10.2 For Fig. 10.2, evaluate k_t and k_f.
Fig. A12(c) applies: $d_1 = 7.62$, $\rho = 0.25$ mm
$d_2 = 7.62 + 2(1.59) = 10.8$ mm

$\frac{d_2}{d_1} = 1.42$, $\frac{\rho}{d_1} = 0.0328$, $k_t \approx 3.1$ (just off the graph)

$\alpha = 0.51$ mm, $k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho} = 1.69$

k_t agrees, but k_f differs from the 2.2 value. The k_f equations are intended for relatively mild notches, but this ρ indicates a quite sharp notch.

Alternate Solution; Use Neuber equation.

$$\log\beta = -9.402\times10^{-9}\sigma_u^3 + 1.422\times10^{-5}\sigma_u^2 - 8.249\times10^{-3}\sigma_u + 1.451$$

$$\beta, \text{mm} = 10^{\log\beta} \qquad \text{(T-series Al)}$$

$$k_f = 1 + \frac{k_t - 1}{1 + \sqrt{\beta/\rho}}$$

2024-T4 Al (Table 4.2)

σ_u, MPa	ρ, mm	k_t	$\log\beta$	β, mm	k_f
476	0.25	3.10	-0.2676	0.540	**1.85**

This k_f is still not in agreement with the experimental 2.2. The comments above apply here also.

10.3 For Fig. 10.11, evaluate k_t and k_f, and estimate notch strength at $N_f = 10^6$ cyc.

Fig. A.11(b) applies: $w_2 = 25.4$, $\rho = 2.54$ mm

$w_1 = w_2 - 2\rho = 20.32$ mm

$\frac{w_2}{w_1} = 1.25$, $\frac{\rho}{w_1} = 0.125$, $k_t = 2.4$ ◀

$$\log\alpha = 2.654\times10^{-7}\sigma_u^2 - 1.309\times10^{-3}\sigma_u + 0.01103$$

$$\alpha, \text{mm} = 10^{\log\alpha} \qquad (345 \le \sigma_u \le 2070\,\text{MPa})$$

$$k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho}$$

AISI 4340 steel (Fig. 10.11)

σ_u, MPa	ρ, mm	k_t	log α	α, mm	k_f
786	2.54	2.40	-0.8539	0.140	**2.33**

◀

At $N_f = 10^6$, smooth specimen curve gives $\sigma_a = 390$ MPa. Estimate for notch case is $S_a = \sigma_a / k_f = 167$ MPa, which is close to $S_a \approx 175$ MPa from notch curve. ◀

Alternate Solution: Use Neuber equation.

$$\log\beta = -1.079\times10^{-9}\sigma_u^3 + 2.740\times10^{-6}\sigma_u^2 - 3.740\times10^{-3}\sigma_u + 0.6404$$

$$\beta, \text{mm} = 10^{\log\beta} \qquad (345 \le \sigma_u \le 1725\,\text{MPa})$$

$$k_f = 1 + \frac{k_t - 1}{1 + \sqrt{\beta/\rho}}$$

(10.3, p. 2)

AISI 4340 steel (Fig. 10.11)

σ_u, MPa	ρ, mm	k_t	log β	β, mm	k_f
786	2.54	2.40	-1.1304	7.406E-02	**2.20**

At $N_f = 10^6$, $\sigma_a = 390$ MPa (smooth spec.)
$S_a = \sigma_a / k_f = 177$ MPa, very close to $S_a \approx$ 175 MPa from notch curve.

10.4 AISI 4130 steel, $\sigma_u = 817$ MPa. Fig. 9.24 for σ_e. Double-edge-notch plate, load P, as Fig. A.11(b). $w_1 = 38.10$, $w_2 = 57.15$, $\rho = 8.06$, $t = 5.0$ mm. (a) k_t, $k_f = ?$, (b) $\hat{P}_a = ?$, 10^6 cyc., $X_S = 2.5$

(a) $w_2/w_1 = 1.5$, $\rho/w_1 = 0.212$, $k_t = 2.13$ ◀

$$\log\alpha = 2.654\times10^{-7}\sigma_u^2 - 1.309\times10^{-3}\sigma_u + 0.01103$$

$$\alpha,\ \text{mm} = 10^{\log\alpha} \qquad (345 \le \sigma_u \le 2070\,\text{MPa})$$

$$k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho}$$

σ_u, MPa	ρ, mm	k_t	log α	α, mm	k_f
817	8.06	2.13	-0.8813	0.1314	**2.112**

◀

(b)

$$\sigma_e = 0.5\sigma_u \qquad \text{(Fig. 9.24)}$$

$$S_a = \sigma_e / k_f$$

$$S_a = \frac{P_a}{w_1 t}, \qquad P_a = S_a w_1 t$$

$$\hat{P}_a = P_a / X_S$$

σ_e, MPa	S_a, MPa	w_1, mm	t, mm	P_a, kN	X_S	$\hat{P}_a$, kN
408.5	193.4	38.10	5.00	36.85	2.50	**14.74**

◀

(10.4, p. 2)

Alternate Solution; Proceed as above except use k_f from Neuber, Eqs. 10.10 and 10.11.

σ_u, MPa	ρ, mm	k_t	$\log \beta$	β, mm	k_f
817	8.06	2.13	-1.1747	0.0669	**2.036**

σ_e, MPa	S_a, MPa	w_1, mm	t, mm	P_a, kN	X_S	$\hat{P}_a$, kN
408.5	200.7	38.10	5.00	38.23	2.50	**15.29**

10.5 Shaft with diameter step in bending, Fig A.12 (b). $d_1 = 50$, $d_2 = 55$, $\rho = 1.3$ mm. $\sigma_u = 1100$ MPa, σ_e from Fig. 9.24. Q & T steel. (a) k_t, k_f = ? (b) $\hat{M}_a$ = ?, 10^6 cycles, $X_S = 1.8$.

(a) $d_2/d_1 = 1.1$, $\rho/d_1 = 0.026$, $k_t = 2.1$ ◀

$$\log\alpha = 2.654\times10^{-7}\sigma_u^2 - 1.309\times10^{-3}\sigma_u + 0.01103$$

$$\alpha, \text{mm} = 10^{\log\alpha} \qquad (345 \le \sigma_u \le 2070\,\text{MPa})$$

$$k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho}$$

σ_u, MPa	ρ, mm	k_t	log α	α, mm	k_f
1100	1.30	2.10	-1.1077	0.0780	**2.038**

◀

$$\sigma_e = 0.5\sigma_u \qquad \text{(Fig. 9.24)}$$

$$S_a = \sigma_e / k_f$$

$$S_a = \frac{32M_a}{\pi d_1^3}, \qquad M_a = \frac{S_a \pi d_1^3}{32}$$

$$\hat{M}_a = M_a / X_S$$

σ_e, MPa	S_a, MPa	d_1, mm	M_a, N-m	X_S	$\hat{M}_a$, N·m
550	269.9	50.00	3312	1.80	**1840**

◀

(10.5, p.2)

Alternate Solution: Proceed as above except use k_f from Neuber, Eqs. 10.10 and 10.11.

σ_u, MPa	ρ, mm	k_t	$\log \beta$	β, mm	k_f
1100	1.30	2.10	-1.5943	0.0254	**1.965**

σ_e, MPa	S_a, MPa	d_1, mm	M_a, N-m	X_S	$\hat{M}_a$, N·m
550	279.9	50.00	3435	1.80	**1908**

10.9 2024-T4 Al in bending, Fig. 9.27. Use tensile data from Table 4.2 to estimate S_a at 10^7 cycles ($S_m = 0$) for no notch and $\rho = 1.59$ and 0.25 mm. $\sigma_u = 476$ MPa.

For no notch and bending, Fig. 9.25 gives $\sigma_a = 130$ MPa at $N_f = 5 \times 10^8$ cycles. Use this as a conservative estimate for $N_f = 10^7$. ◀

For the notches in Fig. 9.27:
$d_1 = 7.62$ mm, $d_2 = d_1 + 2(1.59) = 10.80$ mm
$d_2/d_1 = 1.42$. Then determine k_t and k_f for the two cases. Use Fig. A.12(c) for k_t.

$$k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho}, \quad \alpha = 0.51 \text{ mm}$$

ρ, mm	ρ/d_1	k_t	k_f	Calc. S_a, MPa	Data S_a, MPa
1.59	0.209	1.6	1.45	89.7	110
0.25	0.033	3.1	1.69	76.9	60

◀

$$S_a = \frac{\sigma_a}{k_f} = \frac{130 \text{ MPa}}{k_f}$$

The calculated S_a agree only roughly with the Fig. 9.26 data, being lower for the blunt notch, but higher for the sharp one. For no notch, $\sigma_a \approx 140$ MPa from Fig. 9.27 ◀

(10.9, p. 2)

Alternate Solution: Proceed as above except use k_f from Neuber, Eqs. 10.10 and 10.11.

