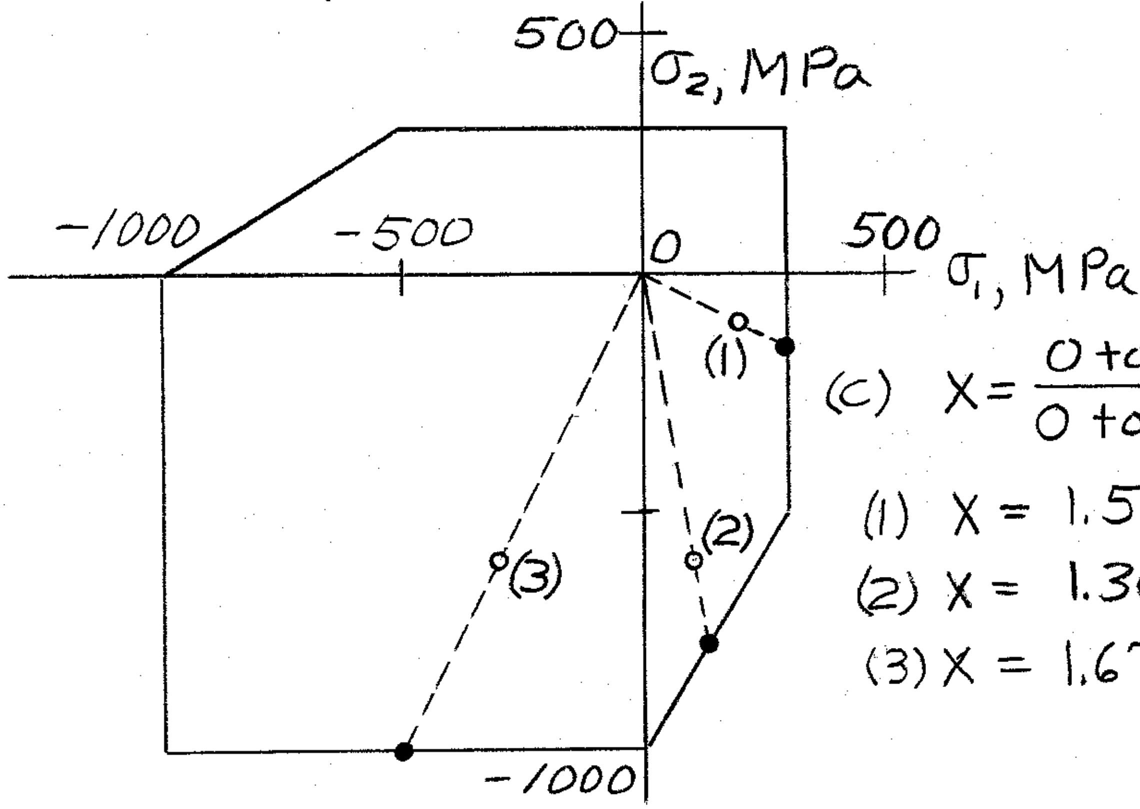
$\sigma_{ut} = 214$, $\sigma_{uc} = -770$, $\sigma_i = -393$ MPa



(1) $X = \dfrac{OA_2}{OA_1} = 2.3$, (2) $X = \dfrac{OB_2}{OB_1} = 2.9$

7.44 $\sigma_{ut} = 300,\ \sigma_{uc} = -1000,\ \sigma_i = -500$ MPa

(a) Mod. Mohr failure locus for σ_1 vs. σ_2



(c) $X = \dfrac{0 \text{ to } (\bullet)}{0 \text{ to } (\circ)}$

(1) $X = 1.5$

(2) $X = 1.30$

(3) $X = 1.67$ ◀

(b) Envelope on σ vs. $|\tau|$

$$m = \frac{|\sigma_{uc}| - \sigma_{ut} + \sigma_i}{|\sigma_{uc}| + \sigma_{ut} + \sigma_i} = 0.250 \qquad (\sigma'_{uc} \approx \sigma_{uc})$$

