11.1 Estimate C and m for straight line portion in Fig. 11.3. Use $MPa\sqrt{m}$, $\frac{mm}{cycle}$.

$\frac{da}{dN} = C(\Delta K)^m$, $\log \frac{da}{dN} = \log C + m \log(\Delta K)$

Two points on line are $(\Delta K, da/dN) =$

$(5, 3 \times 10^{-7})$, $(100, 4.2 \times 10^{-3})$

$$m = \frac{\log(4.2 \times 10^{-3}) - \log(3 \times 10^{-7})}{\log 100 - \log 5} = 3.19$$

$$C = 3 \times 10^{-7} / 5^{3.19} = 1.77 \times 10^{-9} \quad \frac{mm/cycle}{(MPa\sqrt{m})^m}$$

$$\frac{da}{dN} = 1.77 \times 10^{-9} (\Delta K)^{3.19} \quad (10 < \Delta K < 100)$$ ◀

(For units of $MPa\sqrt{m}$, mm/cycle)

11.2 For Fig. 11.25(rt.), MgO-PSZ ceramic in air, evaluate C and m for $da/dN = C(\Delta K)^m$.
Two points on the straight line are:
(ΔK, MPa$\sqrt{m}$; da/dN, mm/cycle)
$= (3.0, 10^{-7}), \quad (4.0, 10^{-4})$

$$\left(\frac{da}{dN}\right)_A = C\,(\Delta K_A)^m, \quad \left(\frac{da}{dN}\right)_B = C\,(\Delta K_B)^m$$

$$m = \frac{\log(da/dN_A) - \log(da/dN_B)}{\log(\Delta K_A) - \log(\Delta K_B)} \qquad \text{(Ex. 11.1)}$$

$$m = \frac{\log 10^{-7} - \log 10^{-4}}{\log 3.0 - \log 4.0} = 24$$

$$C = \frac{10^{-7}}{3^{24}} = 3.54 \times 10^{-19} \ \frac{\text{mm/cycle}}{(\text{MPa}\sqrt{m})^m}$$

$$\frac{da}{dN} = 3.54 \times 10^{-19} (\Delta K)^{24}$$ ◀

(For MPa$\sqrt{m}$, mm/cycle)

The very high value of m = 24 indicates that the growth rates in this material are extremely sensitive to stress level and crack size. ◀

11.3 Center-cracked plate: 7075-T651 Al, $b = 19.05$, $t = 6.60$ mm, $P = 18.8$ kN at $R = 0$. For given a, da/dN, calculate ΔK, plot; fit $da/dN = C(\Delta K)^m$.
(a) approximate, (b) least squares.

$$\Delta K = F \Delta S \sqrt{\pi a}, \quad \Delta S = \frac{\Delta P}{2bt} \qquad \text{(Fig. 8.12(a))}$$

$$F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}}, \quad \alpha = \frac{a}{b}$$

b, mm	t, mm	ΔP, N
19.05	6.6	18,800

a mm	da/dN mm/cyc	α = a/b	F	ΔK $MPa\sqrt{m}$
7.32	1.76E-04	0.384	1.091	12.37
9.53	5.08E-04	0.500	1.176	15.22
12.07	1.27E-03	0.634	1.345	19.58
14.94	3.18E-03	0.784	1.740	28.19

(a) Points form a reasonable straight line on log-log plot, so power-law equation is OK. Use eyeball line, two points; see Ex. 11.1.

$$m = \frac{\log(da/dN_A) - \log(da/dN_B)}{\log(\Delta K_A) - \log(\Delta K_B)}$$

(11.3, p.2)

$$m = \frac{\log 1\times10^{-2} - \log 1\times10^{-4}}{\log 37.5 - \log 10} = 3.484 \quad \blacktriangleleft$$

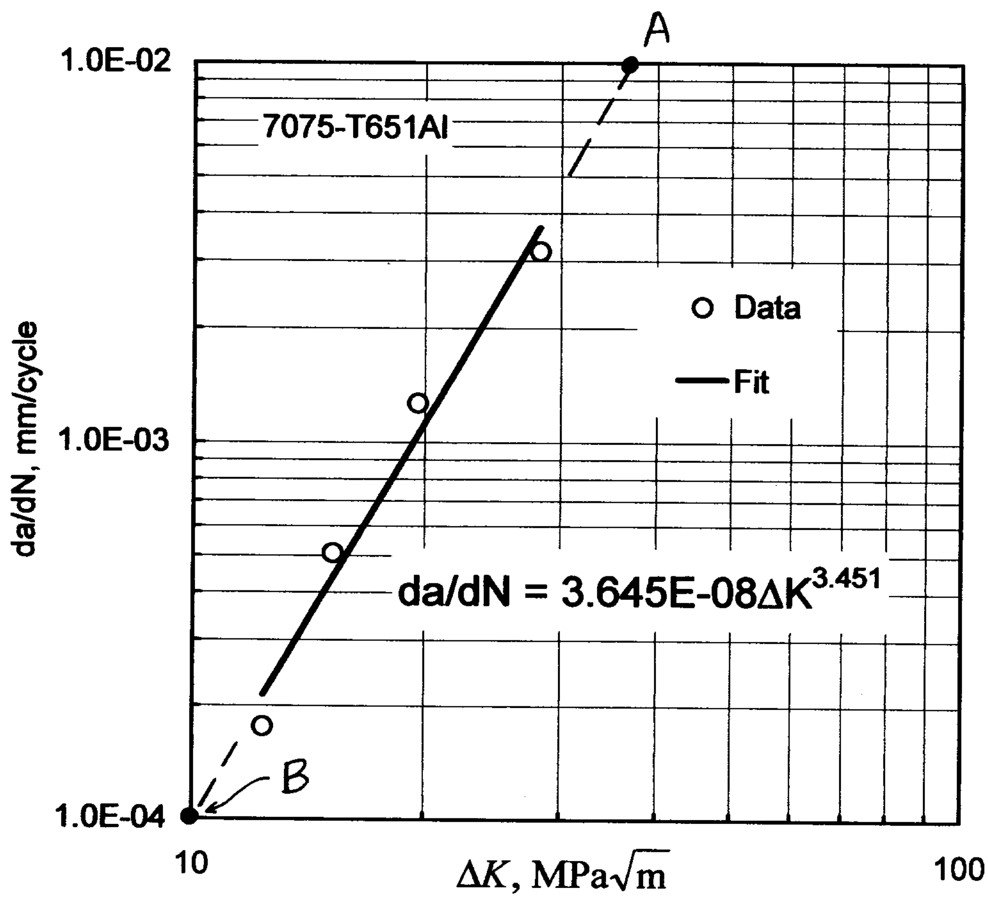
$$C = \frac{da/dN}{(\Delta K)^m} = \frac{1\times10^{-2}}{37.5^{3.484}} = 3.28\times10^{-8} \quad \blacktriangleleft$$

(mm/cycle, $MPa\sqrt{m}$)

(b) $$\underbrace{\log \frac{da}{dN}}_{y} = \underbrace{\log C}_{b} + \underbrace{m}_{m} \underbrace{\log(\Delta K)}_{x}$$

$C = 3.65\times10^{-8}$, $m = 3.451$ ◀

(mm/cycle, $MPa\sqrt{m}$)



11.4

For given data, make a log-log plot and fit to Eq. 11.10: (a) using two points on an approximate line, (b) using least squares.

da/dN mm/cyc	ΔK $MPa\sqrt{m}$
1.26E-06	2.99
2.41E-06	3.64
4.84E-06	5.02
1.02E-05	6.04
1.99E-05	7.68
3.74E-05	9.95
6.69E-05	12.0
1.77E-04	15.9

2124-T851 Al

R = 0.1

(a) The data form an approximate straight line on the log-log plot, so that a fit to Eq. 11.10 is reasonable. The data points with the highest and lowest growth rates seem to represent the line well.

$$da/dN = C(\Delta K)^m$$

$$m = \frac{\log(da/dN_A) - \log(da/dN_B)}{\log(\Delta K_A) - \log(\Delta K_B)}, \quad C = \frac{da/dN_A}{(\Delta K_A)^m} = \frac{da/dN_B}{(\Delta K_B)^m}$$

Point	da/dN	ΔK
A	1.77E-04	15.9
B	1.26E-06	2.99

m = 2.959

C = 4.929E-08 $\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$ ◀

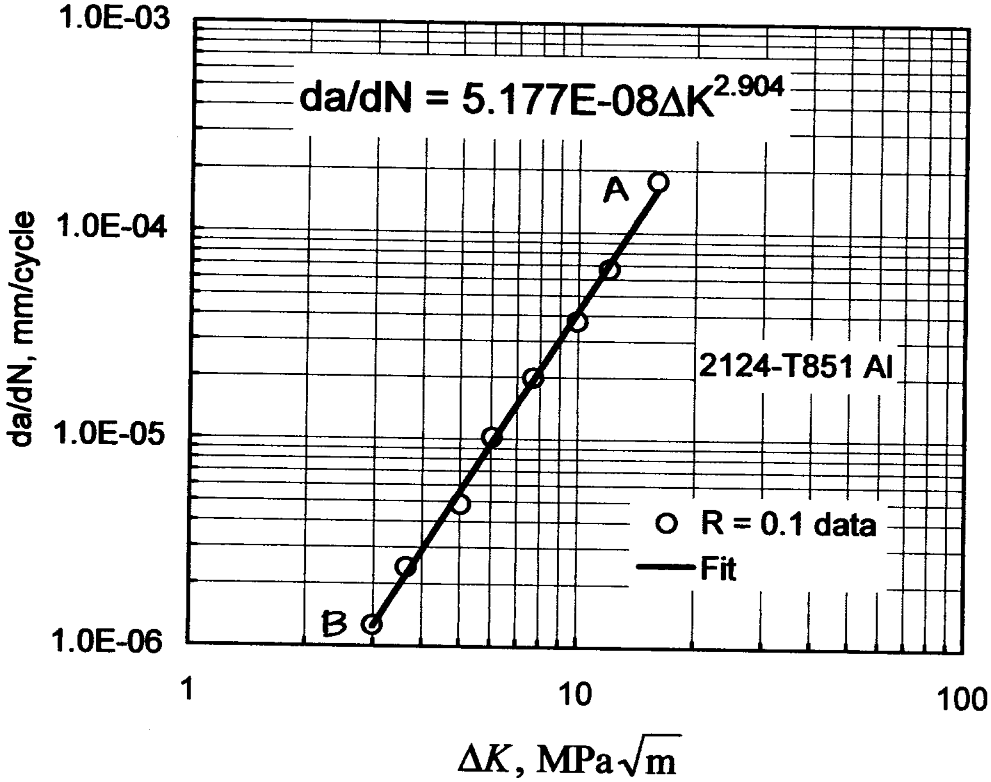
(b) Do a least squares fit $y = mx + b$, where the dependent variable is $y = \log da/dN$ and the independent variable is $x = \log \Delta K$

$$\log(da/dN) = m\log(\Delta K) + \log C$$

(11.4, p. 2)

The result is shown on the graph and is seen to represent the data well.

m = 2.904 $\qquad$ C = 5.18E-08 $\frac{\text{mm/cycle}}{\left(\text{MPa}\sqrt{\text{m}}\right)^m}$



11.5

For given data, make a log-log plot and fit to Eq. 11.10: (a) using two points on an approximate line, (b) using least squares.

da/dN mm/cyc	ΔK $\text{MPa}\sqrt{\text{m}}$
1.25E-06	2.18
2.00E-06	2.75
3.57E-06	3.53
6.62E-06	4.52
1.49E-05	5.48
3.38E-05	6.97
8.16E-05	9.20
1.65E-04	11.7

2124-T851 Al

R = 0.5

(a) The data form an approximate straight line on the log-log plot, so that a fit to Eq. 11.10 is reasonable. The data points with the highest and second lowest growth rates seem to represent the line well.

$$da/dN = C(\Delta K)^m$$

$$m = \frac{\log(da/dN_A) - \log(da/dN_B)}{\log(\Delta K_A) - \log(\Delta K_B)}, \quad C = \frac{da/dN_A}{(\Delta K_A)^m} = \frac{da/dN_B}{(\Delta K_B)^m}$$

Point	da/dN	ΔK
A	1.65E-04	11.7
B	2.00E-06	2.75

m = 3.048

C = 9.165E-08 $\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$

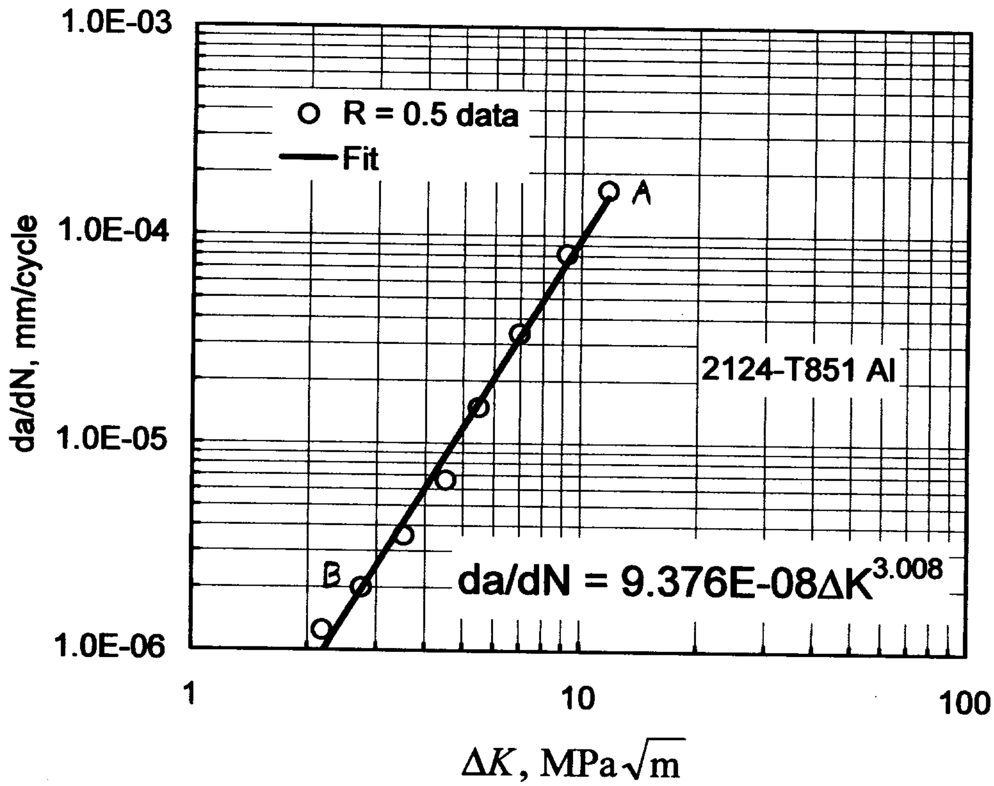
(b) Do a least squares fit $y = mx + b$, where the dependent variable is $y = \log da/dN$ and the independent variable is $x = \log \Delta K$

$$\log(da/dN) = m\log(\Delta K) + \log C$$

(11.5, p.2)

The result is shown on the graph and is seen to represent the data well.

$m = 3.008$ $\qquad$ $C = 9.38E\text{-}08 \ \frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$



11.6

For given data, make a log-log plot and fit to Eq. 11.10: (a) using two points on an approximate line, (b) using least squares.

da/dN mm/cyc	ΔK $MPa\sqrt{m}$
9.98E-06	11.2
2.84E-05	15.6
8.03E-05	22.4
1.83E-04	32.9
3.18E-04	42.3
7.92E-04	65.1
1.60E-03	90.8
3.68E-03	124.0

17-4 PH stainless steel

R = 0.04

(a) The data form an approximate straight line on the log-log plot, so that a fit to Eq. 11.10 is reasonable. The data points with the highest and second lowest growth rates seem to represent the line well.

$$da/dN = C(\Delta K)^m$$

$$m = \frac{\log(da/dN_A) - \log(da/dN_B)}{\log(\Delta K_A) - \log(\Delta K_B)}, \quad C = \frac{da/dN_A}{(\Delta K_A)^m} = \frac{da/dN_B}{(\Delta K_B)^m}$$

Point	da/dN	ΔK
A	3.68E-03	124.0
B	2.84E-05	15.6

m = 2.346

C = 4.505E-08 $\frac{\text{mm/cycle}}{(MPa\sqrt{m})^m}$ ◀

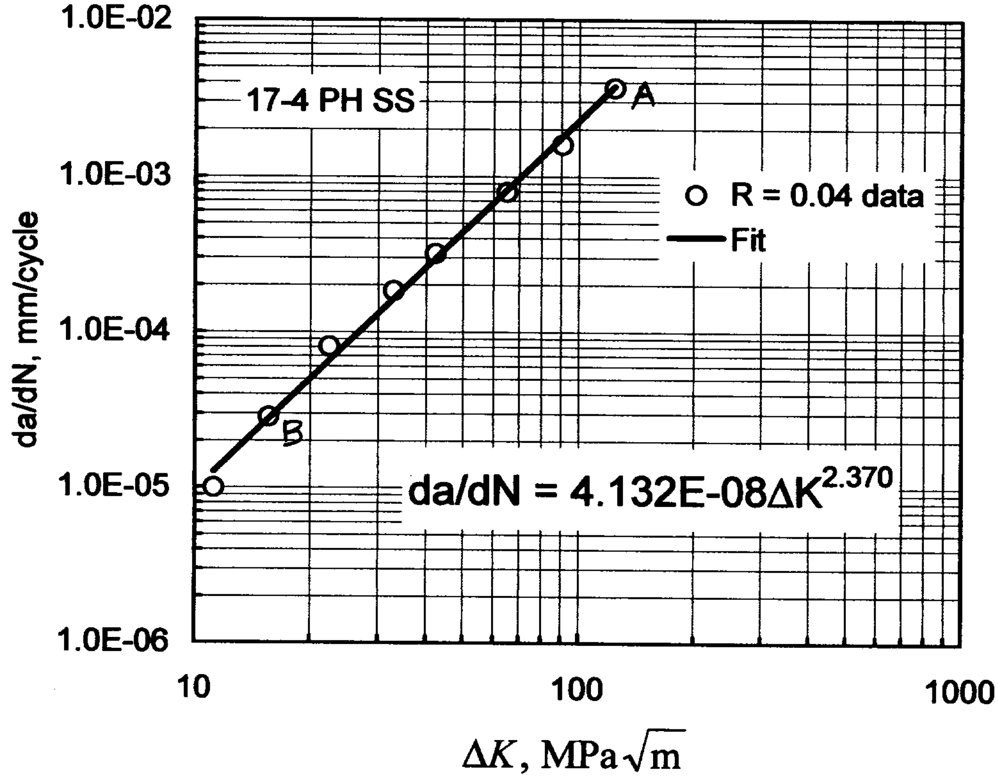
(b) Do a least squares fit $y = mx + b$, where the dependent variable is $y = \log da/dN$ and the independent variable is $x = \log \Delta K$

$$\log(da/dN) = m\log(\Delta K) + \log C$$

(11.6, p.2)

The result is shown on the graph and is seen to represent the data well.

$m = 2.370$ $\qquad$ $C = 4.13\text{E-}08 \dfrac{\text{mm/cycle}}{\left(\text{MPa}\sqrt{\text{m}}\right)^m}$ ◀



11.7

A center-cracked plate of 7075-T6 aluminum is cycled between P_{min} = 48.1 and P_{max} = 96.2 kN, with crack growth data as given. Dimensions as in Fig. 8.12(a) are b = 152.4 and t = 2.29 mm. Determine *da/dN* and ΔK, plot, and fit to Eq. 11.10.

Calculate ΔS based on Fig. 8.12(a), which also gives an expression for F. Then number the data points starting with j = 1, and calculate *da/dN* and ΔK for the intervals between data points by applying the equations below for j = 2, 3, 4, etc.

$$\Delta P = P_{max} - P_{min}, \qquad \Delta S = \frac{\Delta P}{2bt}, \qquad R = \frac{P_{min}}{P_{max}}$$

$$\left(\frac{da}{dN}\right)_j = \frac{a_j - a_{j-1}}{N_j - N_{j-1}}, \qquad a_{avg} = \frac{a_j + a_{j-1}}{2}, \qquad \alpha_{avg} = \frac{a_{avg}}{b}$$

$$F_j = \frac{1 - 0.5\alpha_{avg} + 0.326\alpha_{avg}^2}{\sqrt{1 - \alpha_{avg}}}, \qquad \Delta K_j = F_j \, \Delta S \sqrt{\pi a_{avg}}$$

Calculation results and graph are given on the next page. Using these results, do a least squares fit $y = mx + b$, where the dependent variable is y = log *da/dN* and the independent variable is x = log ΔK.

$$\log(da/dN) = m\log(\Delta K) + \log C$$

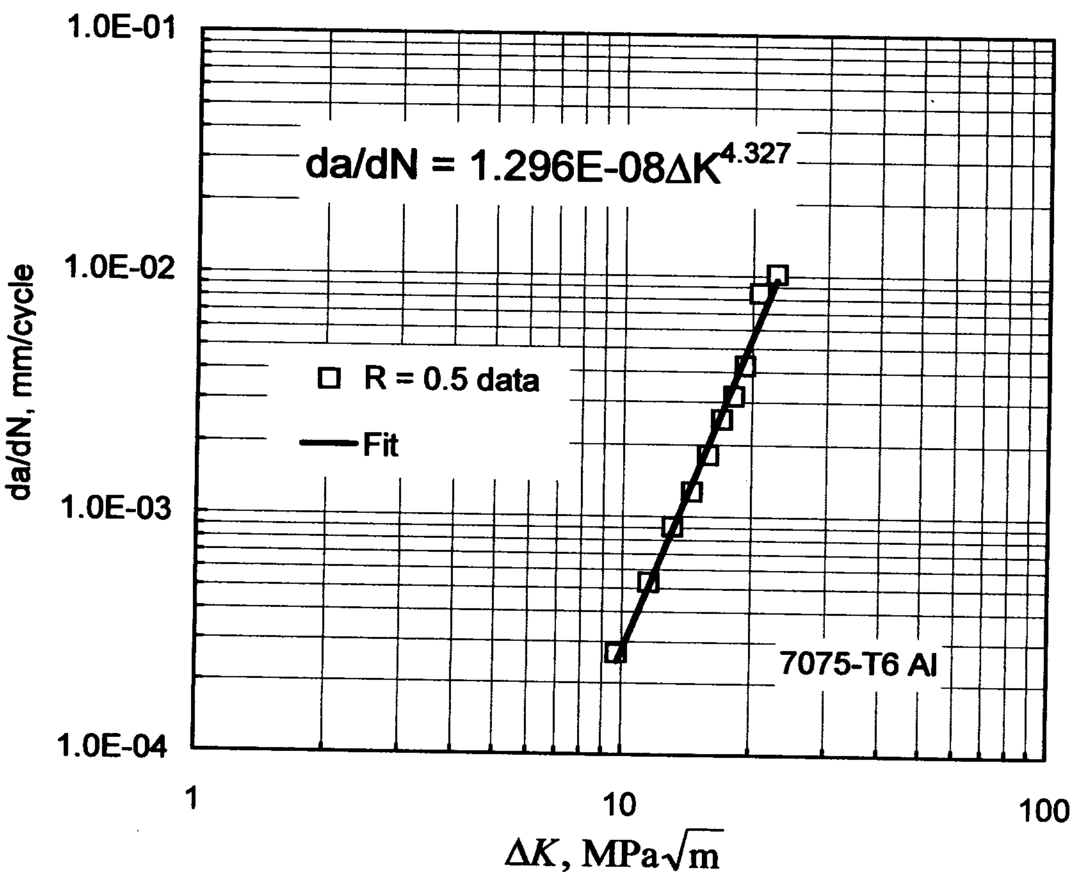
The fit is shown on the graph and is seen to represent the data well.

m = 4.327 $\qquad$ C = 1.296E-08 $\dfrac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$ ◀

(11.7, p.2)

b, mm	t, mm	P_{max}, N	P_{min}, N	ΔP, N	ΔS, MPa	R
152.40	2.29	96,200	48,100	48,100	68.91	0.50

j	a mm	N cycles	da/dN mm/cycle	a_{avg} mm	α_{avg}	F	ΔK $MPa\sqrt{m}$
1	5.08	0	XXX	XXX	XXX	XXX	XXX
2	7.62	9,500	2.674E-04	6.3500	0.0417	1.001	9.74
3	10.16	14,300	5.292E-04	8.8900	0.0583	1.002	11.53
4	12.70	17,100	9.071E-04	11.4300	0.0750	1.003	13.09
5	15.24	19,100	1.270E-03	13.9700	0.0917	1.004	14.49
6	17.78	20,500	1.814E-03	16.5100	0.1083	1.006	15.78
7	20.32	21,500	2.540E-03	19.0500	0.1250	1.008	16.99
8	22.86	22,300	3.175E-03	21.5900	0.1417	1.010	18.13
9	25.40	22,900	4.233E-03	24.1300	0.1583	1.013	19.21
10	30.48	23,500	8.467E-03	27.9400	0.1833	1.017	20.77
11	35.56	24,000	1.016E-02	33.0200	0.2167	1.025	22.74



11.8

A compact specimen of a hard tool steel is cycled between $P_{min} = 44.5$ and $P_{max} = 1379$ N, with crack growth data as given. Dimensions as in Fig. 8.16 are $b = 50.8$ and $t = 6.35$ mm. Determine *da/dN* and ΔK, plot, and fit to Eq. 11.10.