Stresses in MPa

σ_u	ρ, mm	k_t	log β	β, mm	k_f	σ_e	Calc S_a	Data S_a
476.0	1.59	1.60	-0.2676	0.540	1.38	130	**94.3**	110
476.0	0.25	3.10	-0.2676	0.540	1.85	130	**70.3**	60

Calculated and data values of S_a are again only in rough agreement, but the differences are less than before

10.10 Circular rod in bending, with diameter step, Fig. A.12(b). $d_1 = 16$, $d_2 = 20$, $\rho = 1.2$ mm. AISI 4142 (450 HB) steel, constants in Table 9.1. Fillet radius ground. Estimate M_a for 10^6 cycles.

$d_2/d_1 = 1.25$, $\rho/d_1 = 0.075$, $k_t = 1.8$

$$\log\alpha = 2.654\times10^{-7}\sigma_u^2 - 1.309\times10^{-3}\sigma_u + 0.01103$$

$$\alpha,\ \text{mm} = 10^{\log\alpha} \qquad (345 \le \sigma_u \le 2070\,\text{MPa})$$

$$k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho}$$

σ_u, MPa	ρ, mm	k_t	log α	α, mm	k_f
1757	1.20	1.80	-1.4696	0.0339	**1.778**

Calculate σ_a at 10^6 cycles from Table 9.1 constants. Use d_1 with Fig. 10.9 to get m_d, and 450 HB with Fig. 10.10 to get m_s for ground finish. Combine these with k_f to get S_{er}, and from this M_a. Assume a safety factor $X_S = 1.5$ is adequate, which gives $\hat{M}_a$ allowed in service. Details are on the next page.

(10.10, p.2)

$$\sigma_a = \sigma'_f (2N_f)^b, \qquad N_f = 10^6 \text{ cycles}$$

$$S_{er} = \frac{\sigma_{er}}{k_f} = \frac{m_d m_s \sigma_a}{k_f}$$

$$S_{er} = \frac{32M_a}{\pi d_1^3}, \qquad M_a = \frac{S_{er}\pi d_1^3}{32}$$

$$\hat{M}_a = M_a / X_S$$

σ'_f MPa	b	σ_a at 10^6 MPa	m_d	m_s	S_{er} MPa
1937	-0.0762	641.2	0.95	0.79	270.6

d_1 mm	M_a N-m	X_S	$\hat{M}_a$ N-m
16.00	**108.8**	1.50	**72.6**

◀

10.14 S–N data for AISI steel, Fig. 10.11, notched member with half-circular cutouts, $w_2 = 25.4$, $\rho = 2.54$, $t = 6.35$ mm

$w_1 = w_2 - 2\rho = 20.32$ mm

(a) Obtain equation for S–N curve, $N_f = 10^2$ to 10^6 cycles. For straight line on semilog plot, use

$S_a = C + D \log N_f$ MPa

Two points are;

$(S_a, N_f) = (700, 10^2)$, $(230, 10^5)$

$$\left.\begin{aligned} 700 &= C + D \log 10^2 = C + 2D \\ 230 &= C + D \log 10^5 = C + 5D \end{aligned}\right\} \text{solve}$$

$470 = -3D$, $D = -156.7$ MPa

$C = 700 - 2D = 1013$

$S_a = 1013 - 156.7 \log N_f$ MPa ◀

(b) $N_f = ?$ for $P_a = 32$, $P_m = 25$ kN

$S = \dfrac{P}{w_1 t}$ Net area as in Fig. A.11(b)

$$S_a = \frac{P_a}{w_1 t} = \frac{32{,}000 \text{ N}}{(20.32)(6.35) \text{ mm}^2} = 248 \text{ MPa}$$

$$S_m = \frac{P_m}{w_1 t} = \frac{25{,}000}{(20.32)(6.35)} = 193.75 \text{ MPa}$$

(10.14, p.2)

Material is ductile, %RA = 68.

$$S_{ar} = \frac{S_a}{1 - S_m/\sigma_u}, \quad \sigma_u = 786 \text{ MPa} \quad (\text{Fig. } 10.11)$$

$$S_{ar} = \frac{248 \text{ MPa}}{1 - \frac{193.75 \text{ MPa}}{786 \text{ MPa}}} = 329.1 \text{ MPa}$$

Since Fig 10.11 data are for $S_m = 0$ $(R = -1)$, interpret S_a in (a) as S_{ar}.

$S_{ar} = 1013 - 156.7 \log N_f = 329.1$ MPa

$\log N_f = 4.364$, $N_f = 23{,}140$ cycles ◀

Alternate Solution: Use SWT equation.

$S_{ar} = \sqrt{S_{max} S_a} = \sqrt{(248 + 193.75)(248)}$

$S_{ar} = 331.0$ MPa

$S_{ar} = 1013 - 156.7 \log N_f = 331.0$

$\log N_f = 4.352$, $N_f = 22{,}500$ cycles ◀

Comment: The close agreement of the two solutions is fortuitous.

10.15 For notched AISI 4340 steel S-N curve in Fig. 10.11, find S_{max} at $R=0$ for $N_f=10^5$ cycles by (a) Goodman (ductile), (b) SWT.

(a) At $N_f = 10^5$, curve gives $S_{ar} = 230$ MPa

$$S_{ar} = \frac{S_a}{1 - S_m/\sigma_u}, \quad S_a = S_m = S_{max}/2$$

$$230 \text{ MPa} = \frac{S_{max}/2}{1 - S_{max}/(2\sigma_u)}, \quad \sigma_u = 786 \text{ MPa}$$

$S_{max} = 356$ MPa ◀

(b) $S_{ar} = \sqrt{S_{max} S_a}$

$$230 \text{ MPa} = \sqrt{S_{max}(S_{max}/2)}$$

$S_{max} = 325$ MPa ◀

10.16 For S-N curve for 2024-T4 Al, $\rho =$ 1.59 mm, of Fig. 9.27, what M_m with $M_a =$ 7.5 N·m gives $N_f = 10^5$ cycles?

$S = \dfrac{32M}{\pi d_1^3}$ (Fig. A.12(d)), $d_1 = 7.62$ mm

$\sigma_u = 476$ MPa (Table 4.2)

From S-N curve, $S_{ar} = 225$ MPa

$$S_a = \frac{32 M_a}{\pi d_1^3} = \frac{32(7500\ \text{N}\cdot\text{mm})}{\pi(7.62\ \text{mm})^3} = 172.7\ \text{MPa}$$

$$S_{ar} = \sqrt{S_{max} S_a} = \sqrt{(S_m + S_a) S_a}$$

$$225\ \text{MPa} = \sqrt{(S_m + 172.7)\, 172.7}$$

$$S_m = 120.5\ \text{MPa}$$

$$M_m = \pi d_1^3 S_m / 32 = \pi(7.62)^3(120.5)/32$$

$$M_m = 5236\ \text{N}\cdot\text{mm} = 5.236\ \text{N}\cdot\text{m}$$ ◀

Alternate Solution: Use Goodman (ductile)

$$S_{ar} = \frac{S_a}{1 - S_m/\sigma_u}, \quad 225\ \text{MPa} = \frac{172.7}{1 - S_m/476}$$

$$S_m = 110.7\ \text{MPa}$$

$$M_m = \pi(7.62)^3(110.7)/32 = 4810\ \text{N}\cdot\text{mm}$$

$$M_m = 4.810\ \text{N}\cdot\text{m}$$ ◀

10.17 2024-T4 AL, Fig. A.11(a) geometry

$w = 50,\ d = 10,\ t = 20$ mm, $\rho = d/2 = 5$ mm

$P_a = ?$ for $N_f = 10^7$ and $P_m = 80$ kN

$\frac{d}{w} = 0.2,\ k_t = 2.5$

$\alpha = 0.51$ mm, $k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho} = 2.36$

$\sigma_o = 303,\ \sigma_u = 476,\ A = 839$ MPa, $B = -0.102$

$\sigma_{ar} = A (N_f)^B = 162.1$ MPa (Table 9.1)

$$S_m = \frac{P_m}{(w-d)t} = \frac{80{,}000\ \text{N}}{(40\ \text{mm})(20\ \text{mm})} = 100\ \text{MPa}$$

(a) $S_{ar} = \frac{S_a}{1 - S_m/\sigma_u} = \frac{\sigma_{ar}}{k_f}$ Goodman (ductile)

$$S_a = \frac{\sigma_{ar}}{k_f}\left(1 - \frac{S_m}{\sigma_u}\right) = \frac{162.1}{2.36}\left(1 - \frac{100}{476}\right) = 54.26\ \text{MPa}$$

$S_a = \frac{P_a}{(w-d)t}$, $P_a = S_a (w-d) t$

$P_a = (54.26\ \text{MPa})(50-10)(20)\ \text{mm}^2 = 43{,}400$ N ◀

(b) $S_{ar} = \sqrt{S_{max} S_a} = \sigma_{ar}/k_f$

$\sqrt{(100 + S_a) S_a} = 162.1/2.36$, $S_a = 34.96$ MPa

(Solve quadratic equation; take + root)

