9.3 $\sigma_a = C + D \log N_f$, with (N_1, σ_1), (N_2, σ_2)

(a) $C, D = ?$

$$\sigma_1 = C + D \log N_1, \quad \sigma_2 = C + D \log N_2$$

$$\sigma_1 - \sigma_2 = D(\log N_1 - \log N_2)$$

$$D = \frac{\sigma_1 - \sigma_2}{\log(N_1/N_2)}, \quad C = \sigma_1 - D \log N_1$$ ◀

(b) Find C, D for the Fig 9.5 data, $N_f < 10^6$

The data do approximate a straight line on the log-linear plot, so the form is appropriate. Read two pts.

$(N_1, S_1) = (10^4, 770\ \text{MPa})$

$(N_2, S_2) = (10^6, 455\ \text{MPa})$

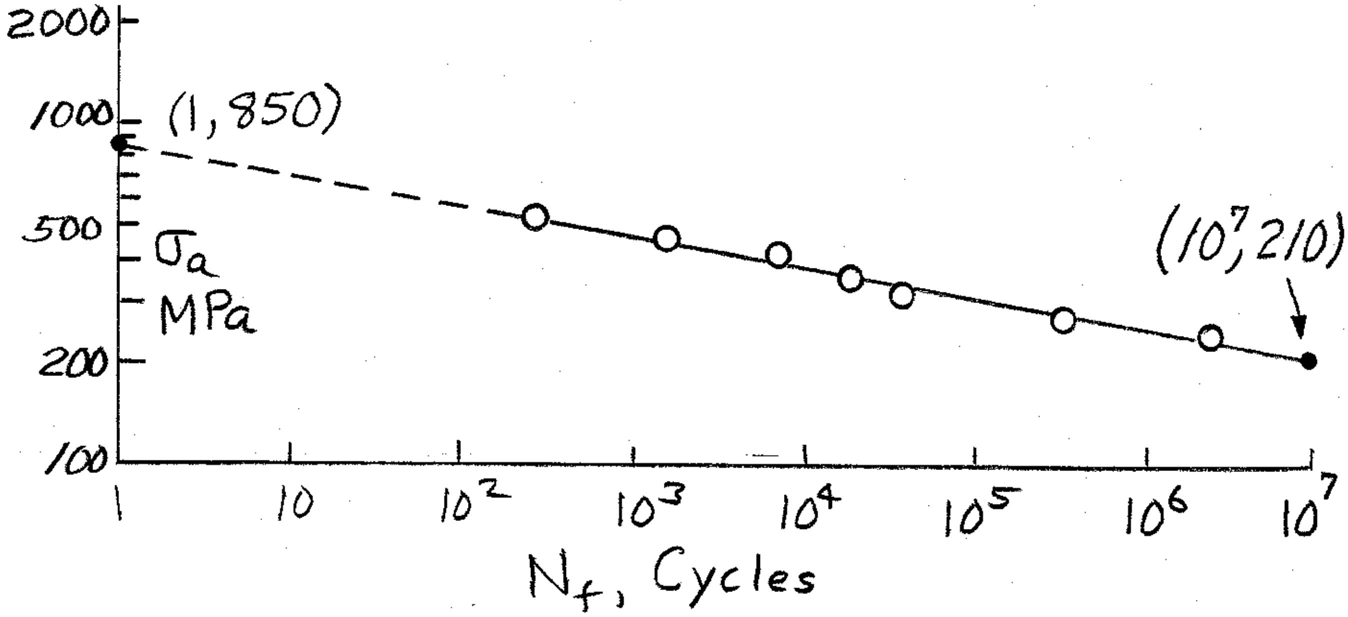
$$D = \frac{770 - 455}{\log(10^4/10^6)} = -157.5\ \text{MPa}$$

$$C = 770 - (-157.5)\log(10^4) = 1400\ \text{MPa}$$

$$S_a = 1400 - 157.5 \log N_f\ \text{MPa}$$ ◀

9.4 $\sigma_a = A N_f^B$ HR & Norm AISI 1045

(a) Plot data on log-log coordinates and draw a straight line through the data.



$A = 850$ is σ_a intercept at $N_f = 1$,
Calculate B from a second pt. on line.

$$\frac{\sigma_a}{A} = N_f^{\,B}, \quad B = \frac{\log(\sigma_a/A)}{\log N_f}$$

$$B = \frac{\log(210/850)}{\log(10^7)} = -0.0867$$

$$\sigma_a = 850\,(N_f)^{-0.0867} \ \text{MPa}$$ ◀

(b) Do a least-squares fit; N_f dependent

$$N_f = \left(\frac{\sigma_a}{A}\right)^{1/B}$$

(9.4, p.2)

$$\underbrace{\log N_f}_{y} = \frac{1}{B} \underbrace{\log \sigma_a}_{x} + \underbrace{\log \frac{1}{A^{1/B}}}_{c}$$

$$= m \quad x \quad + \quad c$$

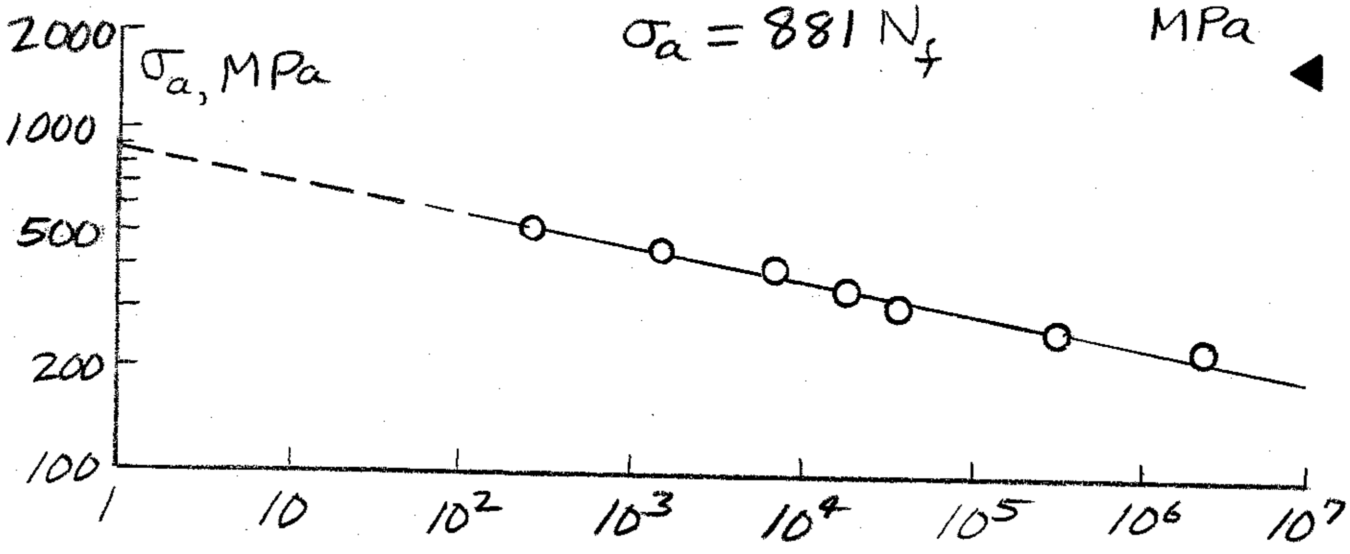
The resulting constants are

$m = -10.916$, $B = 1/m = -0.09161$

$c = 32.15 = \log \frac{1}{A^{1/B}}$

$10^{-cB} = A = 881$ MPa

$\sigma_a = 881\, N_f^{-0.0916}$ MPa ◀



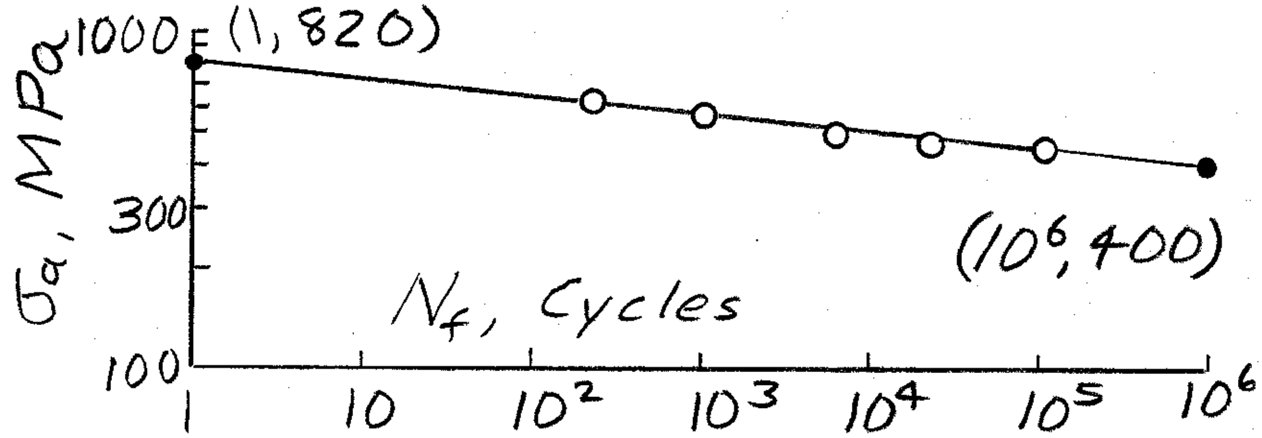
(c) $\sigma_f' = A/2^b = 881/2^{-0.09161} = 939$ MPa

$b = B = -0.0916$

$\sigma_a = 939\,(2N_f)^{-0.0916}$ MPa ◀

9.5 $\sigma_a = A N_f^B$ RQC-100 steel

(a) Plot data on log-log coordinates and draw a straight line through the data.



$A = 820$ is intercept at $N_f = 1$.

Calculate B from a second point.

$$\frac{\sigma_a}{A} = N_f^B, \quad B = \frac{\log(\sigma_a/A)}{\log N_f}$$

$$B = \frac{\log(400/820)}{\log(10^6)} = -0.0520$$

$$\sigma_a = 820 N_f^{-0.0520} \text{ MPa}$$

(b) Least squares fit; N_f dependent.

$$N_f = \left(\frac{\sigma_a}{A}\right)^{1/B}$$

$$\underbrace{\log N_f}_{y} = \underbrace{\frac{1}{B}}_{m} \underbrace{\log \sigma_a}_{x} + \underbrace{\left(-\frac{1}{B}\log A\right)}_{c}$$

(9.5, p.2)

The resulting constants are

$m = -17.592, \quad c = 51.543$

$B = 1/m = -0.056845$

$c = -\frac{1}{B} \log A, \quad A = 10^{-cB} = 851.0 \text{ MPa}$

$\sigma_a = 851 N_f^{-0.0568} \text{ MPa}$ ◀

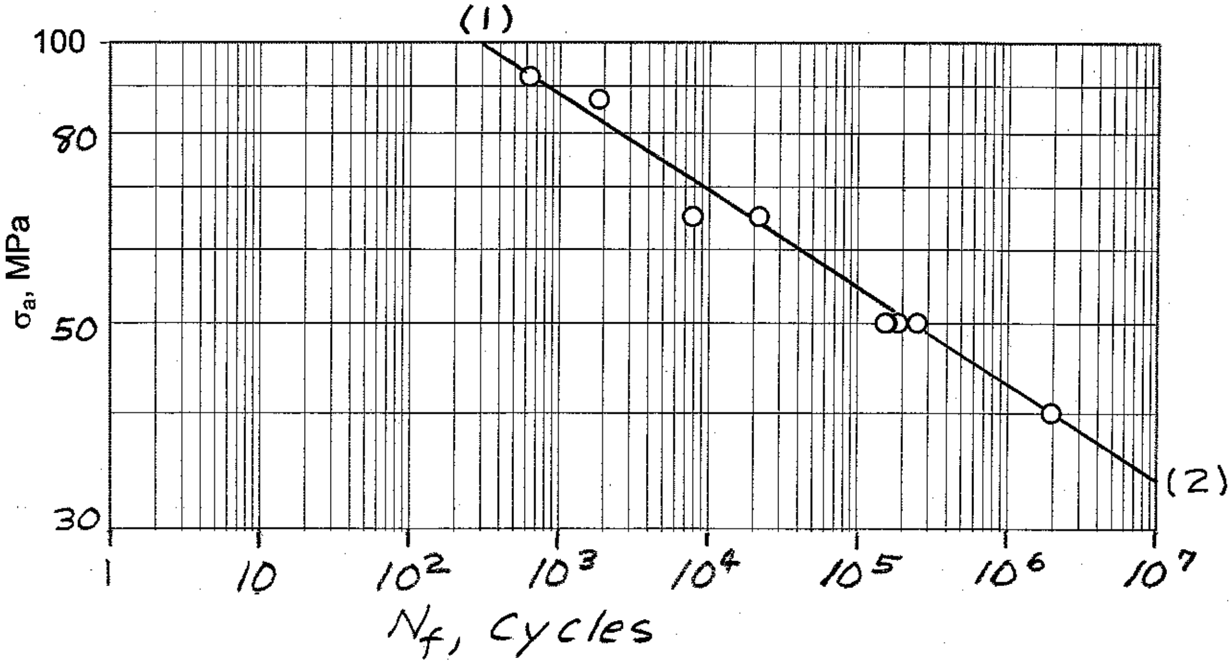
(c) $\sigma_f' = A/2^b = 885 \text{ MPa}$

$b = B = -0.0568$

$\sigma_a = 885 (2N_f)^{-0.0568} \text{ MPa}$ ◀

9.6 $\sigma_a = A N_f^B$, SRIM composite

(a) Plot data on log-log coordinates and draw a straight line through it.



Read two points from the line: (N_f, σ_a)

(1) (300, 100) (2) $(10^7, 34)$

$\sigma_1 = A N_1^B$, $\sigma_2 = A N_2^B$, Use Ex. 9.1(a)

$$B = \frac{\log \sigma_1 - \log \sigma_2}{\log N_1 - \log N_2} = \frac{\log 100 - \log 34}{\log 300 - \log 10^7}$$

$B = -0.1036$

$$A = \frac{\sigma_1}{N_1^B} = \frac{100 \text{ MPa}}{300^{-0.1036}} = 180.6 \text{ MPa}$$

$$\sigma_a = 180.6 \, N_f^{-0.1036} \text{ MPa}$$

(9.6, p. 2)

(b) Least-squares fit; N_f dependent

$$N_f = \left(\frac{\sigma_a}{A}\right)^{1/B}$$

$$\underbrace{\log N_f}_{y} = \underbrace{\frac{1}{B}}_{m} \underbrace{\log \sigma_a}_{x} \underbrace{- \frac{1}{B} \log A}_{+\ c}$$

The resulting constants are

$m = -9.2043$, $c = 20.918$

$B = 1/m = -0.108645$

$c = -\frac{1}{B} \log A$, $A = 10^{-cB} = 187.34$ MPa

$\sigma_a = 187.3\, N_f^{-0.1086}$ MPa ◀

(c) $\sigma_f' = A/2^b = 202.0$ MPa

$b = B = -0.1086$

$\sigma_a = 202(2N_f)^{-0.1086}$ MPa ◀

9.7

For given stress-life data on 2024-T3 aluminum: (a) Plot on log-log coordinates and find approximate constants A, B. (b) Find improved values A, B by least squares; calculate σ'_f, b.

$$\sigma_a = AN_f^B, \qquad \sigma_a = \sigma'_f(2N_f)^b$$

Data

σ_a, MPa	N_f, cycles	$x = \log \sigma_a$	$y = \log N_f$
379	8,000	2.579	3.903
345	13,100	2.538	4.117
276	53,000	2.441	4.724
207	306,000	2.316	5.486
172	1,169,000	2.236	6.068

(a) Read two points from line drawn through data on log-log coordinates.