$$m = \sin\phi, \quad \phi = 14.48°$$

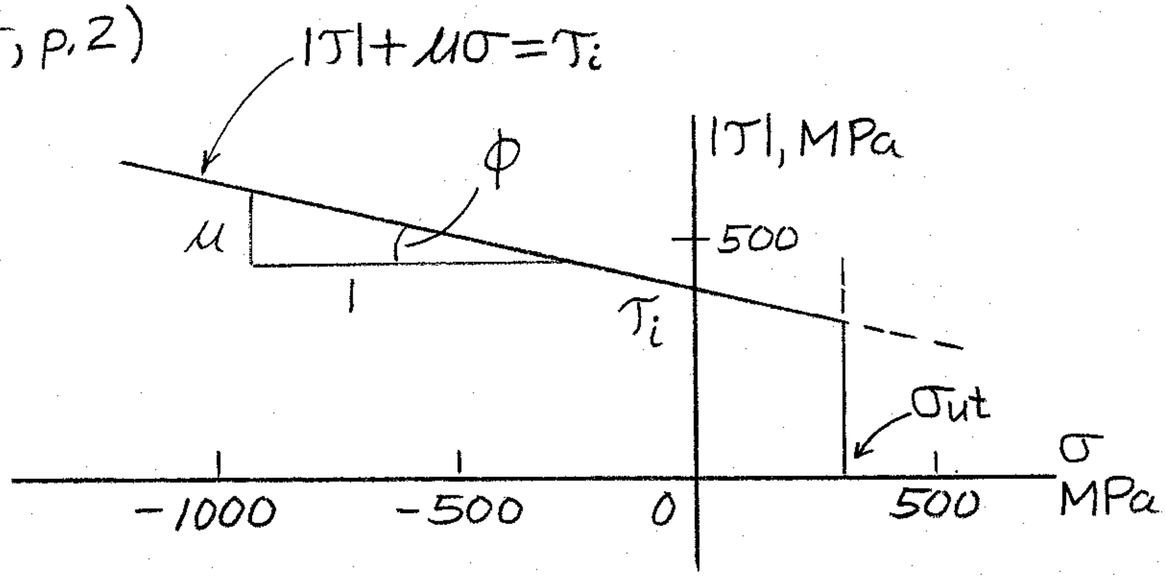
$$\mu = \tan\phi = 0.258$$

$$|\sigma'_{uc}| = 2\tau_i \sqrt{\frac{1+m}{1-m}}, \quad \tau_i = \frac{|\sigma'_{uc}|}{2}\sqrt{\frac{1-m}{1+m}}$$

$$\tau_i = \frac{1000}{2}\sqrt{\frac{1-0.250}{1+0.250}} = 387 \text{ MPa}$$

$$|\tau| + \mu\sigma = \tau_i, \quad |\tau| + 0.258\,\sigma = 387 \text{ MPa}$$ ◀

(7.44, p.2)



(d)

For the given principal stresses, calculate

$$C_{12} = \frac{1}{1-m}\left[\,|\sigma_1 - \sigma_2| + m(\sigma_1 + \sigma_2)\right]$$

$$C_{23} = \frac{1}{1-m}\left[\,|\sigma_2 - \sigma_3| + m(\sigma_2 + \sigma_3)\right]$$

$$C_{31} = \frac{1}{1-m}\left[\,|\sigma_3 - \sigma_1| + m(\sigma_3 + \sigma_1)\right]$$

Then calculate effective stresses and safety factors

$$\bar{\sigma}_{NP} = \mathrm{MAX}(\sigma_1, \sigma_2, \sigma_3), \qquad \frac{1}{X_{NP}} = \frac{\bar{\sigma}_{NP}}{\sigma_{ut}}$$

$$\bar{\sigma}_{CM} = \mathrm{MAX}(C_{12}, C_{23}, C_{31}), \qquad \frac{1}{X_{CM}} = \frac{\bar{\sigma}_{CM}}{|\sigma'_{uc}|}$$

$$X_{MM} = \frac{1}{\mathrm{MAX}\,(1/X_{CM}, 1/X_{NP})}$$

(7.44, p. 3)