$P_a = S_a (w-d) t = 27{,}970$ N ◀

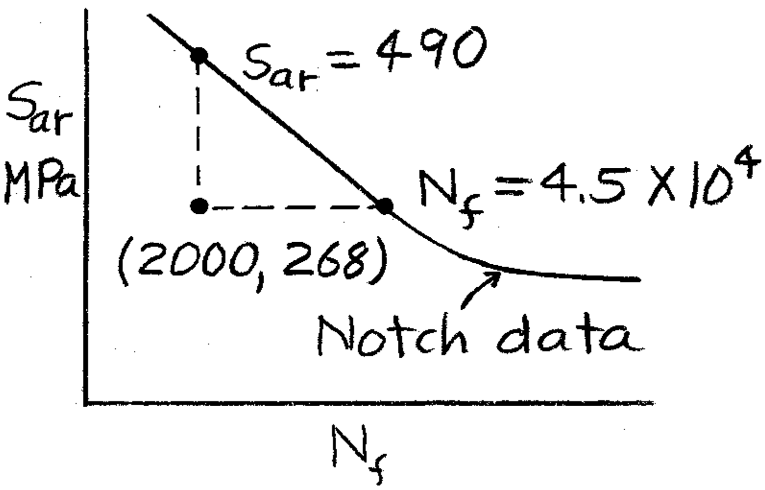
10.18 Fig. 10.11 notched, $X_S, X_N = ?$ for

$\hat{N} = 2000$ cycles, $\hat{S}_a = 220$, $\hat{S}_m = 140$ MPa

$$\hat{S}_{ar} = \frac{\hat{S}_a}{1 - \frac{S_m}{\sigma_u}} = 268 \text{ MPa}$$

(Goodman)

$\sigma_u = 786$ MPa



Read S_{ar} and N_f from graph.

$$X_S = \frac{S_{ar}}{\hat{S}_{ar}} = 1.83$$ ◀

$$X_N = \frac{N_f}{\hat{N}} = 22.5$$ ◀

Alternate Solution: Use SWT

$$\hat{S}_{ar} = \sqrt{\hat{S}_{max} \hat{S}_a} = \sqrt{(\hat{S}_m + \hat{S}_a)\hat{S}_a} = 281 \text{ MPa}$$

Reading from graph at $\hat{S}_{ar} = 281$ MPa gives $N_f = 4.0 \times 10^4$

$$X_S = \frac{S_{ar}}{\hat{S}_{ar}} = \frac{490}{281} = 1.74$$ ◀

$$X_N = \frac{N_f}{\hat{N}} = \frac{4.0 \times 10^4}{2000} = 20.0$$ ◀

10.20 ASTM A514 steel plate with a width reduction, Fig. A.11(c). $N_f = 10^6$ cycles req'd. at $P_a = 16$, $P_m = 9$ kN. $w_1 = 20$, $w_2 = 24$, $t = 10$, $\rho = 0.5$ mm, notch ground finish. Find: (a) X_S, (b) new ρ for $X_S = 1.8$.

$w_2/w_1 = 1.20$, $\rho/w_1 = 0.025$, $k_t = 2.8$

$$\log\alpha = 2.654\times10^{-7}\sigma_u^2 - 1.309\times10^{-3}\sigma_u + 0.01103$$

$$\alpha,\ \text{mm} = 10^{\log\alpha} \qquad (345 \le \sigma_u \le 2070\,\text{MPa})$$

$$k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho}$$

σ_u, MPa	ρ, mm	k_t	log α	α, mm	k_f
807	0.5	2.80	-0.8725	0.1341	**2.419**

$$S_{er} = \frac{\sigma_{er}}{k_f} = \frac{m\sigma_u}{k_f}, \qquad \sigma_u = 807\text{ MPa (Tbl. 4.2)}$$

$m = m_e m_d m_s$, $m_e = 0.5$ (Fig. 9.24)

$m_d = 0.95$ (Fig. 10.9 with w_1)

$m_s = 0.9$ (Fig. 10.10 with $\sigma_u = 117$ ksi)

$$S_{er} = \frac{0.5 \times 0.95 \times 0.9\,(807\text{ MPa})}{2.42} = 142.6\text{ MPa}$$

$$\hat{S}_a = \frac{P_a}{w_1 t} = \frac{16{,}000\text{N}}{20(10)\text{ mm}^2} = 80\text{ MPa}$$

$$\hat{S}_m = \frac{P_m}{w_1 t} = \frac{9000}{20(10)} = 45\text{ MPa}$$

(10.20, p.2)

$$\hat{S}_{ar} = \sqrt{\hat{S}_{max}\,\hat{S}_a} = \sqrt{(45+80)\,80} = 100\ \text{MPa}$$

$$X_S = S_{er}/\hat{S}_{ar} = 142.6/100 = 1.43$$

(b) Apply the following procedure:
Pick ρ, obtain k_t, calculate k_f, S_{er}, X_S.
$\sigma_{er} = m_e m_d m_s \sigma_u = 345.0$ is useful

w_2/w_1	α, mm	σ_{er}, MPa	$\hat{S}_{ar}$, MPa
1.200	0.1341	345.0	100.0

ρ, mm	ρ/w_1	k_t	k_f	S_{er}, MPa	X_S
0.50	0.0250	2.80	2.419	142.6	1.43
1.00	0.0500	2.25	2.102	164.1	1.64
2.00	0.1000	1.90	1.843	187.1	**1.87**

Use $\rho = 2.0$ mm, $X_S = 1.87$

10.21

For Ex. 10.4 data on notched plates of 2024-T3 Al at various S_m: (a) Calculate and plot S_{ar} vs. N_f for the Goodman equation ($k_{fm} = 1$), and include the fitted line for $S_m = 0$. (b, c) Repeat for equations of SWT and Goodman with varying k_{fm}.

$$S_{ar} = \frac{S_a}{1 - S_m / \sigma_u} \qquad \text{(Goodman, } k_{fm} = 1\text{)}$$

$$S_{ar} = \sqrt{S_{\max} S_a} \qquad \text{(SWT)}$$

$$S_{ar} = \frac{S_a}{1 - k_{fm} S_m / \sigma_u} \qquad \text{(Goodman, } k_{fm} \text{ varies)}$$

$$k_{fm} = k_f \qquad (k_f S_{\max} < \sigma_o)$$

$$k_{fm} = \frac{\sigma_o - k_f S_a}{S_m} \qquad (k_f S_{\max} > \sigma_o)$$

$$k_{fm} = 0 \qquad (k_f S_a > \sigma_o)$$

ρ, mm	k_t	σ_u	log β	β, mm	k_f
8.06	2.15	503	-0.2970	0.505	1.92

For (c), calculate k_f from Eqs. 10.10 and 10.11 as above. Numerical values of the various S_{ar} and plots are on pages that follow. Goodman with $k_{fm} = 1$ gives a poor correlation, being overly conservative at short lives, and tending to be nonconservative at long lives. SWT correlates the data somewhat better. Goodman with varying k_{fm} gives the best correlation of the three, but not as good as Walker in Ex. 10.4. ◀

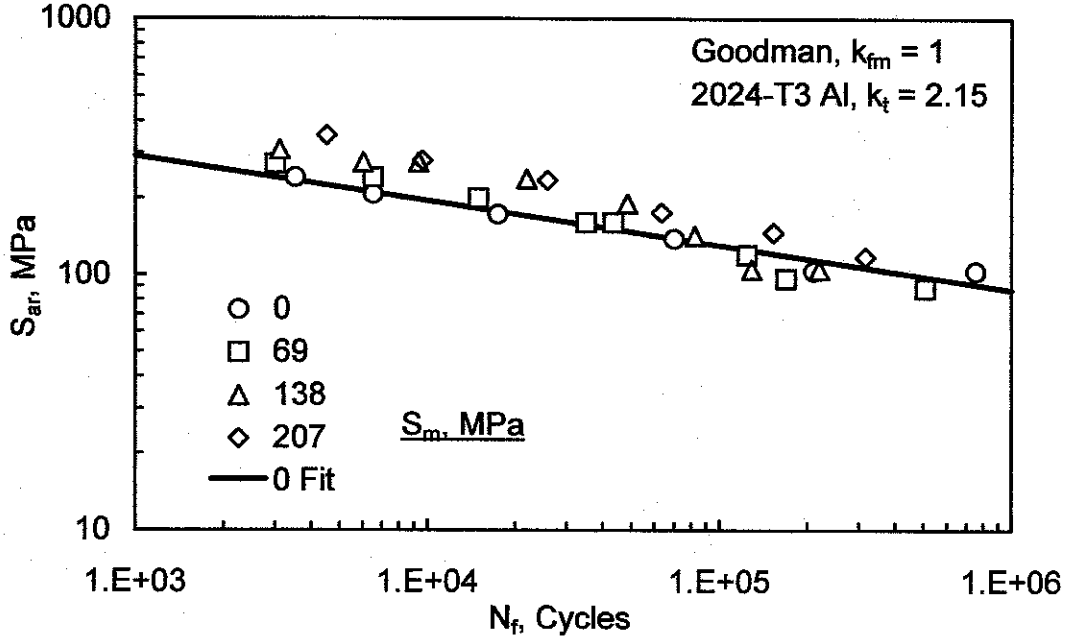
(10.21, p. 2)