Figure. 8.16 gives expressions for F_P and *K*. Number the data points starting with $j = 1$, and calculate *da/dN* and ΔK for the intervals between data points by applying the equations below for $j = 2, 3, 4$, etc.

$$\Delta P = P_{max} - P_{min}\,, \qquad R = \frac{P_{min}}{P_{max}}\,, \qquad \Delta K_j = F_{Pj}\frac{\Delta P}{t\sqrt{b}}$$

$$\left(\frac{da}{dN}\right)_j = \frac{a_j - a_{j-1}}{N_j - N_{j-1}}\,, \qquad a_{avg} = \frac{a_j + a_{j-1}}{2}\,, \qquad \alpha_{avg} = \frac{a_{avg}}{b}$$

$$F_{Pj} = \frac{(2+\alpha_{avg})(0.886 + 4.64\alpha_{avg} - 13.32\alpha_{avg}^2 + 14.72\alpha_{avg}^3 - 5.6\alpha_{avg}^4)}{(1-\alpha_{avg})^{1.5}}$$

Calculation results and graph are given on pages that follow. Using these results, do a least squares fit $y = mx + b$, where the dependent variable is $y = \log da/dN$ and the independent variable is $x = \log \Delta K$.

$$\log(da/dN) = m\log(\Delta K) + \log C$$

The fit is shown on the graph and is seen to represent the data well.

m = 3.525 C = 4.59E-09 $\frac{\text{mm/cycle}}{\left(\text{MPa}\sqrt{\text{m}}\right)^m}$ ◀

(11.8, p. 2)

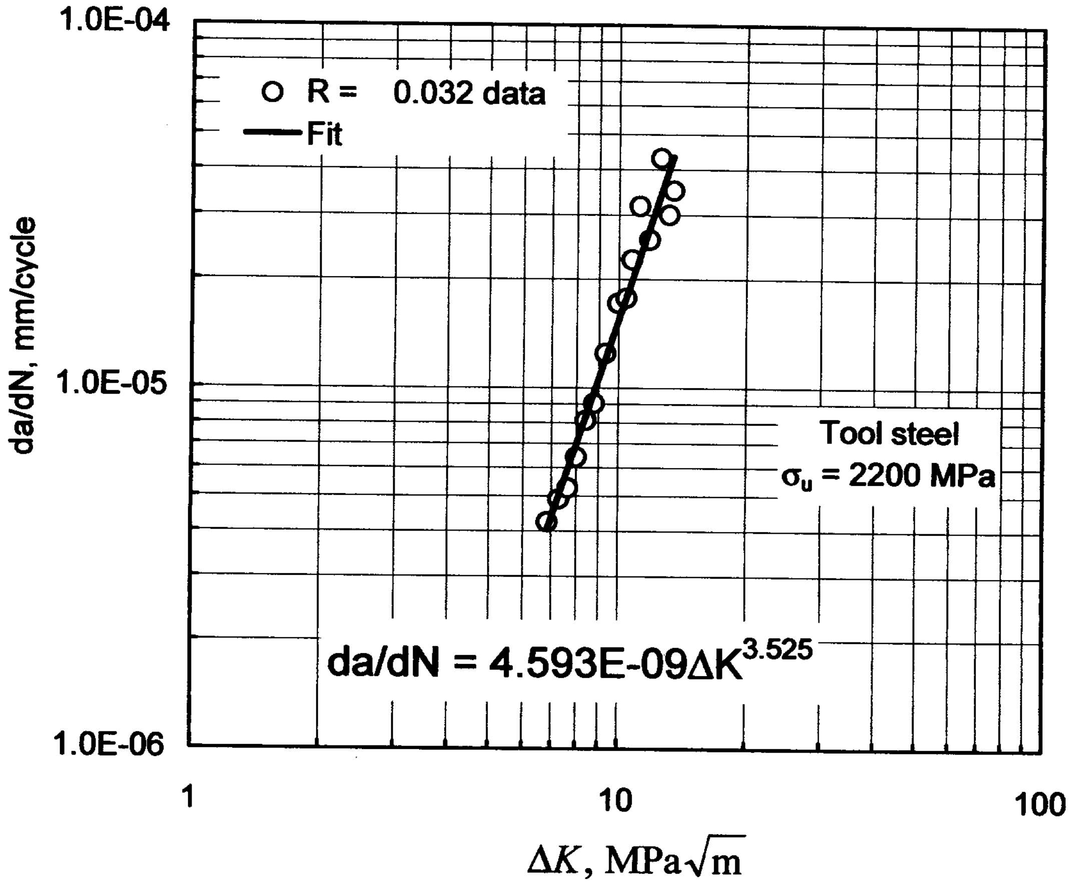
b, mm	t, mm	P_{max}, N	P_{min}, N	ΔP, N	R
50.80	6.35	1379	44.5	1334.5	0.032

j	a mm	N cycles	da/dN mm/cycle	a_{avg} mm	α_{avg}	F_P	ΔK $MPa\sqrt{m}$
1	19.86	0	XXX	XXX	XXX	XXX	XXX
2	21.13	300,000	4.233E-06	20.495	0.4034	7.35	6.85
3	22.15	508,000	4.904E-06	21.640	0.4260	7.80	7.28
4	22.94	658,000	5.267E-06	22.545	0.4438	8.20	7.64
5	23.80	792,000	6.418E-06	23.370	0.4600	8.58	8.00
6	24.61	892,000	8.100E-06	24.205	0.4765	9.00	8.39
7	25.37	976,000	9.048E-06	24.990	0.4919	9.42	8.79
8	26.54	1,070,000	1.245E-05	25.955	0.5109	9.99	9.32
9	27.43	1,122,000	1.712E-05	26.985	0.5312	10.67	9.95
10	27.89	1,148,000	1.769E-05	27.660	0.5445	11.15	10.40
11	28.32	1,167,000	2.263E-05	28.105	0.5532	11.49	10.72
12	29.08	1,191,000	3.167E-05	28.700	0.5650	11.98	11.17
13	29.85	1,221,000	2.567E-05	29.465	0.5800	12.65	11.80
14	30.71	1,241,000	4.300E-05	30.280	0.5961	13.45	12.54
15	31.01	1,251,000	3.000E-05	30.860	0.6075	14.06	13.11
16	31.29	1,259,000	3.500E-05	31.150	0.6132	14.39	13.42

Note: As a convenience for handling units, the equation for ΔK was employed in the mathematically equivalent form below, with units of newtons for ΔP, meters for b in the numerator, and mm for t and b in the denominator. This procedure gives the desired units for ΔK.

$$\Delta K_j = F_{Pj} \frac{\Delta P \sqrt{b}}{t\,b}$$

(11.8, p. 3)



1.0E-04
1.0E-05
1.0E-06
da/dN, mm/cycle
O R = 0.032 data
—Fit
Tool steel
σu = 2200 MPa
da/dN = 4.593E-09ΔK3.525
1
10
100
ΔK, MPa√m

11.9

A compact specimen of 7075-T651 aluminum is cycled between P_{min} = 270 and P_{max} = 2700 N, with crack growth data as given. Dimensions as in Fig. 8.16 are b = 50.8 and t = 6.477 mm. Determine *da/dN* and ΔK, plot, and fit to Eq. 11.10.

Note: Early printings of the text give t = 6.447 mm, which value should be 6.477 mm.

Figure. 8.16 gives expressions for F_P and K. Number the data points starting with j = 1, and calculate *da/dN* and ΔK for the intervals between data points by applying the equations below for j = 2, 3, 4, etc.

$$\Delta P = P_{max} - P_{min}, \qquad R = \frac{P_{min}}{P_{max}}, \qquad \Delta K_j = F_{Pj} \frac{\Delta P}{t\sqrt{b}}$$

$$\left(\frac{da}{dN}\right)_j = \frac{a_j - a_{j-1}}{N_j - N_{j-1}}, \qquad a_{avg} = \frac{a_j + a_{j-1}}{2}, \qquad \alpha_{avg} = \frac{a_{avg}}{b}$$

$$F_{Pj} = \frac{(2+\alpha_{avg})(0.886 + 4.64\alpha_{avg} - 13.32\alpha_{avg}^2 + 14.72\alpha_{avg}^3 - 5.6\alpha_{avg}^4)}{(1-\alpha_{avg})^{1.5}}$$

Calculation results and graph are given on pages that follow. Using these results, do a least squares fit $y = mx + b$, where the dependent variable is y = log *da/dN* and the independent variable is x = log ΔK.

$$\log(da/dN) = m\log(\Delta K) + \log C$$

The fit is shown on the graph and is seen to represent the data well.

m = 3.295 C = 6.42E-08 $\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$ ◀

(11.9, p.2)

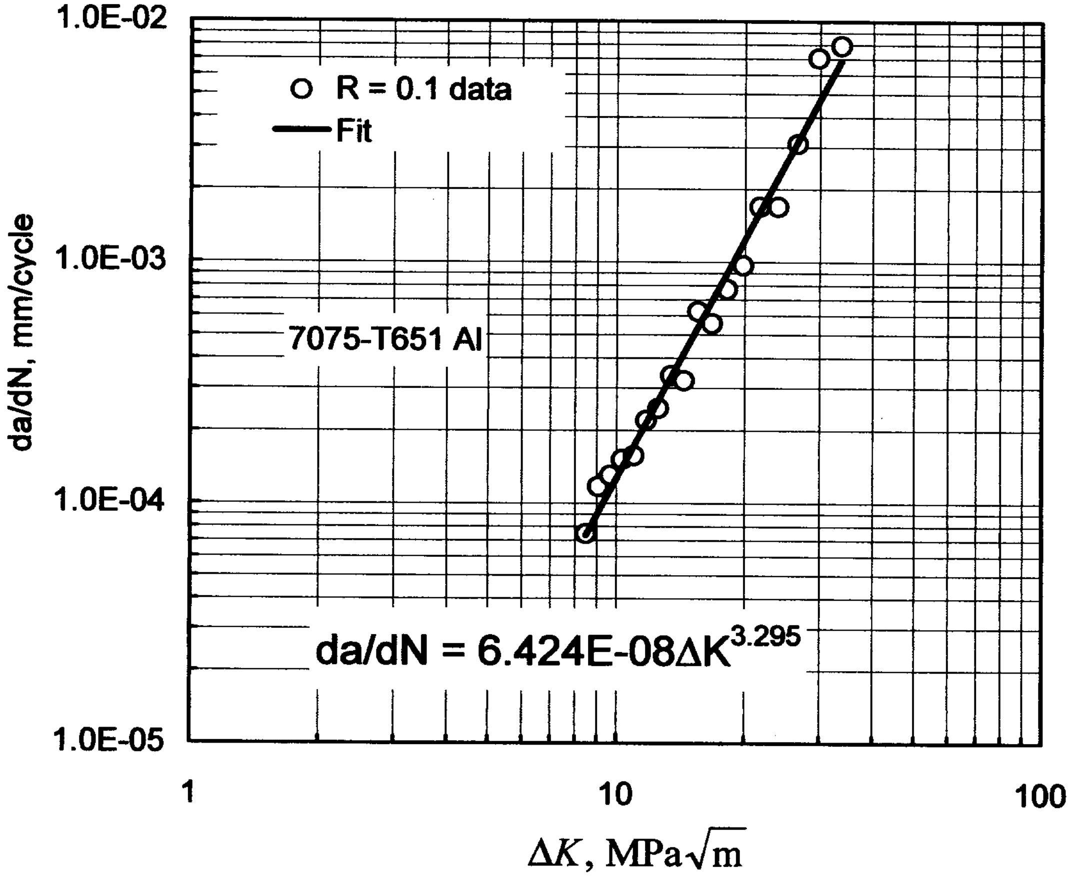
b, mm	t, mm	P_{max}, N	P_{min}, N	ΔP, N	R
50.80	6.477	2700	270	2430	0.100

j	a mm	N cycles	da/dN mm/cycle	a_{avg} mm	α_{avg}	F_P	ΔK $MPa\sqrt{m}$
1	12.70	0	XXX	XXX	XXX	XXX	XXX
2	13.97	17,000	7.471E-05	13.335	0.2625	5.09	8.48
3	15.24	27,840	1.172E-04	14.605	0.2875	5.44	9.06
4	16.51	37,600	1.301E-04	15.875	0.3125	5.81	9.66
5	17.78	45,970	1.517E-04	17.145	0.3375	6.19	10.30
6	19.05	54,050	1.572E-04	18.415	0.3625	6.60	10.99
7	20.32	59,820	2.201E-04	19.685	0.3875	7.04	11.72
8	21.59	64,950	2.476E-04	20.955	0.4125	7.52	12.53
9	22.86	68,710	3.378E-04	22.225	0.4375	8.05	13.41
10	24.13	72,650	3.223E-04	23.495	0.4625	8.64	14.38
11	25.40	74,680	6.256E-04	24.765	0.4875	9.30	15.48
12	26.67	76,960	5.570E-04	26.035	0.5125	10.04	16.72
13	27.94	78,610	7.697E-04	27.305	0.5375	10.89	18.13
14	29.21	79,920	9.695E-04	28.575	0.5625	11.87	19.76
15	30.48	80,670	1.693E-03	29.845	0.5875	13.01	21.66
16	31.75	81,420	1.693E-03	31.115	0.6125	14.35	23.89
17	33.02	81,830	3.098E-03	32.385	0.6375	15.95	26.54
18	34.29	82,010	7.056E-03	33.655	0.6625	17.86	29.73
19	35.56	82,170	7.938E-03	34.925	0.6875	20.19	33.61

Note: As a convenience for handling units, the equation for ΔK was employed in the mathematically equivalent form below, with units of newtons for ΔP, meters for b in the numerator, and mm for t and b in the denominator. This procedure gives the desired units for ΔK.

$$\Delta K_j = F_{Pj}\frac{\Delta P\sqrt{b}}{t\,b}$$

(11.9, p. 3)



1.0E-02
1.0E-03
1.0E-04
1.0E-05
da/dN, mm/cycle
O R = 0.1 data
Fit
7075-T651 Al
da/dN = 6.424E-08ΔK^3.295
1
10
100
ΔK, MPa√m

11.10 2024-T3 Al, Table 11.2, Walker eqn.

$C_0 = 1.42 \times 10^{-8} \frac{mm/cyc}{(MPa\sqrt{m})^m}$, $m = 3.59$

$\gamma = 0.680$ $(R \geq 0)$

(a) Estimate $da/dN = C(\Delta K)^m$ equations for $R = 0.5$ and 0.8.

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}}$$

$C_{0.5} = 3.15 \times 10^{-8}$, $\frac{da}{dN} = 3.15 \times 10^{-8} (\Delta K)^{3.59}$ ◀ $(R = 0.5)$

$C_{0.8} = 9.02 \times 10^{-8}$, $\frac{da}{dN} = 9.02 \times 10^{-8} (\Delta K)^{3.59}$ ◀ $(R = 0.8)$

(Both for units of mm/cyc and $MPa\sqrt{m}$)

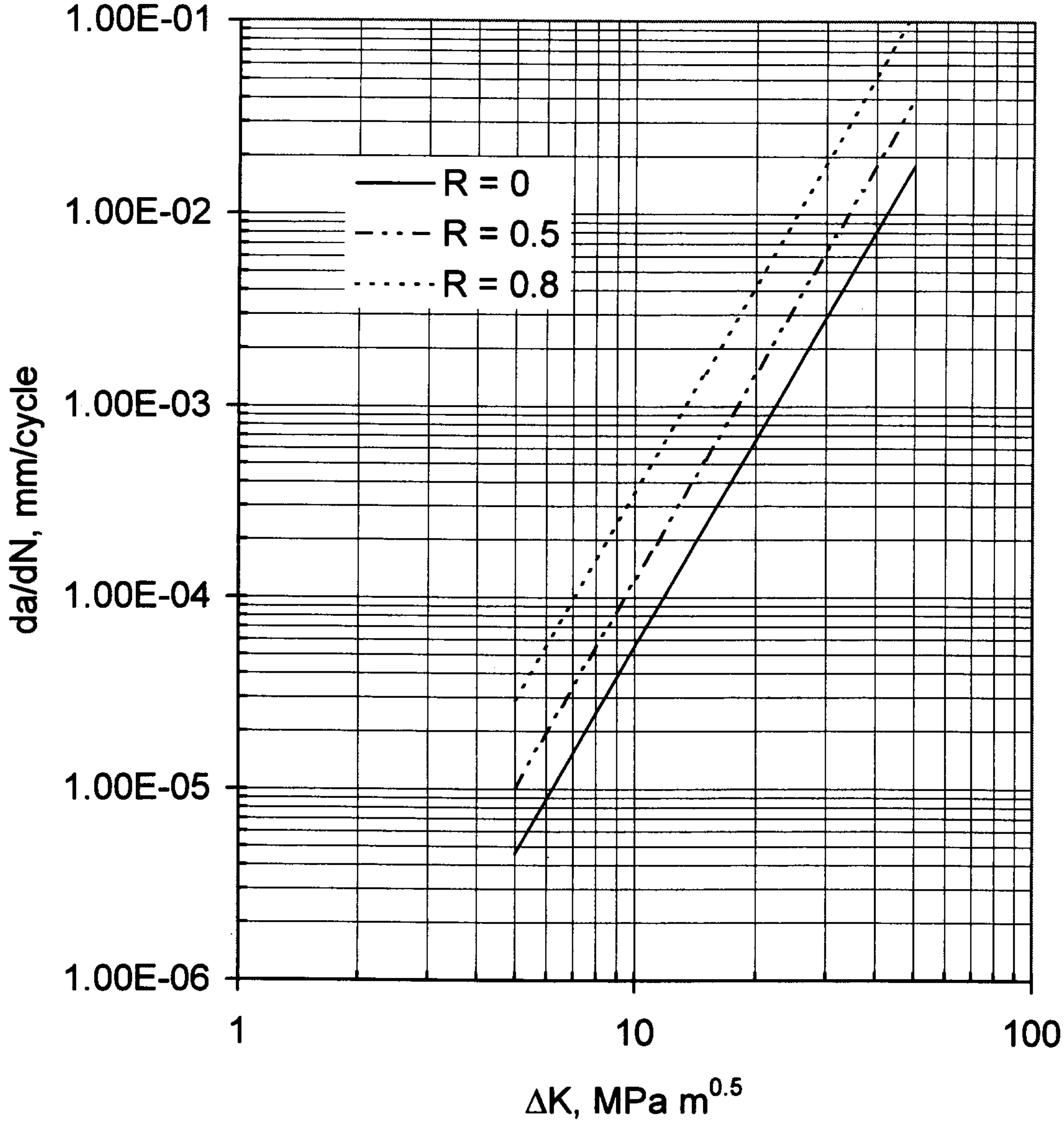
(b) Plot both with $R = 0$ eqn. For each of $R = 0$, 0.5, and 0.8, calculate da/dN for $\Delta K = 5$, 15, and 50 $MPa\sqrt{m}$.

C_0	m	γ
1.42E-08	3.59	0.68

R =	0	0.5	0.8
C =	1.42E-08	3.15E-08	9.02E-08
ΔK $MPa\ m^{0.5}$	da/dN mm/cyc	da/dN mm/cyc	da/dN mm/cyc
5	4.59E-06	1.02E-05	2.91E-05
15	2.37E-04	5.25E-04	1.50E-03
50	1.78E-02	3.96E-02	1.13E-01

(11.10, p, 2)

Connect these points to form the resulting three straight lines on a log-log plot.



(c) The lines are parallel, indicating constant factors for any ΔK, as determined by C.

$$\frac{(da/dN)_R}{(da/dN)_0} = \frac{C_R}{C_0} = \begin{cases} 2.22 & (R = 0.5) \\ 6.35 & (R = 0.8) \end{cases}$$

11.11 Plot da/dN vs. ΔK for Prob. 11.7 data on 7075-T6 Al, $R = 0.5$, and line expected from Table 11.2 constants.

$$\frac{da}{dN} = C(\Delta K)^m, \quad m = m, \quad C = \frac{C_0}{(1-R)^{m(1-\gamma)}}$$

$$C_0 = 2.71 \times 10^{-8} \ \frac{mm/cyc}{(MPa\sqrt{m})^m}, \quad m = 3.70$$

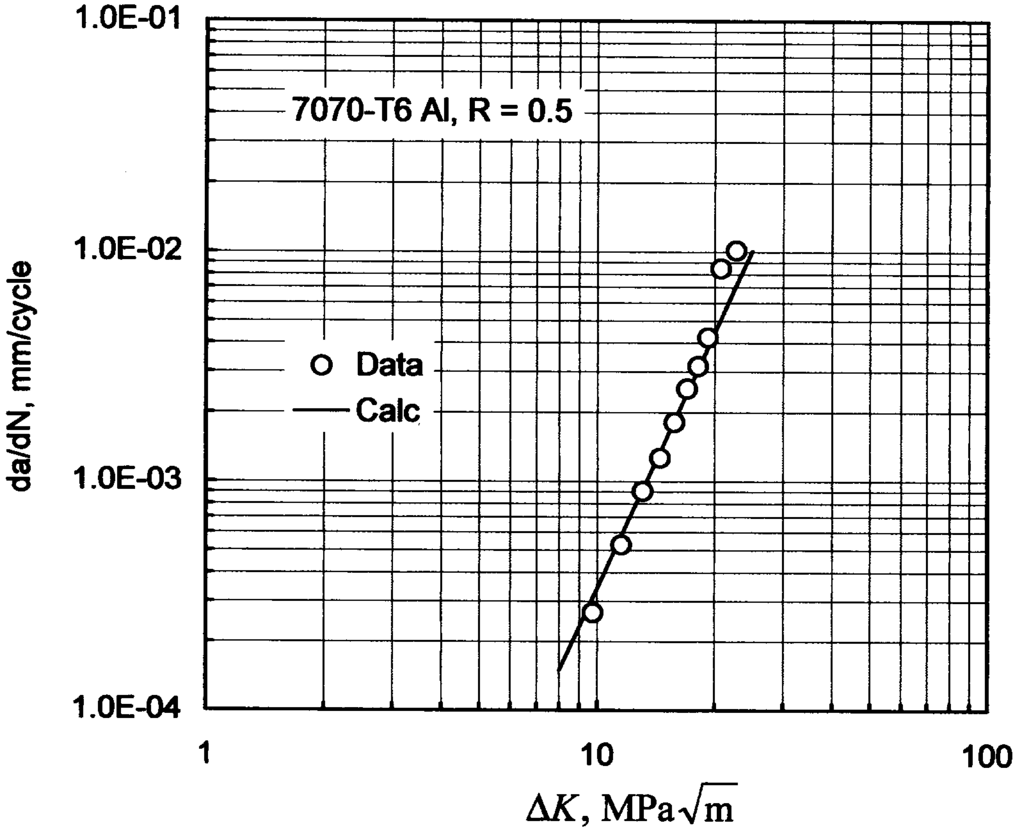
$$\gamma = 0.641$$

For $R = 0.5$, $C = 6.80 \times 10^{-8} \ \frac{mm/cyc}{(MPa\sqrt{m})^m}$

$$\frac{da}{dN} = 6.80 \times 10^{-8} (\Delta K)^{3.70}$$ ◀

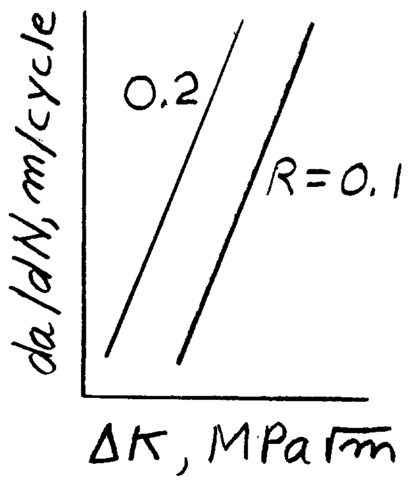
Plot this equation along with the data of da/dN vs. ΔK resulting from Pr. 11.7 as shown on the next page. Data and calculated line agree quite well. ◀

(11.11, p. 2)



7070-T6 Al, R = 0.5
Data
Calc
da/dN, mm/cycle
1.0E-01
1.0E-02
1.0E-03
1.0E-04
1
10
100
ΔK, MPa√m

11.12 Granite rock: $\frac{da}{dN} = C(\Delta K)^m$



R	C	m
0.1	2×10^{-10}	11.9
0.2	7.8×10^{-10}	11.9

C_0 m, γ = ? (Walker)

$$\frac{da}{dN} = C_0\left[\frac{\Delta K}{(1-R)^{1-\gamma}}\right]^m = \frac{C_0}{(1-R)^{m(1-\gamma)}}(\Delta K)^m$$

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}} \quad \text{(known for } R = 0.1, 0.2)$$

$$2 \times 10^{-10} = \frac{C_0}{(1-0.1)^x}, \quad \text{where } x = m_1(1-\gamma)$$

$$m = m_1 = 11.9$$ ◀

$$7.8 \times 10^{-10} = \frac{C_0}{(1-0.2)^x}, \quad \frac{2 \times 10^{-10}}{7.8 \times 10^{-10}} = \left(\frac{0.8}{0.9}\right)^x$$

$$x = \frac{\log(2/7.8)}{\log(0.8/0.9)} = 11.555, \quad 1-\gamma = 0.971, \quad \gamma = 0.029$$ ◀

$$C_1 = 2 \times 10^{-10}(0.9)^{11.555} = 5.92 \times 10^{-11} \frac{m/\text{cycle}}{(\text{MPa}\sqrt{m})^m}$$ ◀

γ small compared to one indicates a very high sensitivity to R, unlike metals, where $\gamma \approx 0.5$ is more typical. ◀

11.13

For the given data on RQC-100 steel at $R = 0.1$, 0.5, and 0.8: (a) Plot *da/dN* vs. ΔK and note if a set of parallel lines seem reasonable. (b) If so, obtain approximate C_0, m, and γ for the Walker equation. (c) Obtain C_0, m, and γ from multiple regression. (d) Plot *da/dN* vs $\overline{\Delta K}$ for data and fitted line, and comment on the success of the fit.

(a) The requested plot is given on the next page. A set of parallel lines does reasonably represent the data.