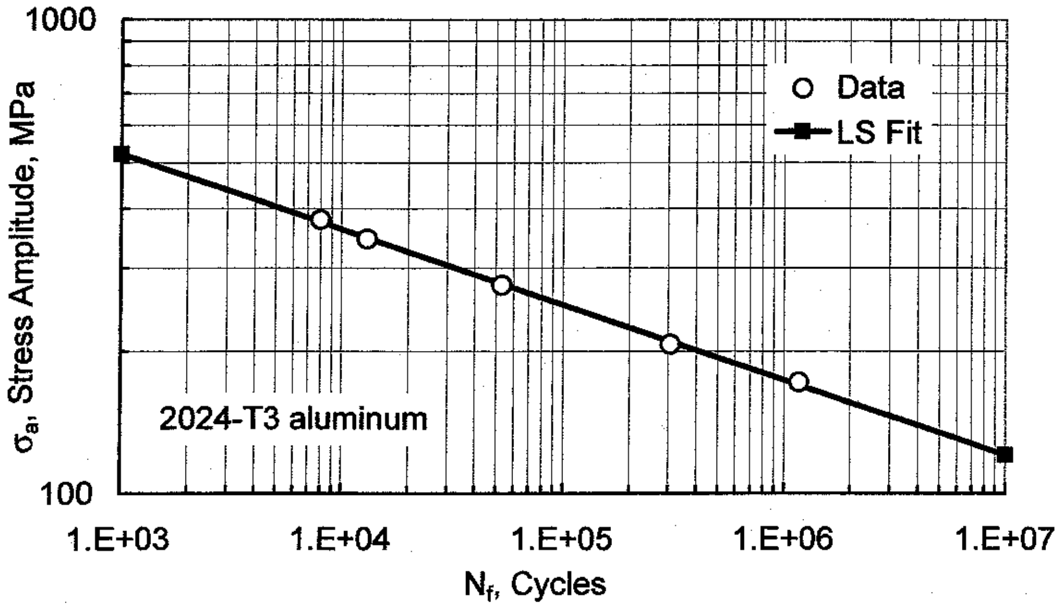
$$B = \frac{\log \sigma_1 - \log \sigma_2}{\log N_1 - \log N_2}, \qquad A = \frac{\sigma_1}{N_1^B}$$

Point No.	σ_a, MPa	N_f, cycles	$\log \sigma_a$	$\log N_f$
1	520	1,000	2.716	3.000
2	120	10,000,000	2.079	7.000

B	A, MPa
-0.1592	**1562**

(b) Do a least squares fit $y = mx + c$, where the dependent variable is $y = \log N_f$ and the independent variable is $x = \log \sigma_a$.

(9.7, p, 2)



$$N_f = \left(\frac{\sigma_a}{A}\right)^{1/B}, \qquad \log N_f = \frac{1}{B}\log\sigma_a - \frac{1}{B}\log A$$

$$B = 1/m, \qquad A = 10^{-cB}, \qquad b = B, \qquad \sigma'_f = A/2^b$$

slope	intercept	B, b	A, MPa	σ'_f, MPa
m	c	1/m	10^{-cB}	$A/2^b$
-6.286	20.083	**-0.1591**	**1566**	**1749**

Also calculate two points on the fitted line and show it on the graph.

LS Fit	
σ_a, MPa	N_f, cycles
521.9	1000
120.6	10,000,000

9.8

For given stress-life data on 2014-T6 aluminum: (a) Plot on log-log coordinates and find approximate constants A, B. (b) Find improved values A, B by least squares; calculate σ'_f, b.

$$\sigma_a = AN_f^B, \qquad \sigma_a = \sigma'_f (2N_f)^b$$

Data

σ_a, MPa	N_f, cycles	$x = \log \sigma_a$	$y = \log N_f$
395	1,800	2.597	3.255
336	16,300	2.526	4.212
256	82,700	2.408	4.918
220	281,000	2.342	5.449
178	1,130,000	2.250	6.053
172	3,490,000	2.236	6.543

(a) Read two points from line drawn through data on log-log coordinates.

$$B = \frac{\log \sigma_1 - \log \sigma_2}{\log N_1 - \log N_2}, \qquad A = \frac{\sigma_1}{N_1^B}$$

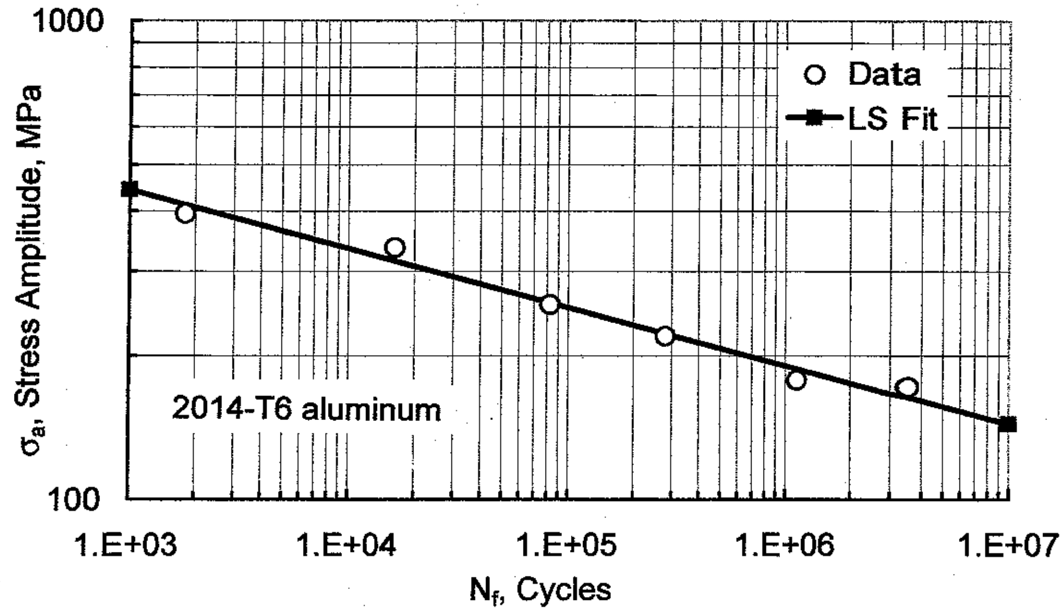
Point No.	σ_a, MPa	N_f, cycles	$\log \sigma_a$	$\log N_f$
1	440	1,000	2.643	3.000
2	150	10,000,000	2.176	7.000

B	A, MPa
-0.1168	**986**

◀

(b) Do a least squares fit $y = mx + c$, where the dependent variable is $y = \log N_f$ and the independent variable is $x = \log \sigma_a$.

(9.8, p.2)



$$N_f = \left(\frac{\sigma_a}{A}\right)^{1/B}, \qquad \log N_f = \frac{1}{B}\log\sigma_a - \frac{1}{B}\log A$$

$$B = 1/m, \qquad A = 10^{-cB}, \qquad b = B, \qquad \sigma'_f = A/2^b$$

slope	intercept	B, b	A, MPa	σ'_f, MPa
m	c	1/m	10^{-cB}	$A/2^b$
-8.189	24.670	**-0.1221**	**1029**	**1120**

Also calculate two points on the fitted line and show it on the graph.

LS Fit	
σ_a, MPa	N_f, cycles
442.8	1000
143.8	10,000,000

9.9

For given stress-life data on SAE 1015 steel: (a) Plot on log-log coordinates and find approximate constants A, B. (b) Find improved values A, B by least squares; calculate σ'_f, b.

$$\sigma_a = AN_f^B, \qquad \sigma_a = \sigma'_f (2N_f)^b$$

Data

σ_a, MPa	N_f, cycles	$x = \log \sigma_a$	$y = \log N_f$
558	52	2.747	1.716
455	242	2.658	2.384
362	1,650	2.559	3.217
245	15,750	2.389	4.197
228	30,000	2.358	4.477
207	90,000	2.316	4.954
172	393,000	2.236	5.594
158	800,000	2.199	5.903

(a) Read two points from line drawn through data on log-log coordinates.

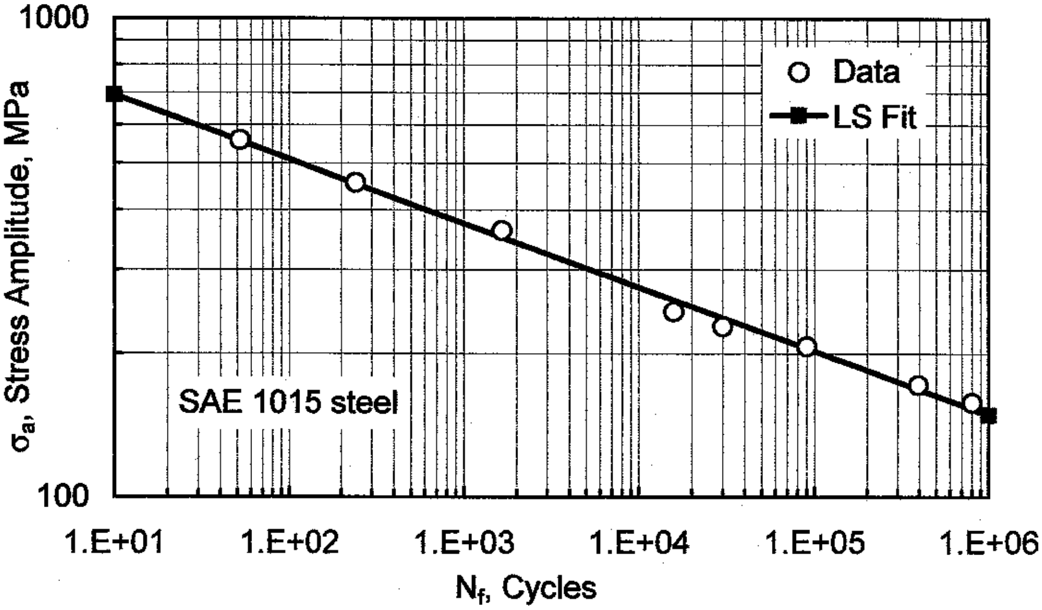
$$B = \frac{\log \sigma_1 - \log \sigma_2}{\log N_1 - \log N_2}, \qquad A = \frac{\sigma_1}{N_1^B}$$

Point No.	σ_a, MPa	N_f, cycles	$\log \sigma_a$	$\log N_f$
1	700	10	2.845	1.000
2	150	1,000,000	2.176	6.000

B	A, MPa
-0.1338	**953**

◀

(9.9, p.2)



(b) Do a least squares fit $y = mx + c$, where the dependent variable is $y = \log N_f$ and the independent variable is $x = \log \sigma_a$.

$$N_f = \left(\frac{\sigma_a}{A}\right)^{1/B}, \qquad \log N_f = \frac{1}{B}\log \sigma_a - \frac{1}{B}\log A$$

$$B = 1/m, \qquad A = 10^{-cB}, \qquad b = B, \qquad \sigma'_f = A/2^b$$

slope	intercept	B, b	A, MPa	σ'_f, MPa
m	c	1/m	10^{-cB}	$A/2^b$
-7.484	22.260	**-0.1336**	**943**	**1034**

Also calculate two points on the fitted line and show it on the graph.

LS Fit	
σ_a, MPa	N_f, cycles
693.2	10
148.8	1,000,000

9.10 In Ex. 9.2, $\sigma_a = 500$ MPa gives $N_f = 1.942 \times 10^5$ cycles. Replace parts at $N_f/3$. (a) X_N, X_S = ? (b) Comment.

(a)

$$X_N = \frac{N_{f2}}{\hat{N}} = \frac{N_{f2}}{N_{f2}/3} = 3.0 \quad ◀$$

$$X_S = X_N^{-B} = 3.0^{-(-0.0977)} = 1.113 \quad ◀$$

(b) The suggestion is a poor one as the $X_N = 3$ gives only an 11% margin of safety in stress. ◀

9.11 Part of 2024-T4 Al subjected to $\hat{\sigma}_a =$ 250 MPa, with $\hat{N} = 30{,}000$ cycles.

(a) X_S, $X_N = ?$ Reasonable? (Table 9.1)

$\sigma_a = \sigma_f'(2N_f)^b$, $\sigma_f' = 900$ MPa, $b = -0.102$

$$N_{f2} = \frac{1}{2}\left(\frac{\hat{\sigma}_a}{\sigma_f'}\right)^{1/b} = \frac{1}{2}\left(\frac{250}{900}\right)^{1/(-0.102)}$$

$N_{f2} = 1.422 \times 10^5$ cycles

$X_N = N_{f2}/\hat{N} = 1.422 \times 10^5 / 30{,}000 = 4.74$ ◀

$X_S = X_N^{-b} = 4.74^{-(-0.102)} = 1.172$ ◀

The safety factor in stress is too small.

(b) $X_S = 1.6$, $\hat{N} = ?$

$X_N = X_S^{-1/b} = 1.6^{-1/(-0.102)} = 100.3$

$\hat{N} = N_{f2}/X_N = 1.422 \times 10^5 / 100.3 = 1418$ cycles ◀

9.21 AISI 4142 (450HB) steel, $\sigma_a = 800\,MPa$

$N_f = ?$ for (a) $\sigma_m = 0$, (b) $\sigma_m = 200$, and (c) $\sigma_m = -200\,MPa$. Use Morrow.

$\sigma_f' = 1937\,MPa$, $b = -0.0762$ (Table 9.1)

$$\sigma_a = \sigma_f'(2N_f)^b \qquad (\sigma_m = 0)$$

$$\sigma_{ar} = \sigma_f'(2N_f)^b, \quad \sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/\sigma_f'} \qquad (\sigma_m \neq 0)$$

(a) $\sigma_m = 0$

$$N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f'}\right)^{1/b} = \frac{1}{2}\left(\frac{800}{1937}\right)^{1/-0.0762} = 5.48 \times 10^4 \text{ cycles}$$

(b) $\sigma_m = 200\,MPa$

$$\sigma_{ar} = \frac{800}{1 - 200/1937} = 892.1$$

$$N_f = \frac{1}{2}\left(\frac{\sigma_{ar}}{\sigma_f'}\right)^{1/b} = \frac{1}{2}\left(\frac{892.1}{1937}\right)^{1/-0.0762} = 1.31 \times 10^4 \text{ cycles}$$

(c) $\sigma_m = -200\,MPa$

$$\sigma_{ar} = \frac{800}{1 - \frac{(-200)}{1937}} = 725.1$$

$$N_f = \frac{1}{2}\left(\frac{\sigma_{ar}}{\sigma_f'}\right)^{1/b} = \frac{1}{2}\left(\frac{725.1}{1937}\right)^{1/-0.0762} = 1.99 \times 10^5 \text{ cycles}$$

Comment: Eq. 9.22 could be employed directly in fewer steps.

9.22

For AISI 4142 (450 HB) steel, with $\sigma_a = 800$ MPa, estimate the life for σ_m of (a) zero, (b) 200 MPa tension, and (c) 200 MPa compression. Use the SWT equation.

$$\sigma_{max} = \sigma_a + \sigma_m\,, \qquad \sqrt{\sigma_{max}\sigma_a} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma'_f}\right)^{1/b}$$

σ'_f, MPa	b
1937	-0.0762

	Stresses in MPa			SWT
	σ_a	σ_m	σ_{max}	N_f, cycles
(a)	800	0	800	5.481E+04
(b)	800	200	1000	1.268E+04
(c)	800	-200	600	3.620E+05

9.23

For Ti-6Al-4V, with $\sigma_a = 600$ MPa, estimate the life for σ_m of (a) zero, (b) 300 MPa tension, and (c) 300 MPa compression. Use the Morrow equation with the true fracture strength.