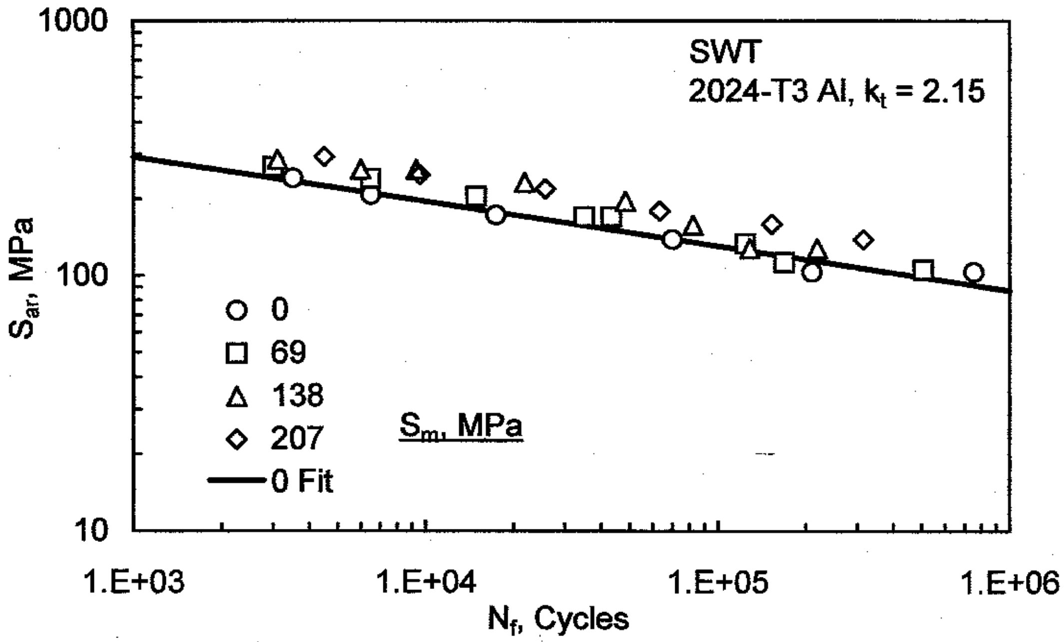
A	B	σ_o	σ_u	k_f
976	-0.1750	372	503	1.92

All σ, S in MPa

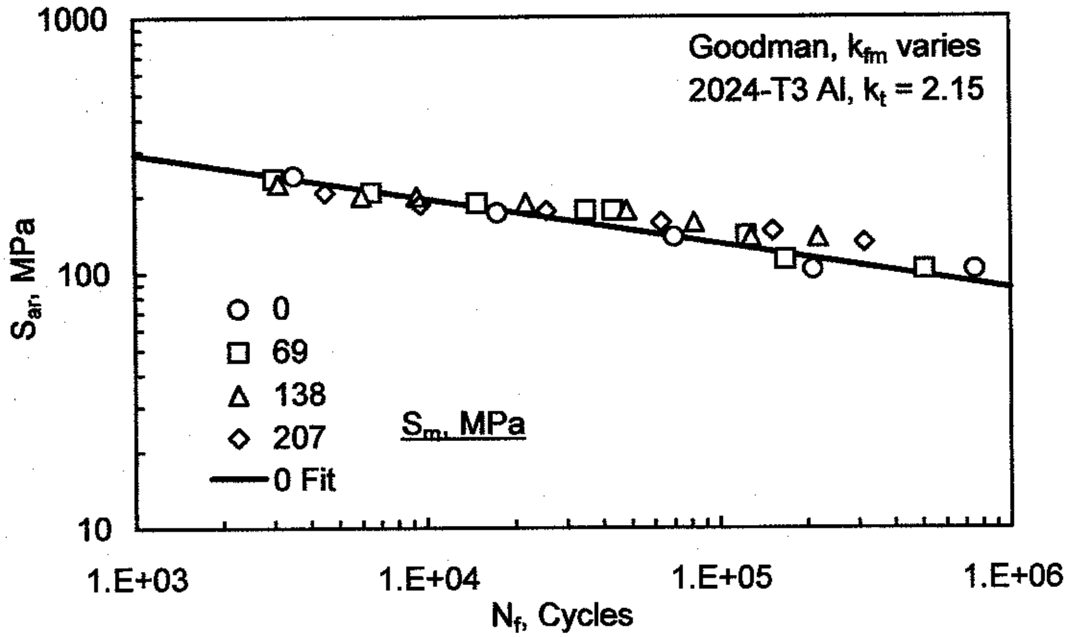
				Goodman $k_{fm} = 1$	SWT	Goodman k_{fm} varies	
S_{max}	S_m	N_f, cycles	S_a	S_{ar}	S_{ar}	k_{fm}	S_{ar}
241	0	3,500	241	241.0	241.0	0.00	241.0
207	0	6,500	207	207.0	207.0	0.00	207.0
172	0	17,400	172	172.0	172.0	1.92	172.0
138	0	70,000	138	138.0	138.0	1.92	138.0
103	0	754,000	103	103.0	103.0	1.92	103.0
103	0	210,000	103	103.0	103.0	1.92	103.0
303	69	3,000	234	271.2	266.3	0.00	234.0
276	69	6,500	207	239.9	239.0	0.00	207.0
241	69	14,900	172	199.3	203.6	0.61	187.6
207	69	35,000	138	159.9	169.0	1.55	175.3
207	69	43,400	138	159.9	169.0	1.55	175.3
172	69	124,200	103	119.4	133.1	1.92	139.8
152	69	168,700	83	96.2	112.3	1.92	112.7
145	69	507,400	76	88.1	105.0	1.92	103.2
362	138	3,100	224	308.7	284.8	0.00	224.0
338	138	9,300	200	275.6	260.0	0.00	200.0
338	138	6,000	200	275.6	260.0	0.00	200.0
310	138	21,800	172	237.0	230.9	0.30	187.6
276	138	48,300	138	190.2	195.2	0.78	175.3
241	138	82,200	103	141.9	157.6	1.26	157.6
214	138	128,500	76	104.7	127.5	1.64	138.1
214	138	218,700	76	104.7	127.5	1.64	138.1
414	207	4,500	207	351.8	292.7	0.00	207.0
372	207	9,600	165	280.4	247.7	0.27	185.4
345	207	25,700	138	234.5	218.2	0.52	175.3
310	207	63,500	103	175.0	178.7	0.84	157.6
293	207	152,900	86	146.1	158.7	1.00	146.1
276	207	315,500	69	117.3	138.0	1.16	131.7
$S_m = 0$		1,000		291.4	291.4		291.4
Fit		1,000,000		87.0	87.0		87.0

(10.21, p. 3)


Goodman, k_{fm} = 1
2024-T3 Al, k_t = 2.15
1000
100
10
S_{ar}, MPa
1.E+03
1.E+04
1.E+05
1.E+06
N_f, Cycles
0
69
138
207
0 Fit
S_m, MPa


SWT
2024-T3 Al, k_t = 2.15
1000
100
10
S_{ar}, MPa
1.E+03
1.E+04
1.E+05
1.E+06
N_f, Cycles
0
69
138
207
0 Fit
S_m, MPa

(10.21, p. 4)



Goodman, k_{fm} varies
2024-T3 Al, k_t = 2.15
1000
100
10
S_{ar}, MPa
0
69
138
207
0 Fit
S_m, MPa
1.E+03
1.E+04
1.E+05
1.E+06
N_f, Cycles

10.22

For given fatigue data on notched plates of 7075-T6 Al at various S_m:
(a) Calculate and plot S_{ar} vs. N_f for the Goodman equation, and include the fitted line for $S_m = 0$. (b) Repeat for equation of SWT.

$$S_{ar} = \frac{S_a}{1 - S_m / \sigma_u} \qquad \text{(Goodman)}$$

$$S_{ar} = \sqrt{S_{max} S_a} \qquad \text{(SWT)}$$

Numerical values of the various S_{ar} and plots are on pages that follow. Goodman gives a poor correlation, being somewhat conservative at short lives, and grossly nonconservative at long lives. SWT correlates the data very well. ◀