(b) Pick two points that represent the $R = 0.1$ data, and a third that lies on the parallel trend for $R = 0.8$. Then use the first two points to calculate constants $C_{0.1}$ and m for the $R = 0.1$ line. Next, using the same value of m for $R = 0.8$, calculate $C_{0.8}$.

$$da/dN = C_{0.1}(\Delta K)^m$$

$$m = \frac{\log(da/dN_1) - \log(da/dN_2)}{\log(\Delta K_1) - \log(\Delta K_2)}, \quad C_{0.1} = \frac{da/dN_1}{(\Delta K_1)^m}$$

$$da/dN = C_{0.8}(\Delta K)^m, \quad C_{0.8} = \frac{da/dN_3}{(\Delta K_3)^m}$$

Point	da/dN	ΔK	R
1	4.87E-02	114	0.1
2	3.10E-05	20.1	0.1
3	1.64E-04	20.0	0.8

m =	4.241
$C_{0.1}$ =	9.227E-11
$C_{0.8}$ =	4.986E-10
	$\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$

(11.13, p.2)

Then apply Eq. 11.20 to both R values, and solve for C_0 and γ.

$$C_R = \frac{C_0}{(1-R)^{m(1-\gamma)}}$$

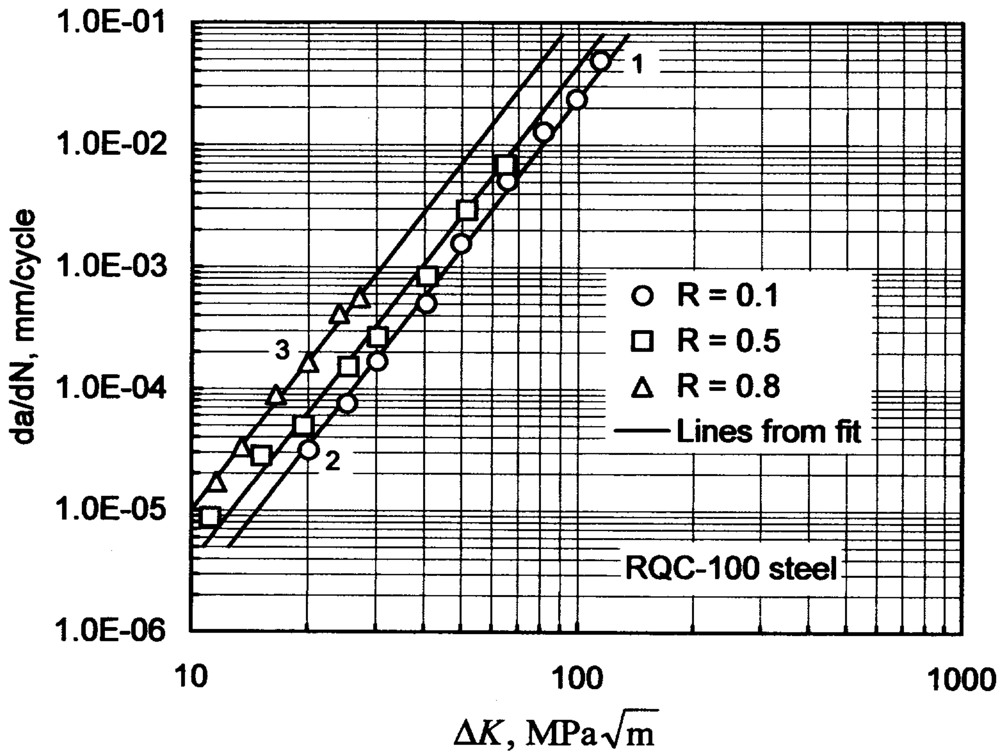
$$C_{0.1} = \frac{C_0}{(1-0.1)^{m(1-\gamma)}}, \qquad C_{0.8} = \frac{C_0}{(1-0.8)^{m(1-\gamma)}}$$

$$\frac{C_{0.1}}{C_{0.8}} = \frac{(1-0.8)^{m(1-\gamma)}}{(1-0.1)^{m(1-\gamma)}} = \left(\frac{0.2}{0.9}\right)^{m(1-\gamma)}, \qquad m(1-\gamma) = \frac{\log(C_{0.1}/C_{0.8})}{\log(0.2/0.9)}$$

$$\gamma = 1 - \frac{1}{m}\left(\frac{\log(C_{0.1}/C_{0.8})}{\log(0.2/0.9)}\right), \qquad C_0 = C_{0.1}(1-0.1)^{m(1-\gamma)}$$

m	γ	C_0
4.241	0.735	8.198E-11

$\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$ ◀



(11.13, p. 3)

(c) Multiple regression then proceeds as below. The values used for fitting are given in a table that follows.

$$\frac{da}{dN} = C_0\left(\frac{\Delta K}{(1-R)^{1-\gamma}}\right)^m$$

$$\log\frac{da}{dN} = m\log\Delta K - m(1-\gamma)\,\log(1-R) + \log C_0$$

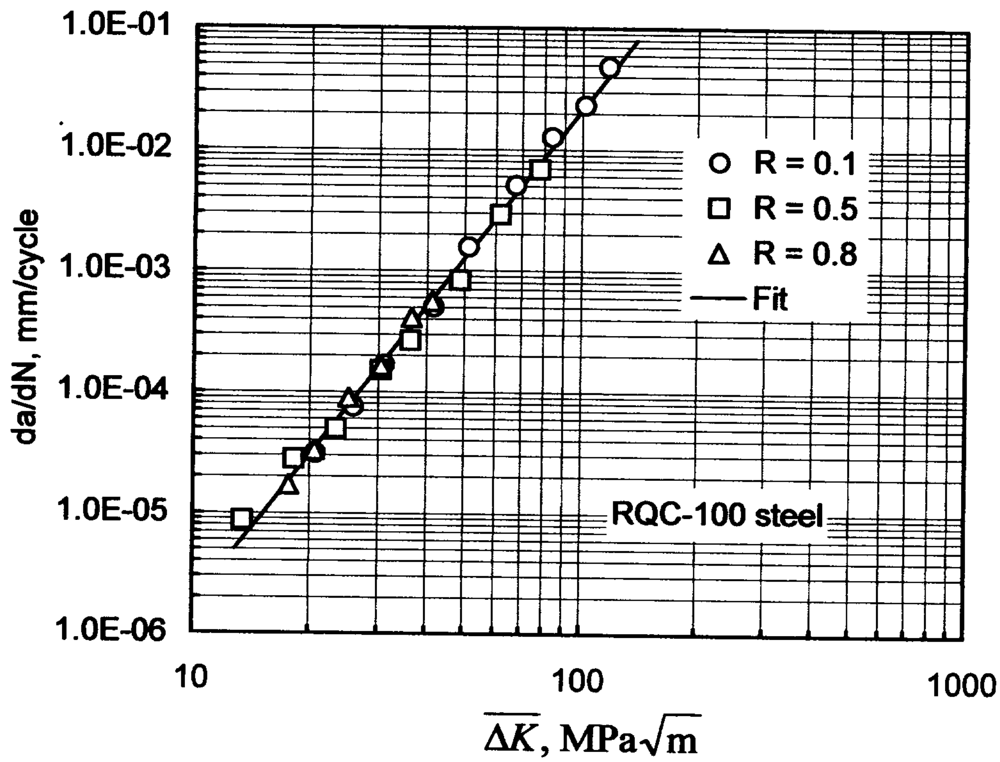
$$y = a_1 \; x_1 \; + \; a_2 \; x_2 \; + \; b$$

$a_1 = m\,,\qquad a_2 = -m(1-\gamma)\,,\qquad \gamma = a_2/a_1 + 1$

$b = \log C_0\,,\qquad C_0 = 10^b$

$a_1 =$ 4.0705
$a_2 =$ -1.06496
$b =$ -9.8135

m	γ	C_0	$\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$
4.070	0.7384	1.536E-10	



(11.13, p.4)

da/dN mm/cyc	ΔK $MPa\sqrt{m}$	R	y log (da/dN)	x_1 log (ΔK)	x_2 log (1 - R)	$\overline{\Delta K}$ $MPa\sqrt{m}$
3.10E-05	20.1	0.1	-4.509	1.3032	-0.0458	20.66
7.54E-05	25.2	0.1	-4.123	1.4014	-0.0458	25.90
1.68E-04	30.2	0.1	-3.775	1.4800	-0.0458	31.04
5.02E-04	40.5	0.1	-3.299	1.6075	-0.0458	41.63
1.56E-03	49.8	0.1	-2.807	1.6972	-0.0458	51.19
5.08E-03	65.7	0.1	-2.294	1.8176	-0.0458	67.54
1.27E-02	81.4	0.1	-1.896	1.9106	-0.0458	83.68
2.34E-02	99.0	0.1	-1.631	1.9956	-0.0458	101.77
4.87E-02	114	0.1	-1.312	2.0569	-0.0458	117.19
8.72E-06	11.2	0.5	-5.059	1.0492	-0.3010	13.43
2.78E-05	15.2	0.5	-4.556	1.1818	-0.3010	18.22
4.94E-05	19.5	0.5	-4.306	1.2900	-0.3010	23.38
1.51E-04	25.4	0.5	-3.821	1.4048	-0.3010	30.45
2.65E-04	30.3	0.5	-3.577	1.4814	-0.3010	36.32
8.33E-04	40.7	0.5	-3.079	1.6096	-0.3010	48.79
2.90E-03	51.5	0.5	-2.538	1.7118	-0.3010	61.74
6.86E-03	64.9	0.5	-2.164	1.8122	-0.3010	77.80
1.70E-05	11.6	0.8	-4.770	1.0645	-0.6990	17.67
3.28E-05	13.5	0.8	-4.484	1.1303	-0.6990	20.57
8.91E-05	16.5	0.8	-4.050	1.2175	-0.6990	25.14
1.64E-04	20.0	0.8	-3.785	1.3010	-0.6990	30.47
4.13E-04	24.0	0.8	-3.384	1.3802	-0.6990	36.57
5.58E-04	27.1	0.8	-3.253	1.4330	-0.6990	41.29

(d) $\overline{\Delta K}$ is calculated using the fitted γ value as listed in the last column above. Plotting these values versus *da/dN* gives the graph on the previous page. Also shown is the line from the fitted constants C_0 and m. The fit is successful as the data all fall together along the fitted line.

$$\overline{\Delta K} = \frac{\Delta K}{(1-R)^{1-\gamma}}$$

11.14

For the data on 2124-T851 aluminum at $R = 0.1$ and 0.5 from Tables P11.4 and P11.5: (a) Plot *da/dN* vs. ΔK and note if a set of parallel lines seem reasonable. (b) If so, obtain approximate C_0, m, and γ for the Walker equation. (c) Obtain C_0, m, and γ from multiple regression. (d) Plot *da/dN* vs. $\overline{\Delta K}$ for data and fitted line, and comment on the success of the fit.

(a) The requested plot is given on the next page. Two parallel lines do reasonably represent the data.

(b) Pick two points that represent the $R = 0.1$ data, and a third that lies on the parallel trend for $R = 0.5$. Then use the first two points to calculate constants $C_{0.1}$ and m for the $R = 0.1$ line. Next, using the same value of m for $R = 0.5$, calculate $C_{0.5}$.

$$da/dN = C_{0.1}(\Delta K)^m$$

$$m = \frac{\log(da/dN_1) - \log(da/dN_2)}{\log(\Delta K_1) - \log(\Delta K_2)}, \qquad C_{0.1} = \frac{da/dN_1}{(\Delta K_1)^m}$$

$$da/dN = C_{0.5}(\Delta K)^m, \qquad C_{0.5} = \frac{da/dN_3}{(\Delta K_3)^m}$$

Point	da/dN	ΔK	R
1	1.77E-04	15.9	0.1
2	1.26E-06	2.99	0.1
3	1.49E-05	5.48	0.5

$m =$	2.959
$C_{0.1} =$	4.929E-08
$C_{0.5} =$	9.704E-08
	$\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$

(11.14, p.2)

Then apply Eq. 11.20 to both R values, and solve for C_0 and γ.

$$C_R = \frac{C_0}{(1-R)^{m(1-\gamma)}}$$

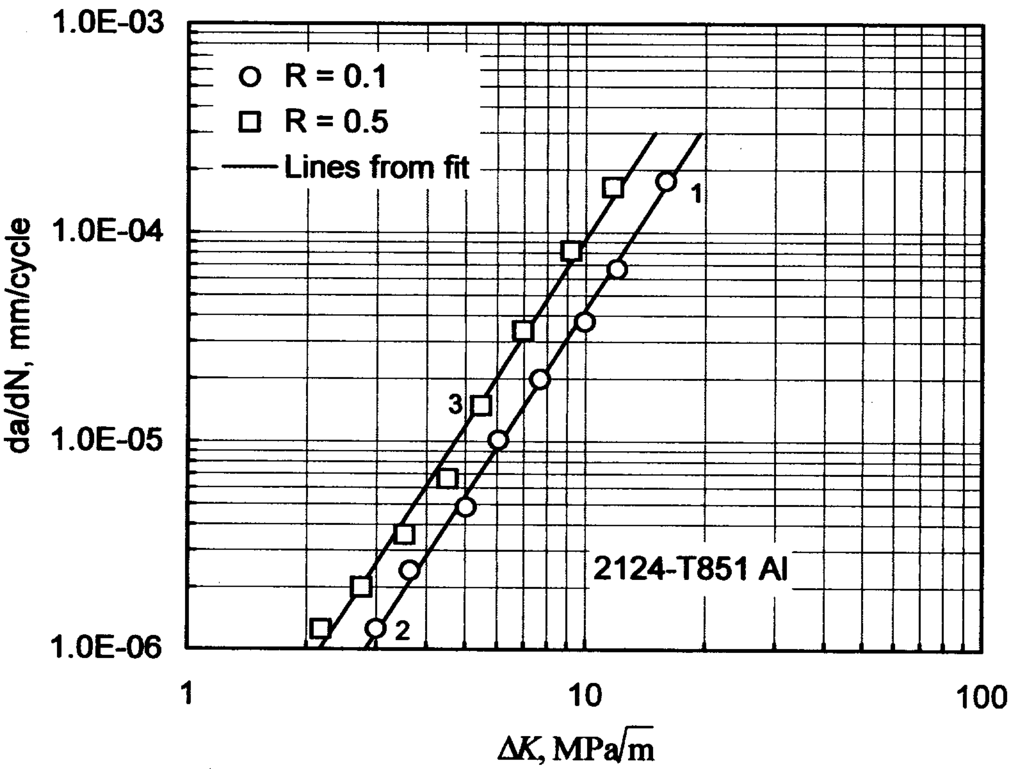
$$C_{0.1} = \frac{C_0}{(1-0.1)^{m(1-\gamma)}}, \qquad C_{0.5} = \frac{C_0}{(1-0.5)^{m(1-\gamma)}}$$

$$\frac{C_{0.1}}{C_{0.5}} = \frac{(1-0.5)^{m(1-\gamma)}}{(1-0.1)^{m(1-\gamma)}} = \left(\frac{0.5}{0.9}\right)^{m(1-\gamma)}, \qquad m(1-\gamma) = \frac{\log(C_{0.1}/C_{0.5})}{\log(0.5/0.9)}$$

$$\gamma = 1 - \frac{1}{m}\left(\frac{\log(C_{0.1}/C_{0.5})}{\log(0.5/0.9)}\right), \qquad C_0 = C_{0.1}(1-0.1)^{m(1-\gamma)}$$

m	γ	C_0	
2.959	0.611	4.365E-08	$\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$

◀



(11, 14, p. 3)

(c) Multiple regression then proceeds as below. The values used for fitting are given in a table that follows.

$$\frac{da}{dN} = C_0\left(\frac{\Delta K}{(1-R)^{1-\gamma}}\right)^m$$

$$\log\frac{da}{dN} = m\log\Delta K - m(1-\gamma)\,\log(1-R) + \log C_0$$

$$y \;=\; a_1 \;\; x_1 \;+\; a_2 \qquad x_2 \qquad +\; b$$

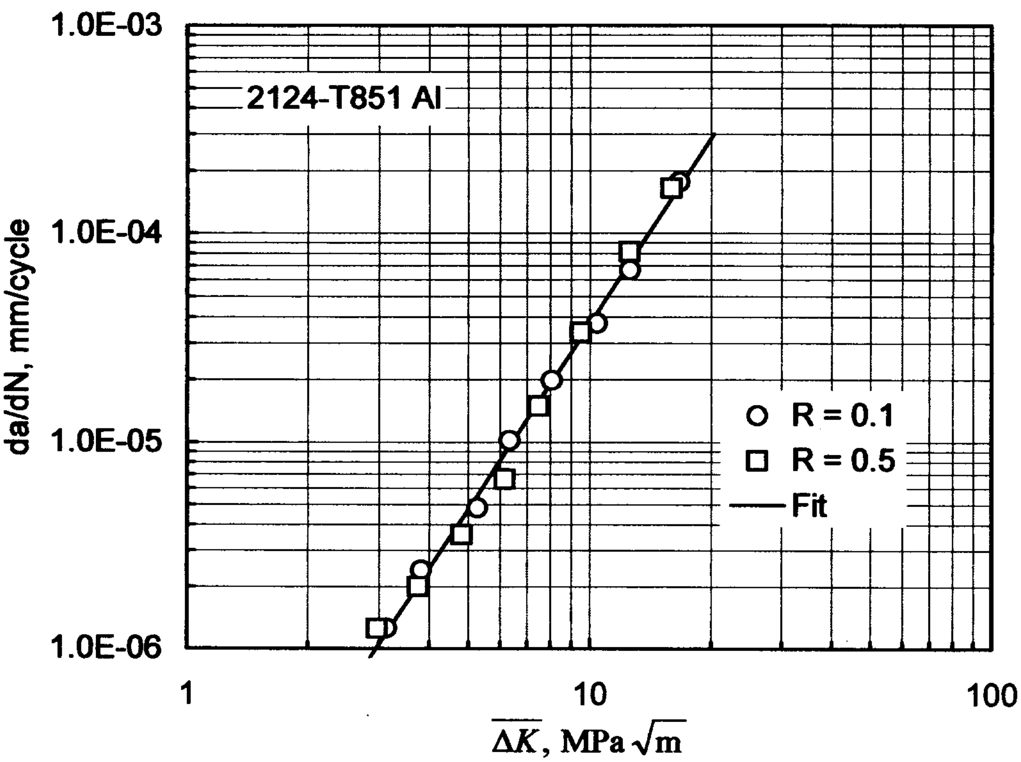
$$a_1 = m\,, \qquad a_2 = -m(1-\gamma)\,, \qquad \gamma = a_2/a_1 + 1$$

$$b = \log C_0\,, \qquad C_0 = 10^b$$

a_1 = 2.9562
a_2 = -1.32061
b = -7.3896

m	γ	C_0
2.956	0.5533	4.078E-08

$\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$ ◀



(11.14, p.4)

da/dN	ΔK	R	y	x_1	x_2	$\overline{\Delta K}$
mm/cyc	$MPa\sqrt{m}$		log (da/dN)	log (ΔK)	log (1 - R)	$MPa\sqrt{m}$
1.26E-06	2.99	0.1	-5.900	0.4757	-0.0458	3.13
2.41E-06	3.64	0.1	-5.618	0.5611	-0.0458	3.82
4.84E-06	5.02	0.1	-5.315	0.7007	-0.0458	5.26
1.02E-05	6.04	0.1	-4.991	0.7810	-0.0458	6.33
1.99E-05	7.68	0.1	-4.701	0.8854	-0.0458	8.05
3.74E-05	9.95	0.1	-4.427	0.9978	-0.0458	10.43
6.69E-05	12.0	0.1	-4.175	1.0792	-0.0458	12.58
1.77E-04	15.9	0.1	-3.752	1.2014	-0.0458	16.67
1.25E-06	2.18	0.5	-5.903	0.3385	-0.3010	2.97
2.00E-06	2.75	0.5	-5.699	0.4393	-0.3010	3.75
3.57E-06	3.53	0.5	-5.447	0.5478	-0.3010	4.81
6.62E-06	4.52	0.5	-5.179	0.6551	-0.3010	6.16
1.49E-05	5.48	0.5	-4.827	0.7388	-0.3010	7.47
3.38E-05	6.97	0.5	-4.471	0.8432	-0.3010	9.50
8.16E-05	9.20	0.5	-4.088	0.9638	-0.3010	12.54
1.65E-04	11.7	0.5	-3.783	1.0682	-0.3010	15.95

(d) $\overline{\Delta K}$ is calculated using the fitted γ value as listed in the last column above. Plotting these values versus *da/dN* gives the graph on the previous page. Also shown is the line from the fitted constants C_0 and m. The fit is successful as the data all fall together along the fitted line.

$$\overline{\Delta K} = \frac{\Delta K}{(1-R)^{1-\gamma}}$$

11.15

For the given data on 17-4 PH stainless steel at $R = 0.04$, 0.67, and 0.8:
(a) Plot *da/dN* vs. ΔK and note if a set of parallel lines seem reasonable.
(b) If so, obtain approximate C_0, m, and γ for the Walker equation.
(c) Obtain C_0, m, and γ from multiple regression. (d) Plot *da/dN* vs. $\overline{\Delta K}$ for data and fitted line, and comment on the success of the fit.

(a) The requested plot is given on the next page. A set of parallel lines does reasonably represent the data.

(b) Pick two points that represent the $R = 0.04$ data, and a third that lies on the parallel trend for $R = 0.8$. Then use the first two points to calculate constants $C_{0.04}$ and m for the $R = 0.04$ line. Next, using the same value of m for $R = 0.8$, calculate $C_{0.8}$.

$$da/dN = C_{0.04}(\Delta K)^m$$

$$m = \frac{\log(da/dN_1) - \log(da/dN_2)}{\log(\Delta K_1) - \log(\Delta K_2)}, \quad C_{0.04} = \frac{da/dN_1}{(\Delta K_1)^m}$$

$$da/dN = C_{0.8}(\Delta K)^m, \quad C_{0.8} = \frac{da/dN_3}{(\Delta K_3)^m}$$

Point	da/dN	ΔK	R
1	3.68E-03	124	0.04
2	9.98E-06	11.2	0.04
3	2.46E-04	28.4	0.8

$m =$ 2.458
$C_{0.04} =$ 2.631E-08
$C_{0.8} =$ 6.585E-08 $\dfrac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$

(11.15, p. 2)

Then apply Eq. 11.20 to both R values, and solve for C_0 and γ.

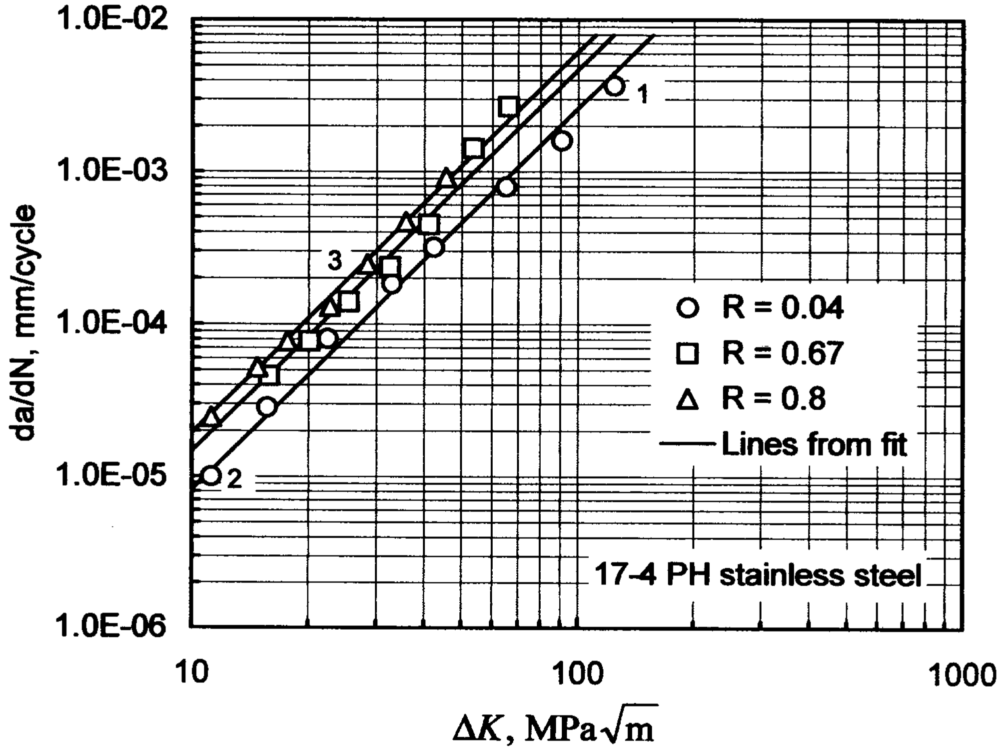
$$C_R = \frac{C_0}{(1-R)^{m(1-\gamma)}}$$

$$C_{0.04} = \frac{C_0}{(1-0.04)^{m(1-\gamma)}}, \qquad C_{0.8} = \frac{C_0}{(1-0.8)^{m(1-\gamma)}}$$

$$\frac{C_{0.04}}{C_{0.8}} = \frac{(1-0.8)^{m(1-\gamma)}}{(1-0.04)^{m(1-\gamma)}} = \left(\frac{0.2}{0.96}\right)^{m(1-\gamma)}, \qquad m(1-\gamma) = \frac{\log(C_{0.04}/C_{0.8})}{\log(0.2/0.96)}$$

$$\gamma = 1 - \frac{1}{m}\left(\frac{\log(C_{0.04}/C_{0.8})}{\log(0.2/0.96)}\right), \qquad C_0 = C_{0.04}(1-0.04)^{m(1-\gamma)}$$

m	γ	C_0	
2.458	0.762	2.569E-08	$\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$



(11.15, p. 3)

(c) Multiple regression then proceeds as below. The values used for fitting are given in a table that follows.