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m / \tilde{\sigma}_{fB}}, \qquad \sigma_{ar} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sigma_{ar}}{\sigma'_f}\right)^{1/b}$$

σ'_f, MPa	b	$\tilde{\sigma}_{fB}$, MPa
2030	-0.104	1717

Stresses in MPa

	σ_a	σ_m	σ_{ar}	N_f, cycles
(a)	600	0	600.0	6.149E+04
(b)	600	300	727.0	9.703E+03
(c)	600	-300	510.8	2.893E+05

9.24

For Ti-6Al-4V, with $\sigma_a = 600$ MPa, estimate the life for σ_m of (a) zero, (b) 300 MPa tension, and (c) 300 MPa compression. Use the SWT equation.

$$\sigma_{max} = \sigma_a + \sigma_m\,, \qquad \sqrt{\sigma_{max}\sigma_a} = \sigma'_f(2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma'_f}\right)^{1/b}$$

σ'_f, MPa	b
2030	-0.104

	Stresses in MPa			SWT
	σ_a	σ_m	σ_{max}	N_f, cycles
(a)	600	0	600	6.149E+04
(b)	600	300	900	8.754E+03
(c)	600	-300	300	1.722E+06

9.25

For AISI 4340 steel ($\sigma_u = 1172$ MPa), with $\sigma_a = 500$ MPa, estimate the life for σ_m of (a) zero, (b) 180 MPa tension, and (c) 180 MPa compression. Use the Morrow equation.

$$\sigma_a = (\sigma'_f - \sigma_m)(2N_f)^b, \qquad N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma'_f - \sigma_m}\right)^{1/b}$$

σ'_f, MPa	b
1758	-0.0977

	Stresses in MPa		Morrow
	σ_a	σ_m	N_f, cycles
(a)	500	0	1.941E+05
(b)	500	180	6.425E+04
(c)	500	-180	5.264E+05

9.26

For AISI 4340 steel ($\sigma_u = 1172$ MPa), with $\sigma_a = 500$ MPa, estimate the life for σ_m of (a) zero, (b) 180 MPa tension, and (c) 180 MPa compression. Use the SWT equation.

$$\sigma_{max} = \sigma_a + \sigma_m\,, \qquad \sqrt{\sigma_{max}\sigma_a} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma'_f}\right)^{1/b}$$

σ'_f, MPa	b
1758	-0.0977

	Stresses in MPa			SWT
	σ_a	σ_m	σ_{max}	N_f, cycles
(a)	500	0	500	1.941E+05
(b)	500	180	680	4.023E+04
(c)	500	-180	320	1.905E+06

9.27

For AISI 4340 steel ($\sigma_u = 1172$ MPa), with $\sigma_a = 500$ MPa, estimate the life for σ_m of (a) zero, (b) 180 MPa tension, and (c) 180 MPa compression. Use the Walker equation with $\gamma = 0.65$.

$$\sigma_{max} = \sigma_a + \sigma_m \, , \qquad \sigma_{ar} = \sigma_{max}^{1-\gamma} \sigma_a^{\gamma} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sigma_{max}^{1-\gamma}\sigma_a^{\gamma}}{\sigma'_f}\right)^{1/b}$$

σ'_f, MPa	b	γ
1758	-0.0977	0.65

Stresses in MPa

	σ_a	σ_m	σ_{max}	Walker N_f, cycles
(a)	500	0	500	1.941E+05
(b)	500	180	680	6.451E+04
(c)	500	-180	320	9.602E+05

9.28 RQC-100 steel. Determine and plot $\sigma_a - N_f$ for various σ_m.

$\sigma_f' = 938$ MPa, $b = -0.0648$ (Table 9.1)

(a) $\sigma_m = 100$ MPa

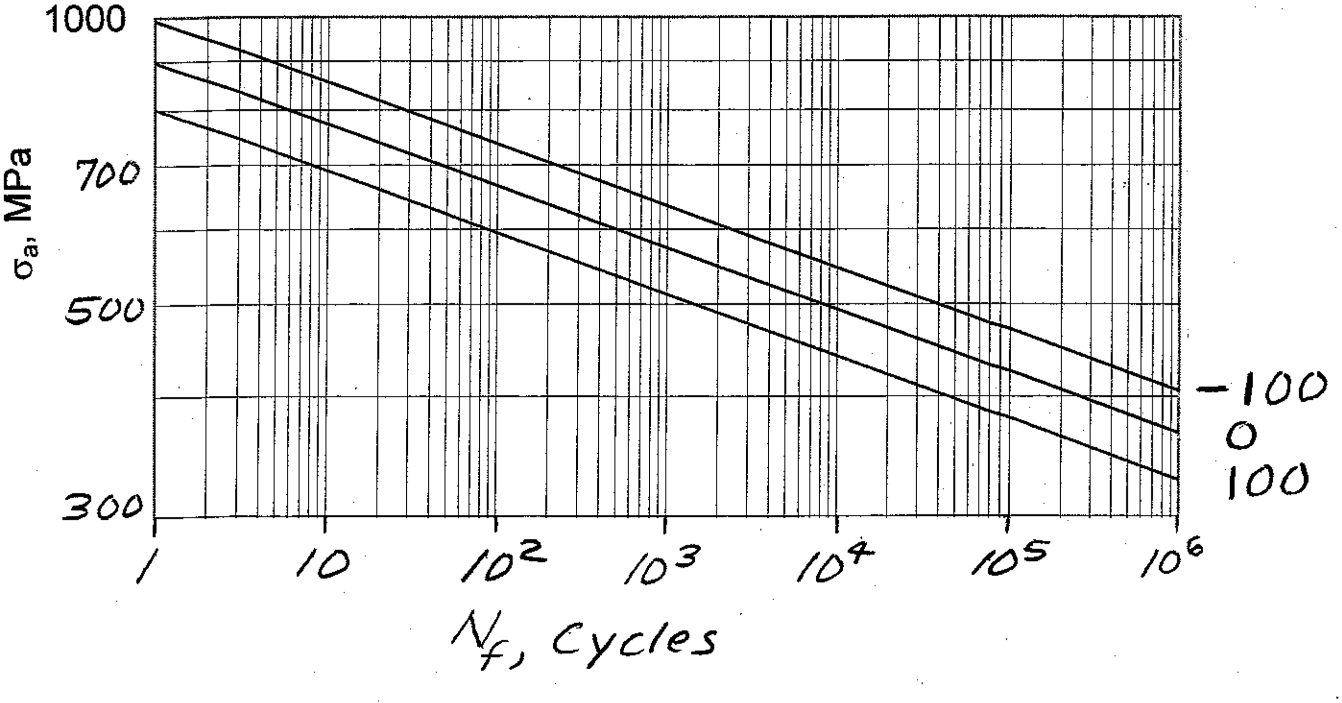
$\sigma_a = (\sigma_f' - \sigma_m)(2N_f)^b$, $\sigma_a = 838(2N_f)^{-0.0648}$ ◀

(b) $\sigma_m = 0$

$\sigma_a = 938(2N_f)^{-0.0648}$ ◀

(c) $\sigma_m = -100$ MPa

$\sigma_a = 1038(2N_f)^{-0.0648}$ ◀



The lines are parallel on a log-log plot, and lower for higher σ_m. ◀

9.29 2024-T4 Al. Determine and plot $\sigma_a - N_f$ for various σ_m, using SWT.

$\sigma_f' = 900$ MPa, $b = -0.102$ (Table 9.1)

$$\sigma_f'(2N_f)^b = \sqrt{\sigma_{max}\sigma_a} = \sqrt{(\sigma_a + \sigma_m)\sigma_a}$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{(\sigma_a + \sigma_m)\sigma_a}}{\sigma_f'}\right)^{1/b}$$

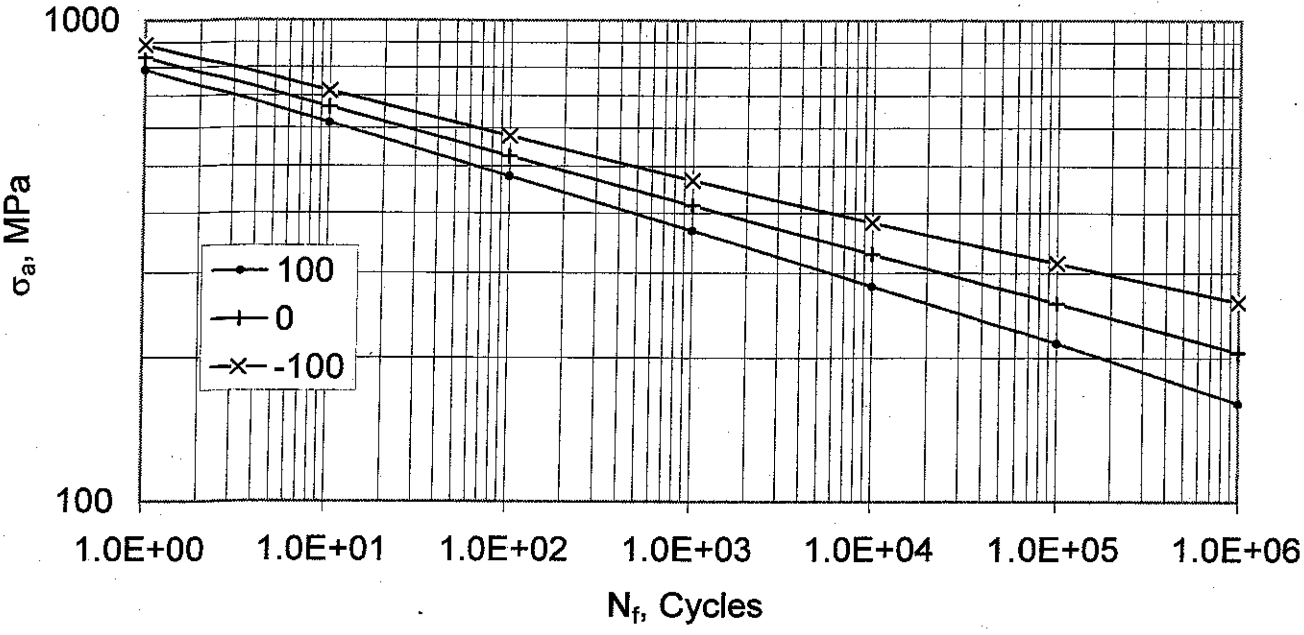
(a) $\sigma_m = 100$ MPa

$$N_f = \frac{1}{2}\left(\frac{\sqrt{(\sigma_a + 100)\sigma_a}}{900}\right)^{1/-0.102}$$ ◀

(b) $\sigma_m = 0$, $N_f = \frac{1}{2}\left(\frac{\sigma_a}{900}\right)^{1/-0.102}$ ◀

(c) $\sigma_m = -100$ MPa

$$N_f = \frac{1}{2}\left(\frac{\sqrt{(\sigma_a - 100)\sigma_a}}{900}\right)^{1/-0.102}$$ ◀



(9.29, p. 2)

The line for $\sigma_m = 0$ is straight on a log-log plot, but that for $\sigma_m = 100$ MPa is lower and a gentle curve. The line for $\sigma_m = -100$ is a gentle curve above the $\sigma_m = 0$ one. ◀

9.30 AISI 4340 steel data at various σ_m in Ex. 9.1 and 9.5. (a) Calculate and plot σ_a/σ_{ar} vs. σ_m. (b) Add lines for Goodman, Gerber, Morrow $\tilde{\sigma}_{fB}$ and σ_f'; comment.

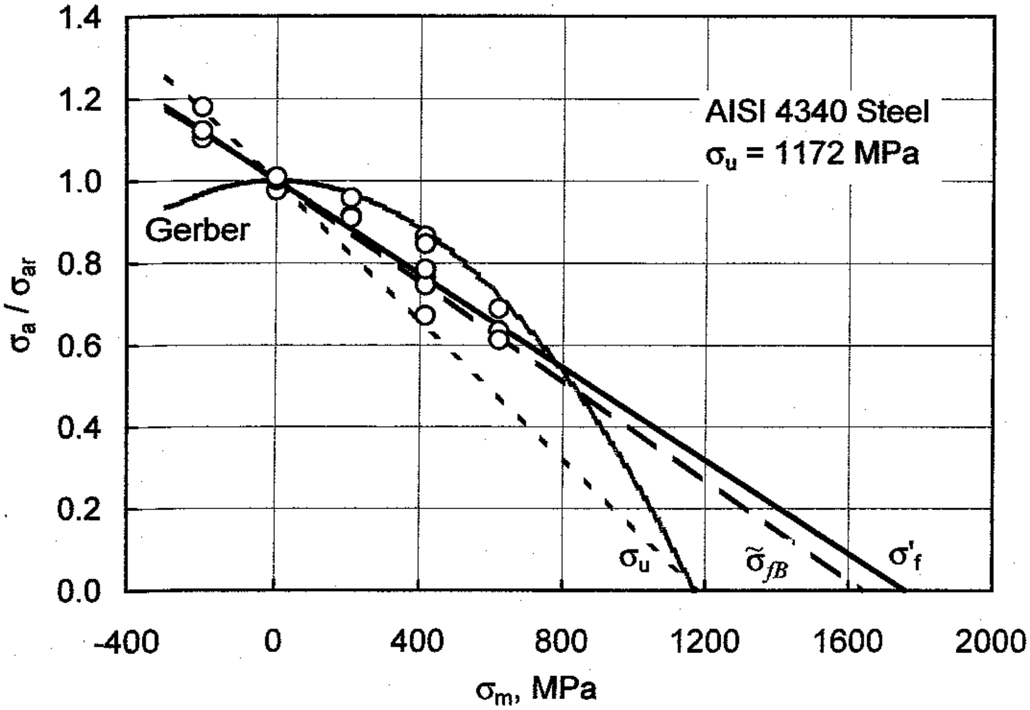
(a) For each N_f in data, calculate:

$\sigma_{ar} = \sigma_f'(2N_f)^b$, and σ_a/σ_{ar}

$\sigma_f' = 1758$ MPa, $b = -0.0971$ (Table 9.1)

σ_a, MPa	σ_m, MPa	N_f, cycles	σ_{ar}, MPa	σ_a/σ_{ar}
379	621	73,780	549.6	0.690
345	621	83,810	542.8	0.636
276	621	567,590	450.2	0.613
517	414	31,280	597.6	0.865
483	414	50,490	570.3	0.847
414	414	84,420	542.4	0.763
345	414	437,170	461.9	0.747
345	414	730,570	439.3	0.785
310	414	445,020	461.1	0.672
552	207	45,490	576.1	0.958
483	207	109,680	528.7	0.914
414	207	510,250	454.9	0.910
586	-207	208,030	496.6	1.180
552	-207	193,220	500.2	1.104
483	-207	901,430	430.3	1.122
948	0	222	969.1	0.978
834	0	992	837.2	0.996
703	0	6,004	702.2	1.001
631	0	14,130	645.9	0.977
579	0	43,860	578.2	1.001
524	0	132,150	519.1	1.009

(9.30, p, 2)



(b) The equations to be plotted are:

$$\sigma_a / \sigma_{ar} = 1 - \sigma_m / \sigma_u \qquad \text{(Goodman)}$$

$$\sigma_a / \sigma_{ar} = 1 - (\sigma_m / \sigma_u)^2 \qquad \text{(Gerber)}$$

$$\sigma_a / \sigma_{ar} = 1 - \sigma_m / \tilde{\sigma}_{fB} \qquad \text{(Morrow } \tilde{\sigma}_{fB}\text{)}$$

$$\sigma_a / \sigma_{ar} = 1 - \sigma_m / \sigma'_f \qquad \text{(Morrow } \sigma'_f\text{)}$$

Two points are calculated to plot the linear relationships, and a number are calculated for the Gerber parabola. See values on next page.

(9.30, p. 3)

σ'_f, MPa	b	σ_o, MPa	σ_u, MPa	$\tilde{\sigma}_{fB}$, MPa
1758	-0.0977	1103	1172	1634

(Table 9.1)

Goodman line

σ_m, MPa	σ_a/σ_{ar}
-300	1.256
1172	0.000

Morrow $\tilde{\sigma}_{fB}$ line

σ_m, MPa	σ_a/σ_{ar}
-300	1.184
1634	0.000

Morrow σ'_f line

σ_m, MPa	σ_a/σ_{ar}
-300	1.171
1758	0.000

Gerber parabola

σ_m, MPa	σ_a/σ_{ar}
-300	0.934
-150	0.984
0	1.000
150	0.984
300	0.934
450	0.853
600	0.738
750	0.590
900	0.410
1050	0.197
1172	0.000

The two variations of Morrow give similar lines that agree with the data as well as its scatter permits. Gerber is nonconservative for $\sigma_m > 0$ and for $\sigma_m < 0$ incorrectly predicts a harmful effect of mean stress. Goodman is conservative for $\sigma_m > 0$ but is quite inaccurate. ◀

9.31 For data of Ex. 9.1 and 9.5, plot σ_{ar} vs. N_f with Table 9.1 line for AISI 4340 steel for (a) SWT and (b) Walker with $\gamma = 0.65$.