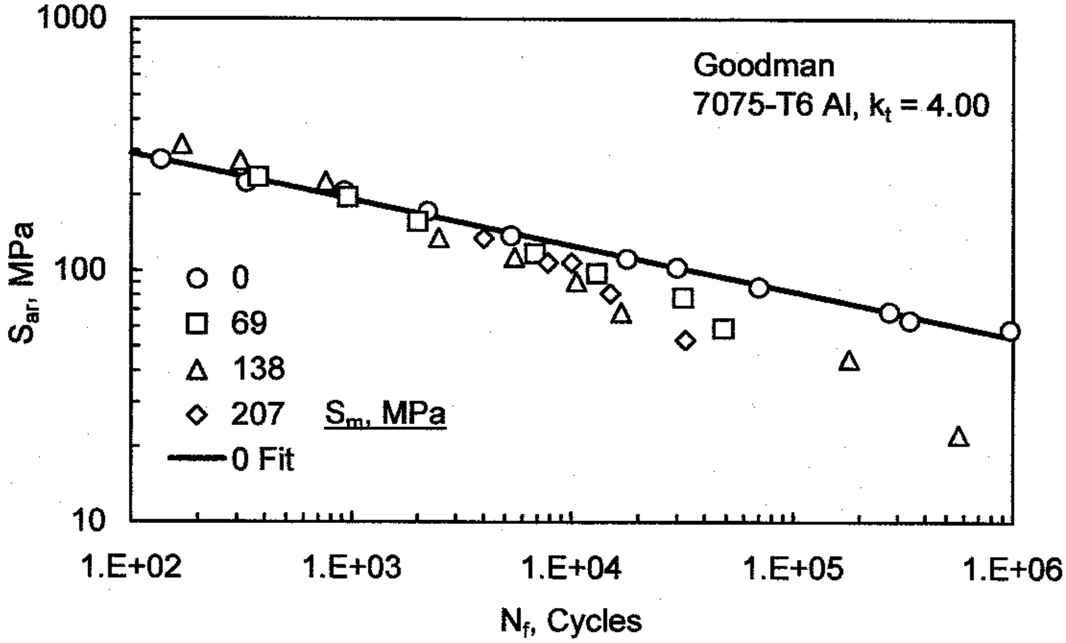
(10.22, p.2)

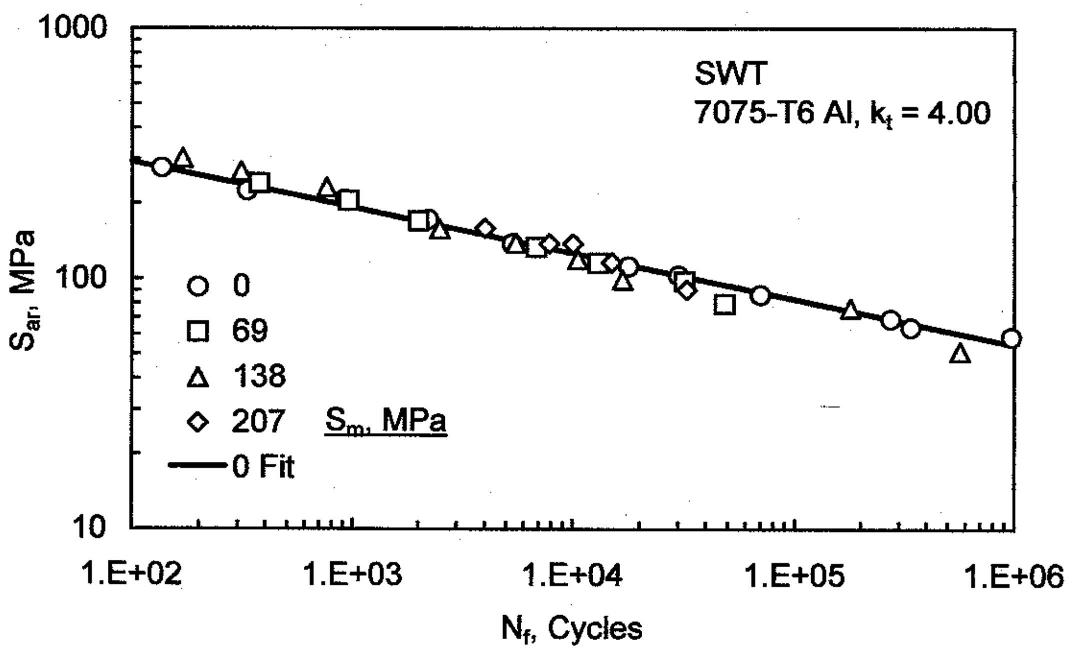
A	B	σ_u
676	-0.1822	572

All σ, S in MPa

				Goodman	SWT
S_{max}	S_m	N_f, cycles	S_a	S_{ar}	S_{ar}
276	0	136	276.0	276.0	276.0
224	0	329	224.0	224.0	224.0
207	0	917	207.0	207.0	207.0
172	0	2,228	172.0	172.0	172.0
138	0	5,300	138.0	138.0	138.0
112	0	17,800	112.0	112.0	112.0
103	0	30,000	103.0	103.0	103.0
86.2	0	70,000	86.2	86.2	86.2
69	0	274,000	69.0	69.0	69.0
63.8	0	339,200	63.8	63.8	63.8
58.6	0	969,200	58.6	58.6	58.6
276	69	374	207.0	235.4	239.0
241	69	955	172.0	195.6	203.6
207	69	2,000	138.0	156.9	169.0
172	69	6,823	103.0	117.1	133.1
155	69	13,000	86.0	97.8	115.5
138	69	32,000	69.0	78.5	97.6
121	69	48,500	52.0	59.1	79.3
379	138	169	241.0	317.6	302.2
345	138	309	207.0	272.8	267.2
310	138	756	172.0	226.7	230.9
241	138	2,500	103.0	135.8	157.6
224	138	5,500	86.0	113.3	138.8
207	138	10,500	69.0	90.9	119.5
190	138	16,800	52.0	68.5	99.4
172	138	179,000	34.0	44.8	76.5
155	138	566,500	17.0	22.4	51.3
293	207	4,000	86.0	134.8	158.7
276	207	7,800	69.0	108.1	138.0
276	207	10,000	69.0	108.1	138.0
259	207	15,000	52.0	81.5	116.1
241	207	32,700	34.0	53.3	90.5
S_m = 0		100		292.1	292.1
Fit		1,000,000		54.5	54.5

(10.22, p.3)


Goodman
7075-T6 Al, k_t = 4.00
1000
100
10
S_{ar}, MPa
O 0
□ 69
Δ 138
◇ 207
S_m, MPa
0 Fit
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
N_f, Cycles


SWT
7075-T6 Al, k_t = 4.00
1000
100
10
S_{ar}, MPa
O 0
□ 69
Δ 138
◇ 207
S_m, MPa
0 Fit
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
N_f, Cycles

10.23

For the given fatigue data on notched plates of 7075-T6 Al at various S_m, fit a stress-life curve of the form of Eq. 10.31, with S_{ar} given by the Walker relationship, Eq. 10.29. A multiple linear regression is needed.

$$S_{ar} = AN_f^B = S_{\max}\left(\frac{1-R}{2}\right)^{\gamma}, \qquad N_f = \left[S_{\max}\left(\frac{1-R}{2}\right)^{\gamma}\frac{1}{A}\right]^{1/B}$$

$$\underbrace{\log N_f}_{y} = \underbrace{\frac{1}{B}}_{m_1}\underbrace{\log S_{\max}}_{x_1} + \underbrace{\frac{\gamma}{B}}_{m_2}\underbrace{\log\left(\frac{1-R}{2}\right)}_{x_2} \underbrace{- \frac{1}{B}\log A}_{c}$$

$$y = m_1 x_1 + m_2 x_2 + c$$

Numerical values for fitting are given on the next page. The result of the fit is given below, and a plot of S_{ar} vs. life showing data and fit follows.

m_1	m_2	c
-5.0091	-2.3996	14.5392

$$B = \frac{1}{m_1}, \qquad \gamma = Bm_2 = \frac{m_2}{m_1}, \qquad A = 10^{-cB} = 10^{-c/m_1}$$

A, MPa	B	γ
799.1	-0.1996	0.4791

$$S_{ar} = 799.1 N_f^{-0.1996}, \qquad S_{ar} = S_{\max}\left(\frac{1-R}{2}\right)^{0.4791} \text{ MPa}$$

Plotting S_{ar} calculated from the fitted γ provides an excellent correlation of the data.

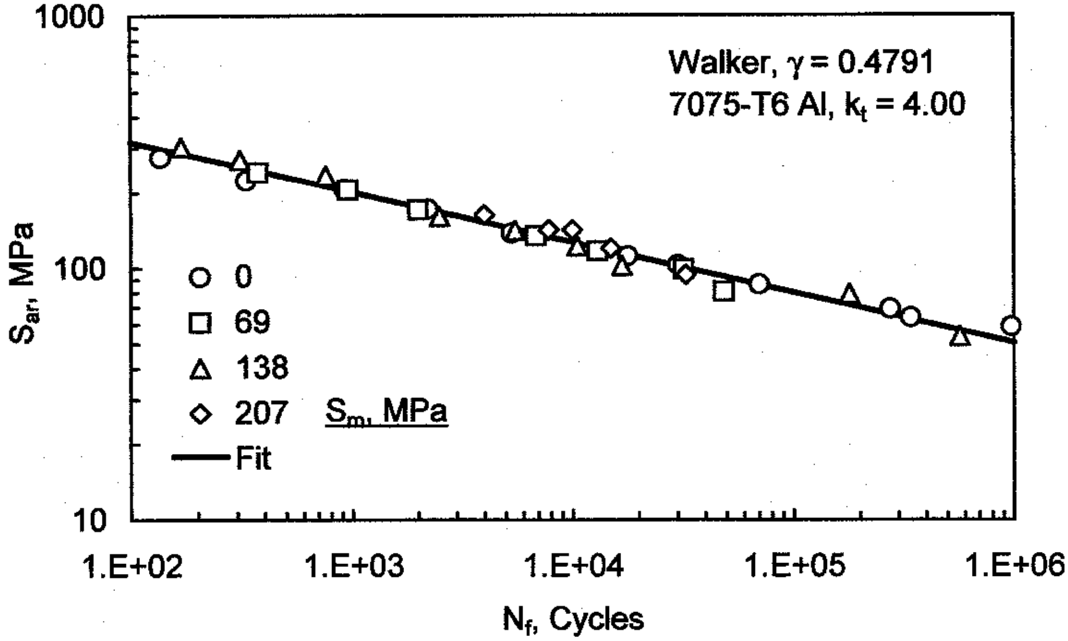
(10.23, p. 2)