$$\frac{da}{dN} = C_0\left(\frac{\Delta K}{(1-R)^{1-\gamma}}\right)^m$$

$$\log\frac{da}{dN} = m\log\Delta K - m(1-\gamma)\log(1-R) + \log C_0$$

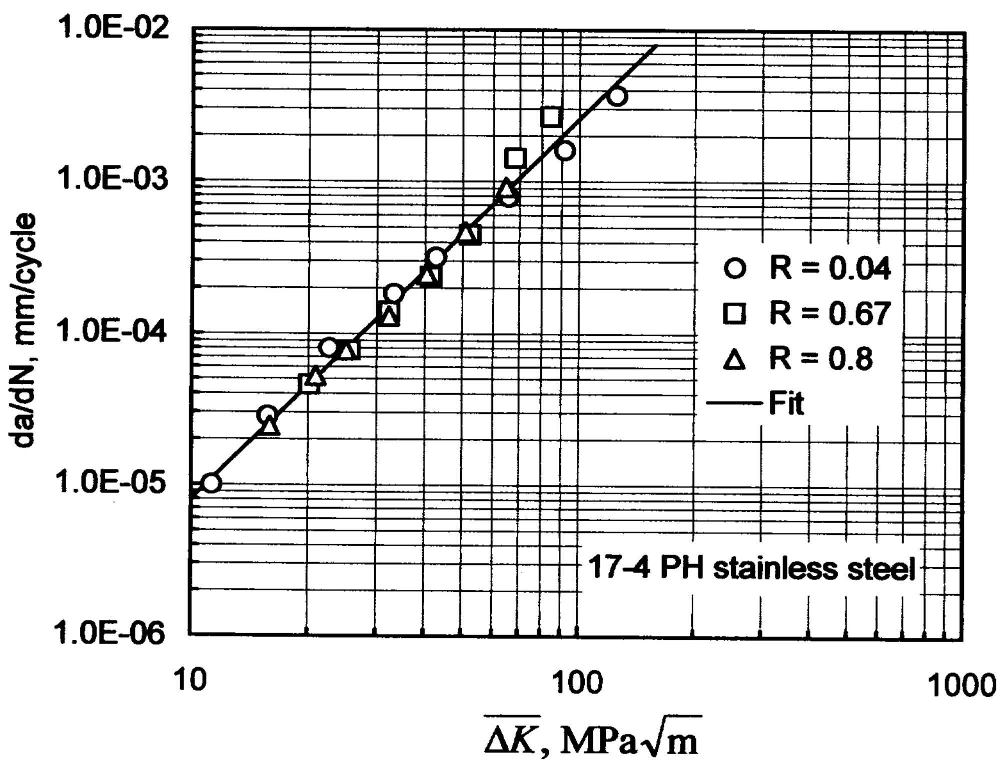
$$y \quad = \quad a_1 \quad x_1 \quad + \quad a_2 \quad\quad x_2 \quad\quad + \quad b$$

$$a_1 = m, \qquad a_2 = -m(1-\gamma), \qquad \gamma = a_2/a_1 + 1$$

$$b = \log C_0, \qquad C_0 = 10^b$$

a_1 = 2.5038
a_2 = -0.54725
b = -7.5991

m	γ	C_0	
2.504	0.7814	2.517E-08	$\frac{\text{mm/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$



(11.15, p.4)

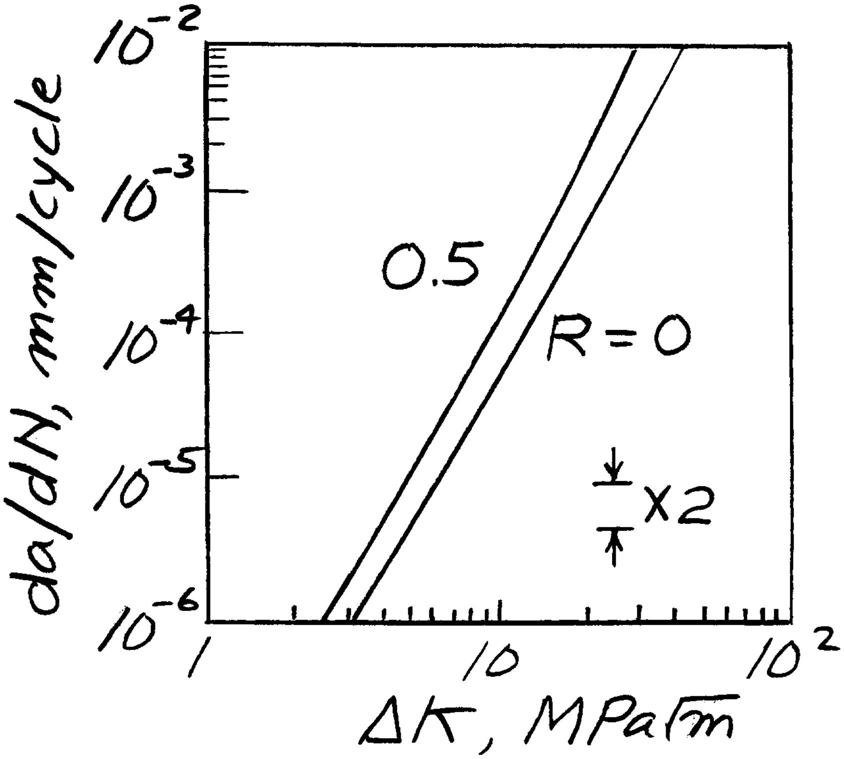
da/dN mm/cyc	ΔK $MPa\sqrt{m}$	R	y log (da/dN)	x_1 log (ΔK)	x_2 log (1 - R)	$\overline{\Delta K}$ $MPa\sqrt{m}$
9.98E-06	11.2	0.04	-5.001	1.0492	-0.0177	11.30
2.84E-05	15.6	0.04	-4.547	1.1931	-0.0177	15.74
8.03E-05	22.4	0.04	-4.095	1.3502	-0.0177	22.60
1.83E-04	32.9	0.04	-3.738	1.5172	-0.0177	33.19
3.18E-04	42.3	0.04	-3.498	1.6263	-0.0177	42.68
7.92E-04	65.1	0.04	-3.101	1.8136	-0.0177	65.68
1.60E-03	90.8	0.04	-2.796	1.9581	-0.0177	91.61
3.68E-03	124	0.04	-2.434	2.0934	-0.0177	125.11
4.57E-05	15.8	0.67	-4.340	1.1987	-0.4815	20.13
7.70E-05	20.1	0.67	-4.114	1.3032	-0.4815	25.61
1.39E-04	25.4	0.67	-3.857	1.4048	-0.4815	32.36
2.36E-04	32.6	0.67	-3.627	1.5132	-0.4815	41.54
4.47E-04	41.1	0.67	-3.350	1.6138	-0.4815	52.37
1.42E-03	53.6	0.67	-2.848	1.7292	-0.4815	68.30
2.67E-03	66.2	0.67	-2.573	1.8209	-0.4815	84.35
2.45E-05	11.2	0.8	-4.611	1.0492	-0.6990	15.92
5.23E-05	14.7	0.8	-4.281	1.1673	-0.6990	20.90
7.75E-05	17.6	0.8	-4.111	1.2455	-0.6990	25.02
1.30E-04	22.7	0.8	-3.886	1.3560	-0.6990	32.27
2.46E-04	28.4	0.8	-3.609	1.4533	-0.6990	40.37
4.67E-04	35.8	0.8	-3.331	1.5539	-0.6990	50.89
9.14E-04	45.5	0.8	-3.039	1.6580	-0.6990	64.68

(d) $\overline{\Delta K}$ is calculated using the fitted γ value as listed in the last column above. Plotting these values versus *da/dN* gives the graph on the previous page. Also shown is the line from the fitted constants C_0 and m. The fit is successful as the data all fall together along the fitted line.

$$\overline{\Delta K} = \frac{\Delta K}{(1-R)^{1-\gamma}}$$

11.16 Forman Eq. for 2024-T3 Al.
Write and plot for $R = 0$ and 0,5
$C_2 = 2.31 \times 10^{-6}$, $m_2 = 3.38$, $K_c = 110$
(for $MPa\sqrt{m}$, mm/cycle, Table 11.3)

$$\frac{da}{dN} = \frac{C_2 (\Delta K)^{m_2}}{(1-R)K_c - \Delta K}$$



The lines curve gently

da/dN typically increases around a factor of two by increasing R from 0 to 0,5. But this ratio increases with ΔK.

11.17 For 7075-T6 Al from Prob. 11.7, data at $R = 0.5$:

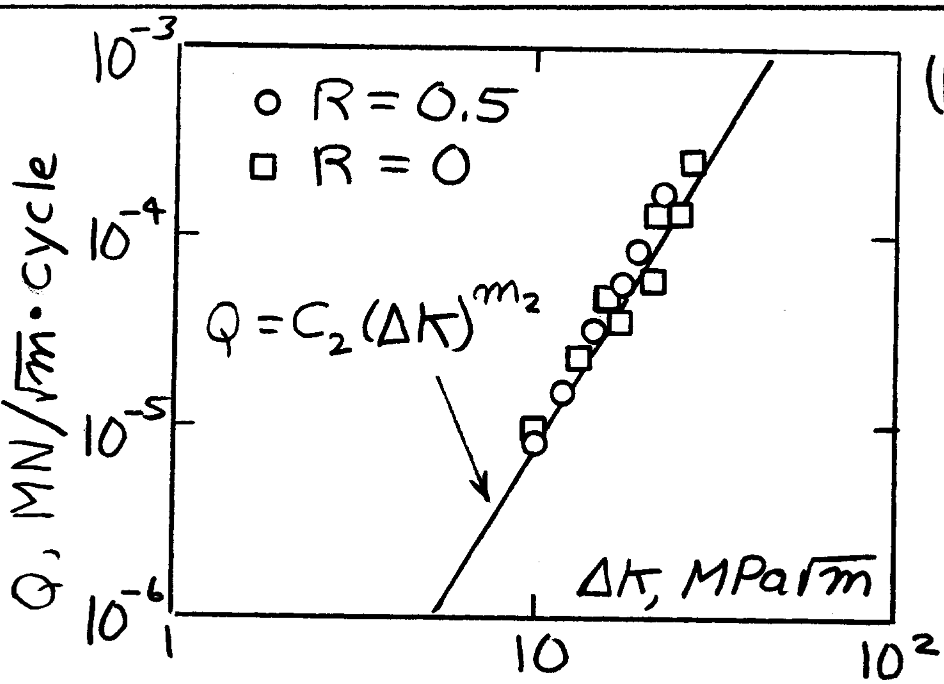
(a) Calculate and plot Q vs. ΔK

$C_2 = 5.29 \times 10^{-9}$, $m_2 = 3.21$, $K_c = 78.7$

(for $MPa\sqrt{m}$, m/cycle, from Table 11.3)

$$Q = \frac{da}{dN}\left[(1-R)K_c - \Delta K\right] \quad \frac{MN}{\sqrt{m}\cdot \text{cycle}} = C_2 (\Delta K)^{m_2}$$

$\frac{da}{dN}$, $\frac{m}{\text{cycle}}$	ΔK, $MPa\sqrt{m}$	Q, $\frac{MN}{\sqrt{m}\cdot\text{cycle}}$
2.67×10^{-7}	9.74	7.92×10^{-6}
5.29×10^{-7}	11.54	1.471×10^{-5}
9.07×10^{-7}	13.10	2.38×10^{-5}
⋮	⋮	⋮
1.016×10^{-5}	22.9	1.675×10^{-4}



(Ex. 11.2 data also shown)

Data and line agree reasonably well. ◀

(b) Add line $Q = C_2(\Delta K)^{m_2} = 5.29 \times 10^{-9}(\Delta K)^{3.21}$

11.18 Ti-5Al-2.5Sn, fit ΔK_{th} vs R data to $\Delta K_{th} = \overline{\Delta K}_{th} (1-R)^{1-\gamma_{th}}$

$$\underbrace{\log(\Delta K_{th})}_{y} = \underbrace{\log(\overline{\Delta K}_{th})}_{b} + \underbrace{(1-\gamma_{th})}_{m} \underbrace{\log(1-R)}_{x}$$

ΔK_{th} MPa m$^{0.5}$	R	Y log(ΔK_{th})	X log(1 - R)
4.8	0	0.6812	0
3.6	0.31	0.5563	-0.1612
3.2	0.5	0.5051	-0.3010
2.4	0.72	0.3802	-0.5528
2.1	0.79	0.3222	-0.6778

A least squares fit gives
$b = 0.6619$, $m = 0.5106$

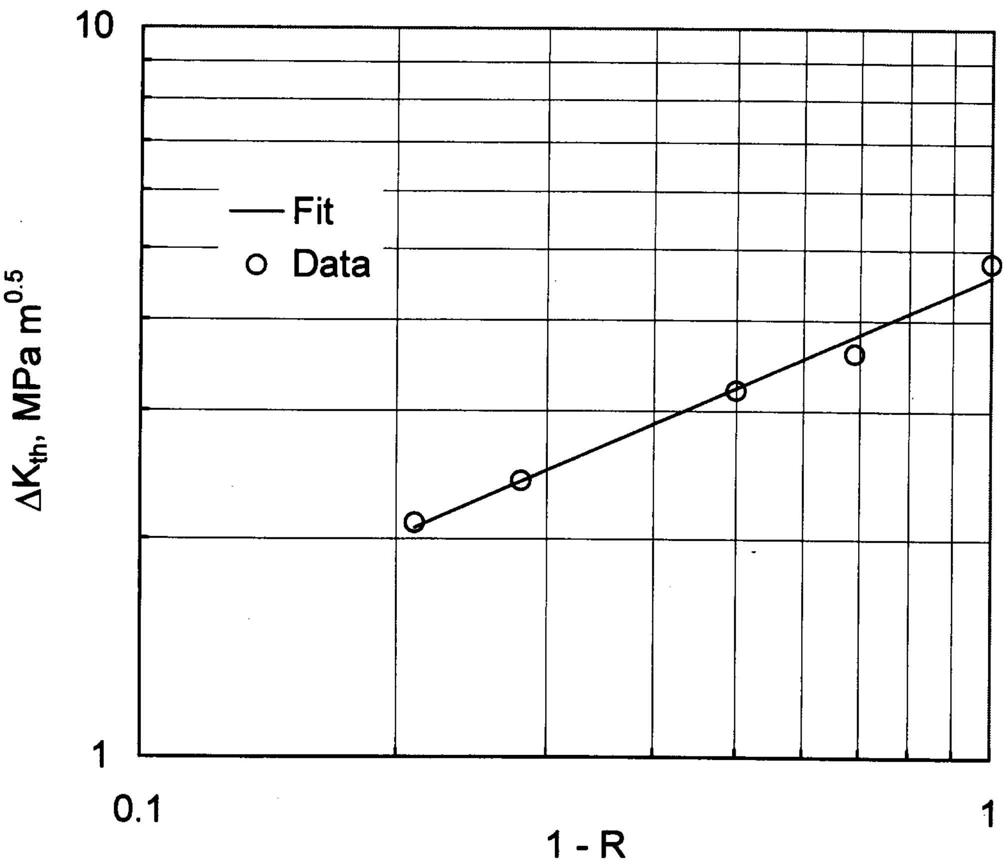
$\overline{\Delta K}_{th} = 10^{b} = 4.59$ MPa$\sqrt{m}$

$\gamma_{th} = 1 - m = 0.489$

$\Delta K_{th} = 4.59\ (1-R)^{1-0.489}$ ◀

Compare data and fitted line by plotting both on a log-log plot of ΔK_{th} vs. $(1-R)$. From the plot, data and line agree nicely. ◀

(11.18, p.2)



11.22 $N_{if} = ?$ for $\Delta K = F \Delta S \sqrt{\pi a}$, $F = \text{constant}$ and $da/dN = C(\Delta K)^2$, that is, $m = 2$.

$$\frac{da}{dN} = C\left(F \Delta S \sqrt{\pi a}\right)^2, \quad dN = \frac{1}{\pi C (F \Delta S)^2} \frac{da}{a}$$

$$N_{if} = \int_{a_i}^{a_f} \frac{1}{\pi C (F \Delta S)^2} \frac{da}{a} = \frac{\ln a_f - \ln a_i}{\pi C (F \Delta S)^2}$$

$$N_{if} = \frac{\ln (a_f / a_i)}{\pi C (F \Delta S)^2} \quad ◀$$

11.23 Find $N_{if} = f(a_i, a_f, \text{etc.})$ for

$$\Delta K = \frac{\Delta P}{t\sqrt{\pi a}}, \quad \frac{da}{dN} = C(\Delta K)^m$$

$$dN = \frac{da}{C(\Delta K)^m} = \frac{a^{m/2}\,da}{C\left[\Delta P/(t\sqrt{\pi})\right]^m}$$

$$N_{if} = \frac{1}{C}\left(\frac{t\sqrt{\pi}}{\Delta P}\right)^m \int_{a_i}^{a_f} a^{m/2}\,da$$

$$N_{if} = \frac{1}{C}\left(\frac{t\sqrt{\pi}}{\Delta P}\right)^m \frac{a_f^{\,1+m/2} - a_i^{\,1+m/2}}{1+m/2}$$ ◀

11.24 Derive $N_{if} = f(C, m, \Delta P, t, b, a_i, a_f)$

$$\Delta K = \frac{0.89\,\Delta P}{t\sqrt{b}}, \quad \frac{da}{dN} = C(\Delta K)^m$$

$$\int_{N_i}^{N_f} dN = \int_{a_i}^{a_f} \frac{da}{C(\Delta K)^m} = \frac{1}{C}\left(\frac{t\sqrt{b}}{0.89\,\Delta P}\right)^m \int_{a_i}^{a_f} da$$

$$N_{if} = \frac{a_f - a_i}{C}\left(\frac{t\sqrt{b}}{0.89\,\Delta P}\right)^m \quad ◀$$

Since $\Delta S = \dfrac{\Delta P}{2bt}$, an alternate form is:

$$N_{if} = \frac{a_f - a_i}{C}\left(\frac{1}{1.78\,\Delta S\sqrt{b}}\right)^m \quad ◁$$

11.25 $N_{if} = ?$ for $K = F\Delta S\sqrt{\pi a}$, $F = \text{const.}$

and
$$\frac{da}{dN} = \frac{C_2(\Delta K)^{m_2}}{(1-R)K_c - \Delta K} = \frac{C_2(F\Delta S\sqrt{\pi a})^{m_2}}{(1-R)K_c - F\Delta S\sqrt{\pi a}}$$

$$dN = \frac{da\left[(1-R)K_c - F\Delta S\sqrt{\pi a}\right]}{C_2(F\Delta S\sqrt{\pi a})^{m_2}}$$

$$N_{if} = \frac{(1-R)K_c}{C_2(F\Delta S\sqrt{\pi})^{m_2}}\int_{a_i}^{a_f}\frac{da}{a^{m_2/2}} - \frac{1}{C_2(F\Delta S\sqrt{\pi})^{m_2-1}}\int_{a_i}^{a_f}\frac{da}{a^{\frac{m_2-1}{2}}}$$

$$N_{if} = \frac{(1-R)K_c\left(a_f^{\frac{2-m_2}{2}} - a_i^{\frac{2-m_2}{2}}\right)}{C_2(F\Delta S\sqrt{\pi})^{m_2}\left(\frac{2-m_2}{2}\right)} - \frac{a_f^{\frac{3-m_2}{2}} - a_i^{\frac{3-m_2}{2}}}{C_2(F\Delta S\sqrt{\pi})^{m_2-1}\left(\frac{3-m_2}{2}\right)}$$ ◀

The above is limited by ($m \neq 2$, $m \neq 3$), but closed form equations are easily obtained for either of these cases, the integral $\int da/a = \ln a$ being needed for the first term if $m = 2$, and for the second if $m = 3$.

11.26 For an infinite colinear array of cracks, $F_P = 1/\sqrt{\sin \pi\alpha}$, $\alpha = a/b$, derive $N_{if} = f(a_i, a_f, C, \Delta P, b)$ for (a) $m = 2$, (b) possible for other m?

$$\frac{da}{dN} = C(\Delta K)^m, \quad m = 2, \quad \Delta K = \frac{\Delta P}{t\sqrt{b}} \frac{1}{\sqrt{\sin \frac{\pi a}{b}}}$$

$$\int_{a_i}^{a_f} dN = \int_{a_i}^{a_f} \frac{da}{C\left[\frac{\Delta P}{t\sqrt{b}} \frac{1}{\sqrt{\sin \frac{\pi a}{b}}}\right]^m}$$

$$N_{if} = \frac{1}{C}\left(\frac{t\sqrt{b}}{\Delta P}\right)^m \int_{a_i}^{a_f} \left(\sin \frac{\pi a}{b}\right)^{m/2} da$$

$$\text{for } m = 2, \quad N_{if} = b\left(\frac{t}{C\Delta P}\right)^2 \left(\frac{b}{\pi}\right) \int_{a_i}^{a_f} \sin \frac{\pi a}{b}\, d\left(\frac{\pi a}{b}\right)$$

$$N_{if} = \frac{1}{\pi}\left(\frac{bt}{C\,\Delta P}\right)^2 \left[\cos \frac{\pi a_i}{b} - \cos \frac{\pi a_f}{b}\right] \blacktriangleleft$$

Looking at a table of integrals indicates that closed form solutions are possible for even integers $m = 2, 4, 6$, etc. ◀

11.27

A center-cracked plate of 2024-T3 aluminum has given dimensions and initial crack length, and is cycled between given minimum and maximum loads. Calculate the number of cycles to grow the crack to failure.

Record the given dimensions and loads, and calculate R.

b	t	a_i	P_{max}	P_{min}	R
mm	mm	mm	N	N	P_{min}/P_{max}
50	4	2.00	60,000	18,000	0.300

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_0	m	γ
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		
353	34	1.420E-11	3.59	0.680

Obtain the definition of S and equations for F from Fig. 8.12(a).

$$S = \frac{P}{2bt}, \qquad \alpha = \frac{a}{b}, \qquad F \approx 1 \quad (\alpha \le 0.4)$$

$$F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} \qquad (\text{any } \alpha)$$

Calculate S_{max} and the crack length a_c to cause brittle fracture. Proceed by picking a trial a, then calculating α, F, and K, varying a until K = K_{Ic} is obtained. Also calculate the crack length a_o to cause fully plastic yielding from Fig. A.16(a).

$$S_{max} = \frac{P_{max}}{2bt}, \qquad K = FS_{max}\sqrt{\pi a}, \qquad a_c \rightarrow K_{Ic}$$

(11.27, p.2)

$$a_o = b\left[1 - \frac{P_{max}}{2bt\sigma_o}\right]$$

S_{max} MPa	a_c mm	α_c a_c / b	F_f	K_{Ic} $MPa\sqrt{m}$	a_o mm
150.0	14.84	0.297	1.0498	34.00	28.75

As F does not vary excessively, the life may be obtained from Eq. 11.32. Organize its input as follows: The smaller of a_c and a_o is the final crack length a_f. Based on the Walker method, calculate C for the applicable R value from Eq. 11.20. Choose an F value near that for a_i. Calculate ΔS, and for convenience, $1 - m/2$. Express all quantities in units of meters, MPa, or combinations thereof.

$$a_f = \text{MIN}(a_c, a_o), \qquad C = \frac{C_0}{(1-R)^{m(1-\gamma)}}, \qquad \Delta S = S_{max}(1-R)$$

a_f m	a_i m	C $\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	F	ΔS MPa	1 - m/2
0.01484	0.0020	2.139E-11	1.00	105.0	-0.795

Finally, calculate the number of cycles to grow the crack to failure.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m (1 - m/2)}$$

N_{if}, cycles = 46,552 ◀

Comment: The final crack length could be calculated by the $F = 1.00$ approximation for $\alpha \leq 0.4$, which gives $a_f = a_c = 16.35$ mm, causing the calculated life to change slightly to $N_{if} = 47{,}400$ cycles. ◀

11.28

A circular shaft of 7075-T6 aluminum has given diameter and initial half-circular surface crack length. It is cycled between given minimum and maximum bending moments. Calculate the number of cycles to grow the crack to failure.

Record the given dimensions and loads, and calculate R.

d	a_i	M_{max}	M_{min}	R
mm	mm	N-mm	N-mm	M_{min}/M_{max}
50	1.00	2,300,000	-460,000	-0.200

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_0	m	γ	γ
MPa	$MPa\sqrt{m}$	$\frac{m/cycle}{(MPa\sqrt{m})^m}$			R < 0
523	29	2.710E-11	3.70	0.641	0

Obtain the definition of S and equations for F from Fig. 8.17(d).

$$S = \frac{32M}{\pi d^3}, \qquad \alpha = \frac{a}{d}, \qquad F \approx 0.728 \quad (\alpha \le 0.35)$$

Calculate S_{max} and the crack length a_c to cause brittle fracture. Check that α_c is such that the F used is valid. Also calculate the crack length a_o to cause fully plastic yielding, very roughly approximating this case as a rectangular section from Fig. A.16(b), with same a and $b = d$, $t = d$.

$$S_{max} = \frac{32M_{max}}{\pi d^3}, \qquad a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{FS_{max}}\right)^2 \quad (F \approx 0.728 \text{ if } \alpha \le 0.35)$$

(11.28, p. 2)

$$a_o = b\left[1 - \frac{2}{b}\sqrt{\frac{M_{max}}{t\sigma_o}}\right], \qquad b, t \rightarrow d$$

S_{max} MPa	F	a_c mm	α_c a_c / d	Is F OK ?	a_o mm
187.4	0.728	14.38	0.288	yes	31.24

As F does not vary excessively, the life may be obtained from Eq. 11.32. Organize its input as follows: The smaller of a_c and a_o is the final crack length a_f. Based on the Walker method, calculate C for the applicable R value from Eq. 11.20. Calculate ΔS, and for convenience, $1 - m/2$. Express all quantities in units of meters, MPa, or combinations thereof.

$$a_f = \text{MIN}(a_c, a_o), \qquad C = \frac{C_0}{(1-R)^{m(1-\gamma)}}, \qquad \Delta S = S_{max}(1-R)$$

a_f m	a_i m	C $\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	F	ΔS MPa	1 - m/2
0.01438	0.0010	1.380E-11	0.728	224.9	-0.85

◀

Finally, calculate the number of cycles to grow the crack to failure.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m(1-m/2)}$$

N_{if}, cycles = 20,942 ◀

11.29

A bending member made of Man-Ten steel has a rectangular cross section with given dimensions and initial full-thicknes crack length. It is cycled between given minimum and maximum bending moments. Calculate the number of cycles to grow the crack to failure.

Record the given dimensions and loads, and calculate R.

b	t	a_i	M_{max}	M_{min}	R
mm	mm	mm	N-mm	N-mm	M_{min}/M_{max}
40	10	0.50	800,000	500,000	0.625

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_0	m	γ
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		
363	200	3.280E-12	3.13	0.928

Obtain the definition of S and equations for F from Fig. 8.13(a).

$$S = \frac{6M}{b^2 t}, \qquad \alpha = \frac{a}{b}, \qquad F \approx 1.12 \quad (\alpha \leq 0.4)$$

$$F = \sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}}\left[\frac{0.923 + 0.199\left(1 - \sin\frac{\pi\alpha}{2}\right)^4}{\cos\frac{\pi\alpha}{2}}\right] \quad (\text{any } \alpha)$$

Calculate S_{max} and the crack length a_c to cause brittle fracture. Proceed by picking a trial a, then calculating α, F, and K, varying a until $K = K_{Ic}$ is obtained. Also calculate the crack length a_o to cause fully plastic yielding from Fig. A.16(b).