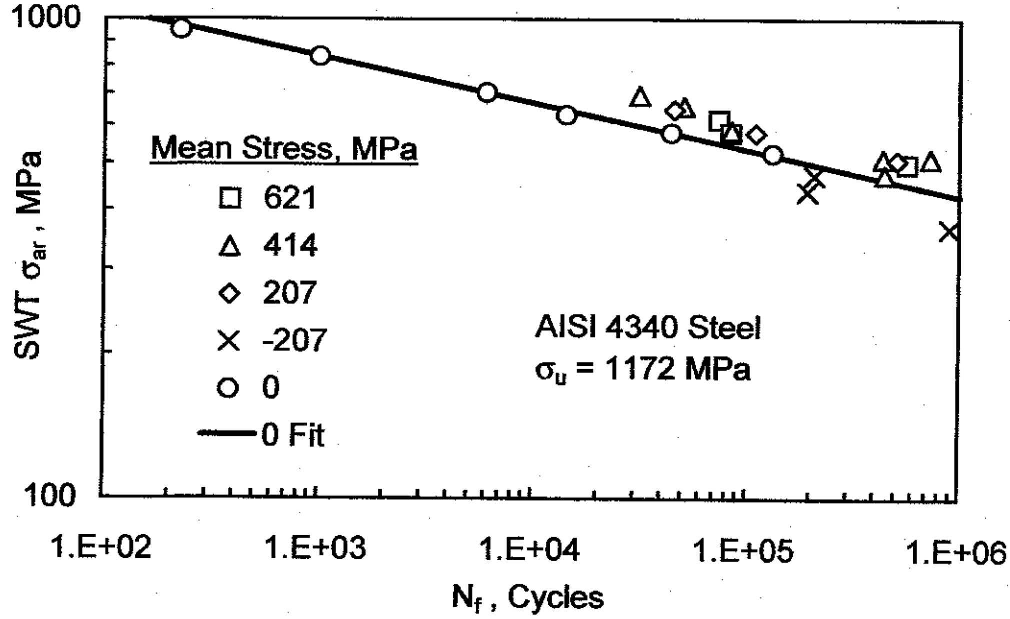
The first value for SWT is:

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a} = \sqrt{(\sigma_m + \sigma_a)\sigma_a}$$

$$\sigma_{ar} = \sqrt{(621 + 379)\,379} = 615.6 \text{ MPa}$$

at $N_f = 73{,}780$ cycles

Then compute remaining values and plot with line $\sigma_{ar} = \sigma_f'(2N_f)^b$



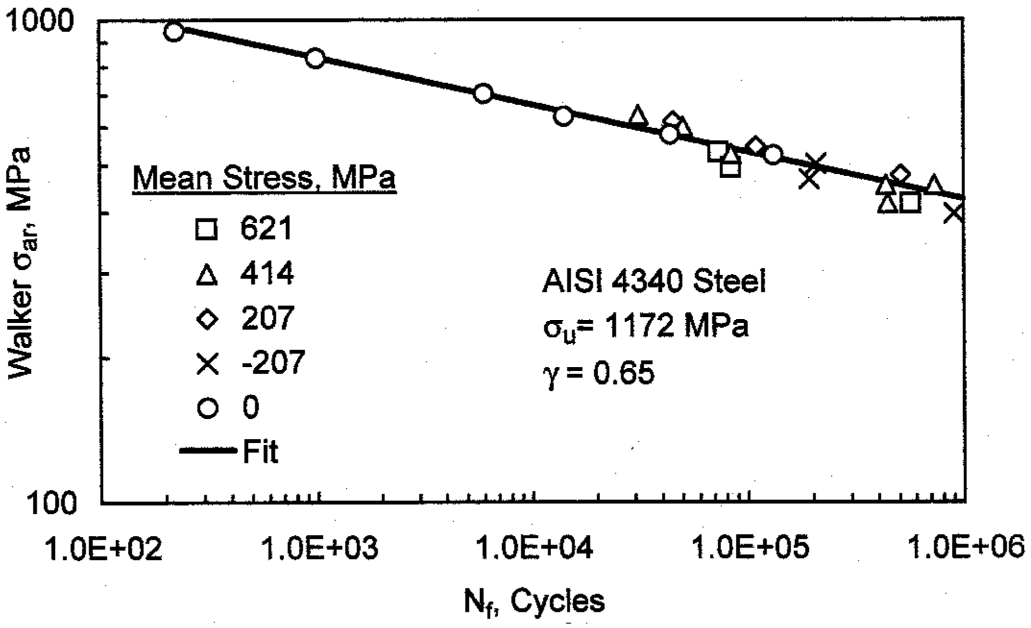
The correlation is reasonable. Compared to Ex. 9.5, it is better than Goodman, but not as good as Morrow.

(9.31, p.2) (b) The first value for Walker is:

$$\sigma_{ar} = \sigma_{max}^{1-\gamma} \sigma_a^{\gamma} = (\sigma_m + \sigma_a)^{1-\gamma} \sigma_a^{\gamma}$$

$$\sigma_{ar} = (621 + 379)^{1-0.65}\, 379^{0.65} = 532.2 \text{ MPa}$$

at $N_f = 73{,}780$ cycles



The correlation is excellent, better than Goodman, Morrow, or SWT. ◀

A detailed table giving all values plotted is on the next page.

(9.31, p.3)

σ'_f, MPa	b	γ
1758	-0.0977	0.650

				SWT	Walker
σ_a, MPa	σ_m, MPa	N_f, cycles	σ_{max}, MPa	σ_{ar}, MPa	σ_{ar}, MPa
379	621	73,780	1000	615.6	532.2
345	621	83,810	966	577.3	494.7
276	621	567,590	897	497.6	416.9
517	414	31,280	931	693.8	635.2
483	414	50,490	897	658.2	599.9
414	414	84,420	828	585.5	527.7
345	414	437,170	759	511.7	454.6
345	414	730,570	759	511.7	454.6
310	414	445,020	724	473.8	417.2
552	207	45,490	759	647.3	617.1
483	207	109,680	690	577.3	547.2
414	207	510,250	621	507.0	477.1
586	-207	208,030	379	471.3	503.1
552	-207	193,220	345	436.4	468.3
483	-207	901,430	276	365.1	397.1
948	0	222	948	948.0	948.0
834	0	992	834	834.0	834.0
703	0	6,004	703	703.0	703.0
631	0	14,130	631	631.0	631.0
579	0	43,860	579	579.0	579.0
524	0	132,150	524	524.0	524.0

Line from Table 9.1 constants

161	1000.0	1000.0
1,000,000	426.0	426.0

9.32

For given data on 2024-T3 Al at various R, and for data of Prob. 9.7: (a) Calculate and plot σ_{ar} vs. N_f for the SWT equation, and include the fitted line for $R = -1$. (b, c, d) Repeat for equations of Goodman, Morrow with $\tilde{\sigma}_{fB}$, and Morrow with σ'_f.

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a} \qquad \text{(SWT)}$$

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m / \sigma_u} \qquad \text{(Goodman)}$$

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m / \tilde{\sigma}_{fB}} \qquad \text{(Morrow } \tilde{\sigma}_{fB}\text{)}$$

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m / \sigma'_f} \qquad \text{(Morrow } \sigma'_f\text{)}$$

Numerical values and plots are on pages that follow. SWT correlates the data very well. Goodman gives a poor correlation, being overly conservative for positive R. Morrow with $\tilde{\sigma}_{fB}$ is reasonable but inferior to SWT. Morrow with σ'_f is grossly nonconservative. ◀

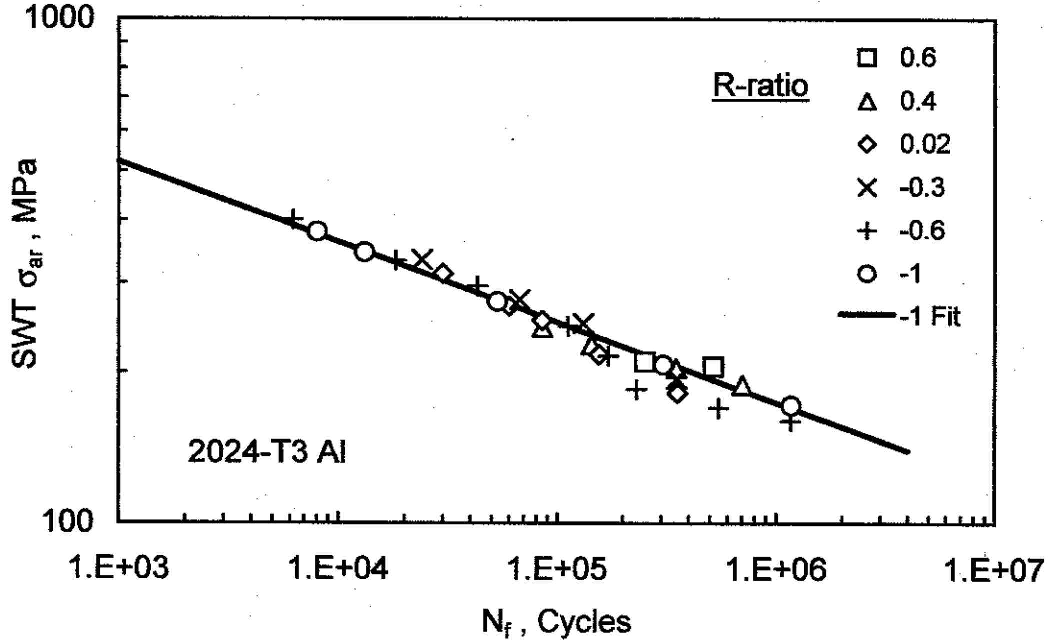
(9.32, p.2)

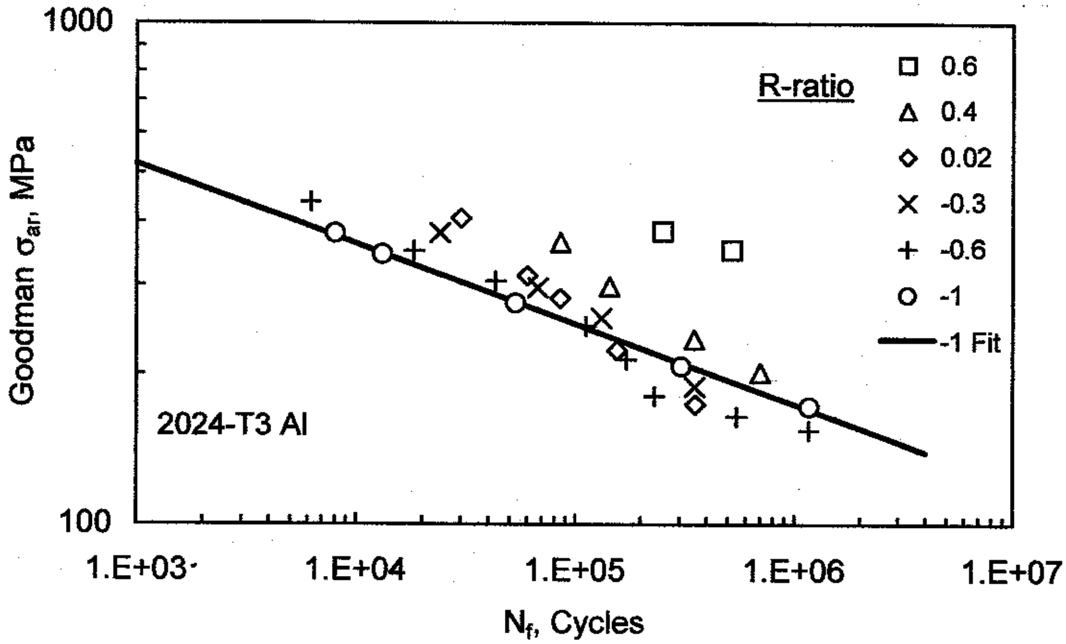
σ'_f	b	σ_o	σ_u	$\tilde{\sigma}_{fB}$
1749	-0.1591	359	497	610

All σ in MPa

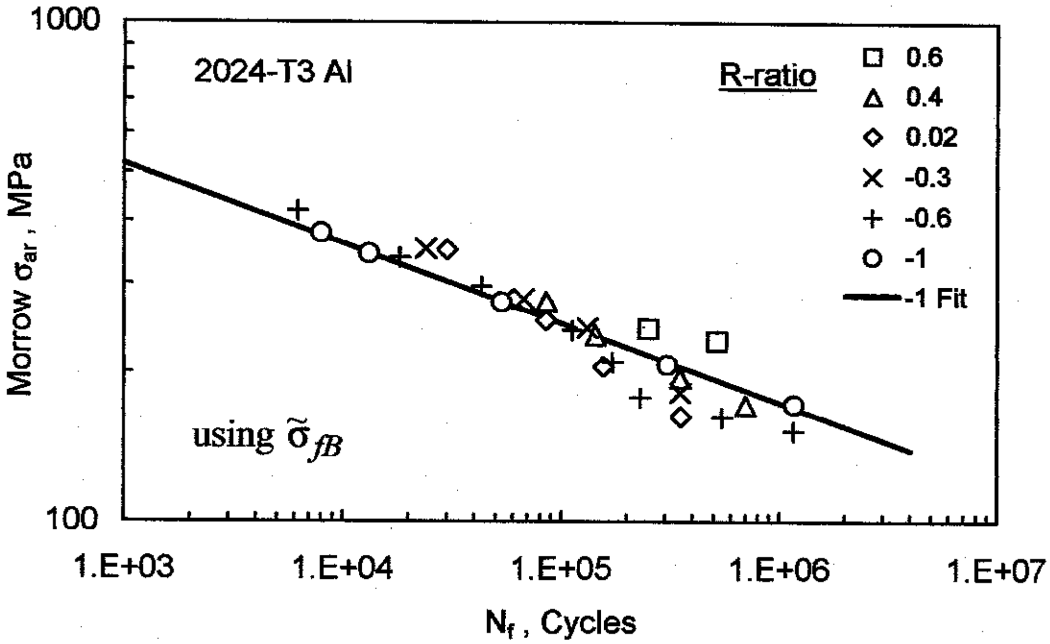
					SWT	Goodman	Morrow ($\tilde{\sigma}_{fB}$)	Morrow (σ'_f)
R	N_f, cycles	σ_{max}	σ_a	σ_m	σ_{ar}	σ_{ar}	σ_{ar}	σ_{ar}
0.60	252,000	469	93.8	375.2	209.7	382.7	243.7	119.4
0.60	520,000	459	91.8	367.2	205.3	351.5	230.6	116.2
0.40	85,000	448	134.4	313.6	245.4	364.2	276.6	163.8
0.40	144,000	414	124.2	289.8	226.8	297.9	236.6	148.9
0.40	351,000	372	111.6	260.4	203.8	234.4	194.7	131.1
0.40	701,000	345	103.5	241.5	189.0	201.3	171.3	120.1
0.02	30,000	448	219.5	228.5	313.6	406.3	351.0	252.5
0.02	60,000	386	189.1	196.9	270.2	313.2	279.3	213.1
0.02	85,000	362	177.4	184.6	253.4	282.2	254.4	198.3
0.02	156,000	310	151.9	158.1	217.0	222.8	205.0	167.0
0.02	355,000	260	127.4	132.6	182.0	173.8	162.8	137.9
-0.30	24,000	414	269.1	144.9	333.8	379.8	352.9	293.4
-0.30	67,000	345	224.3	120.8	278.1	296.2	279.6	240.9
-0.30	132,000	310	201.5	108.5	249.9	257.8	245.1	214.8
-0.30	353,000	241	156.7	84.35	194.3	188.7	181.8	164.6
-0.60	6,200	448	358.4	89.6	400.7	437.2	420.1	377.8
-0.60	18,200	372	297.6	74.4	332.7	350.0	338.9	310.8
-0.60	43,000	331	264.8	66.2	296.1	305.5	297.0	275.2
-0.60	112,000	276	220.8	55.2	246.9	248.4	242.8	228.0
-0.60	172,000	241	192.8	48.2	215.6	213.5	209.3	198.3
-0.60	231,000	207	165.6	41.4	185.1	180.6	177.7	169.6
-0.60	546,000	190	152.0	38.0	169.9	164.6	162.1	155.4
-0.60	1,165,000	179	143.2	35.8	160.1	154.3	152.1	146.2
-1.00	8,000	379	379.0	0	379.0	379.0	379.0	379.0
-1.00	13,100	345	345.0	0	345.0	345.0	345.0	345.0
-1.00	53,000	276	276.0	0	276.0	276.0	276.0	276.0
-1.00	306,000	207	207.0	0	207.0	207.0	207.0	207.0
-1.00	1,169,000	172	172.0	0	172.0	172.0	172.0	172.0
R = -1	1,000				521.9	521.9	521.9	521.9
Fit	4,000,000				139.5	139.5	139.5	139.5

(9.32, p.3)

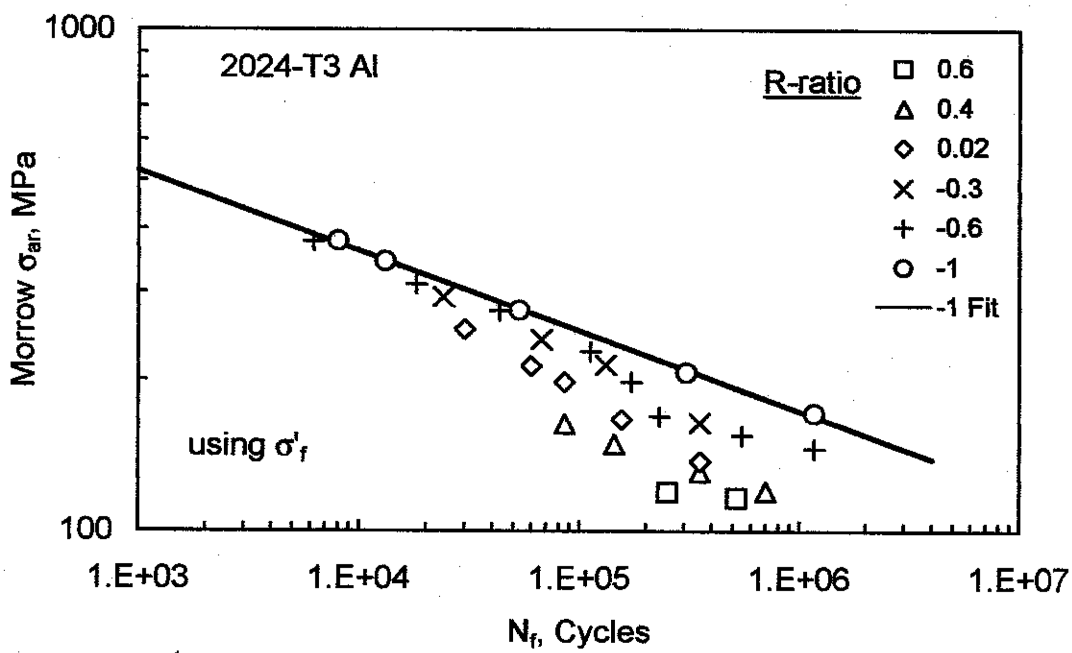

1000
100
SWT σ_{ar} , MPa
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
N_f , Cycles
R-ratio
0.6
0.4
0.02
-0.3
-0.6
-1
-1 Fit
2024-T3 Al