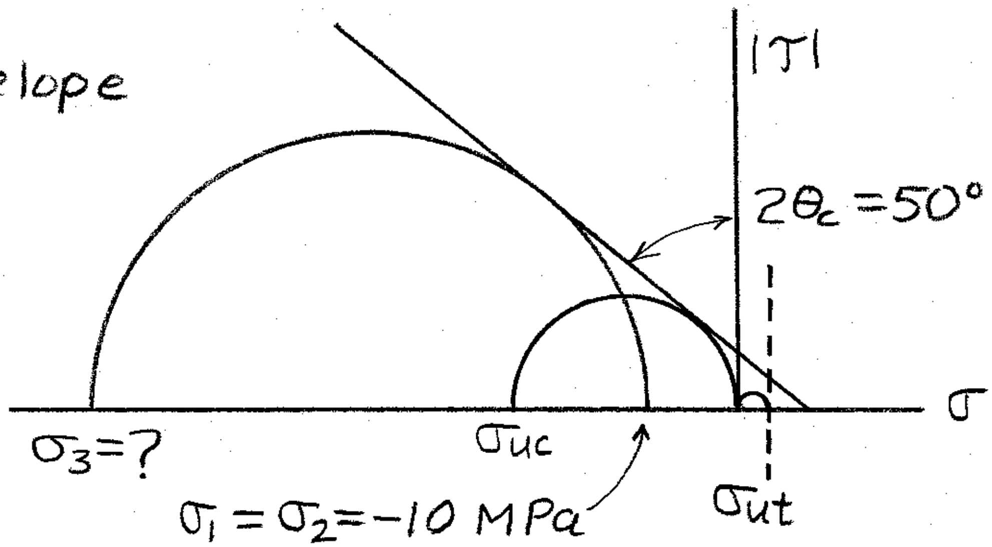
Case	σ_1 MPa	σ_2 MPa	σ_3 MPa	C_{12} MPa	C_{23} MPa	C_{31} MPa
(1)	200.00	-100.00	0.00	433.33	100.00	333.33
(2)	100.00	-600.00	0.00	766.67	600.00	166.67
(3)	-300.00	-600.00	0.00	100.00	600.00	300.00

Case	$\bar{\sigma}_{NP}$ MPa	$1/X_{NP}$	$\bar{\sigma}_{CM}$ MPa	$1/X_{CM}$	X_{MM}
(1)	200.00	0.6667	433.33	0.4333	**1.500**
(2)	100.00	0.3333	766.67	0.7667	**1.304**
(3)	0.00	0.0000	600.00	0.6000	**1.667**

7.45 Concrete: $|\sigma_{uc}| = 27.2$ MPa, $\theta_c = 25°$; $\sigma_{ut} \approx 2.72$ MPa. Find σ_3 to fail if $\sigma_1 = \sigma_2 = -10$ MPa.

C-M envelope controls.

$\sigma'_{uc} \approx \sigma_{uc}$



$\phi = 90° - 2\theta_c = 40°$, $m = \sin\phi = 0.6428$

$\bar{\sigma}_{CM} = MAX(C_{12}, C_{23}, C_{31}) = |\sigma'_{uc}|$ (at frac.)

$C_{12} < 0$, $C_{23} = C_{31}$, so $\bar{\sigma}_{CM} = C_{31} = |\sigma'_{uc}|$

$$C_{31} = \frac{1}{1-m}\left[|\sigma_3 - \sigma_1| + m(\sigma_3 + \sigma_1)\right]$$

$$2.7995\left[\pm(\sigma_3 + 10) + 0.6428(\sigma_3 - 10)\right] = 27.2 \text{ MPa}$$

$\sigma_3 = +3.74$ must be (−); not possible

$\sigma_3 = -73.2$ valid solution ◀

7.46 Mortar of Table 7.1; $m = 0.497$, $\tau_i = 9.86$ MPa. $X = ?$ for $\sigma_z = -50$, $\sigma_x = \sigma_y = -12$ MPa.