(11.29, p.2)

$$S_{max} = \frac{6M_{max}}{b^2 t}, \qquad K = FS_{max}\sqrt{\pi a}, \qquad a_c \to K_{Ic}$$

$$a_o = b\left[1 - \frac{2}{b}\sqrt{\frac{M_{max}}{t\sigma_o}}\right]$$

S_{max} MPa	a_c mm	α_c a_c / b	$\pi\alpha_c / 2$	F_c	K_{Ic} $MPa\sqrt{m}$	a_o mm
300.0	26.40	0.660	1.037	2.3151	200.00	10.31

The smaller of a_c and a_o is the final crack length a_f. As F does not vary excessively up to the controlling a_o, as $a_o / b = 0.258$, the life may be obtained from Eq. 11.32. Organize its input as follows: Based on the Walker method, calculate C for the applicable R value from Eq. 11.20. Choose an F value near that for a_i. Calculate ΔS, and for convenience, $1 - m/2$. Express all quantities in units of meters, MPa, or combinations thereof.

$$a_f = \text{MIN}(a_c, a_o), \qquad C = \frac{C_0}{(1-R)^{m(1-\gamma)}}, \qquad \Delta S = S_{max}(1-R)$$

a_f m	a_i m	C $\frac{\text{m/cycle}}{(MPa\sqrt{m})^m}$	F	ΔS MPa	1 - m/2
0.01031	0.00050	4.091E-12	1.12	112.5	-0.565

Finally, calculate the number of cycles to grow the crack to failure.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m(1 - m/2)}$$

N_{if}, cycles = 1,154,249 ◀

11.30

A tension member made of 7075-T6 aluminum has a rectangular cross section with given dimensions and initial quarter-circular corner crack length. It is cycled between given minimum and maximum stresses. Calculate the number of cycles to grow the crack to failure.

Record the given dimensions and stresses, and calculate R.

b	t	a_i	S_{max}	S_{min}	R
mm	mm	mm	MPa	MPa	S_{min}/S_{max}
40	15	0.50	336	-68	-0.202

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_0	m	γ	γ
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			R < 0
523	29	2.710E-11	3.70	0.641	0

Obtain the definition of S and equations for F from Fig. 8.17(c).

$$S = \frac{P}{bt}, \qquad F \approx 0.722 \quad (a/t \le 0.35,\ a/b \le 0.2)$$

Calculate the crack length a_c to cause brittle fracture. Check that a_c is such that the F used is valid. Also calculate the crack length a_o to cause fully plastic yielding, conservatively approximating this case as a through-thickness crack with same a from Fig. A.16(d).

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{FS_{\max}}\right)^2, \qquad (F \approx 0.722 \text{ if } a/t \le 0.35,\ a/b \le 0.2)$$

$$P' = \frac{P_{\max}}{bt\sigma_o} = \frac{S_{\max}}{\sigma_o}, \qquad a_o = b\left[P' + 1 - \sqrt{2P'(P'+1)}\right]$$

(11.30, p. 2)

F	a_c mm	a_c / t	a_c / b	Is F OK ?	P'	a_o mm
0.722	4.549	0.303	0.114	yes	0.6424	7.589

As F does not vary excessively, the life may be obtained from Eq. 11.32. Organize its input as follows: The smaller of a_c and a_o is the final crack length a_f. Based on the Walker method, calculate C for the applicable R value from Eq. 11.20. Choose an F value near that for a_i. Calculate ΔS, and for convenience, $1 - m/2$. Express all quantities in units of meters, MPa, or combinations thereof.

$$a_f = \text{MIN}(a_c, a_o), \qquad C = \frac{C_0}{(1-R)^{m(1-\gamma)}}, \qquad \Delta S = S_{max} - S_{min}$$

a_f m	a_i m	C $\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	F	ΔS MPa	1 - m/2
0.004549	0.0005	1.370E-11	0.722	404.0	-0.85

Finally, calculate the number of cycles to grow the crack to failure.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m (1-m/2)}$$

N_{if}, cycles = 4,242 ◀

11.31 Bending member, AISI 4340 steel

$b = 60,\ t = 12$ mm, $M_{min} = 0.8,\ M_{max} = 4.0$ kN·m

$N_{if} = 60,000$ cycles, $a_f = 14$ mm

$a_i = ?$

$C_1 = 5.11 \times 10^{-10}$

$m_1 = 3.24,\ \gamma = 0.42$

(for MPa$\sqrt{m}$ and mm/cycle, $R = 0$)

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m (1-m/2)}$$

$$C = \frac{C_1}{(1-R)^{m_1(1-\gamma)}} = \frac{5.11 \times 10^{-13}}{(0.8)^{3.24(1-0.42)}} = 7.77 \times 10^{-13}$$

(for m/cycle)

$R = M_{min}/M_{max} = 0.20,\ m = m_1$

$$\Delta S = \frac{6\,\Delta M}{b^2 t} = \frac{6\,(4.0 - 0.8) \times 1000 \text{ N·m}}{(60 \text{ mm})^2\,(0.012 \text{ m})} = 444 \text{ MPa}$$

$a_f/b = 0.25 < 0.4$, so $F \approx 1.12$ (Fig. 8.13)

Substitute $a_f = 0.014$ m, and solve for a_i:

$a_i = 0.000475$ m $= 0.475$ mm ◀

11.32

A circular rod with 80 mm diameter is made of the ferritic-pearlitic steel ASTM A572 with given σ_o and K_{Ic}. It is subjected to a cyclic axial force ΔP at $R = 0.6$. Determine the ΔP that will cause a circumferential crack to grow from $a_i = 0.5$ mm to $a_f = 10$ mm in two million cycles.

Record the given dimensions, R, and life.

d	b = d / 2	a_i	a_f	R	N_{if}
mm	mm	mm	mm	P_{min}/P_{max}	cycles
80	40	0.50	10.00	0.600	2,000,000

Record the given materials properties and obtain generic crack growth constants from Table 11.1.

σ_o	K_{Ic}	C_0	m	γ
MPa	MPa $\sqrt{m}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		
345	200	6.890E-12	3.00	0.500

Obtain the definition of S and equations for F from Fig. 8.14(a).

$$S = \frac{P}{\pi b^2}, \qquad \alpha = \frac{a}{b}, \qquad \beta = 1 - \alpha, \qquad F \approx 1.12 \quad (\alpha \le 0.21)$$

$$F = \frac{1}{2\beta^{1.5}}\left[1 + \frac{1}{2}\beta + \frac{3}{8}\beta^2 - 0.363\beta^3 + 0.731\beta^4\right] \quad (\text{any } \alpha)$$

Calculate F for the given final crack length.

α_f	$\beta = 1 - \alpha$	F_f
a_f / b		
0.250	0.750	1.2810

(11.32, p. 2)

As F does not vary excessively, the life may be obtained from Eq. 11.32. Organize its input as follows: Based on the Walker method, calculate C for the applicable R value from Eq. 11.20. Choose an F value near that for a_i. For convenience, calculate $1 - m/2$. Then vary ΔS to obtain the given life, and calculate the corresponding ΔP. Express all quantities in units of meters, MPa, or combinations thereof.

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}}, \qquad \Delta P = \pi b^2 \Delta S$$

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\,\sqrt{\pi})^m (1-m/2)}$$

a_f	a_i	C	F	1 - m/2	ΔS
m	m	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			MPa
0.01	0.0005	2.724E-11	1.12	-0.5	54.62

N_{if}, cycles = 2,000,000 ΔP, kN = 274.6 ◀

Comment: An iterative solution can be avoided, but it is then necessary to algebraically solve Eq. 11.32 for ΔS.

11.33

For the center-cracked plate of 2024-T3 aluminum of Prob. 11.27, failure occurs due to brittle fracture at $a_f = 16.35$ mm and $N_{if} = 47{,}400$ cycles. (a) Determine the largest initial crack size a_i that can be permitted so that the life is at least 100,000 cycles. (b) Assuming that this a_i can be found by inspection, what inspection interval in cycles would give a reasonable safety factor in life?

(a) Record the given dimensions, loads, final crack length, and life.

b	t	P_{max}	P_{min}	R	a_f	N_{if}
mm	mm	N	N	P_{min}/P_{max}	mm	cycles
50	4	60,000	18,000	0.300	16.35	100,000

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_o	m	γ
MPa	MPa $\sqrt{m}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		
353	34	1.420E-11	3.59	0.680

Obtain the definition of S and F from Fig. 8.12(a).

$$S = \frac{P}{2bt}, \qquad F \approx 1 \quad (\alpha = a/b \leq 0.4)$$

Calculate S_{max} from P_{max} and check that the approximate F is valid.

S_{max}	α_f	Is F OK ?
MPa	a_f / b	
150.0	0.327	yes

(11.33, p. 2)

As F does not vary excessively, Eq. 11.32 applies for life calculation. Organize its input as follows: Based on the Walker method, calculate C for the applicable R value from Eq. 11.20. Choose the above approximate F value for small a_i. Calculate ΔS, and for convenience, $1 - m/2$. Vary a_i to obtain the desired N_{if}. Express all quantities in units of meters, MPa, or combinations thereof.

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}}, \qquad \Delta S = S_{\max}(1-R)$$

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m (1-m/2)}$$

a_i	a_f	C	F	ΔS	1 - m/2
m	m	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		MPa	
0.000892	0.01635	2.139E-11	1.00	105.0	-0.795

N_{if}, cycles = 100,000 a_i, mm = 0.892 ◀

Comment: An iterative solution can be avoided, but it is then necessary to algebraically solve Eq. 11.32 for a_i.

(b) A reasonable safety factor in life would be in the range $X_N = 3$ to 5. Choosing the latter gives an inspection interval $N_p = N_{if} / X_N$ as below.

N_{if} cycles	X_N	N_p cycles
100,000	5.00	20,000

◀

11.34

A bending member made of 7075-T6 aluminum has a rectangular cross section with given dimensions and initial full-thicknes crack length. It is cycled between zero and a given maximum bending moment. (a) What is the safety factor in life if the expected service life is 80,000 cycles? (b) What inspection period gives a safety factor of five on life?

Record the given dimensions, loads, and R.

b	t	a_i	M_{max}	M_{min}	R
mm	mm	mm	N-mm	N-mm	M_{min}/M_{max}
50	12	0.25	500,000	0	0.00

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_0	m	γ
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		
523	29	2.710E-11	3.70	0.641

Obtain the definition of S and equations for F from Fig. 8.13(a).

$$S = \frac{6M}{b^2 t}, \qquad \alpha = \frac{a}{b}, \qquad F \approx 1.12 \quad (\alpha \le 0.4)$$

$$F = \sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}}\left[\frac{0.923 + 0.199\left(1-\sin\frac{\pi\alpha}{2}\right)^4}{\cos\frac{\pi\alpha}{2}}\right] \qquad (\text{any } \alpha)$$

Calculate S_{max} and the crack length a_c to cause brittle fracture. Proceed by picking a trial a, then calculating α, F, and K, varying a until $K = K_{Ic}$ is obtained. Also calculate the crack length a_o to cause fully plastic yielding from Fig. A.16(b).

(11.34, p.2)

$$S_{max} = \frac{6M_{max}}{b^2 t}, \qquad K = FS_{max}\sqrt{\pi a}, \qquad a_c \rightarrow K_{Ic}$$

$$a_o = b\left[1 - \frac{2}{b}\sqrt{\frac{M_{max}}{t\sigma_o}}\right]$$

S_{max}	a_c	α_c	$\pi\alpha_c / 2$	F_c	K_{Ic}	a_o
MPa	mm	a_c / b			$\text{MPa}\sqrt{\text{m}}$	mm
100.0	18.79	0.376	0.590	1.1936	29.00	32.15

As F does not vary excessively, the life may be obtained from Eq. 11.32. Organize its input as follows: The smaller of a_c and a_o is the final crack length a_f. Note that $C = C_0$ as $R = 0$. Choose an F value near that for a_i. Calculate ΔS, and for convenience, $1 - m/2$. Express all quantities in units of meters, MPa, or combinations thereof.

$$a_f = \text{MIN}(a_c, a_o), \qquad C = \frac{C_0}{(1-R)^{m(1-\gamma)}}, \qquad \Delta S = S_{max}(1-R)$$

a_f	a_i	C	F	ΔS	1 - m/2
m	m	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		MPa	
0.01879	0.00025	2.710E-11	1.15	100.0	-0.85

Calculate the number of cycles to grow the crack to failure.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\,\sqrt{\pi})^m (1 - m/2)}$$

N_{if}, cycles = 139,273 ◁

(11.34, p.3)

(a) Calculate actual safety factor in life, and (b) the required inspection perion N_p for the desired safety factor in life.

$$(a)\ X_N = \frac{N_{if}}{\hat{N}}, \qquad (b)\ N_p = \frac{N_{if}}{X_N}$$

N_{if} cycles	$\hat{N}$ cycles	X_N (actual)
139,273	80,000	1.741
		(a)

N_{if} cycles	X_N (desired)	N_p cycles
139,273	5.000	27,855
		(b)

11.35

A sheet of 2024-T3 aluminum has a row of rivet holes as in Fig. 8.24 with given dimensions and initial crack lengths on both sides of each hole. It is cycled between given minimum and maximum stresses. Calculate the number of cycles to grow the cracks to $\ell = 4$ mm length.

Record the given dimensions and stresses and calculate R.

2b	b	d	t	S_{max}	S_{min}	R
mm	mm	mm	mm	MPa	MPa	S_{min}/S_{max}
24	12	4.0	3.0	28.0	8.5	0.304

ℓ_i	a_i	ℓ_f	a_f
mm	mm	mm	mm
0.50	2.50	4.00	6.00

$$a = \ell + \frac{d}{2}$$

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_0	m	γ
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		
353	34	1.420E-11	3.59	0.680

Obtain the definition of S and equations for K from Fig. 8.23(b), K_2 case.

$$S = \frac{P}{2bt}, \qquad \alpha = \frac{a}{b}, \qquad K_2 = \frac{0.89P}{t\sqrt{b}} \quad (0.12 \le \alpha \le 0.65)$$

Calculate P_{max}. Check $K_{max} < K_{Ic}$ at the given ℓ_f, and also that α_i and α_f are such that the K_2 used is valid. Also calculate the crack length a_o to cause fully plastic yielding, noting that Fig. A.16(a) applies.

(11.35, p.2)

$$P_{max} = 2btS_{max}\,, \qquad K_{max} = \frac{0.89P_{max}}{t\sqrt{b}} = \frac{0.89P_{max}\sqrt{b}}{tb}$$

$$a_o = b\left[1 - \frac{P_{max}}{2bt\sigma_o}\right]$$

P_{max}	K_{max}	α_i	α_f	$K_{max} < K_{Ic}$?	a_o	$a_f < a_o$?
N	MPa $\sqrt{m}$	a_i / b	a_f / b	α OK ?	mm	
2016	5.46	0.208	0.500	yes/yes	11.05	yes

As the K approximation is valid over the range of α that occurs, the result of Prob. 11.24 can be applied. Organize its input as follows: Based on the Walker method, calculate C for the applicable R value from Eq. 11.20. Calculate ΔS and use the second form for N_{if}, which arises from the first by substitution of the definition of S. Express all quantities in units of meters, MPa, or combinations thereof.

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}}\,, \qquad \Delta S = S_{max}(1-R)$$

$$N_{if} = \frac{a_f - a_i}{C}\left(\frac{t\sqrt{b}}{0.89\,\Delta P}\right)^m = \frac{a_f - a_i}{C}\left(\frac{1}{1.78\,\Delta S\sqrt{b}}\right)^m$$

a_f	a_i	C	ΔS	b
m	m	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$	MPa	m
0.00600	0.00250	2.152E-11	19.5	0.01200

N_{if}, cycles = 1,345,594 ◀

11.36

The structural member of Prob. 8.16 is made of the ferritic-pearlitic steel ASTM A572, with σ_o and low temperature, dynamic loading K_{Ic} as given. The member is loaded cyclically between zero and a moment that may be as large as M_{max} = 176 kN·m. (a) At what crack length is brittle fracture expected? (b) For a_i = 2.5 mm and a safety factor of 5.0 on life, how many cycles can be permitted before inspection?

Model the flange as a single-edge-notched member in tension. This should be reasonably accurate for small cracks, but for longer cracks becomes increasingly conservative due to to the real member having stiffening from the web. Dimensions and loads are then as follows:

b	t	a_i	M_{max}	M_{min}	R
mm	mm	mm	N-mm	N-mm	M_{min}/M_{max}
203	13.1	2.50	1.76E+08	0	0.00

Record the given materials properties and obtain generic crack growth constants from Table 11.1.

σ_o	K_{Ic}	C_0	m	γ	γ
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			R < 0
345	40	6.890E-12	3.00	0.500	0

Obtain F from Fig. 8.12(c).

$$F \approx 1.12 \quad (\alpha \le 0.13), \qquad \alpha = \frac{a}{b}$$

(11.36, p. 2)

Approximate S_{max} as the bending stress at mid-thickness of the flange, using I_x for bending about the member x-axis from Fig. P8.16. Calculate the crack length a_c to cause brittle fracture, and check that α_c is such that the F used is valid. Also, conservatively approximate the crack length a_o to cause fully plastic yielding of the flange by using Fig. A.16(d).

$$S_{max} = \frac{M_{max}\, y}{I}$$

$$S_{max} = \frac{(176{,}000{,}000\,\mathrm{N\cdot mm})(303/2 - 13.1/2)\,\mathrm{mm}}{(1.29\times10^{-4}\,\mathrm{m}^4)(1000\,\mathrm{mm/m})^4} = 197.8\,\mathrm{MPa}$$

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{F S_{max}}\right)^2 \qquad (F \approx 1.12 \text{ if } \alpha \le 0.13)$$

$$P' = \frac{P_{max}}{bt\sigma_o} = \frac{S_{max}}{\sigma_o}, \qquad a_o = b\left[P' + 1 - \sqrt{2P'(P'+1)}\right]$$

S_{max} MPa	F	a_c mm	α_c a_c/b	Is F OK ?	P'	a_o mm
197.76	1.120	10.38	0.051	yes	0.5732	46.7

As F does not vary excessively, the life may be obtained from Eq. 11.32. Organize its input as follows: The smaller of a_c and a_o is the final crack length a_f. Based on the Walker method, $C = C_0$ due to $R = 0$. Calculate ΔS, and for convenience, $1 - m/2$. Express all quantities in units of meters, MPa, or combinations thereof.

(11.36, p. 3)

$$a_f = \text{MIN}(a_c, a_o), \qquad C = \frac{C_0}{(1-R)^{m(1-\gamma)}}, \qquad \Delta S = S_{\max}(1-R)$$

a_f	a_i	C	F	ΔS	1 - m/2
m	m	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		MPa	
0.01038	0.00250	6.890E-12	1.120	197.76	-0.5

Finally, calculate the number of cycles to grow the crack to failure and the required inspection period.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m(1-m/2)}, \qquad N_p = \frac{N_{if}}{X_N}$$

N_{if}	X_N	N_p
cycles		cycles
48,864	5.0	9,773

11.37

A center-cracked plate of AISI 4340 steel has given dimensions and initial crack length, and it is cycled between given minimum and maximum loads. (a) Estimate the crack length at failure. (b) Calculate the life in cycles from Eq. 11.32, and comment on whether this result is expected to be accurate. (c) Calculate the life from numerical integration and compare to (b).

Record the given dimensions and loads, and calculate R.

b	t	a_i	P_{max}	P_{min}	R
mm	mm	mm	N	N	P_{min}/P_{max}
38	6	1.00	120,000	40,000	0.3333

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_0	m	γ
MPa	MPa $\sqrt{m}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		
1255	130	5.110E-13	3.24	0.420

Obtain the definition of S and equations for F from Fig. 8.12(a).

$$S = \frac{P}{2bt}, \qquad \alpha = \frac{a}{b}, \qquad F \approx 1 \quad (\alpha \leq 0.4)$$

$$F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} \qquad \text{(any } \alpha\text{)}$$

(a) Calculate S_{max} and the crack length a_c to cause brittle fracture. Proceed by picking a trial a, then calculating α, F, and K, varying a until $K = K_{Ic}$ is obtained. Also calculate the crack length a_o to cause fully plastic yielding from Fig. A.16(a).

(11.37, p.2)

$$S_{\max} = \frac{P_{\max}}{2bt}, \qquad K = FS_{\max}\sqrt{\pi a}\,, \qquad a_c \rightarrow K_{Ic}$$

$$a_o = b\left[1 - \frac{P_{\max}}{2bt\sigma_o}\right]$$

S_{max}	a_c	α_c	F_f	K_{Ic}	a_o
MPa	mm	a_c / b		MPa $\sqrt{m}$	mm
263.2	28.80	0.758	1.6424	130.00	30.03

As F changes excessively, the life from Eq. 11.32 is expected to be inaccurate. But proceed as requested and organize the input as follows: The smaller of a_c and a_o is the final crack length a_f. Based on the Walker method, calculate C for the applicable R value from Eq. 11.20. Guess at an F value nearer to that for a_i than for a_f. Calculate ΔS, and for convenience, $1 - m/2$. Express all quantities in units of meters, MPa, or combinations thereof.

$$a_f = \mathrm{MIN}(a_c, a_o), \qquad C = \frac{C_0}{(1-R)^{m(1-\gamma)}}, \qquad \Delta S = S_{\max}(1-R)$$

a_f	a_i	C	F	ΔS	1 - m/2
m	m	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		MPa	
0.02880	0.0010	1.095E-12	1.10	175.4	-0.62

◀

(b) Finally, calculate the number of cycles to grow the crack to failure.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m(1-m/2)}$$

N_{if}, cycles = 575,528 ◀

(11.37, p. 3)

(c) To proceed with numerical integration, express a_f, a_i, and b, in meters. Pick a number of increments n such that the increment factor is around $r = 1.1$.

$$r = (a_f / a_i)^{1/n}$$

a_f, m	a_i, m	b, m	n	r
0.02880	0.00100	0.03800	36	1.097835

For each crack length starting with a_i, calculate α, F, ΔK, and $y = dN/da$. Increment crack lengths by the factor r until a_f is reached. Then, for each group of three points, 0-1-2, 2-3-4, 4-5-6, etc., apply Eq. 11.38 to obtain ΔN values, and also calculate the cumulative sum, the last value of which is the calculated life N_{if}.

$$\alpha_j = \frac{a_j}{b}, \qquad F_j = F(\alpha_j), \qquad \Delta K_j = F_j \Delta S \sqrt{\pi a_j}$$

$$y_j = \frac{dN}{da} = \frac{1}{C(\Delta K_j)^m}, \qquad a_{j+1} = r a_j$$

$$\Delta N_{j+2} = \frac{a_j(r^2-1)}{6r}\left[y_j r(2-r) + y_{j+1}(r+1)^2 + y_{j+2}(2r-1)\right].$$

j	a m	α = a/b	F	ΔK $MPa\sqrt{m}$	y = dN/da cycles/m	ΔN cycles	Σ(ΔN) cycles
0	0.001000	0.0263	1.0003	9.84	5.545E+08	0	0
1	0.001098	0.0289	1.0004	10.31	4.766E+08	0	0
2	0.001205	0.0317	1.0005	10.80	4.096E+08	97,717	97,717
3	0.001323	0.0348	1.0006	11.32	3.520E+08	0	0
4	0.001453	0.0382	1.0007	11.86	3.025E+08	86,988	184,705
5	0.001595	0.0420	1.0008	12.43	2.599E+08	0	0

(11.37, p.4)

6	0.001751	0.0461	1.0010	13.02	2.233E+08	77,416	262,121
7	0.001922	0.0506	1.0012	13.65	1.918E+08	0	0
8	0.002110	0.0555	1.0014	14.30	1.648E+08	68,870	330,991
9	0.002317	0.0610	1.0017	14.99	1.415E+08	0	0
10	0.002543	0.0669	1.0021	15.71	1.215E+08	61,234	392,225
11	0.002792	0.0735	1.0026	16.47	1.043E+08	0	0
12	0.003065	0.0807	1.0031	17.27	8.952E+07	54,398	446,623
13	0.003365	0.0886	1.0038	18.11	7.680E+07	0	0
14	0.003694	0.0972	1.0046	18.99	6.585E+07	48,265	494,888
15	0.004056	0.1067	1.0055	19.91	5.643E+07	0	0
16	0.004452	0.1172	1.0067	20.89	4.833E+07	42,745	537,633
17	0.004888	0.1286	1.0081	21.92	4.135E+07	0	0
18	0.005366	0.1412	1.0099	23.00	3.535E+07	37,752	575,384
19	0.005891	0.1550	1.0121	24.16	3.018E+07	0	0
20	0.006468	0.1702	1.0147	25.38	2.573E+07	33,204	608,588
21	0.007100	0.1869	1.0180	26.67	2.189E+07	0	0
22	0.007795	0.2051	1.0220	28.06	1.858E+07	29,022	637,609
23	0.008558	0.2252	1.0269	29.54	1.572E+07	0	0
24	0.009395	0.2472	1.0331	31.14	1.326E+07	25,124	662,734
25	0.010314	0.2714	1.0407	32.87	1.113E+07	0	0
26	0.011323	0.2980	1.0502	34.75	9.288E+06	21,431	684,165
27	0.012431	0.3271	1.0622	36.83	7.697E+06	0	0
28	0.013647	0.3591	1.0774	39.14	6.320E+06	17,862	702,027
29	0.014982	0.3943	1.0967	41.74	5.129E+06	0	0
30	0.016448	0.4328	1.1216	44.73	4.100E+06	14,344	716,371
31	0.018057	0.4752	1.1540	48.22	3.213E+06	0	0
32	0.019824	0.5217	1.1970	52.41	2.454E+06	10,832	727,203
33	0.021763	0.5727	1.2553	57.59	1.808E+06	0	0
34	0.023892	0.6287	1.3368	64.25	1.268E+06	7,353	734,556
35	0.026230	0.6903	1.4558	73.31	8.269E+05	0	0
36	0.028796	0.7578	1.6424	86.67	4.809E+05	4,072	738,628

N_{if}, cycles = 738,628 ◀

This result is noted to differ considerably from the calculation by Eq. 11.32, where the value of constant F to assume is problematical.