1000
100
Goodman σ_{ar}, MPa
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
N_f, Cycles
R-ratio
0.6
0.4
0.02
-0.3
-0.6
-1
-1 Fit
2024-T3 Al

(9.32, p.4)



1000
100
2024-T3 Al
R-ratio
0.6
0.4
0.02
-0.3
-0.6
-1
-1 Fit
Morrow σ_{ar} , MPa
using $\tilde{\sigma}_{fB}$
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
N_f , Cycles



1000
100
2024-T3 Al
R-ratio
0.6
0.4
0.02
-0.3
-0.6
-1
-1 Fit
Morrow σ_{ar}, MPa
using σ'_f
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
N_f, Cycles

9.33

For given data on SAE 1015 steel at various mean stresses, and for the data of Prob. 9.9: (a) Calculate and plot σ_{ar} vs. N_f for the Goodman equation, and include the line for $\sigma_m = 0$ from Table 9.1 constants. (b, c, d) Repeat for equations of Morrow with σ'_f, SWT, and Walker with $\gamma = 0.71$.

$$\sigma_{ar} = \frac{\sigma_a}{1-\sigma_m/\sigma_u} \qquad \text{(Goodman)}$$

$$\sigma_{ar} = \frac{\sigma_a}{1-\sigma_m/\sigma'_f} \qquad \text{(Morrow } \sigma'_f\text{)}$$

$$\sigma_{ar} = \sqrt{\sigma_{\max}\sigma_a} \qquad \text{(SWT)}$$

$$\sigma_{ar} = \sigma_{\max}^{1-\gamma}\sigma_a^{\gamma} \qquad \text{(Walker)}$$

Numerical values and plots are on pages that follow. Goodman does not correlate the data very well, being overly conservative for positive mean stress. Morrow gives a good correlation. SWT is reasonable but inferior to Morrow. Walker gives an excellent correlation which is the best of those examined. Thes data at zero mean stress do not fit the line from Table 9.1 costants very closely at the longer lives, likely due to batch-to-batch variation of the material. ◀

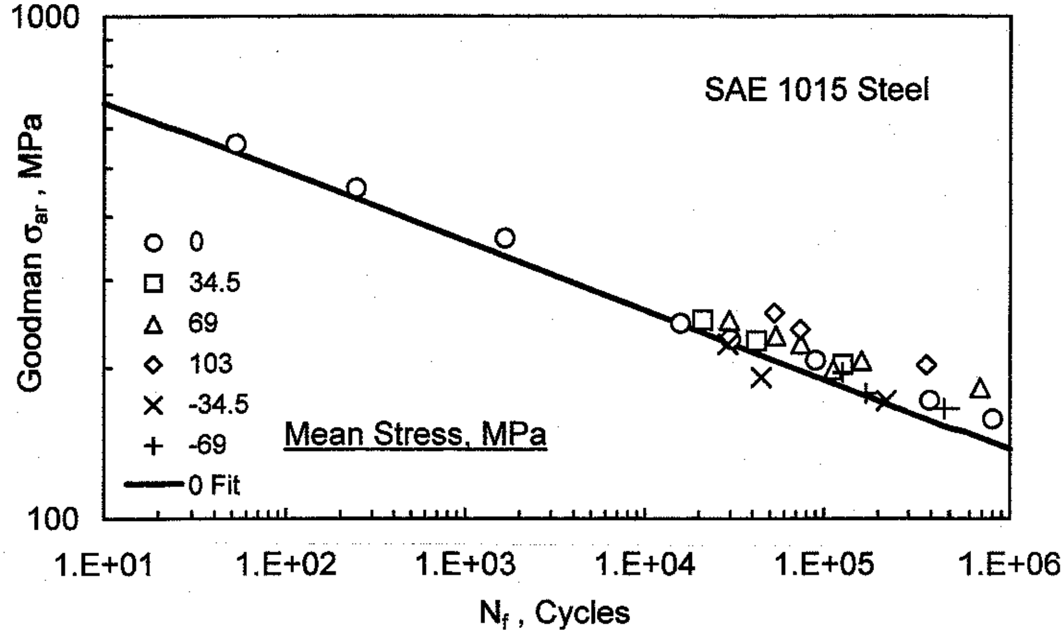
(9.33, p. 2)

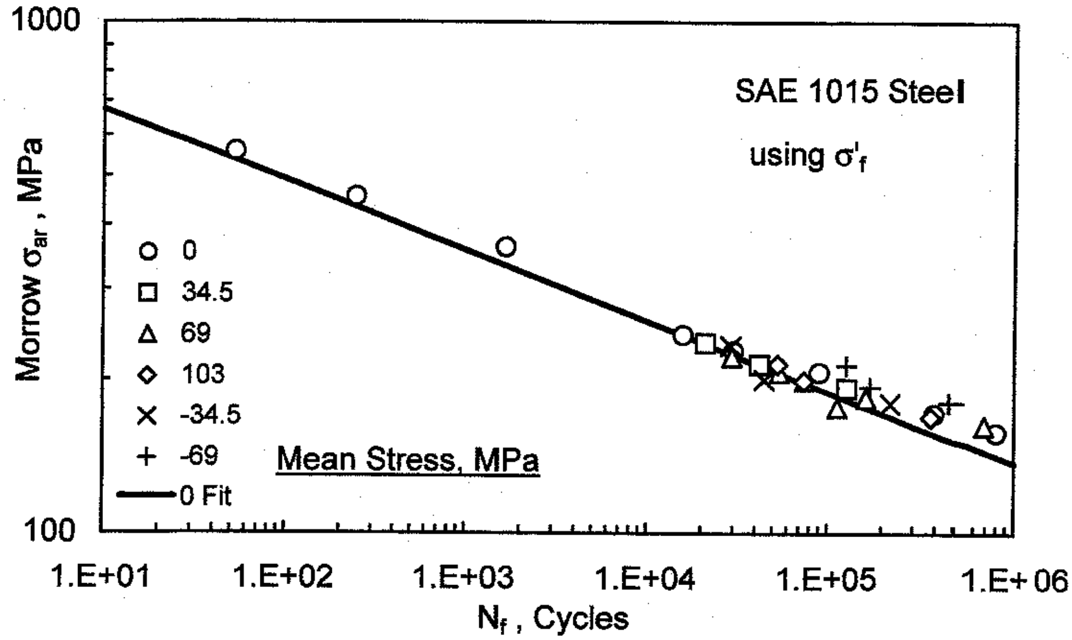
σ'_f	b	σ_o	σ_u	$\tilde{\sigma}_{fB}$	γ
1020	-0.138	228	415	726	0.71

All σ in MPa

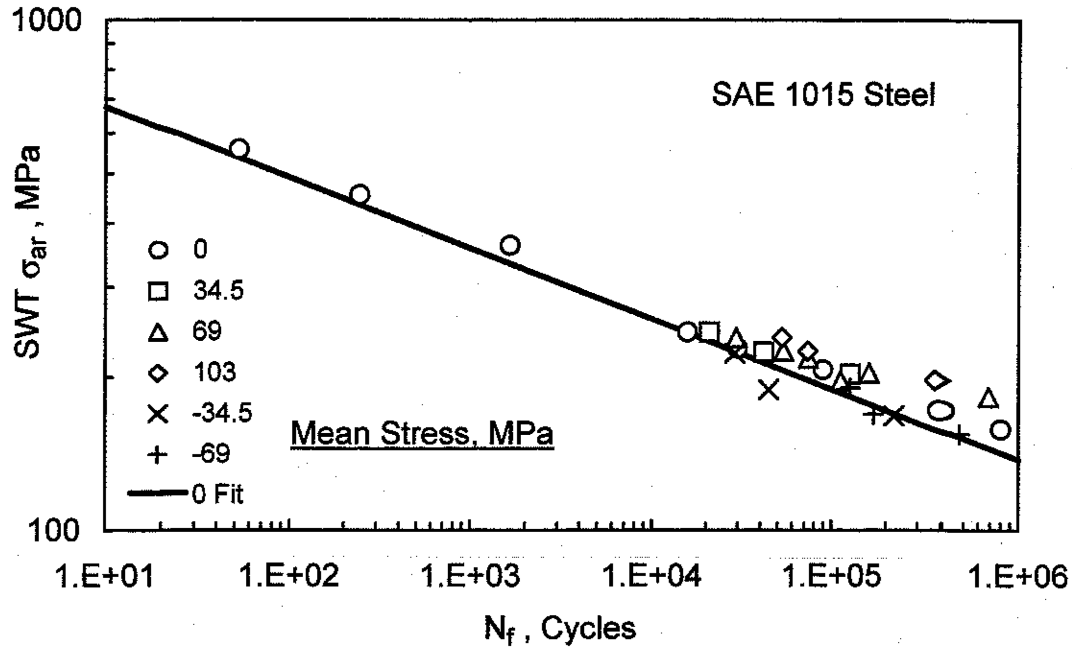
				Goodman	Morrow (σ'_f)	SWT	Walker
σ_a	σ_m	N_f, cycles	σ_{max}	σ_{ar}	σ_{ar}	σ_{ar}	σ_{ar}
558	0	52	558	558.0	558.0	558.0	558.0
455	0	242	455	455.0	455.0	455.0	455.0
362	0	1,650	362	362.0	362.0	362.0	362.0
245	0	15,750	245	245.0	245.0	245.0	245.0
228	0	30,000	228	228.0	228.0	228.0	228.0
207	0	90,000	207	207.0	207.0	207.0	207.0
172	0	393,000	172	172.0	172.0	172.0	172.0
158	0	800,000	158	158.0	158.0	158.0	158.0
228	34.5	21,000	262.5	248.7	236.0	244.6	237.5
207	34.5	42,000	241.5	225.8	214.2	223.6	216.5
186	34.5	128,000	220.5	202.9	192.5	202.5	195.4
207	69.0	29,500	276	248.3	222.0	239.0	225.0
193	69.0	54,000	262	231.5	207.0	224.9	210.9
186	69.0	74,480	255	223.1	199.5	217.8	203.8
172	69.0	162,800	241	206.3	184.5	203.6	189.7
165	69.0	113,000	234	197.9	177.0	196.5	182.6
152	69.0	683,970	221	182.3	163.0	183.3	169.4
193	103.4	52,880	296.4	257.0	214.8	239.2	218.6
179	103.4	73,940	282.4	238.4	199.2	224.8	204.3
152	103.4	377,000	255.4	202.4	169.1	197.0	176.7
241	-34.5	29,000	206.5	222.5	233.1	223.1	230.4
207	-34.5	44,700	172.5	191.1	200.2	189.0	196.3
186	-34.5	223,600	151.5	171.7	179.9	167.9	175.3
228	-69.0	127,380	159	195.5	213.6	190.4	205.4
207	-69.0	172,250	138	177.5	193.9	169.0	184.0
193	-69.0	473,250	124	165.5	180.8	154.7	169.8
$\sigma_m = 0$		10		674.6	674.6	674.6	674.6
(Table 9.1)		1,000,000		137.7	137.7	137.7	137.7

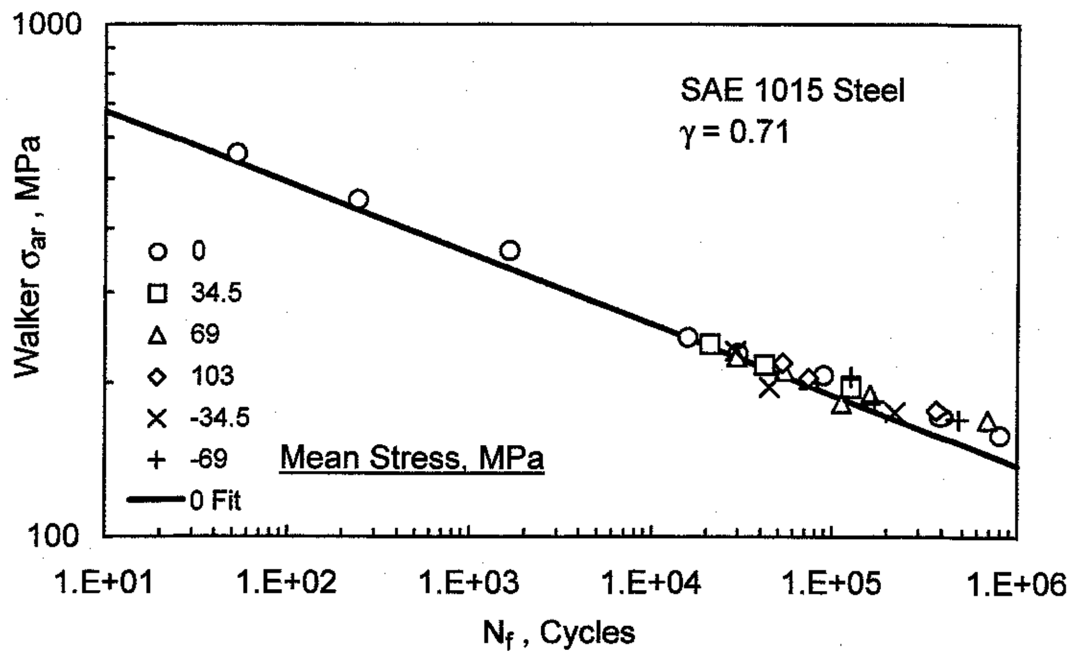
(9.33, p.3)


SAE 1015 Steel
Goodman σ_{ar} , MPa
1000
100
0
34.5
69
103
-34.5
-69
0 Fit
Mean Stress, MPa
1.E+01
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
N_f , Cycles


SAE 1015 Steel
using σ'_f
Morrow σ_{ar} , MPa
1000
100
0
34.5
69
103
-34.5
-69
0 Fit
Mean Stress, MPa
1.E+01
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
N_f , Cycles

(9.33, p.4)


1000
SAE 1015 Steel
SWT σ_{ar} , MPa
0
34.5
69
103
-34.5
-69
0 Fit
Mean Stress, MPa
100
1.E+01
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
N_f , Cycles


1000
SAE 1015 Steel
$\gamma = 0.71$
Walker σ_{ar} , MPa
0
34.5
69
103
-34.5
-69
0 Fit
Mean Stress, MPa
100
1.E+01
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
N_f , Cycles

9.34

For given data on 2014-T6 Al at various R, and for the data of Prob. 9.8: (a) Calculate and plot σ_{ar} vs. N_f for the SWT equation, and include the fitted line for $R = -1$. (b, c, d) Repeat for equations of Goodman, Morrow with $\tilde{\sigma}_{fB}$, and Morrow with σ'_f.

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a} \qquad \text{(SWT)}$$

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m / \sigma_u} \qquad \text{(Goodman)}$$

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m / \tilde{\sigma}_{fB}} \qquad \text{(Morrow } \tilde{\sigma}_{fB}\text{)}$$

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m / \sigma'_f} \qquad \text{(Morrow } \sigma'_f\text{)}$$

Numerical values and plots are on pages that follow. SWT correlates the data very well. Goodman gives a poor correlation, being overly conservative for positive R. Morrow with $\tilde{\sigma}_{fB}$ is reasonable but inferior to SWT. Morrow with σ'_f is grossly nonconservative.