A	B	γ
799.1	-0.1996	0.4791

Stresses in MPa

				y	x_1	x_2	
S_{max}	S_m	N_f, cycles	R	log N_f	log S_{max}	log[(1-R)/2]	S_{ar}
276	0	136	-1.0000	2.134	2.441	0.0000	276.0
224	0	329	-1.0000	2.517	2.350	0.0000	224.0
207	0	917	-1.0000	2.962	2.316	0.0000	207.0
172	0	2,228	-1.0000	3.348	2.236	0.0000	172.0
138	0	5,300	-1.0000	3.724	2.140	0.0000	138.0
112	0	17,800	-1.0000	4.250	2.049	0.0000	112.0
103	0	30,000	-1.0000	4.477	2.013	0.0000	103.0
86.2	0	70,000	-1.0000	4.845	1.936	0.0000	86.2
69.0	0	274,000	-1.0000	5.438	1.839	0.0000	69.0
63.8	0	339,200	-1.0000	5.530	1.805	0.0000	63.8
58.6	0	969,200	-1.0000	5.986	1.768	0.0000	58.6
276	69	374	-0.5000	2.573	2.441	-0.1249	240.5
241	69	955	-0.4274	2.980	2.382	-0.1465	205.0
207	69	2,000	-0.3333	3.301	2.316	-0.1761	170.5
172	69	6,823	-0.1977	3.834	2.236	-0.2227	134.5
155	69	13,000	-0.1097	4.114	2.190	-0.2558	116.9
138	69	32,000	0.0000	4.505	2.140	-0.3010	99.0
121	69	48,500	0.1405	4.686	2.083	-0.3668	80.7
379	138	169	-0.2718	2.228	2.579	-0.1966	305.1
345	138	309	-0.2000	2.490	2.538	-0.2218	270.1
310	138	756	-0.1097	2.879	2.491	-0.2558	233.8
241	138	2,500	0.1452	3.398	2.382	-0.3692	160.4
224	138	5,500	0.2321	3.740	2.350	-0.4157	141.6
207	138	10,500	0.3333	4.021	2.316	-0.4771	122.3
190	138	16,800	0.4526	4.225	2.279	-0.5628	102.1
172	138	179,000	0.6047	5.253	2.236	-0.7040	79.1
155	138	566,500	0.7806	5.753	2.190	-0.9599	53.8
293	207	4,000	0.4130	3.602	2.467	-0.5324	162.9
276	207	7,800	0.5000	3.892	2.441	-0.6021	142.1
276	207	10,000	0.5000	4.000	2.441	-0.6021	142.1
259	207	15,000	0.5985	4.176	2.413	-0.6973	120.0
241	207	32,700	0.7178	4.515	2.382	-0.8505	94.3
Fit		100					318.7
		1,000,000					50.7

(10.23, p.3)



Walker, γ = 0.4791
7075-T6 Al, k_t = 4.00
1000
100
10
S_{ar}, MPa
0
69
138
207
S_m, MPa
Fit
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
N_f, Cycles

10.24 Given three $\sigma_{max}-R-N_f$ data points, find A, B, γ for:

$$\sigma_{ar} = A N_f^{B} = \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} \qquad \text{MPa}$$

Above applied to each data point gives 3 equations; solve for A, B, γ.

$$A(13,100)^{B} = 345(1)^{\gamma} = 345$$
$$A(1,169,000)^{B} = 172\,(1)^{\gamma} = 172$$
$$A(351,000)^{B} = 372\,(0.3)^{\gamma}$$

$$\left(\frac{13,100}{1,169,000}\right)^{B} = \frac{345}{172}, \quad B = \frac{\log(345/172)}{\log(13,100/1,169,000)}$$

$$B = -0.1550$$ ◀

$$A = \frac{345}{13,100^{B}} = 1499 \text{ MPa}$$ ◀

$$1499\,(351,000)^{-0.1550} = 372\,(0.3)^{\gamma}$$

$$\log\frac{1499}{372} - 0.1550 \log 351,000 = \gamma \log 0.3$$

$$\gamma = 0.486$$ ◀

10.25

For the given fatigue data on unnotched specimens of 2024-T3 Al at various R, fit a power-law stress-life curve, with σ_{ar} given by the Walker relationship, Eq. 9.19. A multiple linear regression is needed.

$$\sigma_{ar} = AN_f^B = \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma}, \qquad N_f = \left[\sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma}\frac{1}{A}\right]^{1/B}$$

$$\underbrace{\log N_f}_{y} = \underbrace{\frac{1}{B}}_{m_1}\underbrace{\log\sigma_{max}}_{x_1} + \underbrace{\frac{\gamma}{B}}_{m_2}\underbrace{\log\left(\frac{1-R}{2}\right)}_{x_2}\underbrace{-\frac{1}{B}\log A}_{c}$$

$$y = m_1 x_1 + m_2 x_2 + c$$

Numerical values for fitting are given on the next page. The result of the fit is given below, and a plot of σ_{ar} vs. life showing data and fit follows.

m_1	m_2	c
-5.4218	-2.8755	17.9387

$$B = \frac{1}{m_1}, \qquad \gamma = Bm_2 = \frac{m_2}{m_1}, \qquad A = 10^{-cB} = 10^{-c/m_1}$$

A	B	γ
2035	-0.1844	0.5304

$$\sigma_{ar} = 2035\,N_f^{-0.1844}, \qquad \sigma_{ar} = \sigma_{max}\left(\frac{1-R}{2}\right)^{0.5304} \text{ MPa}$$ ◀

Plotting σ_{ar} calculated from the fitted γ provides an excellent correlation of the data. ◀

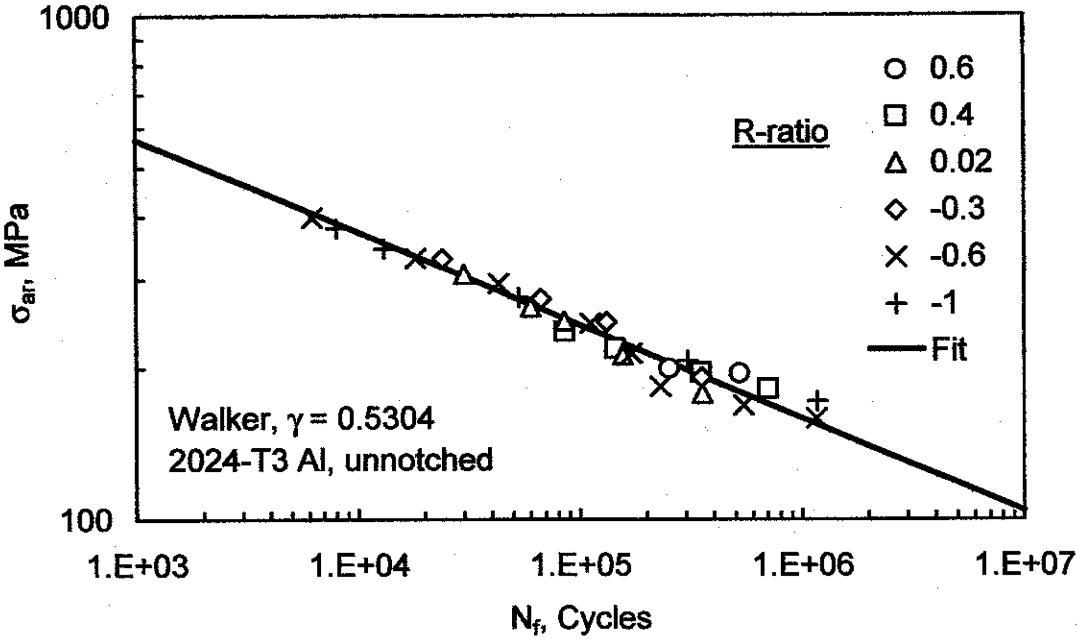
(10.25, p.2)