$\sigma_1, \sigma_2, \sigma_3 = \sigma_x, \sigma_y, \sigma_z \quad (\tau\text{'s} = 0)$

$\sigma_1, \sigma_2, \sigma_3 = -12, -12, -50$

$C_{31} = C_{23}$, $C_{12} < 0$, so $C_{31} = C_{23}$ controls

$$C_{31} = \frac{1}{1-m}\left[|\sigma_3 - \sigma_1| + m(\sigma_3 + \sigma_1)\right] = 14.286 \text{ MPa}$$

$$\bar{\sigma}_{CM} = MAX(C_{12}, C_{23}, C_{31}) = C_{31} = |\sigma_{uc}'| / X_{CM}$$

$$\sigma_{uc}' = -2\tau_i\sqrt{\frac{1+m}{1-m}} = -34.02 \text{ MPa}$$

$14.286 = 34.02 / X_{CM}$, $X_{CM} = 2.38$ ◁

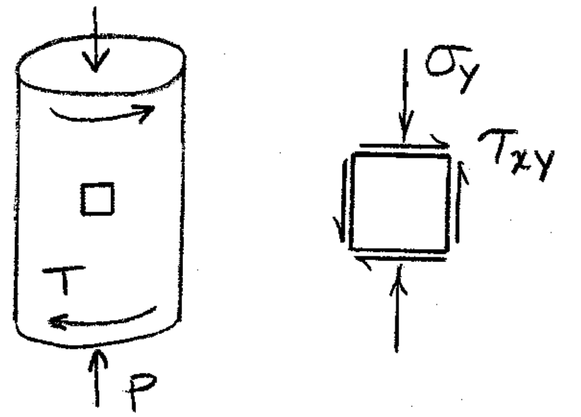
$\bar{\sigma}_{NP} = MAX(\sigma_1, \sigma_2, \sigma_3) = -12$ MPa, $X_{NP} = \infty$ ◁

$X_{MM} = MIN(X_{CM}, X_{NP}) = 2.38$ ◀

7.47 Building column, 400 mm diameter.

Sandstone, $\sigma_{ut} = 3$, $|\sigma_{uc}| = 100$, $\tilde{T}_i = 23.35$ MPa,

$m = 0.700$ (Table 7.1)



$c = 200$ mm radius

$$\sigma_y = -\frac{P}{A} = -\frac{P}{\pi c^2}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{Tc}{\pi c^4/2} = \frac{2T}{\pi c^3}$$

(a) $X_{MM} = ?$ for $P = 1250$ kN, $T = 0$, $\tau_{xy} = 0$

$$\sigma_y = -\frac{1.25 \times 10^6 \text{ N}}{\pi (200 \text{ mm})^2} = -9.947 \text{ MPa}$$

$$\sigma_{uc}' = -2\tilde{T}_i\sqrt{\frac{1+m}{1-m}} = -111.2 \text{ MPa}$$

$$\sigma_1, \sigma_2, \sigma_3 = 0, -9.947, 0 \text{ MPa} \qquad (\text{all } \tau = 0)$$

$$\bar{\sigma}_{CM} = MAX(C_{12}, C_{23}, C_{31}) = C_{12}$$

$$C_{12} = \frac{1}{1-m}\left[|\sigma_1 - \sigma_2| + m(\sigma_1 + \sigma_2)\right] = 9.947 \text{ MPa}$$

$$X_{CM} = |\sigma_{uc}'|/\bar{\sigma}_{CM} = 11.18 \quad \triangleleft$$

$$\bar{\sigma}_{NP} = MAX(\sigma_1, \sigma_2, \sigma_3) = 0, \quad X_{NP} = \infty \quad \triangleleft$$

$$X_{MM} = MIN(X_{CM}, X_{NP}) = 11.18 \quad \blacktriangleleft$$

(b) $X_{MM} = ?$ for $P = 0$, $T = 20{,}000$ N·m, $\sigma_y = 0$

$$\tau_{xy} = \frac{2(20 \times 10^6 \text{ N·mm})}{\pi (200 \text{ mm})^3} = 1.592 \text{ MPa}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \quad \sigma_3 = 0$$

(7.47, p.2) $\sigma_1, \sigma_2 = 1.592, -1.592$ MPa

$\bar{\sigma}_{CM} = MAX(C_{12}, C_{23}, C_{31}) = C_{12}$

$C_{12} = \frac{1}{1-m}\left[|\sigma_1 - \sigma_2| + m(\sigma_1 + \sigma_2)\right] = 10.61$ MPa