(9.34, p.2)

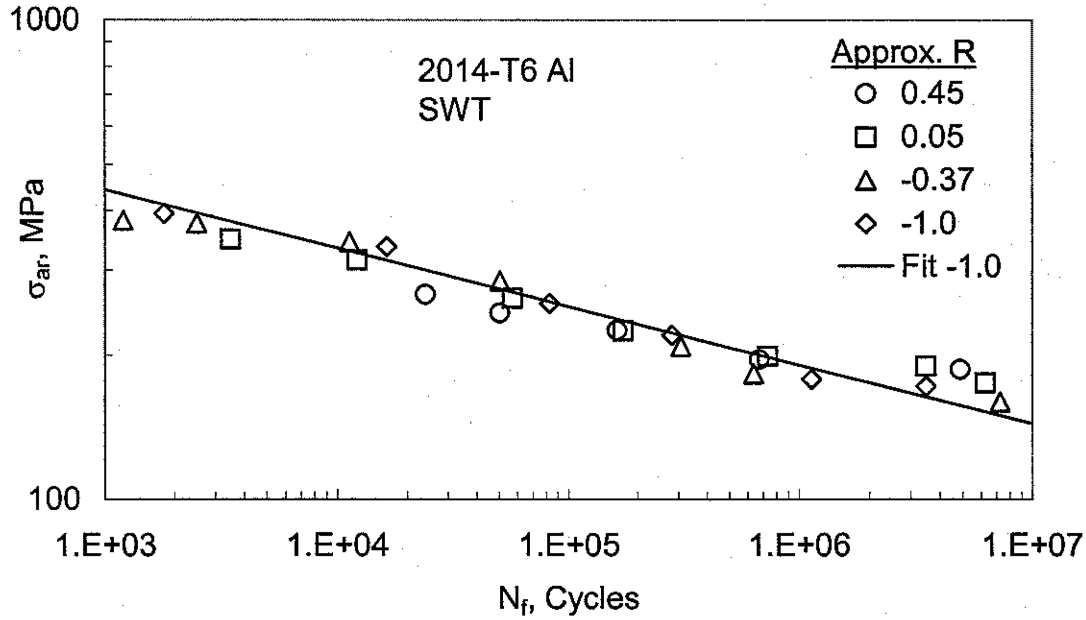
σ'_f	b	σ_o	σ_u	$\tilde{\sigma}_{fB}$
1120	-0.122	438	494	580

	N_f	σ_a
$\sigma_m = 0$	1.E+03	442.8
Fit	1.E+07	143.8

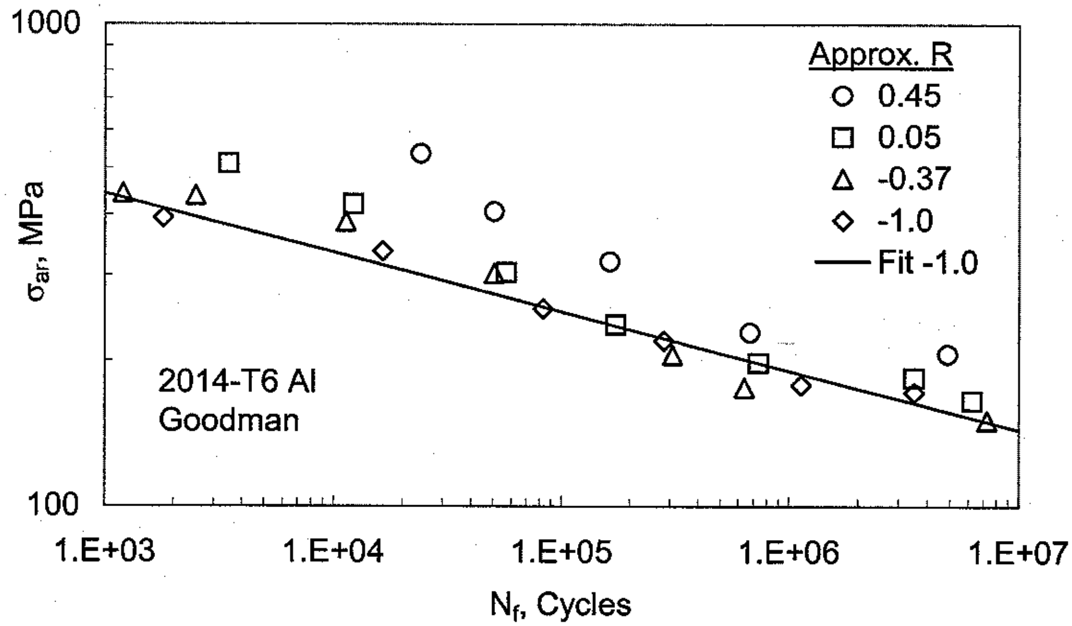
All σ in MPa

					SWT	Goodman	Morrow ($\tilde{\sigma}_{fB}$)	Morrow (σ'_f)
σ_{max}	R-ratio	N_f, cycles	σ_a	σ_m	σ_{ar}	σ_{ar}	σ_{ar}	σ_{ar}
505	0.440	24,000	141.4	363.6	267.2	535.7	379.0	209.4
466	0.449	50,500	128.4	337.6	244.6	405.6	307.2	183.8
430	0.451	163,000	118.0	312.0	225.3	320.3	255.4	163.6
376	0.458	675,000	101.9	274.1	195.7	228.9	193.2	134.9
359	0.460	4,890,000	96.9	262.1	186.5	206.5	176.8	126.5
502	0.036	3,500	242.0	260.0	348.5	510.9	438.6	315.1
456	0.045	12,200	217.7	238.3	315.1	420.6	369.5	276.6
381	0.051	57,100	180.8	200.2	262.4	304.0	276.1	220.1
327	0.055	173,000	154.5	172.5	224.8	237.4	219.9	182.6
290	0.058	734,000	136.6	153.4	199.0	198.1	185.7	158.3
276	0.058	3,490,000	130.0	146.0	189.4	184.5	173.7	149.5
255	0.058	6,260,000	120.1	134.9	175.0	165.2	156.5	136.6
458	-0.388	1,200	317.9	140.1	381.5	443.7	419.1	363.3
455	-0.369	2,500	311.4	143.6	376.4	439.0	413.9	357.2
415	-0.373	11,300	284.9	130.1	343.8	386.8	367.3	322.3
345	-0.365	50,400	235.5	109.5	285.0	302.5	290.3	261.0
252	-0.371	308,000	172.7	79.3	208.6	205.8	200.1	185.9
221	-0.367	637,000	151.1	69.9	182.7	176.0	171.8	161.1
193	-0.367	7,260,000	131.9	61.1	159.6	150.5	147.4	139.5
395	-1.000	1,800	395.0	0.0	395.0	395.0	395.0	395.0
336	-1.000	16,300	336.0	0.0	336.0	336.0	336.0	336.0
256	-1.000	82,700	256.0	0.0	256.0	256.0	256.0	256.0
220	-1.000	281,000	220.0	0.0	220.0	220.0	220.0	220.0
178	-1.000	1,130,000	178.0	0.0	178.0	178.0	178.0	178.0
172	-1.000	3,490,000	172.0	0.0	172.0	172.0	172.0	172.0

(9,34, p.3)

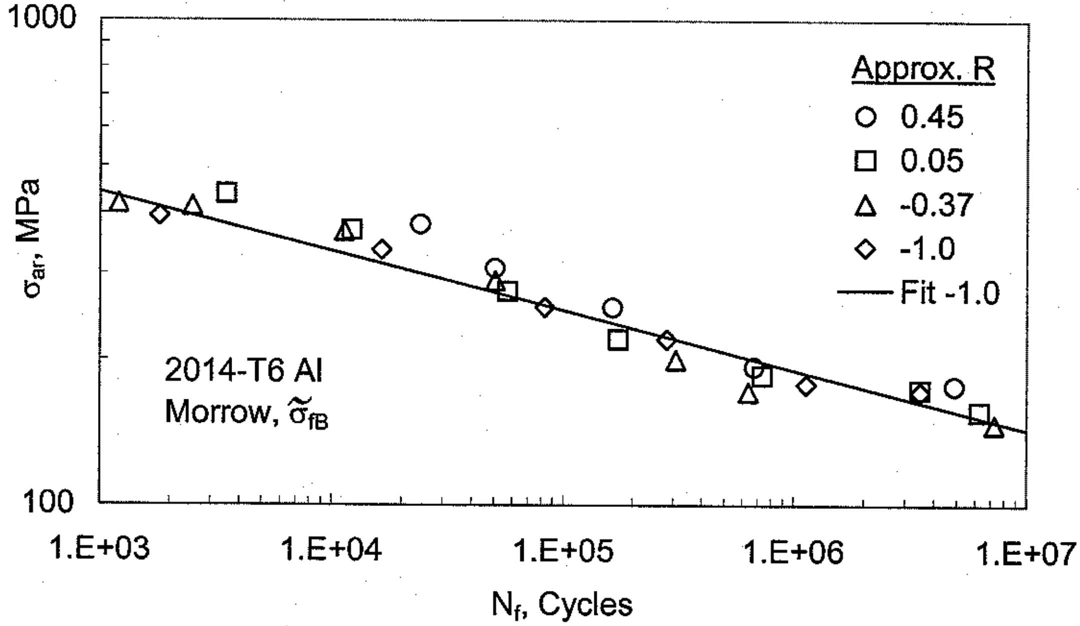


1000
100
2014-T6 Al
SWT
Approx. R
0.45
0.05
-0.37
-1.0
Fit -1.0
σ_{ar}, MPa
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
N_f, Cycles

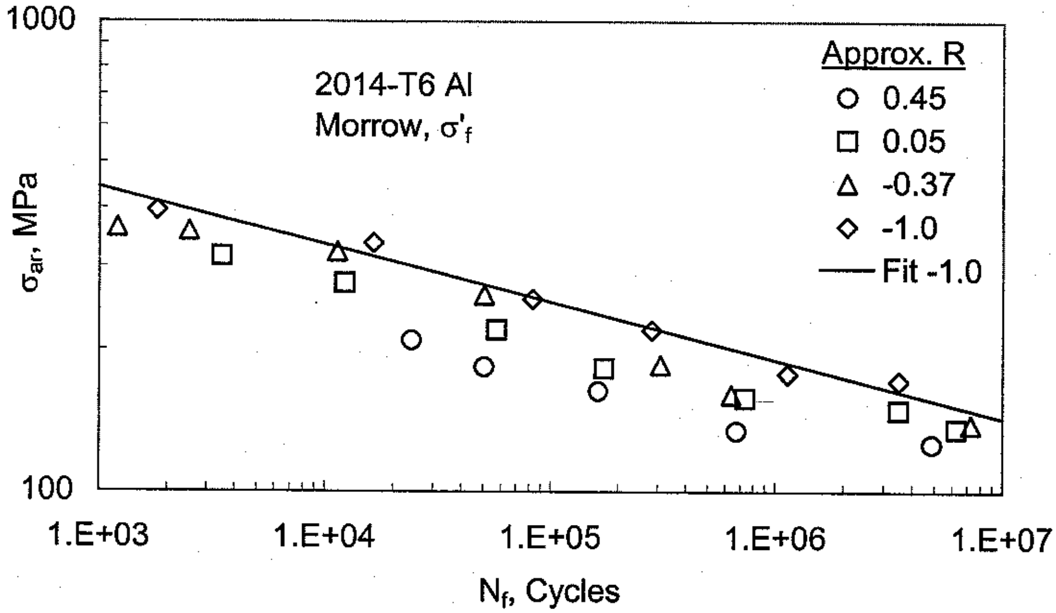


1000
100
Approx. R
0.45
0.05
-0.37
-1.0
Fit -1.0
2014-T6 Al
Goodman
σ_{ar}, MPa
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
N_f, Cycles

(9.34, p. 4)



1000
100
σ_{ar}, MPa
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
N_f, Cycles
Approx. R
0.45
0.05
-0.37
-1.0
Fit -1.0
2014-T6 Al
Morrow, $\tilde{\sigma}_{fB}$



1000
100
σ_{ar}, MPa
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
N_f, Cycles
Approx. R
0.45
0.05
-0.37
-1.0
Fit -1.0
2014-T6 Al
Morrow, σ'_f

9.35 AISI 4142 steel (450 HB) is subjected to stresses in service of $\hat{\sigma}_a = 500$, $\hat{\sigma}_m = 250$ MPa, with $\hat{N} = 3000$ cycles desired.

$X_S, X_N = ?$ $\quad \sigma_f' = 1937$ MPa, $b = -0.0762$

$$\hat{\sigma}_{ar} = \frac{\hat{\sigma}_a}{1 - \hat{\sigma}_m/\sigma_f'} = \frac{500}{1 - 250/1937} = 574.1 \text{ MPa}$$

$$\hat{\sigma}_{ar} = \sigma_f' (2N_{f2})^b, \quad N_{f2} = \frac{1}{2}\left(\frac{\hat{\sigma}_{ar}}{\sigma_f'}\right)^{1/b}$$

$$N_{f2} = \frac{1}{2}\left(\frac{574.1}{1937}\right)^{1/(-0.0762)} = 4.266 \times 10^6 \text{ cycles}$$

$$X_N = N_{f2}/\hat{N} = 4.266 \times 10^6 / 3000 = 1422 \quad \blacktriangleleft$$

$$X_S = X_N^{-b} = 1422^{0.0762} = 1.739 \quad \blacktriangleleft$$

9.36 Alloy 2024-T4 Al is subjected in service to $\hat{\sigma}_a = 160$, $\hat{\sigma}_m = 70$ MPa, with $\hat{N} = 5000$ cycles desired. $X_S, X_N = ?$

$\sigma_f' = 900$ MPa, $b = -0.102$ (Table 9.1)

$$\hat{\sigma}_{ar} = \sqrt{\hat{\sigma}_{max}\hat{\sigma}_a} = \sqrt{(160+70)160} = 191.8 \text{ MPa}$$

$$\hat{\sigma}_{ar} = \sigma_f' (2N_{f2})^b, \quad N_{f2} = \frac{1}{2}\left(\frac{\hat{\sigma}_{ar}}{\sigma_f'}\right)^{1/b}$$

$$N_{f2} = \frac{1}{2}\left(\frac{191.8}{900}\right)^{1/(-0.102)} = 1.908 \times 10^6 \text{ cycles}$$

$$X_N = N_{f2}/\hat{N} = 1.908 \times 10^6 / 5000 = 381.6 \quad \blacktriangleleft$$

$$X_S = X_N^{-b} = 381.6^{0.102} = 1.834 \quad \blacktriangleleft$$

9.37 Alloy Ti-6Al-4V in service has fixed value $\hat{\sigma}_a = 400$ MPa, with a mean stress that can vary expected to be $\hat{\sigma}_m = 250$ MPa $\hat{N} = 10{,}000$ cycles is desired. (a) $X_N = ?$, (b) $Y_m = ?$ $\sigma_f' = 2030$ MPa, $b = -0.104$

$$\hat{\sigma}_{ar} = \sqrt{\hat{\sigma}_{max}\hat{\sigma}_a} = \sqrt{(400+250)400} = 509.9 \text{ MPa}$$

$$\hat{\sigma}_{ar} = \sigma_f'(2N_{f2})^b, \quad N_{f2} = \frac{1}{2}\left(\frac{\hat{\sigma}_{ar}}{\sigma_f'}\right)^{1/b}$$

$$N_{f2} = \frac{1}{2}\left(\frac{509.9}{2030}\right)^{1/(-0.104)} = 2.940 \times 10^5 \text{ cyc.}$$

$$X_N = N_{f2}/\hat{N} = 29.40$$ ◀

$$\sigma_{ar1}' = \sqrt{(Y_m\hat{\sigma}_m + Y_a\hat{\sigma}_a)Y_a\hat{\sigma}_a}, \quad Y_a = 1$$

$$\sigma_{ar1}' = \sigma_f'(2\hat{N})^b = \sqrt{(Y_m\sigma_m + \sigma_a)\sigma_a}$$

$$\left[\sigma_f'(2\hat{N})^b\right]^2 = Y_m\sigma_m\sigma_a + \sigma_a^2$$

$$Y_m = \frac{\left[\sigma_f'(2\hat{N})^b\right]^2 - \sigma_a^2}{\sigma_m\sigma_a}$$

$$Y_m = \frac{\left[2030(2 \times 10{,}000)^{-0.104}\right]^2 - 400^2}{250 \times 400} = 3.653$$ ◀

9.38 Man-Ten steel is subjected to stress in service of $\hat{\sigma}_a = 120$, $\hat{\sigma}_m = 190$ MPa, with $\hat{N} = 5000$ cycles desired. $Y_a = 2Y_m$.
(a) $X_N = ?$, (b) $Y_a = 2Y_m = ?$

$$\hat{\sigma}_{ar} = \frac{\hat{\sigma}_a}{1 - \hat{\sigma}_m / \sigma_f'} = \frac{120}{1 - 190/1089} = 145.4 \text{ MPa}$$

$\sigma_f' = 1089$ MPa, $b = -0.115$ (Table 9.1)

$$\hat{\sigma}_{ar} = \sigma_f' (2N_{f2})^b, \quad N_{f2} = \frac{1}{2}\left(\frac{\hat{\sigma}_{ar}}{\sigma_f'}\right)^{1/b}$$

$$N_{f2} = \frac{1}{2}\left(\frac{145.4}{1089}\right)^{1/(-0.115)} = 2.014 \times 10^7 \text{ cycles}$$

$$X_N = N_{f2} / \hat{N} = 4027 \quad \blacktriangleleft$$

$$\sigma_{ar1}' = \sigma_f' (2\hat{N})^b = 1089 \, (2 \times 5000)^{-0.115}$$

$$\sigma_{ar1}' = 377.6 \text{ MPa}$$

$$\sigma_{ar1}' = \frac{Y_a \hat{\sigma}_a}{1 - Y_m \hat{\sigma}_m / \sigma_f'} = \frac{2Y_m (120)}{1 - Y_m (190)/1089}$$

Solving gives $Y_m = 1.234$

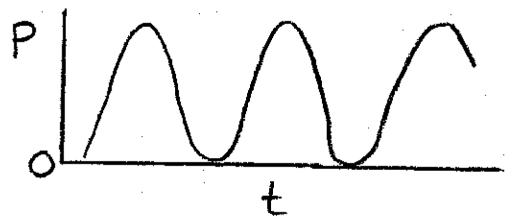
$$Y_a = 2Y_m = 2.469 \quad \blacktriangleleft$$

9.39 Ti-6Al-4V (Table 9.1), cylindrical pressure vessel $d_1 = 250$, $t = 2.5$ mm

(a) $p = ?$ for 10^5 cycles. $\sigma_o = 1185$ MPa

$\sigma_{ar} = \sigma_f'(2N_f)^b$, $\sigma_f' = 2030$ MPa, $b = -0.104$

$r = d_1/2$



At max. pressure:

$$\sigma_1 = \frac{pr}{t}, \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{1a} = \frac{1}{2}\left(\frac{pr}{t}\right) = \frac{1}{2}\,\frac{p\,(125\text{ mm})}{(2.5\text{ mm})} = 25p \text{ MPa}$$

$$\sigma_{2a} = \frac{1}{2}\left(\frac{pr}{2t}\right) = 12.5p \text{ MPa}, \quad \sigma_{3a} = 0$$

$$\sigma_{1m} = 25p, \quad \sigma_{2m} = 12.5p \text{ MPa}, \quad \sigma_{3m} = 0$$

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{1a}-\sigma_{2a})^2 + (\sigma_{2a}-\sigma_{3a})^2 + (\sigma_{3a}-\sigma_{1a})^2} = 21.65p$$

$$\bar{\sigma}_m = \sigma_{1m} + \sigma_{2m} + \sigma_{3m} = 37.5p \text{ MPa}$$

$$\sigma_{ar} = \sqrt{(\bar{\sigma}_a + \bar{\sigma}_m)\,\bar{\sigma}_a} \qquad \text{(from SWT)}$$

$$\sigma_{ar} = \sqrt{(21.65 + 37.5)\,37.5}\;p = 47.1p \text{ MPa}$$

$$47.1p = 2030\,(2\times10^5)^{-0.104} \text{ MPa}$$

$$p = 12.11 \text{ MPa}$$ ◀

(b) Calculate X_o at peak stresses.