A	B	γ
2035	-0.1844	0.5304

Stresses in MPa

			y	x_1	x_2	
σ_{max}	R	N_f, cycles	log N_f	log σ_{max}	log [(1-R)/2]	σ_{ar}
469	0.6	252,000	5.401	2.671	-0.6990	199.7
459	0.6	520,000	5.716	2.662	-0.6990	195.5
448	0.4	85,000	4.929	2.651	-0.5229	236.6
414	0.4	144,000	5.158	2.617	-0.5229	218.6
372	0.4	351,000	5.545	2.571	-0.5229	196.4
345	0.4	701,000	5.846	2.538	-0.5229	182.2
448	0.02	30,000	4.477	2.651	-0.3098	306.9
386	0.02	60,000	4.778	2.587	-0.3098	264.4
362	0.02	85,000	4.929	2.559	-0.3098	248.0
310	0.02	156,000	5.193	2.491	-0.3098	212.3
260	0.02	355,000	5.550	2.415	-0.3098	178.1
414	-0.3	24,000	4.380	2.617	-0.1871	329.4
345	-0.3	67,000	4.826	2.538	-0.1871	274.5
310	-0.3	132,000	5.121	2.491	-0.1871	246.7
241	-0.3	353,000	5.548	2.382	-0.1871	191.8
448	-0.6	6,200	3.792	2.651	-0.0969	398.0
372	-0.6	18,200	4.260	2.571	-0.0969	330.5
331	-0.6	43,000	4.633	2.520	-0.0969	294.1
276	-0.6	112,000	5.049	2.441	-0.0969	245.2
241	-0.6	172,000	5.236	2.382	-0.0969	214.1
207	-0.6	231,000	5.364	2.316	-0.0969	183.9
190	-0.6	546,000	5.737	2.279	-0.0969	168.8
179	-0.6	1,165,000	6.066	2.253	-0.0969	159.0
379	-1	8,000	3.903	2.579	0.0000	379.0
345	-1	13,100	4.117	2.538	0.0000	345.0
276	-1	53,000	4.724	2.441	0.0000	276.0
207	-1	306,000	5.486	2.316	0.0000	207.0
172	-1	1,169,000	6.068	2.236	0.0000	172.0
Fit		1,000				569.3
		10,000,000				104.2

(10.25, p.3)



1000
100
σar, MPa
R-ratio
0.6
0.4
0.02
-0.3
-0.6
-1
Fit
Walker, γ = 0.5304
2024-T3 Al, unnotched
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
Nf, Cycles

10.26

For the given fatigue data on unnotched specimens of AISI 4340 steel at various σ_m, fit a power-law stress-life curve, with σ_{ar} given by the Walker relationship, Eq. 9.19. A multiple linear regression is needed.

$$\sigma_{ar} = AN_f^B = \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma}, \qquad N_f = \left[\sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma}\frac{1}{A}\right]^{1/B}$$

$$\underbrace{\log N_f}_{y} = \underbrace{\frac{1}{B}}_{m_1}\underbrace{\log\sigma_{max}}_{x_1} + \underbrace{\frac{\gamma}{B}}_{m_2}\underbrace{\log\left(\frac{1-R}{2}\right)}_{x_2} \underbrace{- \frac{1}{B}\log A}_{c}$$

$$y = m_1 x_1 + m_2 x_2 + c$$

Numerical values for fitting are given on the next page. The result of the fit is given below, and a plot of σ_{ar} vs. life showing data and fit follows.

m_1	m_2	c
-9.3096	-6.0714	30.3295

$$B = \frac{1}{m_1}, \qquad \gamma = Bm_2 = \frac{m_2}{m_1}, \qquad A = 10^{-cB} = 10^{-c/m_1}$$

A	B	γ
1811	-0.1074	0.6522

$$\sigma_{ar} = 1811 N_f^{-0.1074}, \qquad \sigma_{ar} = \sigma_{max}\left(\frac{1-R}{2}\right)^{0.6522} \text{ MPa}$$

Plotting σ_{ar} calculated from the fitted γ provides an excellent correlation of the data.

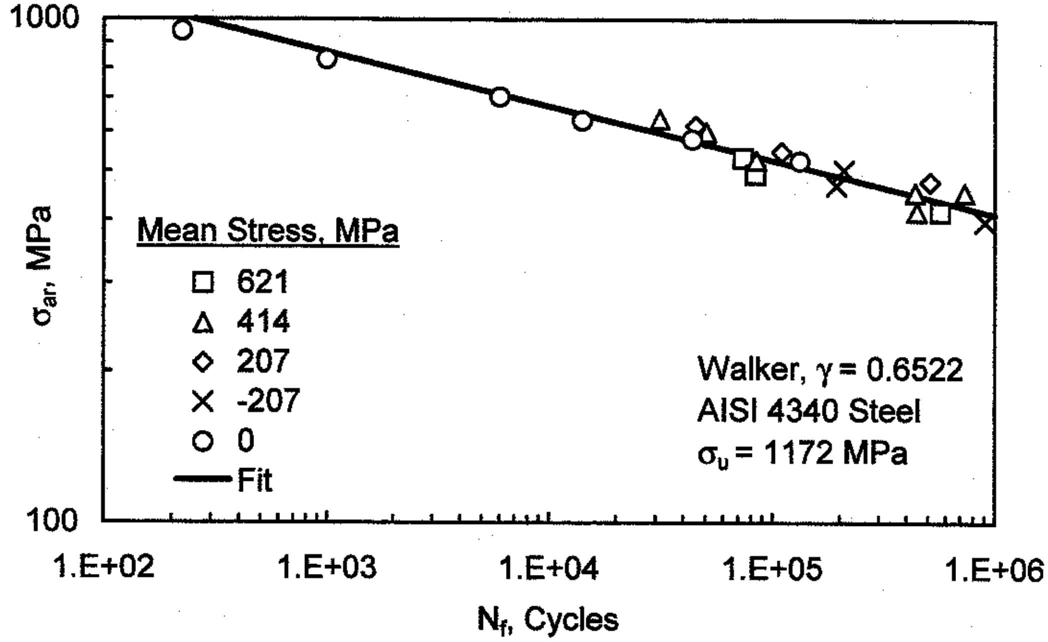
(10.26, p.2)

A	B	γ
1811	-0.1074	0.6522

$$R = 1 - 2\sigma_a / \sigma_{max}$$

Stresses in MPa					y	x_1	x_2	
σ_a	σ_m	N_f, cycles	σ_{max}	R	log N_f	log σ_{max}	log [(1-R)/2]	σ_{ar}
379	621	73,780	1000	0.242	4.868	3.000	-0.4214	531.1
345	621	83,810	966	0.286	4.923	2.985	-0.4472	493.6
276	621	567,590	897	0.385	5.754	2.953	-0.5119	415.9
517	414	31,280	931	-0.111	4.495	2.969	-0.2555	634.4
483	414	50,490	897	-0.077	4.703	2.953	-0.2688	599.0
414	414	84,420	828	0.000	4.926	2.918	-0.3010	526.9
345	414	437,170	759	0.091	5.641	2.880	-0.3424	453.9
345	414	730,570	759	0.091	5.864	2.880	-0.3424	453.9
310	414	445,020	724	0.144	5.648	2.860	-0.3684	416.4
552	207	45,490	759	-0.455	4.658	2.880	-0.1383	616.7
483	207	109,680	690	-0.400	5.040	2.839	-0.1549	546.8
414	207	510,250	621	-0.333	5.708	2.793	-0.1761	476.7
586	-207	208,030	379	-2.092	5.318	2.579	0.1893	503.6
552	-207	193,220	345	-2.200	5.286	2.538	0.2041	468.8
483	-207	901,430	276	-2.500	5.955	2.441	0.2430	397.6
948	0	222	948	-1.000	2.346	2.977	0.0000	948.0
834	0	992	834	-1.000	2.997	2.921	0.0000	834.0
703	0	6,004	703	-1.000	3.778	2.847	0.0000	703.0
631	0	14,130	631	-1.000	4.150	2.800	0.0000	631.0
579	0	43,860	579	-1.000	4.642	2.763	0.0000	579.0
524	0	132,150	524	-1.000	5.121	2.719	0.0000	524.0
Fit		252						1000
		1,000,000						410.7

(10.26, p. 3)



1000
100
σ_{ar}, MPa
Mean Stress, MPa
621
414
207
-207
0
Fit
Walker, γ = 0.6522
AISI 4340 Steel
σ_u = 1172 MPa
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
N_f, Cycles

10.27

For the given fatigue data on unnotched AISI 1015 steel at various σ_m, fit a power-law stress-life curve, with σ_{ar} given by the Walker relationship, Eq. 9.19. A multiple linear regression is needed.

$$\sigma_{ar} = AN_f^B = \sigma_{\max}\left(\frac{1-R}{2}\right)^{\gamma}, \qquad N_f = \left[\sigma_{\max}\left(\frac{1-R}{2}\right)^{\gamma}\frac{1}{A}\right]^{1/B}$$

$$\underbrace{\log N_f}_{y} = \underbrace{\frac{1}{B}}_{m_1}\underbrace{\log\sigma_{\max}}_{x_1} + \underbrace{\frac{\gamma}{B}}_{m_2}\underbrace{\log\left(\frac{1-R}{2}\right)}_{x_2} \underbrace{-\frac{1}{B}\log A}_{c}$$

$$y = m_1 \; x_1 + m_2 \; x_2 + c$$

Numerical values for fitting are given on the next page. The result of the fit is given below, and a plot of σ_{ar} vs. life showing data and fit follows.

m_1	m_2	c
-7.4791	-5.2961	22.2004

$$B = \frac{1}{m_1}, \qquad \gamma = Bm_2 = \frac{m_2}{m_1}, \qquad A = 10^{-cB} = 10^{-c/m_1}$$

A	B	γ
929.7	-0.1337	0.7081

$$\sigma_{ar} = 929.7\,N_f^{-0.1337}, \qquad \sigma_{ar} = \sigma_{\max}\left(\frac{1-R}{2}\right)^{0.7081} \text{ MPa}$$

Plotting σ_{ar} calculated from the fitted γ provides an excellent correlation of the data.