$X_{CM} = |\sigma_{uc}'| / \bar{\sigma}_{CM} = 10.48$ ◁

$\bar{\sigma}_{NP} = MAX(\sigma_1, \sigma_2, \sigma_3) = 1.592$ MPa

$X_{NP} = \sigma_{ut} / \bar{\sigma}_{NP} = 1.88$ ◁

$X_{MM} = MIN(X_{CM}, X_{NP}) = 1.88$ ◀

(c) $X_{MM} = ?$ for $P = 1250$ kN, $T = 20{,}000$ N·m

$\sigma_y = -9.947$, $\tau_{xy} = 1.592$

$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$, $\sigma_3 = 0$

$\sigma_1, \sigma_2 = -4.974 \pm 5.222 = 0.248, -10.196$ MPa

$\bar{\sigma}_{CM} = MAX(C_{12}, C_{23}, C_{31}) = C_{12}$

$C_{12} = \frac{1}{1-m}\left[|\sigma_1 - \sigma_2| + m(\sigma_1 + \sigma_2)\right] = 11.60$ MPa

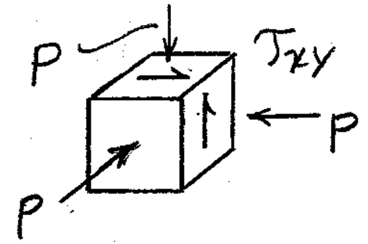
$X_{CM} = |\sigma_{uc}'| / \bar{\sigma}_{CM} = 9.59$ ◁

$\bar{\sigma}_{NP} = MAX(\sigma_1, \sigma_2, \sigma_3) = 0.248$ MPa

$X_{NP} = \sigma_{ut} / \bar{\sigma}_{NP} = 12.1$ ◁ $X_{MM} = 9.59$ ◀

(d) Fracture in (b) is easy due to tension from τ_{xy} and low σ_{ut}. But the tension is much reduced in (c) due to the compressive P, so that the fracture is shifted to C–M mode, with safety factor almost as large as in (a). ◀

7.48 Concrete of Table 7.1, with pressure p and $\tau_{xy} = 30$ MPa. (a) Fracture if $p = 40$ MPa? (b) Smallest p for no fracture.

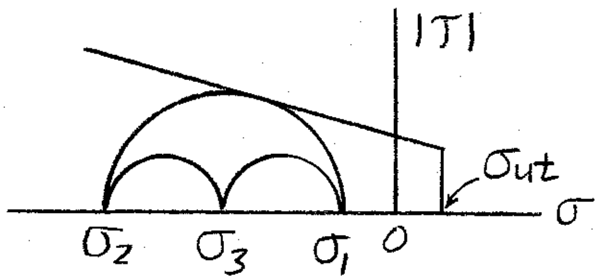


$m = 0.631$, $\tau_i = 11.54$ MPa

$$|\sigma'_{uc}| = 2\tau_i \sqrt{\frac{1+m}{1-m}} = 48.52 \text{ MPa}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = -p \pm \tau_{xy}$$

$$\sigma_3 = \sigma_z = -p$$



(a)

$\sigma_1 = -40 + 30 = -10$

$\sigma_2 = -40 - 30 = -70$

$\sigma_3 = -40$ MPa

Controls: $C_{12} = \frac{1}{1-m}\left[|\sigma_1 - \sigma_2| + m(\sigma_1 + \sigma_2)\right]$

$C_{12} = 25.80$, $\bar{\sigma}_{CM} = C_{12} = 25.80$

$X_{CM} = |\sigma'_{uc}| / \bar{\sigma}_{CM} = 1.88$, no fracture ◀

(b) $\sigma_1 = -p + 30$, $\sigma_2 = -p - 30$, $\sigma_3 = -p$

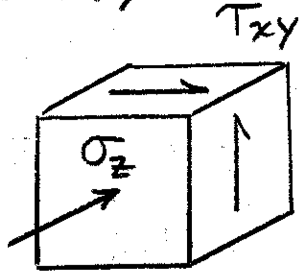
$\bar{\sigma}_{CM} = C_{12}$, $X_{CM} = |\sigma'_{uc}| / \bar{\sigma}_{CM} = 1$, $C_{12} = |\sigma'_{uc}|$