(9.39, p.2)

$$\sigma_1 = \frac{pr}{t} = \frac{(12.11\ \text{MPa})(125\ \text{mm})}{2.5\ \text{mm}} = 606\ \text{MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 303\ \text{MPa}, \quad \sigma_3 = 0$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\bar{\sigma}_H = 525\ \text{MPa}$$

$$X_0 = \sigma_0 / \bar{\sigma}_H = 1185/525 = 2.26 \quad \blacktriangleleft$$

9.40 AISI 4142 steel, 450 HB

$\sigma_o = 1584$, $\sigma_u = 1757$, $A = 1837$ MPa, $B = -0.0762$

Solid circular shaft, $d = 50$ mm, loaded zero to $T = 20$ kN·m.

(a) $N_f = ?$ $\quad \bar{\sigma}_m = 0$

$$\sigma_{ar} = \bar{\sigma}_a = \sqrt{3}\,\tau_{xya} = \sqrt{3}\,\frac{T_a r}{J} = \sqrt{3}\,\frac{(\Delta T/2)\,r}{\pi r^4/2}$$

$$\sigma_{ar} = \frac{\sqrt{3}\,(0.010\ \text{MN}\cdot\text{m})(2)}{\pi\,(0.025\ \text{m})^3} = 705.7\ \text{MPa}$$

$$\sigma_{ar} = A\,N_f^{\,B} = 1837\,(N_f)^{-0.0762} = 705.7\ \text{MPa}$$

$$N_f = \left(\frac{705.7}{1837}\right)^{\frac{1}{-0.0762}} = 283{,}500\ \text{cycles} \blacktriangleleft$$

(b) $X_o = ?$, $\quad \tau_{xy} = \dfrac{T_{max}\,r}{J} = \dfrac{T_{max}\,r}{\pi r^4/2} = \dfrac{2\,T_{max}}{\pi r^3}$

$$\tau_{xy} = \frac{2\,(0.020\ \text{MN}\cdot\text{m})}{\pi\,(0.025\ \text{m})^3} = 814.9\ \text{MPa}$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

$$\bar{\sigma}_H = \sqrt{3}\,\tau_{xy}, \quad X_o = \frac{\sigma_o}{\bar{\sigma}_H} = \frac{1584\ \text{MPa}}{\sqrt{3}\,(814.9\ \text{MPa})} = 1.12 \blacktriangleleft$$

(c) $N_f = X_N \hat{N} = 20\,(100{,}000) = 2 \times 10^6$ cycles

$$\sigma_{ar} = A\,N_f^{\,B} = \sqrt{3}\,\tau_{xya} = \sqrt{3}\,\frac{T_a r}{\pi r^4/2} = \frac{2\sqrt{3}\,T_a}{\pi r^3}$$

$$r = \left(\frac{2\sqrt{3}\,T_a}{\pi A N_f^{\,B}}\right)^{1/3} = \left[\frac{2\sqrt{3}\,(0.010\ \text{MN}\cdot\text{m})}{\pi\left(1837\,\frac{\text{MN}}{\text{m}^2}\right)(2\times10^6)^{-0.0762}}\right]^{1/3}$$

$$r = 0.0263\ \text{m} = 26.3\ \text{mm}, \quad d = 52.5\ \text{mm} \blacktriangleleft$$

(9.40, p.2)

(d) $X_o = \sigma_o / \bar{\sigma}_H = 1.5$, $d = ?$

$$\bar{\sigma}_H = \frac{\sigma_o}{X_o} = \sqrt{3}\,\tau_{xy} = \sqrt{3}\,\frac{2T_{max}}{\pi r^3}$$

$$r = \left[\frac{2\sqrt{3}\,T_{max}\,X_o}{\pi \sigma_o}\right]^{1/3} = \left[\frac{2\sqrt{3}\,(0.02\ \mathrm{MN\cdot m})(1.5)}{\pi\,(1584\ \frac{\mathrm{MN}}{\mathrm{m}^2})}\right]^{1/3}$$

$r = 0.0275\ \mathrm{m}$, $d = 55.1\ \mathrm{mm}$ ◀

(e) $d = MAX\,(52.5, 55.1) = 55.1\ \mathrm{mm}$ ◀

9.41 AISI 4340 steel, $\sigma_u = 1172$ MPa

$$\sigma_a = A N_f^{B} = 1643 N_f^{-0.0977} \text{ MPa} \quad (\sigma_m = 0)$$

$$N_f = \left(\frac{\sigma_a}{1643}\right)^{1/-0.0977}$$

j	N_j	σ_a	σ_m	N_{fj}
1	2000	650	0	13,240
2	10,000	575	0	46,460
3	$N_3 = ?$	700	0	6203

$$\sum \frac{N_j}{N_{fj}} = 1 = \frac{2000}{13{,}240} + \frac{10{,}000}{46{,}460} + \frac{N_3}{6203}$$

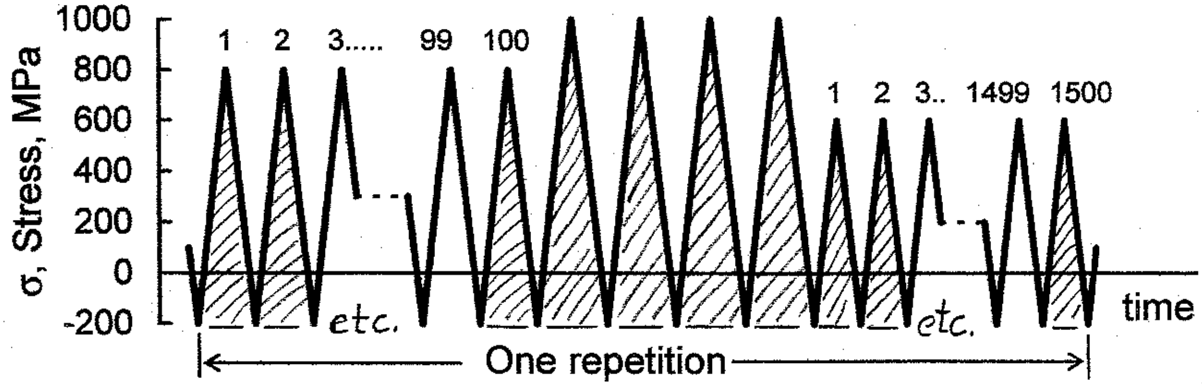
$N_3 = 3930$ cycles ◀

9.42

For given repeating stress history and material, estimate the number of repetitions to fatigue failure. First perform cycle counting on the stress history. Then for each level of cycling, calculate the life N_f using the SWT equation. Finally, apply the P-M rule.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sqrt{\sigma_{max}\sigma_a} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma'_f}\right)^{1/b}, \qquad B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$



AISI 4340 steel, σ_u = 1172 MPa

σ'_f = 1758 b = -0.0977

Stresses in MPa

j	N_j	σ_{min}	σ_{max}	σ_a	SWT N_{fj}	N_j/N_{fj}
1	100	-200	800	500	1.75E+04	5.71E-03
2	4	-200	1000	600	2.20E+03	1.82E-03
3	1500	-200	600	400	2.39E+05	6.27E-03
					Σ =	1.38E-02

B_f = 1/Σ = **72**

(9.42, p.2)

An alternate solution is to proceed similarly except for calculating the life N_f using the Morrow equation.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = (\sigma'_f - \sigma_m)(2N_f)^b, \qquad N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma'_f - \sigma_m}\right)^{1/b}$$

$$B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$

AISI 4340 steel, σ_u = 1172 MPa

Stresses in MPa

σ'_f = 1758 b = -0.0977

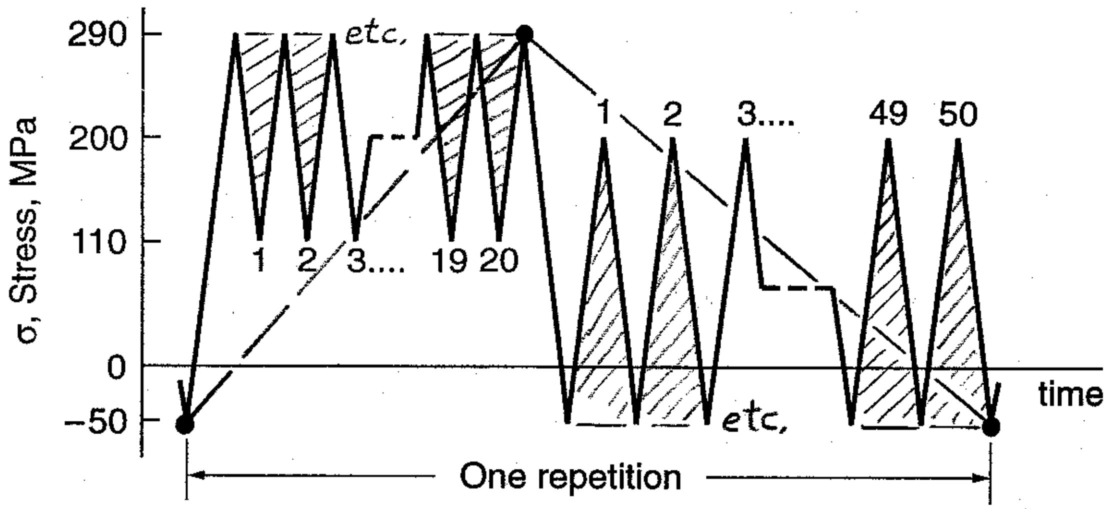
						Morrow	
j	N_j	σ_{min}	σ_{max}	σ_a	σ_m	N_{fj}	N_j/N_{fj}
1	100	-200	800	500	300	2.86E+04	3.50E-03
2	4	-200	1000	600	400	2.14E+03	1.87E-03
3	1500	-200	600	400	200	5.53E+05	2.71E-03
						Σ =	8.08E-03

B_f = 1/Σ = **124**

9.43 Estimate repetitions to failure for given stress history. For each level of cycling, using SWT, calculate N_f.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \quad \text{then apply Eq. 9.20.}$$

$$\sqrt{\sigma_{max}\sigma_a} = \sigma_f'(2N_f)^b, \quad N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma_f'}\right)^{1/b}$$



$$B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep.}} = 1$$

2024-T4 Al   Stresses in MPa

$\sigma'_f = 900$ $b = -0.102$

j	N_j	σ_{min}	σ_{max}	σ_a	N_{fj}	N_j/N_{fj}
1	20	110	290	90	1.03E+07	1.95E-06
2	50	-50	200	125	1.27E+07	3.94E-06
3	1	-50	290	170	4.55E+05	2.20E-06
					Σ =	8.08E-06

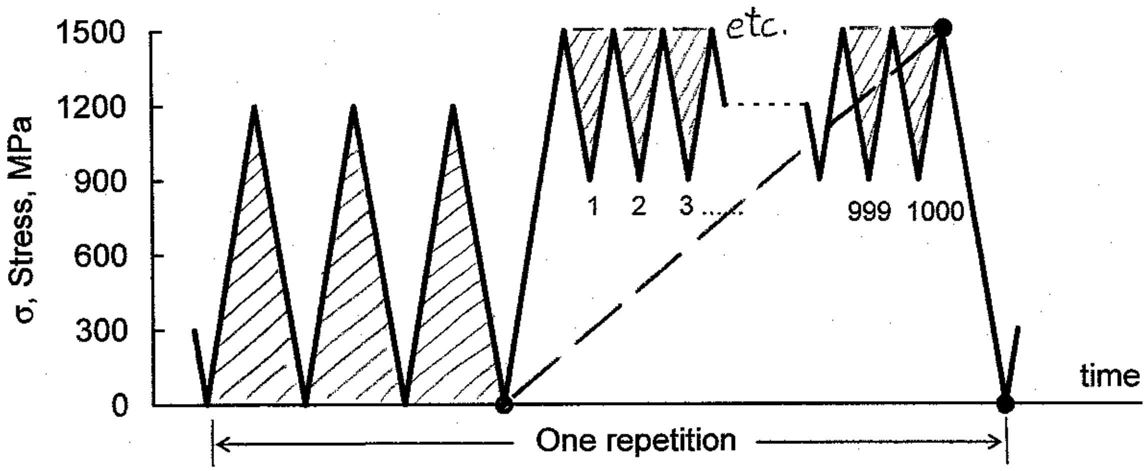
B_f = 124,000 repetitions $B_f = 1/\Sigma =$ 1.24E+05 ◀

9.44

For given repeating stress history and material, estimate the number of repetitions to fatigue failure. First perform cycle counting on the stress history. Then for each level of cycling, calculate the life N_f using the SWT equation. Finally, apply the P-M rule.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sqrt{\sigma_{max}\sigma_a} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma'_f}\right)^{1/b}, \qquad B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$



AISI 4142 steel, 450 HB

σ'_f = 1937 b = -0.0762

Stresses in MPa

j	N_j	σ_{min}	σ_{max}	σ_a	SWT N_{fj}	N_j/N_{fj}
1	3	0	1200	600	2.53E+04	1.19E-04
2	1000	900	1500	300	5.53E+05	1.81E-03
3	1	0	1500	750	1.35E+03	7.39E-04
					Σ =	2.67E-03

B_f = 1/Σ = **375**

(9.44, p.2)

An alternate solution is to proceed similarly except for calculating the life N_f using the Morrow equation.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = (\sigma'_f - \sigma_m)(2N_f)^b, \qquad N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma'_f - \sigma_m}\right)^{1/b}$$

$$B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$

AISI 4142 steel, 450 HB

Stresses in MPa

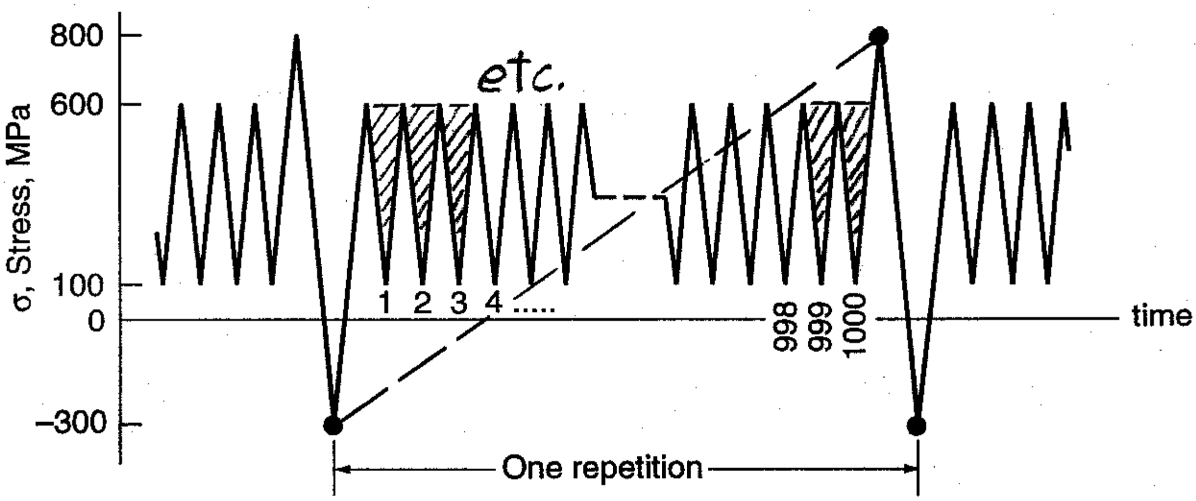
σ'_f = 1937 b = -0.0762

						Morrow	
j	N_j	σ_{min}	σ_{max}	σ_a	σ_m	N_{fj}	N_j/N_{fj}
1	3	0	1200	600	600	1.84E+04	1.63E-04
2	1000	900	1500	300	1200	6.63E+04	1.51E-02
3	1	0	1500	750	750	2.07E+02	4.83E-03
						Σ =	2.01E-02
						B_f = 1/Σ =	**50**