11.38

For the center-cracked plate of 2024-T3 aliminum in Prob. 11.27, estimate the life by numerical integration.

Record the given dimensions and loads, and calculate R.

b	t	a_i	P_{max}	P_{min}	R
mm	mm	mm	N	N	P_{min}/P_{max}
50	4	2.00	60,000	18,000	0.3000

Obtain materials properties from Table 11.2. Based on the Walker method, calculate C for the applicable R value from Eq. 11.20.

σ_o	K_{Ic}	C_0	m	γ	C
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$
353	34	1.420E-11	3.59	0.680	2.139E-11

From Fig. 8.12(a), obtain the definition of S and the equation for F.

$$S = \frac{P}{2bt}, \qquad \alpha = \frac{a}{b}$$

$$F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} \qquad \text{(any } \alpha\text{)}$$

Calculate S_{max} and ΔS. Also, determine the crack length a_c to cause brittle fracture. Proceed by picking a trial a, then calculating α, F, and K, varying a until $K = K_{Ic}$ is obtained. In addition, calculate the crack length a_o to cause fully plastic yielding from Fig. A.16(a).

$$S_{max} = \frac{P_{max}}{2bt}, \qquad \Delta S = S_{max}(1-R)$$

$$K = FS_{max}\sqrt{\pi a}, \qquad a_c \rightarrow K_{Ic}$$

(11.38, p. 2)

$$a_o = b\left[1 - \frac{P_{max}}{2bt\sigma_o}\right]$$

S_{max} MPa	ΔS MPa	a_c mm	α_c a_c/b	F_f	K_{Ic} $MPa\sqrt{m}$	a_o mm
150.0	105.00	14.84	0.297	1.0498	34.00	28.75

The smaller of a_c and a_o is the final crack length a_f. Express a_f, and also a_i and b, in meters. Pick a number of increments n such that the increment factor is around $r = 1.1$.

$$a_f = \text{MIN}(a_c, a_o), \qquad r = (a_f/a_i)^{1/n}$$

a_f, m	a_i, m	b, m	n	r
0.01484	0.00200	0.05000	20	1.105401

For each crack length starting with a_i, calculate α, F, ΔK, and $y = dN/da$. Increment crack lengths by the factor r until a_f is reached. Then, for each group of three points, 0-1-2, 2-3-4, 4-5-6, etc., apply Eq. 11.38 to obtain ΔN values, and also calculate the cumulative sum, the last value of which is the calculated life N_{if}.

$$\alpha_j = \frac{a_j}{b}, \qquad F_j = F(\alpha_j), \qquad \Delta K_j = F_j \Delta S \sqrt{\pi a_j}$$

$$y_j = \frac{dN}{da} = \frac{1}{C(\Delta K_j)^m}, \qquad a_{j+1} = r a_j$$

$$\Delta N_{j+2} = \frac{a_j(r^2-1)}{6r}\left[y_j r(2-r) + y_{j+1}(r+1)^2 + y_{j+2}(2r-1)\right]$$

(11.38, p.3)

j	a m	α = a/b	F	ΔK $MPa\sqrt{m}$	y = dN/da cycles/m	ΔN cycles	Σ(ΔN) cycles
0	0.002000	0.0400	1.0007	8.329	2.316E+07	0	0
1	0.002211	0.0442	1.0009	8.759	1.934E+07	0	0
2	0.002444	0.0489	1.0011	9.210	1.614E+07	8,577	8,577
3	0.002701	0.0540	1.0014	9.686	1.347E+07	0	0
4	0.002986	0.0597	1.0017	10.187	1.124E+07	7,302	15,879
5	0.003301	0.0660	1.0021	10.714	9.379E+06	0	0
6	0.003649	0.0730	1.0025	11.270	7.822E+06	6,211	22,090
7	0.004033	0.0807	1.0031	11.856	6.521E+06	0	0
8	0.004459	0.0892	1.0038	12.474	5.434E+06	5,276	27,366
9	0.004928	0.0986	1.0047	13.126	4.525E+06	0	0
10	0.005448	0.1090	1.0058	13.816	3.765E+06	4,474	31,840
11	0.006022	0.1204	1.0071	14.545	3.131E+06	0	0
12	0.006657	0.1331	1.0088	15.317	2.600E+06	3,782	35,622
13	0.007358	0.1472	1.0108	16.137	2.156E+06	0	0
14	0.008134	0.1627	1.0134	17.009	1.785E+06	3,183	38,805
15	0.008991	0.1798	1.0166	17.939	1.474E+06	0	0
16	0.009939	0.1988	1.0205	18.935	1.214E+06	2,659	41,464
17	0.010987	0.2197	1.0255	20.005	9.969E+05	0	0
18	0.012145	0.2429	1.0318	21.162	8.148E+05	2,197	43,661
19	0.013425	0.2685	1.0397	22.420	6.622E+05	0	0
20	0.014840	0.2968	1.0498	23.800	5.344E+05	1,783	45,444

N_{if}, cycles = 45,444 ◀

11.39

Consider the bending member with a rectangular cross section, made of AISI 4340 steel, in Prob. 11.31. Determine the initial crack length by numerical integration.

Record the given dimensions, moments, final crack length, and life.

b	t	M_{max}	M_{min}	R	a_f	N_{if}
mm	mm	N	N	M_{min}/M_{max}	mm	cycles
60	12	4,000,000	800,000	0.200	14.00	60,000

Obtain materials properties from Table 11.2. Based on the Walker method, calculate C for the applicable R value from Eq. 11.20.

σ_o	K_{Ic}	C_0	m	γ	C
MPa	MPa $\sqrt{m}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$
1255	130	5.110E-13	3.24	0.420	7.772E-13

From Fig. 8.13(a), obtain the definition of S and the equation for F.

$$S = \frac{6M}{b^2 t}, \qquad \alpha = \frac{a}{b}$$

$$F = \sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}}\left[\frac{0.923 + 0.199\left(1 - \sin\frac{\pi\alpha}{2}\right)^4}{\cos\frac{\pi\alpha}{2}}\right] \quad \text{(any } \alpha\text{)}$$

Calculate S_{max} and ΔS.

S_{max}	ΔS
MPa	MPa
555.6	444.44

$$S_{max} = \frac{6M_{max}}{b^2 t}, \qquad \Delta S = S_{max}(1 - R)$$

(11.39, p.2)

Initially estimate $a_i = 0.475$ mm based on the result of Prob. 11.31. Pick a number of increments n such that the increment factor is around r = 1.1. Then vary a_i until the given N_{if} is obtained. For use in the numerical integration calculations, express a_i, a_f, and b in meters.

$$r = (a_f / a_i)^{1/n}$$

	a_i mm	a_i m	a_f m	b m	n	r	N_{if} cycles	
(1)	0.4750	0.000475	0.01400	0.06000	36	1.098544	65,894	
(2)	**0.5476**	0.000548	0.01400	0.06000	36	1.094214	60,000	◀

(1) 1st try (2) final

For each crack length starting with a_i, calculate α, F, ΔK, and $y = dN/da$. Increment crack lengths by the factor r until a_f is reached. Then, for each group of three points, 0-1-2, 2-3-4, 4-5-6, etc., apply Eq. 11.38 to obtain ΔN values, and also calculate the cumulative sum, the last value of which is the calculated life N_{if}.

$$\alpha_j = \frac{a_j}{b}, \qquad F_j = F(\alpha_j), \qquad \Delta K_j = F_j \Delta S \sqrt{\pi a_j}$$

$$y_j = \frac{dN}{da} = \frac{1}{C(\Delta K_j)^m}, \qquad a_{j+1} = r a_j$$

$$\Delta N_{j+2} = \frac{a_j (r^2 - 1)}{6r}\left[y_j r(2-r) + y_{j+1}(r+1)^2 + y_{j+2}(2r-1)\right]$$

j	a m	$\alpha = a/b$	F	ΔK MPa$\sqrt{m}$	y = dN/da cycles/m	ΔN cycles	$\Sigma(\Delta N)$ cycles
0	0.0005476	0.0091	1.1110	20.480	7.257E+07	0	0
1	0.0005992	0.0100	1.1100	21.403	6.291E+07	0	0

(11.39, p.3)

2	0.0006556	0.0109	1.1089	22.367	5.454E+07	6,791	6,791
3	0.0007174	0.0120	1.1077	23.372	4.730E+07	0	0
4	0.0007850	0.0131	1.1065	24.420	4.103E+07	6,113	12,904
5	0.0008589	0.0143	1.1051	25.513	3.561E+07	0	0
6	0.0009398	0.0157	1.1036	26.652	3.091E+07	5,510	18,414
7	0.0010284	0.0171	1.1020	27.838	2.684E+07	0	0
8	0.0011253	0.0188	1.1002	29.073	2.332E+07	4,973	23,388
9	0.0012313	0.0205	1.0983	30.360	2.027E+07	0	0
10	0.0013473	0.0225	1.0963	31.699	1.762E+07	4,496	27,884
11	0.0014742	0.0246	1.0941	33.093	1.533E+07	0	0
12	0.0016131	0.0269	1.0918	34.543	1.334E+07	4,071	31,955
13	0.0017651	0.0294	1.0892	36.050	1.162E+07	0	0
14	0.0019314	0.0322	1.0866	37.617	1.012E+07	3,694	35,649
15	0.0021134	0.0352	1.0837	39.246	8.822E+06	0	0
16	0.0023125	0.0385	1.0807	40.938	7.694E+06	3,359	39,008
17	0.0025304	0.0422	1.0775	42.696	6.715E+06	0	0
18	0.0027688	0.0461	1.0741	44.522	5.863E+06	3,061	42,068
19	0.0030296	0.0505	1.0705	46.418	5.122E+06	0	0
20	0.0033151	0.0553	1.0668	48.387	4.477E+06	2,795	44,864
21	0.0036274	0.0605	1.0630	50.433	3.915E+06	0	0
22	0.0039691	0.0662	1.0590	52.559	3.424E+06	2,558	47,422
23	0.0043431	0.0724	1.0550	54.770	2.996E+06	0	0
24	0.0047523	0.0792	1.0510	57.072	2.622E+06	2,344	49,766
25	0.0052000	0.0867	1.0469	59.472	2.294E+06	0	0
26	0.0056899	0.0948	1.0430	61.979	2.007E+06	2,149	51,915
27	0.0062260	0.1038	1.0393	64.603	1.755E+06	0	0
28	0.0068125	0.1135	1.0360	67.359	1.533E+06	1,968	53,882
29	0.0074544	0.1242	1.0331	70.265	1.337E+06	0	0
30	0.0081567	0.1359	1.0309	73.343	1.163E+06	1,794	55,677
31	0.0089252	0.1488	1.0296	76.622	1.010E+06	0	0
32	0.0097660	0.1628	1.0294	80.139	8.730E+05	1,622	57,299
33	0.0106861	0.1781	1.0308	83.941	7.513E+05	0	0
34	0.0116929	0.1949	1.0341	88.088	6.426E+05	1,445	58,744
35	0.0127946	0.2132	1.0399	92.659	5.454E+05	0	0
36	0.0140000	0.2333	1.0488	97.754	4.586E+05	1,256	60,000

11.40

A single-edge-cracked plate made of Man-Ten steel has given dimensions and initial crack length, and is subjected to a given zero-to-maximum cyclic force. Estimate the life by numerical integration.

Record the given dimensions, cyclic force, and R.

b	t	a_i	P_{max}	P_{min}	R
mm	mm	mm	N	N	P_{min}/P_{max}
75	5.0	4.0	20,000	0	0.00

Obtain materials properties from Table 11.2. Based on the Walker method, $C = C_0$ for the $R = 0$ case.

σ_o	K_{Ic}	C_0	m	γ	C
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$
363	200	3.280E-12	3.13	0.928	3.280E-12

From Fig. 8.12(c), obtain the definition of S and the equation for F.

$$S = \frac{P}{bt}, \qquad \alpha = \frac{a}{b}$$

$$F = 0.265(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{1.5}} \qquad \text{(any } \alpha\text{)}$$

Calculate S_{max} and ΔS. Also, determine the crack length a_c to cause brittle fracture. Proceed by picking a trial a, then calculating α, F, and K, varying a until $K = K_{Ic}$ is obtained. In addition, calculate the crack length a_o to cause fully plastic yielding from Fig. A.16(d).

$$S_{max} = \frac{P_{max}}{bt}, \qquad \Delta S = S_{max}(1-R)$$

$$K = F S_{max}\sqrt{\pi a}, \qquad a_c \rightarrow K_{Ic}$$

(11.40, p.2)

$$P' = \frac{P_{max}}{bt\sigma_o} = \frac{S_{max}}{\sigma_o}, \qquad a_o = b\left[P' + 1 - \sqrt{2P'(P'+1)}\right]$$

S_{max} MPa	ΔS MPa	a_c mm	α_c a_c/b	F_f	K_{Ic} $\text{MPa}\sqrt{\text{m}}$	a_o mm
53.3	53.33	56.83	0.758	8.8747	200.00	42.48

The smaller of a_c and a_o is the final crack length a_f. Express a_f, and also a_i and b, in meters. Pick a number of increments n such that the increment factor is around $r = 1.1$.

$$a_f = \text{MIN}(a_c, a_o), \qquad r = (a_f / a_i)^{1/n}$$

a_f, m	a_i, m	b, m	n	r
0.04248	0.00400	0.07500	24	1.10346

For each crack length starting with a_i, calculate α, F, ΔK, and $y = dN/da$. Increment crack lengths by the factor r until a_f is reached. Then, for each group of three points, 0-1-2, 2-3-4, 4-5-6, etc., apply Eq. 11.38 to obtain ΔN values, and also calculate the cumulative sum, the last value of which is the calculated life N_{if}.

$$\alpha_j = \frac{a_j}{b}, \qquad F_j = F(\alpha_j), \qquad \Delta K_j = F_j \Delta S \sqrt{\pi a_j}$$

$$y_j = \frac{dN}{da} = \frac{1}{C(\Delta K_j)^m}, \qquad a_{j+1} = r a_j$$

$$\Delta N_{j+2} = \frac{a_j(r^2-1)}{6r}\left[y_j r(2-r) + y_{j+1}(r+1)^2 + y_{j+2}(2r-1)\right]$$

(11.40, p.3)

j	a m	α = a/b	F	ΔK MPa√m	y = dN/da cycles/m	ΔN cycles	Σ(ΔN) cycles
0	0.004000	0.0533	1.1586	6.927	7.132E+08	0	0
1	0.004414	0.0589	1.1636	7.308	6.032E+08	0	0
2	0.004870	0.0649	1.1694	7.715	5.091E+08	524,431	524,431
3	0.005374	0.0717	1.1762	8.151	4.286E+08	0	0
4	0.005930	0.0791	1.1840	8.619	3.598E+08	453,719	978,150
5	0.006544	0.0873	1.1932	9.125	3.011E+08	0	0
6	0.007221	0.0963	1.2040	9.671	2.509E+08	388,068	1,366,218
7	0.007968	0.1062	1.2167	10.266	2.082E+08	0	0
8	0.008792	0.1172	1.2316	10.917	1.717E+08	326,692	1,692,911
9	0.009702	0.1294	1.2494	11.633	1.408E+08	0	0
10	0.010706	0.1427	1.2705	12.427	1.145E+08	269,019	1,961,929
11	0.011813	0.1575	1.2957	13.313	9.229E+07	0	0
12	0.013035	0.1738	1.3260	14.311	7.360E+07	214,805	2,176,734
13	0.014384	0.1918	1.3625	15.447	5.795E+07	0	0
14	0.015872	0.2116	1.4068	16.754	4.494E+07	164,287	2,341,022
15	0.017514	0.2335	1.4608	18.275	3.424E+07	0	0
16	0.019326	0.2577	1.5272	20.069	2.554E+07	118,306	2,459,328
17	0.021325	0.2843	1.6095	22.218	1.857E+07	0	0
18	0.023531	0.3138	1.7125	24.833	1.311E+07	78,306	2,537,634
19	0.025966	0.3462	1.8431	28.076	8.930E+06	0	0
20	0.028652	0.3820	2.0112	32.181	5.826E+06	46,031	2,583,664
21	0.031616	0.4216	2.2316	37.509	3.606E+06	0	0
22	0.034887	0.4652	2.5279	44.633	2.093E+06	22,833	2,606,498
23	0.038496	0.5133	2.9393	54.517	1.119E+06	0	0
24	0.042479	0.5664	3.5365	68.902	5.376E+05	8,794	2,615,292

N_{if}, cycles = 2,615,292 ◀

11.41

Consider the sheet of 2024-T3 aluminum with a row of rivet holes (Fig. 8.24) from Prob. 11.35. Use the exact K_2 with numerical integration to calculate the number of cycles to grow the cracks to $\ell = 4$ mm length.

Record the given dimensions and stresses, and calculate R, ΔS, a_i, a_f.

2b	b	d	t	S_{max}	S_{min}	R
mm	mm	mm	mm	MPa	MPa	S_{min}/S_{max}
24	12	4.0	3.0	28.0	8.5	0.3036

ΔS	ℓ_i	a_i	ℓ_f	a_f
MPa	mm	mm	mm	mm
19.5	0.50	2.50	4.00	6.00

$$a = \ell + \frac{d}{2}$$

Obtain materials properties from Table 11.2. Based on the Walker method, calculate C for the applicable R value from Eq. 11.20.

σ_o	K_{Ic}	C_0	m	γ	C
MPa	$\mathrm{MPa}\sqrt{\mathrm{m}}$	$\frac{\mathrm{m/cycle}}{(\mathrm{MPa}\sqrt{\mathrm{m}})^m}$			$\frac{\mathrm{m/cycle}}{(\mathrm{MPa}\sqrt{\mathrm{m}})^m}$
353	34	1.420E-11	3.59	0.680	2.152E-11

Obtain the definition of S and equations for K from Fig. 8.23(b), using the exact K_2 case.

$$S = \frac{P}{2bt}, \qquad K = F_P \frac{P}{t\sqrt{b}}, \qquad \alpha = \frac{a}{b}$$

$$F_P = \frac{1}{2}\left(\frac{1}{\sqrt{\sin \pi\alpha}} + \sqrt{\frac{1}{2}\tan\frac{\pi\alpha}{2}}\right)$$

(11.41, p.2)

Express a_f, a_i, and b in meters. Since K is nearly constant, the ordinary Simpson's rule of Eq. 11.36 is preferrable, and the number of increments need not be very large. Pick a crack length increment that gives $n = 10$.

a_f, m	a_i, m	b, m	n	Δa, m
0.00600	0.00250	0.01200	10	0.00035

$$\Delta a = \frac{a_f - a_i}{n}$$

For each crack length starting with a_i, calculate α, F_P, ΔK, and $y = dN/da$. Increment crack lengths by Δa until a_f is reached. Then, for each group of three points, 0-1-2, 2-3-4, 4-5-6, etc., apply Eq. 11.36 to obtain ΔN values, and also calculate the cumulative sum, the last value of which is the calculated life N_{if}.

$$\alpha_j = \frac{a_j}{b}, \qquad F_{Pj} = F(\alpha_j), \qquad \Delta K_j = F_{Pj}\frac{\Delta P}{t\sqrt{b}} = 2F_{Pj}\,\Delta S\sqrt{b}$$

$$y_j = \frac{dN}{da} = \frac{1}{C(\Delta K_j)^m}, \qquad a_{j+1} = a_j + \Delta a$$

$$\Delta N_{j+2} = \frac{\Delta a}{3}\left[y_j + 4y_{j+1} + y_{j+2}\right]$$

j	a m	α = a/b	F_P	ΔK $MPa\sqrt{m}$	y = dN/da cycles/m	ΔN cycles	Σ(ΔN) cycles
0	0.00250	0.2083	0.8468	3.618	4.596E+08	0	0
1	0.00285	0.2375	0.8281	3.538	4.981E+08	0	0
2	0.00320	0.2667	0.8159	3.486	5.252E+08	347,339	347,339
3	0.00355	0.2958	0.8089	3.456	5.418E+08	0	0
4	0.00390	0.3250	0.8061	3.444	5.486E+08	378,102	725,441
5	0.00425	0.3542	0.8068	3.447	5.469E+08	0	0
6	0.00460	0.3833	0.8106	3.463	5.377E+08	381,978	1,107,419

(11.41, p.3)

7	0.00495	0.4125	0.8173	3.492	5.221E+08	0	0
8	0.00530	0.4417	0.8267	3.532	5.011E+08	364,854	1,472,274
9	0.00565	0.4708	0.8388	3.583	4.757E+08	0	0
10	0.00600	0.5000	0.8536	3.647	4.467E+08	332,564	1,804,837

N_{if}, cycles = 1,804,837 ◀

11.42

A solid circular shaft made of 17-4 PH stainless steel has given dimensions and initial circumferential crack. For a given moment in rotating bending, estimate the life by numerical integration.

Record the given dimensions and cyclic moment, and calculate R.

d	b = d/2	a_i	M_{max}	M_{min}	R
mm	mm	mm	N-mm	N-mm	M_{min}/M_{max}
50	25	0.50	5.0E+06	-5.0E+06	-1.0000

Obtain materials properties from Table 11.2. Assume that the compressive loading does not contribute to crack growth by using $\gamma = 0$. Based on the Walker method, calculate C for the $R = -1$ case from Eq. 11.20.

σ_o	K_{Ic}	C_0	m	γ	C
MPa	MPa $\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		R < 0	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$
1059	120	3.290E-11	2.44	0.00	6.063E-12

From Fig. 8.14(b), obtain the definition of S and the equation for F.

$$S = \frac{4M}{\pi b^3}, \qquad \alpha = \frac{a}{b}, \qquad \beta = 1 - \alpha$$

$$F = \frac{3}{8\beta^{2.5}}\left[1 + \frac{1}{2}\beta + \frac{3}{8}\beta^2 + \frac{5}{16}\beta^3 + \frac{35}{128}\beta^4 + 0.537\beta^5\right] \quad \text{(any } \alpha\text{)}$$

Calculate S_{max} and ΔS. Also, determine the crack length a_c to cause brittle fracture. Proceed by picking a trial a, then calculating $\beta = 1 - \alpha$, F, and K, varying a until $K = K_{Ic}$ is obtained. In addition, calculate the crack length a_o to cause fully plastic yielding by solving for crack length using the expression for M_o from Prob. 8.14.

(11.42, p. 2)

$$S_{max} = \frac{4M_{max}}{\pi b^3}, \qquad \Delta S = S_{max}(1-R)$$

$$K = FS_{max}\sqrt{\pi a}\,, \qquad a_c \rightarrow K_{Ic}$$

$$a_o = b\left[1 - \frac{1}{b}\left(\frac{3M_{max}}{4\sigma_o}\right)^{1/3}\right]$$

S_{max} MPa	ΔS MPa	a_c mm	β_c $1 - a_c/b$	F_f	K_{Ic} MPa$\sqrt{m}$	a_o mm
407.4	814.87	8.494	0.660	1.8029	120.00	9.758

The smaller of a_c and a_o is the final crack length a_f. Express a_f, and also a_i and b, in meters. Pick a number of increments n such that the increment factor is around $r = 1.1$.

$$a_f = \text{MIN}(a_c, a_o)\,, \qquad r = (a_f/a_i)^{1/n}$$

a_f, m	a_i, m	b, m	n	r
0.008494	0.000500	0.02500	30	1.09902

For each crack length starting with a_i, calculate β, F, ΔK, and $y = dN/da$. Increment crack lengths by the factor r until a_f is reached. Then, for each group of three points, 0-1-2, 2-3-4, 4-5-6, etc., apply Eq. 11.38 to obtain ΔN values, and also calculate the cumulative sum, the last value of which is the calculated life N_{if}.

$$\beta_j = 1 - \frac{a_j}{b}, \qquad F_j = F(\beta_j)\,, \qquad \Delta K_j = F_j \Delta S\sqrt{\pi a_j}$$

(11.42, p.3)

$$y_j = \frac{dN}{da} = \frac{1}{C(\Delta K_j)^m}, \qquad a_{j+1} = r a_j$$

$$\Delta N_{j+2} = \frac{a_j(r^2-1)}{6r}\left[y_j r(2-r) + y_{j+1}(r+1)^2 + y_{j+2}(2r-1)\right]$$

j	a	β	F	ΔK	y = dN/da	ΔN	Σ(ΔN)
	m	1 - a/b		$MPa\sqrt{m}$	cycles/m	cycles	cycles
0	0.000500	0.9800	1.1367	36.711	2.507E+07	0	0
1	0.000550	0.9780	1.1380	38.531	2.228E+07	0	0
2	0.000604	0.9758	1.1395	40.447	1.979E+07	2,312	2,312
3	0.000664	0.9735	1.1412	42.465	1.758E+07	0	0
4	0.000729	0.9708	1.1431	44.591	1.560E+07	2,203	4,515
5	0.000802	0.9679	1.1452	46.833	1.384E+07	0	0
6	0.000881	0.9648	1.1476	49.199	1.227E+07	2,095	6,610
7	0.000968	0.9613	1.1503	51.698	1.088E+07	0	0
8	0.001064	0.9574	1.1533	54.339	9.630E+06	1,989	8,599
9	0.001170	0.9532	1.1567	57.135	8.521E+06	0	0
10	0.001285	0.9486	1.1606	60.097	7.532E+06	1,882	10,481
11	0.001413	0.9435	1.1650	63.241	6.651E+06	0	0
12	0.001553	0.9379	1.1700	66.584	5.865E+06	1,774	12,255
13	0.001706	0.9318	1.1757	70.145	5.165E+06	0	0
14	0.001875	0.9250	1.1823	73.948	4.541E+06	1,664	13,919
15	0.002061	0.9176	1.1899	78.019	3.984E+06	0	0
16	0.002265	0.9094	1.1987	82.393	3.488E+06	1,550	15,469
17	0.002489	0.9004	1.2088	87.107	3.045E+06	0	0
18	0.002736	0.8906	1.2206	92.211	2.650E+06	1,431	16,900
19	0.003007	0.8797	1.2345	97.764	2.298E+06	0	0
20	0.003304	0.8678	1.2507	103.841	1.983E+06	1,304	18,204
21	0.003632	0.8547	1.2700	110.535	1.703E+06	0	0
22	0.003991	0.8404	1.2928	117.966	1.453E+06	1,167	19,371
23	0.004386	0.8245	1.3202	126.288	1.230E+06	0	0
24	0.004821	0.8072	1.3532	135.703	1.032E+06	1,018	20,390

(11.42, p.4)

25	0.005298	0.7881	1.3934	146.482	8.567E+05	0	0
26	0.005823	0.7671	1.4426	158.987	7.015E+05	856	21,246
27	0.006399	0.7440	1.5036	173.718	5.651E+05	0	0
28	0.007033	0.7187	1.5800	191.378	4.462E+05	682	21,929
29	0.007729	0.6908	1.6773	212.976	3.437E+05	0	0
30	0.008494	0.6602	1.8029	240.000	2.568E+05	502	22,431

N_{if}, cycles = 22,431 ◀

11.44 For Prob. 11.27 situation, estimate repetitions to failure for given load history and $a_i = 2$ mm. 2024-T3 Al center-cracked plate, $b = 50$, $t = 4$ mm.