$$C_{12} = \frac{1}{1-m}\left[60 + m(-2p)\right] = 48.52 \text{ MPa}$$

$p = 33.36$ MPa ◀

7.49 Mortar, $\sigma_{ut} = 2.8$, $|\sigma_{uc}| = 31.8$ MPa

$m = 0.497$, $\tau_i = 9.86$ MPa (Table 7.1)



$$\sigma_{uc}' = -2\tau_i\sqrt{\frac{1+m}{1-m}} = -34.0 \text{ MPa}$$

(a) $\tau_{xy} = 1.0$, $\sigma_z = 0$, $X_{MM} = ?$

$\sigma_3 = \sigma_z = 0$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1, \sigma_2 = \pm\tau_{xy} = \pm 1.0$ MPa

$\bar{\sigma}_{CM} = MAX(C_{12}, C_{23}, C_{31}) = C_{12}$

$$C_{12} = \frac{1}{1-m}\left[|\sigma_1 - \sigma_2| + m(\sigma_1 + \sigma_2)\right] = 3.976 \text{ MPa}$$

$\bar{\sigma}_{CM} = 3.976$ MPa, $X_{CM} = |\sigma_{uc}'|/\bar{\sigma}_{CM} = 8.55$

$\bar{\sigma}_{NP} = MAX(\sigma_1, \sigma_2, \sigma_3) = 1.0$ MPa

$X_{NP} = \sigma_{ut}/\bar{\sigma}_{NP} = 2.80$

$X_{MM} = MIN(X_{CM}, X_{NP}) = 2.80$

(b) $\tau_{xy} = 1.0$, $\sigma_z = -15$ MPa

$\sigma_1, \sigma_2 = \pm\tau_{xy} = \pm 1.0$ MPa, $\sigma_3 = \sigma_z = -15$ MPa

$\bar{\sigma}_{CM} = MAX(C_{12}, C_{23}, C_{31}) = C_{31}$

$$C_{31} = \frac{1}{1-m}\left[|\sigma_3 - \sigma_1| + m(\sigma_3 + \sigma_1)\right] = 17.98 \text{ MPa}$$

$X_{CM} = |\sigma_{uc}'|/\bar{\sigma}_{CM} = 1.89$

$\bar{\sigma}_{NP} = MAX(\sigma_1, \sigma_2, \sigma_3) = 1.0$ MPa, $X_{NP} = 2.80$

$X_{MM} = MIN(X_{CM}, X_{NP}) = 1.89$

(7.49, p.2) (C) $X_{MM} = 2.5$, $\tau_{xy} = 1.0$ MPa

What is largest allowable σ_z ?

$\sigma_1, \sigma_2 = \pm \tau_{xy} = \pm 1.0$ MPa , $\sigma_3 = \sigma_z$

Assume C-M controls, and $\sigma_3 < \sigma_2 = -1.0$ MPa

$\bar{\sigma}_{CM} = MAX\,(C_{12}, C_{23}, C_{31}) = C_{31} = |\sigma'_{uc}|/X_{CM}$

$$\frac{1}{1-0.497}\left[\,|\sigma_z - 1.0| + 0.497\,(\sigma_z + 1.0)\right] = 34/2.5$$

Since $\sigma_z < -1.0$, $|\sigma_z - 1.0| = -(\sigma_z - 1.0)$

$-\sigma_z + 1.0 + 0.497\,(\sigma_z + 1.0) = (0.503)\,34/2.5$

$\sigma_z = -10.63$ MPa ◁

$\bar{\sigma}_{NP} = MAX(\sigma_1, \sigma_2, \sigma_3) = 1.0$ MPa , $X_{NP} = 2.80$