9.45 Estimate repetitions to failure for given stress history. Use Morrow; calculate N_f for each level of cycling.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \quad \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = (\sigma_f' - \sigma_m)(2N_f)^b, \quad N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f' - \sigma_m}\right)^{1/b}$$



$$B_f \left[\sum \frac{N_j}{N_{fj}}\right]_{one\ rep.} = 1$$

Ti-6Al-4V Stresses in MPa

σ'_f = 2030 b = -0.104

j	N_j	σ_{min}	σ_{max}	σ_a	σ_m	N_{fj}	N_j/N_{fj}
1	1000	100	600	250	350	4.51E+07	2.22E-05
2	1	-300	800	550	250	4.01E+04	2.49E-05
						Σ =	4.71E-05

B_f = 21,200 reps. B_f = 1/Σ = 2.12E+04 ◀

(9.45, p.2)

An alternate solution is to proceed similarly except for calculating the life N_f using the SWT equation.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sqrt{\sigma_{max}\sigma_a} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma'_f}\right)^{1/b}, \qquad B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$

Ti-6Al-4V

σ'_f = 2030 b = -0.104

Stresses in MPa

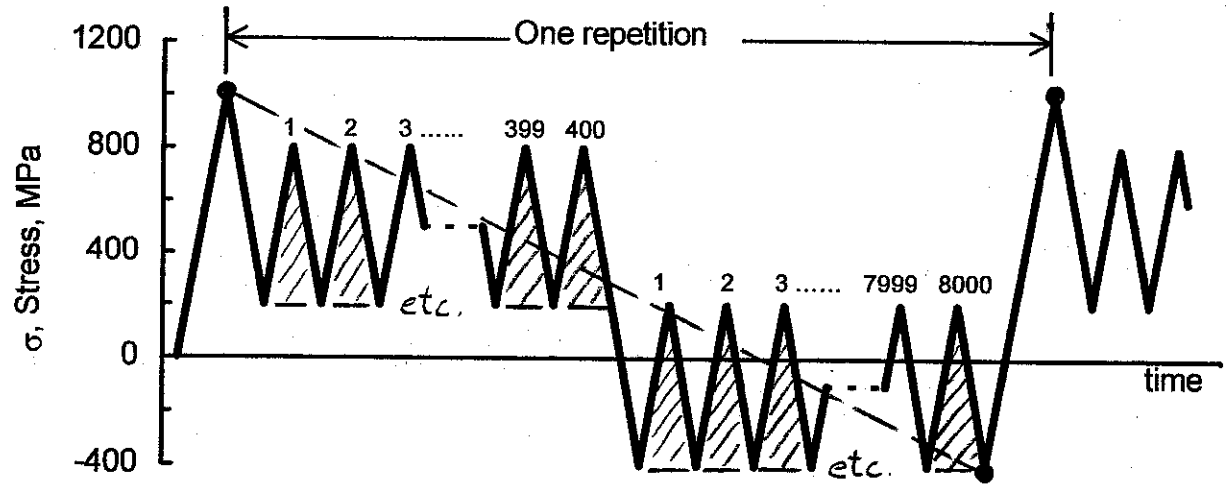
					SWT	
j	N_j	σ_{min}	σ_{max}	σ_a	N_{fj}	N_j/N_{fj}
1	1000	100	600	250	4.14E+06	2.42E-04
2	1	-300	800	550	2.34E+04	4.27E-05
					Σ =	2.84E-04

$B_f = 1/\Sigma =$ **3,517**

9.46 Estimate repetitions to failure for given stress history. Count cycles and calculate N_f for each level of cycling using SWT. Then apply P-M rule.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sqrt{\sigma_{max}\,\sigma_a} = \sigma_f'(2N_f)^b, \quad N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\,\sigma_a}}{\sigma_f'}\right)^{\frac{1}{b}}$$



$$B_f\left[\frac{N_j}{N_{fj}}\right]_{\text{one rep.}} = 1$$

Ti-6Al-4V
σ'_f = 2030 b = -0.104

SWT
Stresses in MPa

j	N_j	σ_{min}	σ_{max}	σ_a	N_{fj}	N_j/N_{fj}
1	400	200	800	300	4.32E+05	9.26E-04
2	8000	-400	200	300	3.39E+08	2.36E-05
3	1	-400	1000	700	2.51E+03	3.98E-04
					Σ =	1.35E-03

B_f = 742 repetitions

$B_f = 1/\Sigma =$ 7.42E+02 ◀

(9.46, p.2)

An alternate solution is to proceed similarly except for calculating the life N_f using the Morrow equation.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = (\sigma'_f - \sigma_m)(2N_f)^b, \qquad N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma'_f - \sigma_m}\right)^{1/b}$$

$$B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$

Ti-6Al-4V

Stresses in MPa

σ'_f = 2030 b = -0.104

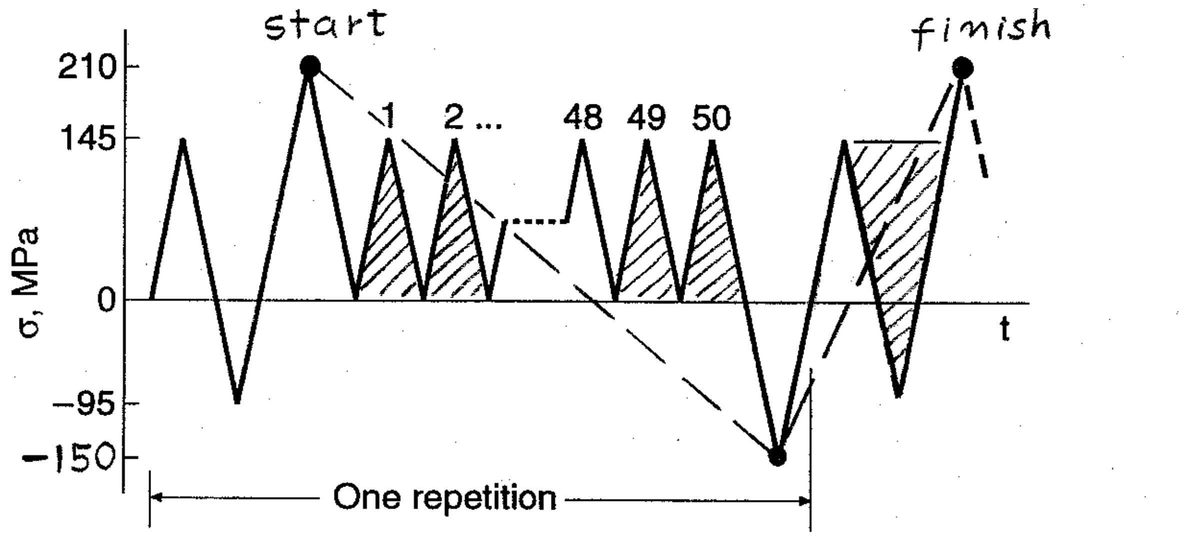
j	N_j	σ_{min}	σ_{max}	σ_a	σ_m	Morrow N_{fj}	N_j/N_{fj}
1	400	200	800	300	500	3.18E+06	1.26E-04
2	8000	-400	200	300	-100	7.66E+07	1.04E-04
3	1	-400	1000	700	300	3.00E+03	3.33E-04
						Σ =	5.63E-04

B_f = 1/Σ = **1775** ◀

9.47 Estimate repetitions to failure for given stress history. Using Morrow, calculate N_f for each level of cycling.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \quad \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = (\sigma_f' - \sigma_m)(2N_f)^b, \quad N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f' - \sigma_m}\right)^{1/b}$$



$$B_f = \left[\sum \frac{N_j}{N_{fj}}\right]_{per\ rep} = 1$$

AISI 1015 steel

Stresses in MPa

$\sigma'_f = 1020$ $\quad b = -0.138$

						Morrow	
j	N_j	σ_{min}	σ_{max}	σ_a	σ_m	N_{fj}	N_j/N_{fj}
1	50	0	145	72.5	72.5	6.13E+07	8.15E-07
2	1	-95	145	120	25	2.27E+06	4.41E-07
3	1	-150	210	180	30	1.16E+05	8.63E-06
						Σ =	9.89E-06

$B_f = 1/\Sigma =$ **101,138** ◀

(9.47, p. 2)

An alternate solution is to proceed similarly except for calculating the life N_f using the SWT equation.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sqrt{\sigma_{max}\sigma_a} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma'_f}\right)^{1/b}, \qquad B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$

AISI 1015 steel

Stresses in MPa

σ'_f = 1020 b = -0.138

SWT

j	N_j	σ_{min}	σ_{max}	σ_a	N_{fj}	N_j/N_{fj}
1	50	0	145	72.5	8.49E+06	5.89E-06
2	1	-95	145	120	1.37E+06	7.31E-07
3	1	-150	210	180	8.23E+04	1.22E-05
					Σ =	1.88E-05

B_f = 1/Σ = **53,271** ◀

9.48

For the repeating stress history of Fig. 9.47(a) applied to 2024-T4 Al, with 1 unit = 60 MPa: (a) Estimate the number of repetitions to fatigue failure. (b) For 1000 repetitions expected in service, find safety factors in stress and life.

(a) Cycle counting of the stress history is available from Fig. 9.47. First, convert the units for the peak and valley of each cycle to stress in MPa. Then for each level of cycling, calculate the life N_f using the SWT equation. Finally, apply the P-M rule.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sqrt{\sigma_{max}\sigma_a} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma'_f}\right)^{1/b}, \qquad B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$

See the next page for details. The result is B_f = **36,300** repetitions. ◀

(b)

$$X_N = \frac{B_{f2}}{\hat{B}}, \qquad X_S = X_N^{-b}$$

$\hat{B}$, reps	B_{f2}, reps	X_N	b	X_S
1000	36,294	**36.29**	-0.102	**1.442**

◀

(9.48, p.2)

Cycle	MIN units	MAX units	σ_{min} MPa	σ_{max} MPa
E-F	-1	3	-60	180
A-B	-2	1	-120	60
H-C	-3	4	-180	240
D-G	-4	5	-240	300

2024-T4 Al

Stresses in MPa

$\sigma'_f = 900$ $b = -0.102$

j	N_j	σ_{min}	σ_{max}	σ_a	SWT N_{fj}	N_j/N_{fj}
1	1	-60	180	120	2.60E+07	3.85E-08
2	1	-120	60	90	2.32E+10	4.30E-11
3	1	-180	240	210	4.08E+05	2.45E-06
4	1	-240	300	270	3.99E+04	2.51E-05
					$\Sigma =$	2.76E-05
					$B_f = 1/\Sigma =$	**36,294**

9.49

Rework Ex. 9.9 using the equivalent stress level (σ_{aq}) method, estimating the number of repetitions to failure for the given repeating stress history and material.

Employ cycle counting of the stress history as already done for Ex. 9.9. Sum the N_j to obtain N_B. Then for each level of cycling, calculate σ_a, σ_{ar}, and the quantity $N_j(\sigma_{arj})^{-1/b}$. Next sum the latter, and from this sum calculate σ_{aq}, which then gives N_f and B_f.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sigma_{ar} = \sqrt{\sigma_{max}\sigma_a}$$

$$N_B = \sum_{j=1}^{k} N_j, \qquad \sigma_{aq} = \left[\sum_{j=1}^{k} N_j (\sigma_{arj})^{-1/b} \Big/ N_B \right]^{-b}$$

$$\sigma_{aq} = \sigma'_f (2N_f)^b, \qquad N_f = \frac{1}{2}\left(\frac{\sigma_{aq}}{\sigma'_f}\right)^{1/b}, \qquad B_f = \frac{N_f}{N_B}$$

Ti-6Al-4V

$\sigma'_f = 2030$ $b = -0.104$ Stresses in MPa

j	N_j	σ_{min}	σ_{max}	σ_a	σ_{ar}	$N_j(\sigma_{arj})^{-1/b}$
1	3	130	950	410	624.1	2.262E+27
2	100	-140	560	350	442.7	2.777E+27
3	1	-250	950	600	755.0	4.704E+27
$N_B =$	104				$\Sigma =$	9.743E+27
					$\sigma_{aq} =$	502.40
					$N_f =$	338,960
					$B_f =$	**3,259**

9.50

For the situation of Ex. 9.9, assume that 500 repetitions of the stress history are expected in service. Find safety factors in stress and life.

$$X_N = \frac{B_{f2}}{\hat{B}}, \qquad X_S = X_N^{-b}$$

$\hat{B}$, reps	B_{f2}, reps	X_N	b	X_S
500	3,259	**6.52**	-0.104	**1.215**

The safety factor in stress is inadequate, as it allows only a 21% margin for variation in materials properties, uncertainties in the service stress, etc.

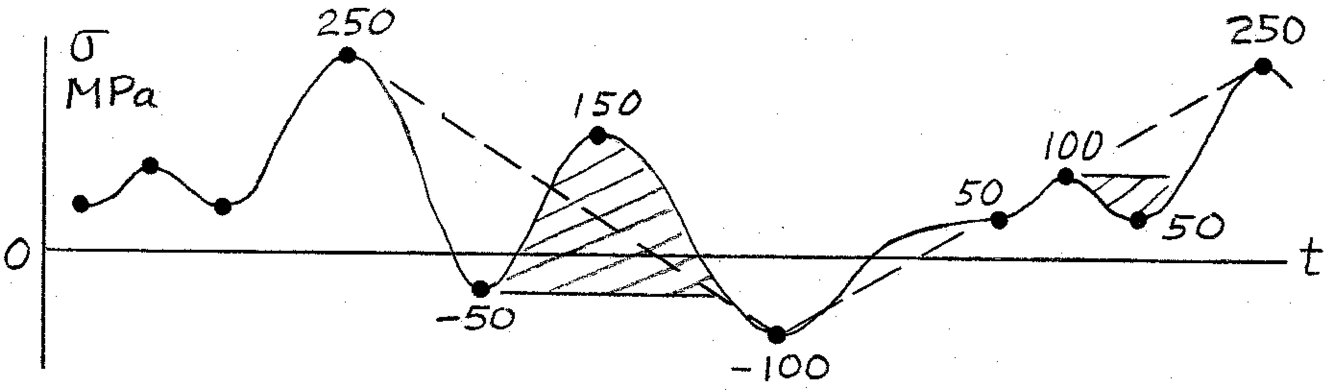
9.51

For the rotor revolution stress history of Fig. 9.9 applied to Ti-6Al-4V, with 1 unit = 50 MPa: (a) Estimate the number of rotor revolutions to fatigue failure, and the number of flight hours if 1 revolution takes 0.3 seconds. (b) For a service life of 2000 hours, is a load factor of 1.5 applied to all stresses satisfied?

(a) First perform cycle counting on the stress history. Then for each level of cycling, calculate the life N_f using the SWT equation. Finally, apply the P-M rule.

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}, \qquad \sqrt{\sigma_{max}\sigma_a} = \sigma'_f (2N_f)^b$$

$$N_f = \frac{1}{2}\left(\frac{\sqrt{\sigma_{max}\sigma_a}}{\sigma'_f}\right)^{1/b}, \qquad B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$



B_f = no. of rotor revs, H_f = no. of hours

$$(B_f, \text{revs})\left(\frac{0.3 \text{ sec/rev}}{3600 \text{ sec/hr}}\right) = H_f, \text{ hours}$$

(9.51, p, 2)

Ti-6Al-4V

Stresses in MPa

σ'_f = 2030   b = -0.104

					SWT	
j	N_j	σ_{min}	σ_{max}	σ_a	N_{fj}	N_j/N_{fj}
1	1	-50	150	100	2.66E+11	3.76E-12
2	1	50	100	25	1.46E+15	6.83E-16
3	1	-100	250	175	1.55E+09	6.47E-10
					Σ =	6.50E-10

B_f, revs = $1/\Sigma$ = **1.538E+09**

H_f, hours = **128,150**

(b) Since (a) uses the SWT equation, a load factor Y applied to all stresses has the same value as a safety factor in stress X_S.

$$X_N = \frac{H_{f2}}{\hat{H}}, \qquad Y = X_S = X_N^{-b}$$

$\hat{H}$, hrs	H_{f2}, hrs	X_N	b	$Y = X_S$
2000	128,150	64.08	-0.104	**1.541**