Due to the same $P_{max} = 60$ kN, the $a_f = 16.35$ mm from Prob. 11.27 solution applies.

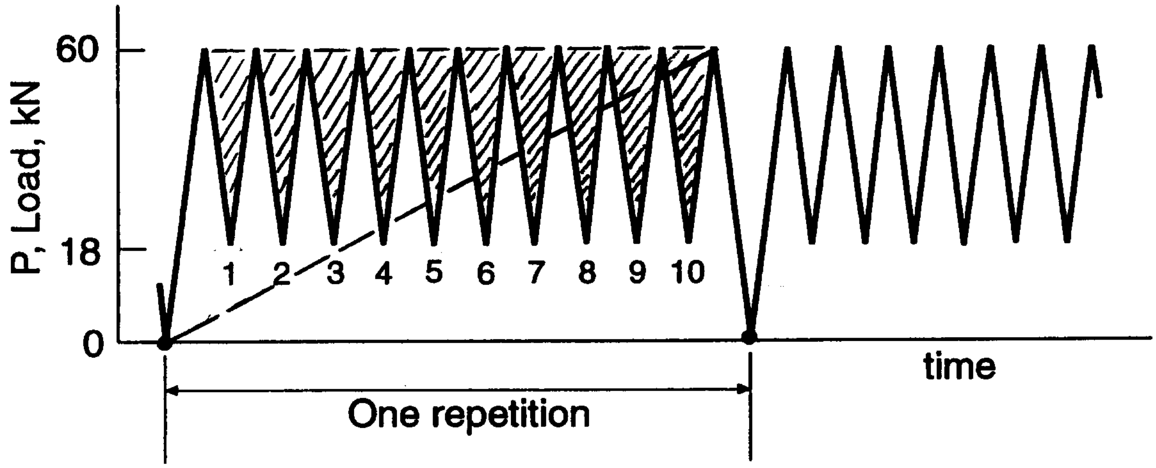
$$\Delta S_q = \left[\frac{\sum^{N_B} (\overline{\Delta S})^m}{N_B} \right]^{1/m}, \quad \overline{\Delta S} = S_{max} (1-R)^{\gamma}$$

For multiple cycles N_j at different levels:

$$\Delta S_q = \left[\frac{\sum^{k} N_j \, (\overline{\Delta S}_j)^m}{N_B} \right]^{1/m}, \quad N_B = \sum N_j$$

For each level, calculate:

$$R = \frac{P_{min}}{P_{max}}, \quad S_{max} = \frac{P_{max}}{2bt}, \quad \overline{\Delta S}, \quad N_j (\overline{\Delta S}_j)^m$$



Use $C = C_0$, m, and γ from Table 11.2.

$F \approx 1.0$ and constant from Prob. 11.20.

Calculations are shown in the table.

(11.44, p. 2)

The sum of the last column in the table and $\Sigma N_j = N_B$ are used to calculate ΔS_q, and then N_{if} and B_{if} follow.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C_0 (F\, \Delta S_q \sqrt{\pi})^m (1-m/2)}, \qquad B_{if} = \frac{N_{if}}{N_B}$$

b	t	a_i	a_f	F	C	m	γ	1 - m/2
mm	mm	m	m		$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			
50	4.0	0.00200	0.01635	1.00	1.42E-11	3.59	0.680	-0.795

j	N_j	P_{max}	P_{min}	R	S_{max}	$\overline{\Delta S}$	$N_j(\overline{\Delta S}_j)^m$
	cycles	kN	kN		MPa	MPa	
1	1	60	0	0	150	150.00	6.489E+07
2	10	60	18	0.3	150	117.69	2.717E+08
Σ	11						3.365E+08

ΔS_q, MPa = 121.66

N_{if}, cycles = 42,116

B_{if}, repetitions = 3,829 ◀

11.45 For Prob. 11.31, $a_i = 0.5$ mm, replace load history as shown. $B_{if} = ?$

Bending member, $b = 60$, $t = 12$ mm

AISI 4340 steel, $\sigma_o = 1255$ MPa, $K_{Ic} = 130$ MPa$\sqrt{m}$

$C_1 = 5.11 \times 10^{-13}$ (m/cycle), $m_1 = 3.24$, $\gamma = 0.42$

$a_f = a_o$ or a_c, a_o from Fig. A.13(b)

$$a_o = b\left[1 - \frac{2}{b}\sqrt{\frac{M_{max}}{t\sigma_o}}\right], \quad M_{max} = 4 \text{ kN·m}$$

$$a_o = (60 \text{ mm})\left[1 - \frac{2}{60 \text{ mm}}\sqrt{\frac{4 \times 10^6 \text{ N·mm}}{(12 \text{ mm})(1255 \text{ MPa})}}\right]$$

$a_o = 27.4$ mm

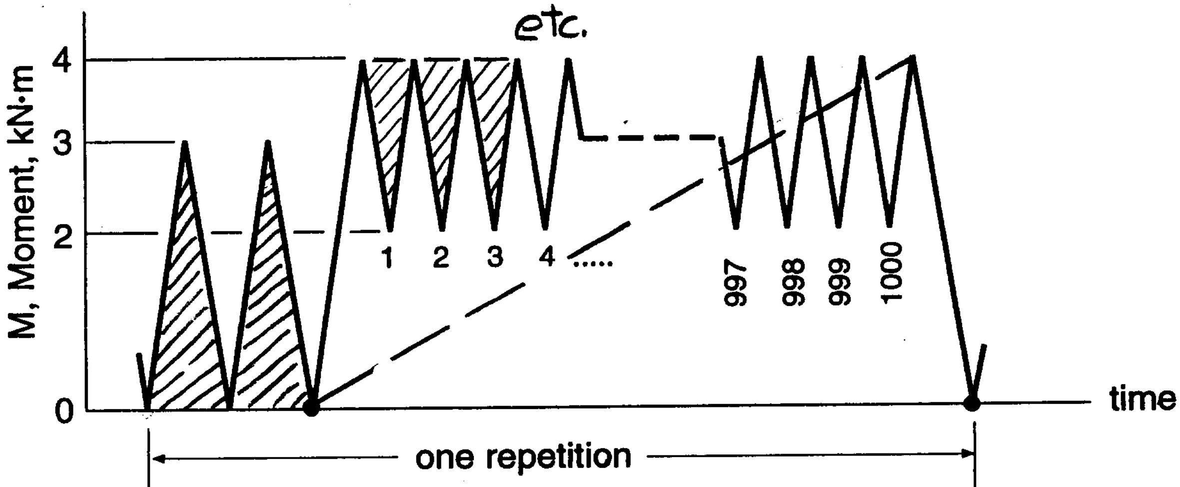
assume $a_f/b < 0.4$, $F = 1.12$ (Fig. 8.13)

$$S_{max} = \frac{6M_{max}}{b^2 t} = \frac{6(4 \times 10^6 \text{ N·mm})}{(60)^2(12) \text{ mm}^3} = 555.6 \text{ MPa}$$

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{F S_{max}}\right)^2 = 0.0139 \text{ m} = 13.9 \text{ mm}$$

$a_c/b = 0.23$, calculation O.K., $F \approx 1.12$

$a_f = a_c = 13.9$ mm controls ◁



(11.45, p.2)

For each level with N_j cycles, calculate:

$$R = \frac{M_{min}}{M_{max}}, \quad S_{max} = \frac{6 M_{max}}{b^2 t}$$

$$\overline{\Delta S} = S_{max} (1-R)^{\gamma}, \quad N_j (\overline{\Delta S_j})^m$$

Then sum to obtain

$$N_B = \Sigma N_j, \quad \Delta S_q = \left[\frac{\sum^{k} N_j (\overline{\Delta S_j})^m}{N_B} \right]^{1/m}$$

Calculation of N_{if} and B_{if} then follow.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C_0 (F \Delta S_q \sqrt{\pi})^m (1-m/2)}, \quad B_{if} = \frac{N_{if}}{N_B}$$

b	t	a_i	a_f	F	C_0	m	γ	1 - m/2
mm	mm	m	m		$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			
60	12	0.00050	0.01390	1.12	5.11E-13	3.24	0.420	-0.62

j	N_j	M_{max}	M_{min}	R	S_{max}	$\overline{\Delta S}$	$N_j(\overline{\Delta S_j})^m$
	cycles	kN-m	kN-m		MPa	MPa	
1	2	3.00	0.00	0.00	416.67	416.67	6.154E+08
2	1000	4.00	2.00	0.50	555.56	415.24	3.043E+11
3	1	4.00	0.00	0.00	555.56	555.56	7.815E+08
Σ	1003						3.057E+11

ΔS_q, MPa =	415.44
N_{if}, cycles =	109,118
B_{if}, repetitions =	108.8

11.46

A bending member made of Man-Ten steel has a rectangular cross section with given dimensions and initial full-thicknes crack. It is repeatedly subjected to a given bending moment history. Estimate the number of repetitions to grow the crack to failure.

Record the given dimensions and maximum moment in the history,

b	t	a_i	M_{MAX}
mm	mm	mm	N-mm
40	20	1.00	1,200,000

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_0	m	γ	γ
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		$(R \geq 0)$	$(R < 0)$
363	200	3.280E-12	3.13	0.928	0.220

Obtain the definition of S and equations for F from Fig. 8.13(a).

$$S = \frac{6M}{b^2 t}, \qquad \alpha = \frac{a}{b}, \qquad \beta = \frac{\pi\alpha}{2}, \qquad F \approx 1.12 \quad (\alpha \leq 0.4)$$

$$F = \sqrt{\frac{1}{\beta}\tan\beta}\left[\frac{0.923 + 0.199(1-\sin\beta)^4}{\cos\beta}\right] \quad (\text{any } \alpha)$$

Calculate S_{MAX} and the crack length a_c to cause brittle fracture. Proceed by picking a trial a, then calculating α, F, and K, varying a until $K = K_{Ic}$ is obtained. Also calculate the crack length a_o to cause fully plastic yielding from Fig. A.16(b).

$$S_{MAX} = \frac{6M_{MAX}}{b^2 t}, \qquad K = FS_{MAX}\sqrt{\pi a}, \qquad a_c \to K_{Ic}$$

(11.46, p. 2)

$$a_o = b\left[1 - \frac{2}{b}\sqrt{\frac{M_{MAX}}{t\sigma_o}}\right]$$

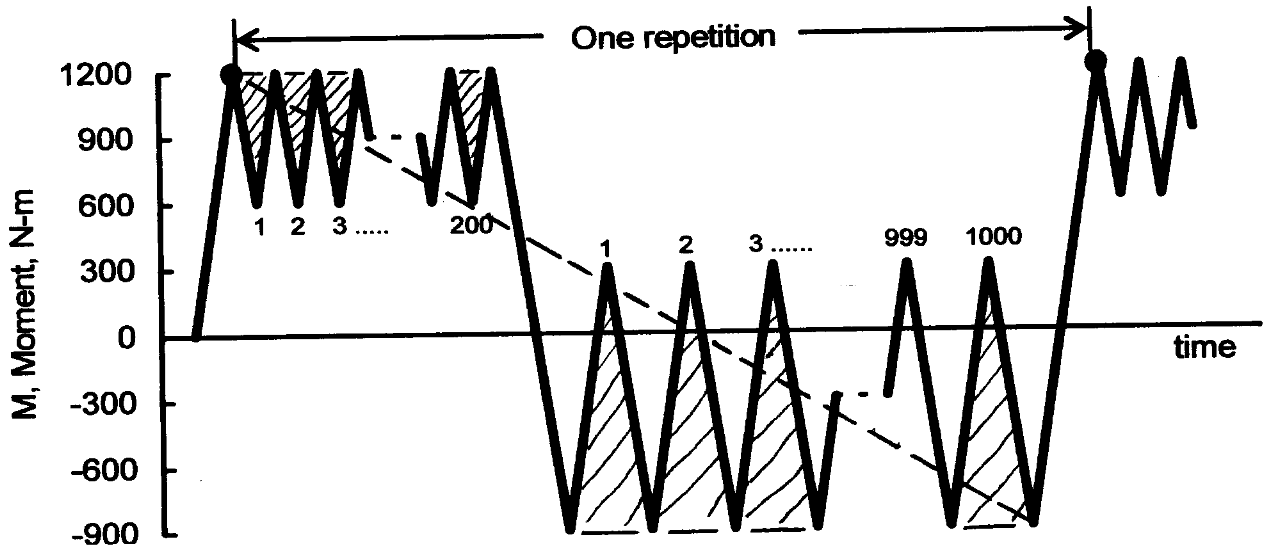
S_{MAX}	a_c	α_c	β_c	F_c	K_{Ic}	a_o
MPa	mm	a_c / b	$\pi\alpha_c / 2$		$\text{MPa}\sqrt{\text{m}}$	mm
225.0	28.76	0.719	1.129	2.9573	200.00	14.29

The smaller of a_c and a_o is the final crack length a_f. Calculating the corresponding F indicates that this quantity does not vary excessively. So the life calculation can be based on Eq. 11.32.

a_f	α_f	β_f	F_f
mm	a_f / b	$\pi\alpha_f / 2$	
14.29	0.357	0.561	1.1656

$$a_f = \text{MIN}(a_c, a_o)$$

Perform cycle counting on the given moment history.



(11.46, p. 3)

Set up a table as below. For each level of cycling in the history, calculate:

$$R = \frac{M_{min}}{M_{max}}, \quad S_{max} = \frac{6M_{max}}{b^2 t}, \quad \overline{\Delta S} = S_{max}(1-R)^{\gamma}, \quad N_j\left(\overline{\Delta S}_j\right)^m$$

Then employ sums from the table to determine N_B and ΔS_q.

$$N_B = \sum N_j, \qquad \Delta S_q = \left[\frac{\sum N_j\left(\overline{\Delta S}_j\right)^m}{N_B}\right]^{1/m}$$

Finally, employ Eq. 11.32 to calculate N_{if}, using C_0, choosing F as a value near that for a_i, and expressing all substituted quantities in units of meters, MPa, or combinations thereof. Then calculate B_{if}.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C_0(F\,\Delta S_q\sqrt{\pi})^m(1-m/2)}, \qquad B_{if} = \frac{N_{if}}{N_B}$$

a_i	a_f	F	C_0	m	γ	γ	1 - m/2
m	m		$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		(R ≥ 0)	(R < 0)	
0.00100	0.01429	1.12	3.280E-12	3.13	0.928	0.220	-0.565

j	N_j	M_{max}	M_{min}	R	S_{max}	$\overline{\Delta S}$	$N_j\left(\overline{\Delta S}_j\right)^m$
	cycles	N-m	N-m		MPa	MPa	
1	200	1200	600	0.50	225.00	118.26	6.1515E+08
2	1000	300	-900	-3.00	56.25	76.31	7.8066E+08
3	1	1200	-900	-0.75	225.00	254.48	3.3860E+07
Σ	1201						1.4297E+09

ΔS_q, MPa = 87.32

N_{if}, cycles = 2,041,541

B_{if}, repetitions = 1700 ◀

11.47

A circular shaft made of 7075-T6 aluminum has given diameter and initial half-circular surface crack. It is repeatedly subjected to a given bending moment history. Estimate the number of repetitions to grow the crack to failure.

Record the given dimensions and maximum moment in the history,

d	a_i	M_{MAX}
mm	mm	N-mm
50	1.00	2,300,000

Obtain materials properties from Table 11.2

σ_o	K_{Ic}	C_0	m	γ	γ
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		$(R \geq 0)$	$(R < 0)$
523	29	2.710E-11	3.70	0.641	0.00

Obtain the definition of S and value of F from Fig. 8.17(d).

$$S = \frac{32M}{\pi d^3}, \qquad \alpha = \frac{a}{d}, \qquad F \approx 0.728 \quad (\alpha \leq 0.35)$$

Calculate S_{MAX} and the crack length a_c to cause brittle fracture. Check that α_c is such that the F used is valid. Also calculate the crack length a_o to cause fully plastic yielding, very roughly approximating this case as a rectangular section from Fig. A.16(b), with same a and $b = d, t = d$.

$$S_{MAX} = \frac{32M_{MAX}}{\pi d^3}, \qquad a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{FS_{MAX}}\right)^2 \qquad (F \approx 0.728 \text{ if } \alpha \leq 0.35)$$

(11.47, p.2)

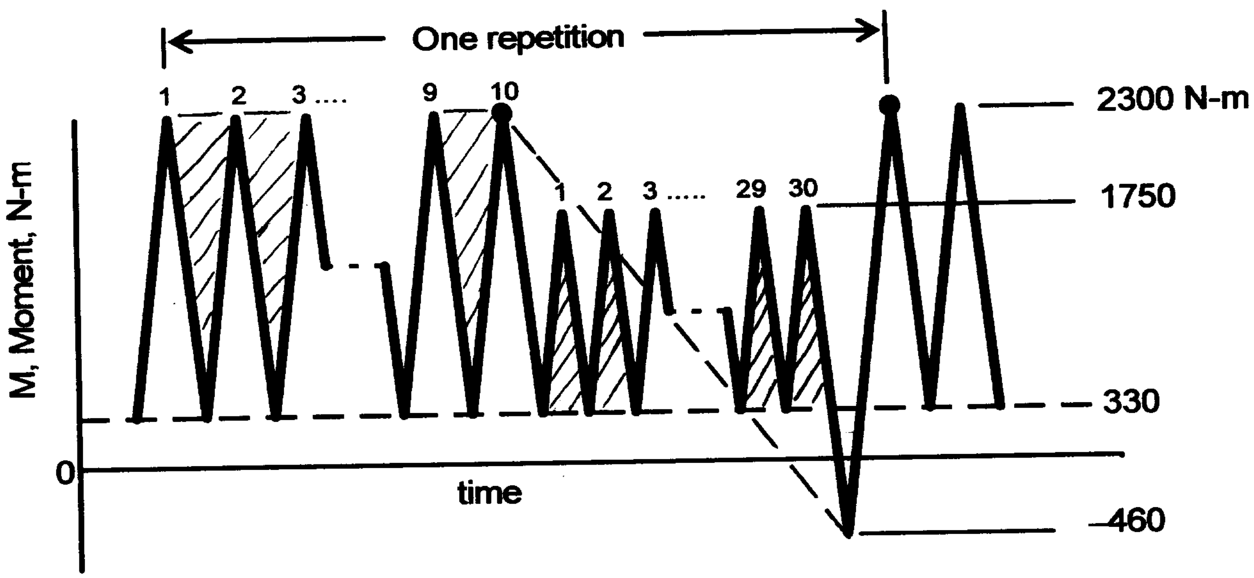
$$a_o = b\left[1 - \frac{2}{b}\sqrt{\frac{M_{MAX}}{t\sigma_o}}\right], \qquad b, t \rightarrow d$$

S_{MAX} MPa	a_c mm	α_c a_c / d	F	Is F OK ?	a_o mm	a_f mm
187.4	14.38	0.288	0.728	yes	31.24	14.38

The smaller of a_c and a_o is the final crack length a_f. As F does not vary excessively, the life calculation can be based on Eq. 11.32.

$$a_f = \text{MIN}(a_c, a_o)$$

Perform cycle counting on the given moment history.



(11.47, p.3)

Set up a table as below. For each level of cycling in the history, calculate:

$$R = \frac{M_{min}}{M_{max}}, \quad S_{max} = \frac{32 M_{max}}{\pi d^3}, \quad \overline{\Delta S} = S_{max}(1-R)^{\gamma}, \quad N_j\left(\overline{\Delta S}_j\right)^m$$

Then employ sums from the table to determine N_B and ΔS_q.

$$N_B = \sum N_j, \qquad \Delta S_q = \left[\frac{\sum N_j\left(\overline{\Delta S}_j\right)^m}{N_B}\right]^{1/m}$$

Finally, employ Eq. 11.32 to calculate N_{if}, using C_0, choosing F as a value near that for a_i, and expressing all substituted quantities in units of meters, MPa, or combinations thereof. Then calculate B_{if}.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C_0(F\,\Delta S_q\sqrt{\pi})^m(1-m/2)}, \qquad B_{if} = \frac{N_{if}}{N_B}$$

a_i	a_f	F	C_0	m	γ	γ	1 - m/2
m	m		$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		(R ≥ 0)	(R < 0)	
0.00100	0.01438	0.728	2.710E-11	3.70	0.641	0.00	-0.85

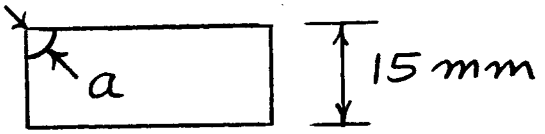
j	N_j	M_{max}	M_{min}	R	S_{max}	$\overline{\Delta S}$	$N_j\left(\overline{\Delta S}_j\right)^m$
	cycles	N-m	N-m		MPa	MPa	
1	9	2300	330	0.143	187.42	169.71	1.6001E+09
2	30	1750	330	0.189	142.60	124.73	1.7067E+09
3	1	2300	-460	-0.200	187.42	187.42	2.5670E+08
Σ	40						3.5635E+09

ΔS_q, MPa = 140.80

N_{if}, cycles = 60,342

B_{if}, repetitions = 1509 ◀

11.48 For Prob. 11.30, replace loading with history of Fig. P10.39 7075-T6 Al part as in Fig. P11.30 with a corner crack.



$a_i = 0.5$ mm

$F \approx 0.722$ (Fig. 8.17)

$a_f = 4.55$ mm from Prob. 11.30 applies due to same $S_{max} = 336$ MPa.

Use cycle counting result from Prob. 10.39 solution.

For each level with N_j cycles, calculate

$$R = \frac{S_{min}}{S_{max}}, \quad \overline{\Delta S} = S_{max}(1-R)^{\gamma}, \quad N_j(\overline{\Delta S}_j)^m$$

Then sum to obtain

$$N_B = \Sigma N_j, \quad \Delta S_q = \left[\frac{\sum^{k} N_j (\overline{\Delta S}_j)^m}{N_B}\right]^{1/m}$$

Use $C = C_0$, m, γ from Table 11.2.
Note $\gamma = 0$ for $R < 0$ is equivalent to no effect of compression, $\overline{\Delta S} = S_{max}$.

See table on the next page for details.
Finally, calculate:

$$N_{if} = \frac{a_f^{\,1-m/2} - a_i^{\,1-m/2}}{C_0 (F \,\Delta S_q \sqrt{\pi})^m (1 - m/2)}, \quad B_{if} = \frac{N_{if}}{N_B}$$

$B_{if} = 49$ repetitions ◄

(11.48, p.2)

a_i	a_f	F	C_0	m	γ	γ	1 - m/2
m	m		$\frac{m/cycle}{(MPa\sqrt{m})^m}$		$(R \geq 0)$	$(R < 0)$	
0.0005	0.00455	0.722	2.71E-11	3.70	0.641	0	-0.85

j	N_j	S_{max}	S_{min}	R	$\overline{\Delta S}$	$N_j\left(\overline{\Delta S}_j\right)^m$
	cycles	MPa	MPa		MPa	
1	11	336	48.3	0.1438	304.18	1.6943E+10
2	35	292	48.3	0.1654	260.04	3.0182E+10
3	88	255	48.3	0.1894	222.89	4.2893E+10
4	180	219	48.3	0.2205	186.67	4.5527E+10
5	300	181	48.3	0.2669	148.34	3.2420E+10
6	510	143	48.3	0.3378	109.80	1.8106E+10
7	780	105	48.3	0.4600	70.74	5.4427E+09
8	1030	67.6	48.3	0.7145	30.27	3.1082E+08
9	15	48.3	-19.3	-0.3996	48.30	2.5509E+07
10	1	48.3	-67.6	-1.3996	48.30	1.7006E+06
11	1	336	-67.6	-0.2012	336.00	2.2256E+09
Σ	2951					1.9408E+11

ΔS_q, MPa =	129.71
N_{if}, cycles =	143,559
B_{if}, repetitions =	48.6

11.49 For center-cracked plate of AISI 4340 steel of Ex. 11.4, periodic inspections at $N_p = 23{,}500$ cycles are done.

$b = 38,\ t = 6,\ a_i = 1.0\ mm = a_d$

$P_{min} = 80,\ P_{max} = 240\ kN$

The following from the Ex. 11.4 soln. apply:

$a_f = 15.8\ mm,\quad C = 1.095 \times 10^{-12} \frac{m/cyc}{(MPa\sqrt{m})^m}$

(for $R = P_{min}/P_{max} = 0.333$)

$m = 3.24,\ \gamma = 0.42$ (Table 11.2)

Also from Ex. 11.4: $S_{max} = 526,\ \Delta S = 351\ MPa$

(a) $X_c, X_o = ?$ just before inspection

The crack size after N_p cycles is needed.

$$N_p = \frac{a_f^{1-m/2} - a_d^{1-m/2}}{C(F\Delta S\sqrt{\pi})^m(1-m/2)},\quad F \approx 1.03\ \ (\text{Ex. } 11.4)$$

$$23{,}500 = \frac{a_f^{-0.62} - 0.001^{-0.62}}{1.095 \times 10^{-12}(1.03 \times 351\sqrt{\pi})^{3.24}(-0.62)}$$

Solving gives $a_f = 0.001673\ m = 1.673\ mm$

$$X_c = \frac{S_c}{S_{max}} = \frac{K_{Ic}}{K_{max}},\quad K_{Ic} = 130\ MPa\sqrt{m}$$

$K_{max} = F S_{max}\sqrt{\pi a},\quad F \approx 1.0\ \ (\text{Fig. } 8.12(a))$

$K_{max} = (1)(526\ MPa)\sqrt{\pi(0.001673\ m)}$

$K_{max} = 38.1\ MPa\sqrt{m}$

$$X_c = \frac{130}{38.1} = 3.41$$ ◀

(11.49, p. 2)

$$X_o = \frac{P_o}{P_{max}}, \quad P_o = 2bt\sigma_o\left(1 - \frac{a}{b}\right) \quad \text{(Fig. A.16)}$$

$$P_o = 2(38\ mm)(6\ mm)(1255\ MPa)\left(1 - \frac{1.673\ mm}{38\ mm}\right)$$

$$P_o = 547{,}000\ N = 547\ kN$$

$$X_o = \frac{547}{240} = 2.28$$ ◀

(b) These safety factors seem to be adequate. It is good that yielding is more likely than sudden brittle fracture, due to $X_o < X_c$. ◀

11.50

A single-edge-cracked plate made of AISI 4340 steel has given dimensions and cyclic loading. (a) For $a_i = a_d = 1.3$ mm, estimate the life. (b) Is the design adequate for a desired service life of 60,000 cycles and safety factor of 3.0 in life? (c) If not, what new a_d would need to be found? (d) If a_d cannot be made smaller, what inspection interval is needed? (e) If neither is possible, what lower stresses with $R = 0.5$ are required?