$X_{MM} = MIN\,(X_{CM}, X_{NP}) = 2.5$

Assumptions are OK

$\sigma_z = -10.63$ MPa ◀

7.50 Gray cast iron shaft, $d = 30$ mm.
$X = 3$, $P = 100$ kN compression, $T = ?$
$\sigma_{ut} = 214$, $|\sigma_{uc}'| = 770$ MPa, $m = 0.276$

$$\sigma_x = \frac{P}{A} = \frac{-100{,}000\text{ N}}{\pi(15\text{ mm})^2} = -141.47\text{ MPa}$$

$$\tau_{xy} = \frac{Tr}{J}, \quad J = \frac{\pi r^4}{2}, \quad \tau_{xy} = \frac{2T}{\pi r^3}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \quad \sigma_3 = 0$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1 > 0$, $\sigma_2 < 0$, so σ_1 and C_{12} control.
Assume $\bar{\sigma}_{CM}$ controls and check $\bar{\sigma}_{NP}$.

$$C_{12} = \frac{1}{1-m}\left[|\sigma_1 - \sigma_2| + m(\sigma_1 + \sigma_2)\right] = \frac{|\sigma_{uc}'|}{X_{CM}}$$

$$\frac{|\sigma_{uc}'|}{X_{CM}} = \frac{1}{1-m}\left[2\sqrt{(\sigma_x/2)^2 + \tau_{xy}^2} + m\,\sigma_x\right]$$

$$\sqrt{(\sigma_x/2)^2 + \tau_{xy}^2} = \frac{1}{2}\left[\frac{|\sigma_{uc}'|(1-m)}{X_{CM}} - m\,\sigma_x\right]$$

$$\tau_{xy}^2 = \frac{1}{4}\left[\frac{|\sigma_{uc}'|(1-m)}{X_{CM}} - m\,\sigma_x\right]^2 - \left(\frac{\sigma_x}{2}\right)^2$$

Substitute values from above and $X_{CM} = 3$.

$\tau_{xy} = 87.40$ MPa ◁

(7.50, p.2)

Check $\bar{\sigma}_{NP}$

$$\sigma_1 = (\sigma_x/2) + \sqrt{(\sigma_x/2)^2 + \tau_{xy}^2} = 41.70 \text{ MPa}$$

$$\bar{\sigma}_{NP} = \text{MAX}\ (\sigma_1, \sigma_2, \sigma_3) = \sigma_1 = 41.70 \text{ MPa}$$

$$X_{NP} = \frac{\sigma_{ut}}{\bar{\sigma}_{NP}} = 5.13, \quad \text{so } X_{CM} = 3 \text{ controls}$$

$$T = \frac{1}{2}\pi r^3 \tau_{xy} = 463,300 \text{ N}\cdot\text{mm}$$

$$T = 463 \text{ N}\cdot\text{m}$$

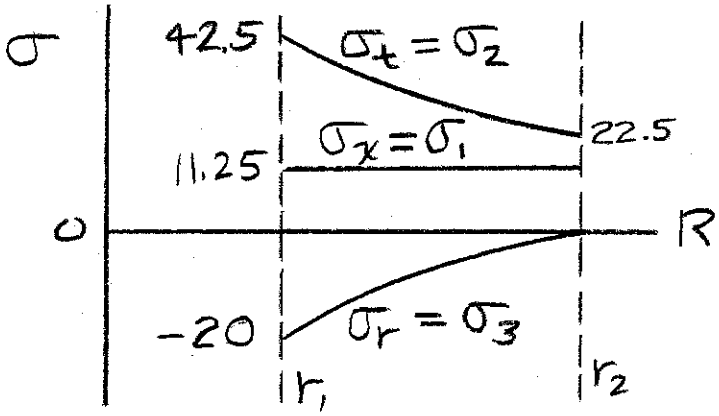
7.51 Gray cast iron, $\sigma_{ut} = 214$ MPa,
$|\sigma_{uc}| = |\sigma'_{uc}| = 770$ MPa, $m = 0.276$
Thick-walled tube $r_1 = 30$, $r_2 = 50$ mm.

(a) $X_{MM} = ?$ for $p = 20$ MPa

$$\sigma_x = C = \frac{p r_1^2}{r_2^2 - r_1^2} = 11.25 \text{ MPa at all } R$$

$$\sigma_t = C\left(\frac{r_2^2}{R^2} + 1\right), \quad \sigma_r = -C\left(\frac{r_2^2}{R^2} - 1\right)$$

At $R = r_1 = 30$ MPa, $\sigma_t = 42.5$, $\sigma_r = -20$ MPa



As $\tau_{xy} = \tau_{yz} = \tau_{zx}$ all zero, then $\sigma_x, \sigma_t, \sigma_r = \sigma_1, \sigma_2, \sigma_3$. $\sigma_t = \sigma_2$ largest, and $\sigma_r = \sigma_3$ smallest, at all R.

$$\bar{\sigma}_{CM} = \text{MAX}(C_{12}, C_{23}, C_{31}) = C_{23} \text{ at all } R$$

$$C_{23} = \frac{1}{1-m}\left[|\sigma_2 - \sigma_3| + m(\sigma_2 + \sigma_3)\right]$$

Checking various R gives largest C_{23} at $R = r_1$, where $C_{23} = 94.90 \text{ MPa} = \bar{\sigma}_{CM}$

$$X_{CM} = |\sigma'_{uc}| / \bar{\sigma}_{CM} = 8.11 \qquad \triangleleft$$

$$\bar{\sigma}_{NP} = \text{MAX}(\sigma_1, \sigma_2, \sigma_3) = 42.5 \text{ MPa}$$

$$X_{NP} = \sigma_{ut} / \bar{\sigma}_{NP} = 5.04 \qquad \triangleleft$$

$$X_{MM} = \text{MIN}(X_{CM}, X_{NP}) = 5.04 \qquad \blacktriangleleft$$

(7.51, p.2) (b) $X_{MM} = ?$ if $F = -700{,}000\,N$ axial compression added to $p = 20\,MPa$.

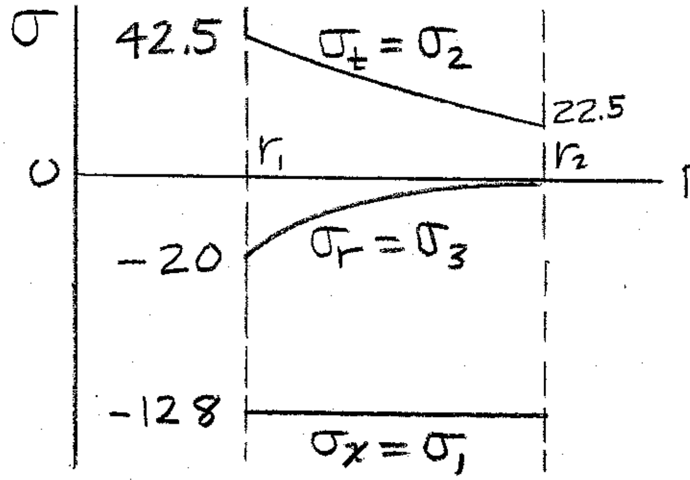
$$\sigma_x = \sigma_{xp} + \sigma_{xF} = 11.25 - \frac{700{,}000\ N}{\pi(50^2 - 30^2)\,mm^2}$$

$\sigma_x = -128.0\ MPa$; σ_t, σ_r as above.

$\sigma_x, \sigma_t, \sigma_r = \sigma_1, \sigma_2, \sigma_3$

$\sigma_t = \sigma_2$ largest, $\sigma_x = \sigma_1$ smallest, at all R.

$\bar{\sigma}_{CM} = MAX(C_{12}, C_{23}, C_{31}) = C_{12}$ at all R



$C = 11.25\ MPa$

$\sigma_t = C\left(\frac{r_2^2}{R^2} + 1\right)$

$\sigma_r = -C\left(\frac{r_2^2}{R^2} - 1\right)$

$\sigma_x = -128.0$ all R

$$C_{12} = \frac{1}{1-m}\left[|\sigma_1 - \sigma_2| + m(\sigma_1 + \sigma_2)\right]$$

Checking various R gives largest C_{12} at $R = r_1$, where $C_{12} = 202.9\ MPa = \bar{\sigma}_{CM}$

$X_{CM} = |\sigma_{uc}'| / \bar{\sigma}_{CM} = 3.80$

$\bar{\sigma}_{NP} = MAX(\sigma_1, \sigma_2, \sigma_3) = 42.5\ MPa$

$X_{NP} = \sigma_{ut} / \bar{\sigma}_{NP} = 5.04$

$X_{MM} = MIN(X_{CM}, X_{NP}) = 3.80$