Record the given dimensions, cyclic force, and R.

b	t	P_{max}	P_{min}	R
mm	mm	MN	MN	P_{min}/P_{max}
250	25.0	3.40	1.70	0.50

Obtain materials properties from Table 11.2. Based on the Walker method, calculate C for the applicable R value from Eq. 11.20.

σ_o	K_{Ic}	C_0	m	γ	C
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$
1255	130	5.110E-13	3.24	0.420	1.880E-12

From Fig. 8.12(c), obtain the definition of S and equations for F.

$$S = \frac{P}{bt}, \qquad \alpha = \frac{a}{b}, \qquad F = 1.12 \quad (\alpha \leq 0.13)$$

$$F = 0.265(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{1.5}} \quad (\text{any } \alpha)$$

Calculate S_{max} and ΔS. Determine the crack length a_c to cause brittle fracture, by picking a trial a and varying a until $K = K_{Ic}$ is obtained. Also, calculate a_o to cause fully plastic yielding from Fig. A.16(d).

(11.50, p. 2)

$$S_{max} = \frac{P_{max}}{bt}, \qquad \Delta S = S_{max}(1-R)$$

$$K = FS_{max}\sqrt{\pi a}, \qquad a_c \to K_{Ic}$$

$$P' = \frac{P_{max}}{bt\sigma_o} = \frac{S_{max}}{\sigma_o}, \qquad a_o = b\left[P' + 1 - \sqrt{2P'(P'+1)}\right]$$

S_{max} MPa	ΔS MPa	a_c mm	α_c a_c / b	F_f	K_{Ic} $\text{MPa}\sqrt{\text{m}}$	a_o mm
544.0	272.0	13.53	0.0541	1.1593	130.00	79.67

As F does not vary excessively, the life may be obtained from Eq. 11.32. The smaller of a_c and a_o is the final crack length a_f. Choose F near that for a small value of a_i. Express all quantities in units of meters, MPa, or combinations thereof.

$$a_f = \text{MIN}(a_c, a_o), \qquad N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m(1-m/2)}$$

$$X_N = N_{if} / \hat{N} \qquad \hat{N}, \text{cycles} = \quad 60{,}000$$

	a_f m	a_i m	F	ΔS MPa	1 - m/2	N_{if} cycles	X_N	
(a)	0.01353	0.001300	1.12	272.0	-0.620	**56,779**	**0.946**	(b) ◀
(c)	0.01353	**0.000268**	1.12	272.0	-0.620	180,000	3.000	◀
(e)	0.02770	0.001300	1.12	196.7	-0.620	180,000	3.000	

(a) Calculate N_{if} for a_i = 1.30 mm. See table above.

(b) Calculate X_N in the table above. It is too small, so the design is not adequate. ◀

(11.50, p. 3)

(c) Vary a_i to get N_{if} such that $X_N = 3.0$. See table above.

(d)

N_{if} cycles	X_N	N_p cycles
56,779	3.00	**18,926**

$$N_p = \frac{N_{if}}{X_N}$$

◀

(e) Vary S_{max} to get N_{if} such that $X_N = 3.0$. Noting that a_f must change, simplify the calculation of a_c by approximating F as constant. See the life calculation in the table above.

S_{max} MPa	ΔS MPa	F	a_c mm	α_c a_c / b	Is F OK?	a_o mm
393.5	196.7	1.12	27.70	0.111	yes	101.50

◀

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{FS_{max}}\right)^2 \qquad (F \approx 1.12 \text{ if } \alpha \le 0.13)$$

11.51

A beam made of Man-Ten steel has a rectangular cross section with given dimensions and cyclic loading, and also given length of inspection-size crack. (a) Estimate the number of cycles to grow the inspection-size crack to failure. (b) What is the safety factor in life for a desired service life of 1,000,000 cycles? (c) For a safety factor in life of 3.00, is inspection necesary? At what interval? (d) What is the largest crack size that can be present just before inspection? (e) For the crack size from(d), what are the safety factors against brittle fracture and fully plastic limit load? Adequate?

Record the given dimensions and loads, and calculate R.

b	t	a_i	M_{max}	M_{min}	R
mm	mm	mm	N-m	N-m	M_{min}/M_{max}
40	20	1.00	1200	480	0.400

Obtain materials properties from Table 11.2. Based on the Walker method, calculate C for the applicable R value from Eq. 11.20.

σ_o	K_{Ic}	C_0	m	γ	C
MPa	$\text{MPa}\sqrt{\text{m}}$	$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$			$\frac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$
363	200	3.280E-12	3.13	0.928	3.680E-12

Obtain the definition of S and equations for F from Fig. 8.13(a).

$$S = \frac{6M}{b^2 t}, \qquad \alpha = \frac{a}{b}, \qquad \beta = \frac{\pi\alpha}{2}, \qquad F \approx 1.12 \quad (\alpha \le 0.4)$$

$$F = \sqrt{\frac{1}{\beta}\tan\beta}\left[\frac{0.923 + 0.199(1-\sin\beta)^4}{\cos\beta}\right] \quad (\text{any } \alpha)$$

(11.51, p. 2)

Calculate S_{max} and the crack length a_c to cause brittle fracture. Proceed by picking a trial a, then calculating α, F, and K, varying a until $K = K_{Ic}$ is obtained. Also calculate the crack length a_o to cause fully plastic yielding from Fig. A.16(b).

$$S_{max} = \frac{6M_{max}}{b^2 t}, \qquad K = FS_{max}\sqrt{\pi a}, \qquad a_c \rightarrow K_{Ic}$$

$$a_o = b\left[1 - \frac{2}{b}\sqrt{\frac{M_{max}}{t\sigma_o}}\right]$$

S_{max} MPa	a_c mm	α_c a_c/b	β_c $\pi\alpha_c/2$	F_c	K_{Ic} $MPa\sqrt{m}$	a_o mm
225.0	28.76	0.719	1.129	2.9573	200.00	14.29

The smaller of a_c and a_o is the final crack length a_f. Calculate the corresponding F.

a_f mm	α_f a_f/b	β_f $\pi\alpha_f/2$	F_f
14.29	0.357	0.561	1.1656

$$a_f = \text{MIN}(a_c, a_o)$$

F is seen not vary excessively, so the crack growth life may be calculated from Eq. 11.32. Choose F near that for a small value of a_i. Express all quantities in units of meters, MPa, or combinations thereof.

$$N_{if} = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(F\,\Delta S\sqrt{\pi})^m(1-m/2)}, \qquad X_N = N_{if}/\hat{N}$$

(11.51, p.3)

$\Delta S = S_{max}(1-R)$

$\hat{N}$, cycles= 1,000,000

	a_f	a_i	F	ΔS	1 - m/2	N_{if}	X_N	
	m	m		MPa		cycles		
(a)	0.01429	0.00100	1.12	135.0	-0.565	**465,274**	**0.465**	(b) ◀
(d)	**0.00170**	0.00100	1.12	135.0	-0.565	155,091	----	◀

(a, b) Calculate N_{if} and X_N. See table above.

(c)

N_{if}	X_N	N_p
cycles		cycles
465,274	3.00	**155,091**

$$N_p = \frac{N_{if}}{X_N}$$ ◀

(d) Vary a_f to make the life equal to N_p. See table above.

(e) For the crack length from (d), calculate K and the resulting X_c. Also calculate M_o from Fig. A.16(b) and X_o.

$$K = FS_{max}\sqrt{\pi a}, \qquad X_c = \frac{K_{Ic}}{K}$$

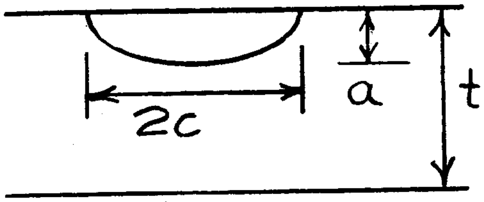
$$M_o = \frac{b^2 t\sigma_o}{4}(1-\alpha)^2, \qquad X_o = \frac{M_o}{M_{max}}$$

S_{max}	a	α	β	F	K	X_c	
MPa	mm	a / b	πα / 2		MPa $\sqrt{m}$		
225.0	1.70	0.043	0.067	1.0772	17.71	**11.29**	◀

M_o	X_o
N-m	
2662	**2.22**

The controlling X_o is adequate, and it is beneficial that yielding is more likely than fracture. ◀

11.52 Spherical pressure vessel, $r_1 = 600$, $t = 50$ mm. Austenitic stainless steel, $K_{Ic} \geq 200$ MPa$\sqrt{m}$. Crack $2c = 8$, $a = 2$ mm.



$$K = F_D S \sqrt{\frac{\pi a}{Q}} \quad \left(\begin{matrix}\text{Fig.} \\ 8.19\end{matrix}\right)$$

$$Q \approx 1 + 1.464 \left(\frac{a}{c}\right)^{1.65}$$

$$F_D = 1.085 + 0.731 \left(\frac{a}{t}\right)^2 - 0.37 \left(\frac{a}{t}\right)^4 \quad \left(\frac{a}{c} = 0.5\right)$$

(a) $N_{if} = ?$ for $p = 20$ MPa cycles

$$S_{max} = \Delta S = \frac{p r_1}{2t} = \frac{(20\text{ MPa})(600\text{ mm})}{2(50\text{ mm})}$$

$$S_{max} = \Delta S = 120\text{ MPa} \qquad \text{(Fig. A.7(b).)}$$

$$C = 5.61 \times 10^{-12} \frac{m/cyc}{(MPa\sqrt{m})^m}, \quad m = 3.25$$

for $R \approx 0$ (Table 11.1)

$$Q \approx 1 + 1.464 \left(\frac{2\text{ mm}}{4\text{ mm}}\right)^{1.65} = 1.4665$$

$$K = F_D S \sqrt{\pi a}, \quad F = F_D / \sqrt{Q}$$

Check K at $a/t = 1$, $a = 50$ mm

$$F_D = 1.085 + 0.731(1)^2 - 0.37(1)^4 = 1.446$$

$$K = 1.446 (120\text{ MPa}) \sqrt{\frac{\pi (0.050\text{ m})}{1.4665}}$$

$$K = 56.8\text{ MPa}\sqrt{m} < K_{Ic}$$

Calculate life to this point; vessel leaks.

(11.52, p.2)

Variation in F requires numerical integration to $a_f = t = 50$ mm; $a_i = 2$ mm.

$$N_{if} = \int_{a_i}^{a_f} \left(\frac{dN}{da}\right) da, \quad \frac{da}{dN} = C(\Delta K)^m$$

Use Simpson's rule in the modified form of Eq. 11.38.

$$r^n = \frac{a_n}{a_i}, \quad 1.1^n = \frac{50}{2}, \quad n \approx 33.7$$

Use $n = 34$ (even integer)

$$r = \left(\frac{a_n}{a_i}\right)^{1/n} = 1.09930$$

To proceed, calculate for $j = 0, 1, 2 \ldots n$:

$$a_j = r^j a_i, \quad \alpha_j = a_j/t, \quad F_{Dj} = F_D(\alpha_j)$$

$$\Delta K_j = F_{Dj}\, \Delta S \sqrt{\frac{\pi a_j}{Q}}, \quad y_j = \frac{1}{C(\Delta K_j)^m}$$

Then, for $j = 0, 2, 4 \ldots (n-2)$, calculate:

$$\Delta N = \int_{a_j}^{a_{j+2}} y\, da \quad \text{from Eq. 11.38}; \quad y = \frac{dN}{da}$$

$\Sigma(\Delta N)$ = cumulative sum

Details are given on pages that follow.

The final $\Sigma(\Delta N)$ is the calculated life.

$N_{if} = 425{,}300$ cycles ◀

(11.52, p.3)

ΔS	C	m	t	a_i	a_f	Q
MPa	m/cyc		m	m	m	
120.0	5.61E-12	3.25	0.050	0.0020	0.0500	1.4665

n = 34 r = 1.09930

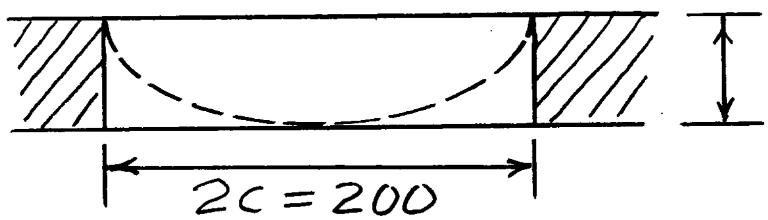
j	a_j m	$\alpha = a_j/t$	F_{Dj}	ΔK MPa $m^{0.5}$	Y = dN/da cycles/m	ΔN cycles	Σ(ΔN) cycles
0	2.000E-03	0.0400	1.086	8.532	1.680E+08		
1	2.199E-03	0.0440	1.086	8.947	1.439E+08		
2	2.417E-03	0.0483	1.087	9.383	1.233E+08	59937	5.994E+04
3	2.657E-03	0.0531	1.087	9.841	1.056E+08		
4	2.921E-03	0.0584	1.087	10.323	9.041E+07	53144	1.131E+05
5	3.211E-03	0.0642	1.088	10.828	7.740E+07		
6	3.530E-03	0.0706	1.089	11.360	6.624E+07	47079	1.602E+05
7	3.880E-03	0.0776	1.089	11.918	5.667E+07		
8	4.265E-03	0.0853	1.090	12.507	4.845E+07	41653	2.018E+05
9	4.689E-03	0.0938	1.091	13.126	4.141E+07		
10	5.155E-03	0.1031	1.093	13.779	3.536E+07	36782	2.386E+05
11	5.666E-03	0.1133	1.094	14.468	3.018E+07		
12	6.229E-03	0.1246	1.096	15.196	2.573E+07	32393	2.710E+05
13	6.848E-03	0.1370	1.099	15.967	2.191E+07		
14	7.528E-03	0.1506	1.101	16.783	1.863E+07	28417	2.994E+05
15	8.275E-03	0.1655	1.105	17.651	1.581E+07		
16	9.097E-03	0.1819	1.109	18.574	1.340E+07	24790	3.242E+05
17	1.000E-02	0.2000	1.114	19.560	1.133E+07		
18	1.099E-02	0.2199	1.119	20.615	9.548E+06	21456	3.457E+05
19	1.208E-02	0.2417	1.126	21.749	8.023E+06		
20	1.328E-02	0.2657	1.135	22.972	6.717E+06	18367	3.640E+05
21	1.460E-02	0.2921	1.145	24.296	5.599E+06		
22	1.605E-02	0.3211	1.156	25.735	4.643E+06	15488	3.795E+05
23	1.765E-02	0.3530	1.170	27.307	3.830E+06		
24	1.940E-02	0.3880	1.187	29.030	3.139E+06	12805	3.923E+05
25	2.133E-02	0.4265	1.206	30.927	2.555E+06		
26	2.344E-02	0.4689	1.228	33.020	2.065E+06	10328	4.026E+05
27	2.577E-02	0.5155	1.253	35.333	1.657E+06		
28	2.833E-02	0.5666	1.282	37.887	1.321E+06	8101	4.107E+05
29	3.115E-02	0.6229	1.313	40.696	1.047E+06		
30	3.424E-02	0.6848	1.346	43.757	8.272E+05	6191	4.169E+05

(11.52, p.4)

31	3.764E-02	0.7528	1.380	47.037	6.541E+05		
32	4.138E-02	0.8275	1.412	50.448	5.210E+05	4681	4.216E+05
33	4.548E-02	0.9097	1.437	53.810	4.224E+05		
34	5.000E-02	1.0000	1.446	56.790	3.545E+05	3664	4.253E+05

N_{if}, cycles = 425,278

(b) The $K < K_{Ic}$ calculation for $a=t$ above implies leaking before fracture. Assume that the crack maintains the $a/c = 0.5$ shape until near the back wall, and then become a through crack as below with $a = t = 0.5c$



For this situation:

$K = FS\sqrt{\pi c} = (1)(120\,MPa)\sqrt{\pi\,(0.100\,m)}$

$K = 67.26\ MPa\sqrt{m}$

$X_c = K_{Ic}/K = 200/67.26 = 2.97$ ◀

(c) $X_N = ?$ for 5 cycles a day for 20 years.

$\hat{N} = (5\ cyc/day)(365.25\ days/yr)(20\ yrs)$

$\hat{N} = 36{,}525$

$$X_N = \frac{N_{if}}{\hat{N}} = \frac{425{,}300}{36{,}525} = 11.64$$ ◀

(11.52, p. 5)

(d) $X_c = ?$ after 20 years, or $\hat{N} = 36{,}525$ cycles. From the numerical integration table, this occurs prior to $j = 2$, where $\Sigma(\Delta N) = 59{,}940$ cycles. In this region, $F_D \approx 1.086$ and constant. Hence, find a_{20} from:

$$N_{if} = \frac{a_{20}^{1-m/2} - a_i^{1-m/2}}{C\,(F\,\Delta S\sqrt{\pi})^m\,(1-m/2)}$$

$$36{,}525 = \frac{a_{20}^{-0.625} - 0.002^{-0.625}}{5.61\times10^{-12}\,(0.897\times120\sqrt{\pi})^{3.25}\,(-0.625)}$$

For $K = F S\sqrt{\pi a}$, $F = \dfrac{F_D}{\sqrt{Q}} = \dfrac{1.086}{\sqrt{1.4665}} = 0.897$

Solving gives $a_{20} = 0.00224\ m$

$K_{20} = F\, S_{max}\sqrt{\pi a_{20}}$

$K_{20} = 0.897\,(120\ MPa)\sqrt{\pi\,(0.00224\ m)}$

$K_{20} = 9.03$, $X_c = K_{Ic}/K_{20} = 200/9.03 = 22.1$ ◀

(e) The vessel seems quite safe from the above calculations. However, one should check that hostile chemical or thermal environments are not present that accelerate crack growth. Also, check that occasion pressure transients do not

(11.52, p.6)

occur, perhaps by monitoring the vessel operation for a period of time. More detailed analysis of crack shape changes during growth could be done based on [Newman 86]. Finally, check that the vessel meets any applicable design codes, such as [ASME 04] on p. 379.

11.53 For data of Fig. 11.38, find A, n for $\dot{a} = AK^n$. Comment on n values. How does $\dot{a}$ increase for 25% increase in K?

Find two points on each line.

$$n = \frac{\log \dot{a}_1 - \log \dot{a}_2}{\log K_1 - \log K_2} = \frac{\log(\dot{a}_1/\dot{a}_2)}{\log(K_1/K_2)}, \quad A = \frac{\dot{a}_1}{K_1^n}$$

Glass	$\dot{a}_1$ m/s	K_1 MPa$\sqrt{m}$	$\dot{a}_2$ m/s	K_2 MPa$\sqrt{m}$	n	A
Soda	10^{-11}	0.28	10^{-4}	0.62	20.3	1.67
Ultra	10^{-11}	0.315	10^{-4}	0.49	36.5	2.05×10^7

These very high values of n indicate an extreme sensitivity of $\dot{a}$ to K.

If $K' = 1.25K$, $\dot{a}'/\dot{a} = 1.25^n = \begin{cases} \times 92 \text{ soda} \\ \times 3400 \text{ ultra} \end{cases}$

11.54 For soda-lime glass, $A = 1.67$, $n = 20.3$, for $\dot{a} = AK^n$ (m/s, MPa$\sqrt{m}$). Half-circular $a_i = 10\,\mu m$, $F = 0.728$ for $K = FS\sqrt{\pi a}$

(a) $$t_{if} = \frac{a_f^{1-\frac{n}{2}} - a_i^{1-\frac{n}{2}}}{A(FS\sqrt{\pi})^m(1-\frac{n}{2})} = f(S) = ?$$

$$t_{if} = \frac{(1/a_i)^{\frac{n}{2}-1} - (1/a_f)^{\frac{n}{2}-1}}{A(FS\sqrt{\pi})^n(\frac{n}{2}-1)} = \frac{1}{a_i^{\frac{n}{2}-1}}\left(\frac{1-(a_i/a_f)^{\frac{n}{2}-1}}{A(FS\sqrt{\pi})^n(\frac{n}{2}-1)}\right)$$

If $a_i << a_f$, then for $n = 20.3$, the quantity $(a_i/a_f)^{n/2-1}$ is small and can be dropped. For example, if $a_i = 0.25\,a_f$, then $(a_i/a_f)^{n/2-1} = 3.1 \times 10^{-6}$. Hence, for this high n, the life is essentially independent of a_f.

$$t_{if} = \frac{1}{a_i^{\frac{n}{2}-1}A(FS\sqrt{\pi})^n(\frac{n}{2}-1)} = \frac{C}{a_i^{\frac{n}{2}-1}S^n}$$

where $$C = \frac{1}{A(F\sqrt{\pi})^n(\frac{n}{2}-1)} = 3.50 \times 10^{-4}$$

for $a_i = 10\,\mu m = 10^{-5}\,m$, $t_{if}S^{20.3} = 1.97 \times 10^{42}$ ◀

(b) $t_{if}S^{20.3} = C/a_i^{9.15} = C'$

a_i, m	C' (for MPa, seconds)	C_2
0.000005	1.18E+45	166.1
0.000010	2.08E+42	121.5
0.000020	3.67E+39	88.9

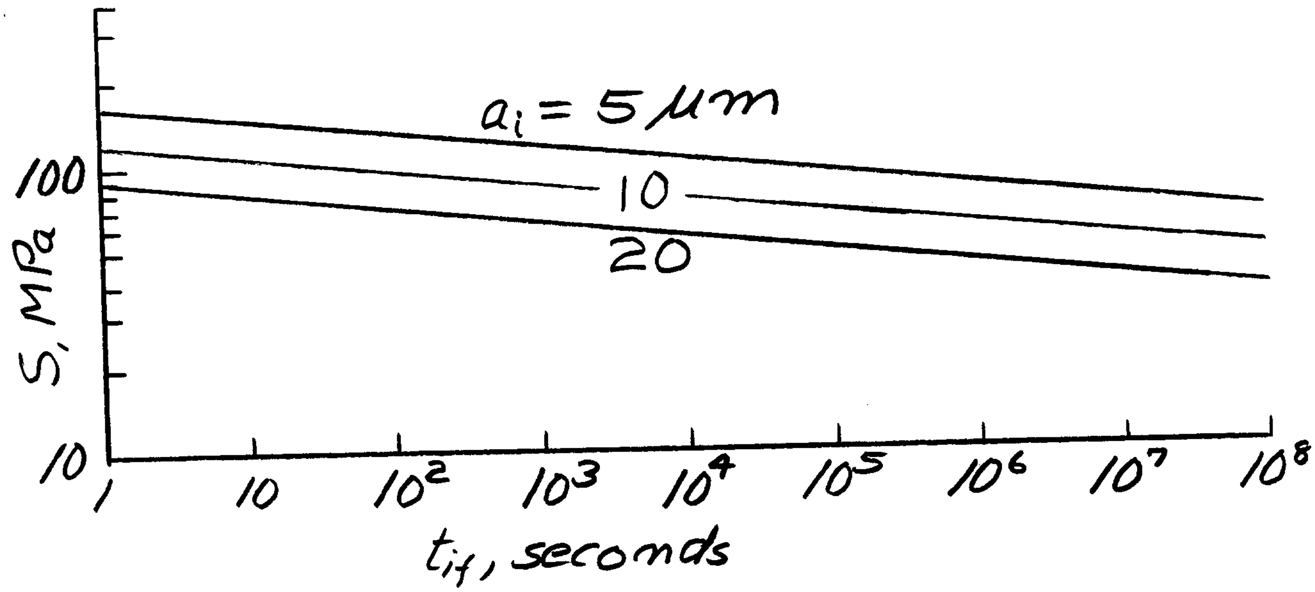
◀

(11.54, p.2)

To avoid large exponents, use

$$t_{if} = \frac{C'}{S^{20.3}} = \left(\frac{C_2}{S}\right)^{20.3}, \text{ where } C_2 = (C')^{1/20.3}$$

Now, for $S = C_2$ (MPa), $t_{if} = 1$ s. Calculate one or more additional t_{if} using S below C_2; do for each case. Straight lines on a log-log plot are formed.



The life is highly sensitive to both a_i and stress. Doubling a_i shortens the life by a factor of $2^{9.15} = 570$ for any given stress. The sensitivity to stress is even greater; doubling S shortens the life by a factor of $2^{20.3} = 1.3 \times 10^6$.