(10.27, p.2)

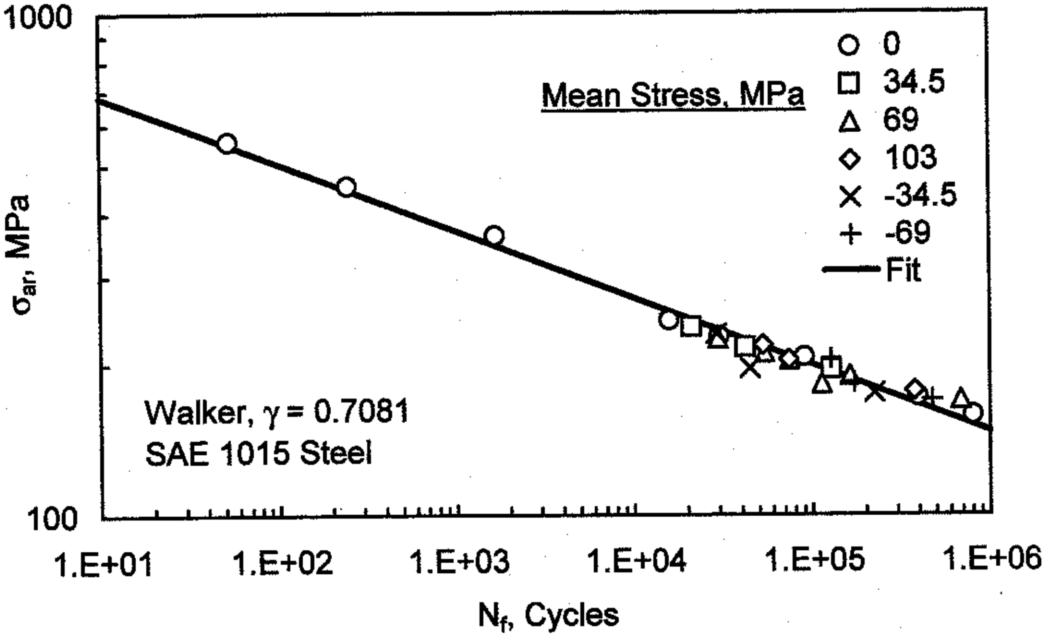
A	B	γ
929.7	-0.1337	0.7081

$$R = 1 - 2\sigma_a / \sigma_{max}$$

Stresses in MPa

					y	x_1	x_2	
σ_a	σ_m	N_f, cycles	σ_{max}	R	log N_f	log σ_{max}	log [(1-R)/2]	σ_{ar}
558	0.0	52	558.0	-1.000	1.716	2.747	0.0000	558.0
455	0.0	242	455.0	-1.000	2.384	2.658	0.0000	455.0
362	0.0	1,650	362.0	-1.000	3.217	2.559	0.0000	362.0
245	0.0	15,750	245.0	-1.000	4.197	2.389	0.0000	245.0
228	0.0	30,000	228.0	-1.000	4.477	2.358	0.0000	228.0
207	0.0	90,000	207.0	-1.000	4.954	2.316	0.0000	207.0
172	0.0	393,000	172.0	-1.000	5.594	2.236	0.0000	172.0
158	0.0	800,000	158.0	-1.000	5.903	2.199	0.0000	158.0
228	34.5	21,000	262.5	-0.737	4.322	2.419	-0.0612	237.6
207	34.5	42,000	241.5	-0.714	4.623	2.383	-0.0669	216.5
186	34.5	128,000	220.5	-0.687	5.107	2.343	-0.0739	195.5
207	69.0	29,500	276.0	-0.500	4.470	2.441	-0.1249	225.1
193	69.0	54,000	262.0	-0.473	4.732	2.418	-0.1327	211.0
186	69.0	74,480	255.0	-0.459	4.872	2.407	-0.1370	203.9
172	69.0	162,800	241.0	-0.427	5.212	2.382	-0.1465	189.8
165	69.0	113,000	234.0	-0.410	5.053	2.369	-0.1517	182.7
152	69.0	683,970	221.0	-0.376	5.835	2.344	-0.1625	169.5
193	103.4	52,880	296.4	-0.302	4.723	2.472	-0.1863	218.7
179	103.4	73,940	282.4	-0.268	4.869	2.451	-0.1980	204.5
152	103.4	377,000	255.4	-0.190	5.576	2.407	-0.2254	176.9
241	-34.5	29,000	206.5	-1.334	4.462	2.315	0.0671	230.4
207	-34.5	44,700	172.5	-1.400	4.650	2.237	0.0792	196.3
186	-34.5	223,600	151.5	-1.455	5.349	2.180	0.0891	175.2
228	-69.0	127,380	159.0	-1.868	5.105	2.201	0.1565	205.2
207	-69.0	172,250	138.0	-2.000	5.236	2.140	0.1761	183.9
193	-69.0	473,250	124.0	-2.113	5.675	2.093	0.1921	169.6
Fit		10						683.3
		1,000,000						146.6

(10.27, p.3)



1000
100
σ_ar, MPa
Mean Stress, MPa
0
34.5
69
103
-34.5
-69
Fit
Walker, γ = 0.7081
SAE 1015 Steel
1.E+01
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
N_f, Cycles

10.28

The notched bending member of Ex. 10.3 is a plate with a width change, Fig. A.11(d), having a machined surface. (a) Estimate the *S-N* curve by Shigley method. (b) Calculate the expected life for given cyclic loading.

First, list the various given values and calculate the equivalent diameter for size effect. Also calculate stresses from given applied loads, and verify k_f from Eqs. 10.7 and 10.9.

$$d_e = 0.81\sqrt{w_1 t}$$

$$S_a = \frac{6M_a}{w_1^2 t}, \qquad S_m = \frac{6M_m}{w_1^2 t}$$

$$S_{ar} = \sqrt{S_{\max} S_a} = \sqrt{(S_m + S_a) S_a}$$

Material: RQC-100 Steel Stresses in MPa

w_1, mm	t, mm	ρ, mm	d_e, mm	Surface
80	10	4.00	22.91	machined

M_a, N-mm	M_m, N-mm	S_a	S_m	SWT S_{ar}
2,000,000	2,500,000	187.5	234.4	281.3

σ_u, MPa	log α	α, mm	k_t	k_f
758	-0.8287	0.1484	1.85	1.82

Next, establish the *S-N* curve point at N_e cycles based on Table 10.1.

(10.28, p.2)

$$m_d = 1.24 d_e^{-0.107}\ , \qquad m_s = 4.51 \sigma_u^{-0.265}$$

$$m = m_e m_t m_d m_s\ , \quad \sigma_{er} = m\sigma_u\ , \quad S_{er} = \frac{\sigma_{er}}{k_f}$$

N_e	m_e	m_t	m_d	m_s	m	σ_{er}	S_{er}
1,000,000	0.504	1.00	0.887	0.778	0.348	263.7	144.9

Then establish the *S-N* curve point at $N' = 1000$ cycles based on Table 10.2. Use estimated σ'_f and b values.

$$\sigma'_f = \sigma_u + 345\ , \qquad b' = -\frac{\log(\sigma'_f/\sigma_{er})}{\log(2N_e)}\ , \qquad m' = \frac{\sigma'_f (2000)^{b'}}{\sigma_u}$$

$$k'_f = 1 + (k_f - 1)(-0.18 + 6.24\times10^{-4}\sigma_u - 9.47\times10^{-8}\sigma_u^2)$$

$$S'_{ar} = \frac{m'\sigma'_u}{k'_f}\ , \qquad \sigma'_u = \sigma_u$$

N'	σ'_f	b'	m'	k'_f	S'_{ar}
1000	1103	-0.0986	0.688	1.196	435.9

Finally, use the above two points to obtain the constants for the estimated *S-N* curve. Substitute S_{ar} for the applied stresses to estimate the life

(10.28, p.3)

$$B = \frac{\log S'_{ar} - \log S_{er}}{\log N' - \log N_e}, \qquad A = \frac{S'_{ar}}{(N')^B}$$

$$S_{ar} = AN_f^B, \qquad N_f = \left(\frac{S_{ar}}{A}\right)^{1/B} \qquad (N' \geq N_f \geq N_e)$$

B	A	S_{ar}	N_f
-0.1594	**1311.4**	281.3	**15,619**

10.29 Axially loaded plate with hole, Pr. 10.17.

2024-T4 Al, $\sigma_o = 303$, $\sigma_u = 476$ MPa

$w = 50$, $d = 10$, $t = 20$ mm, $k_f = 2.36$

(a) Juvinall estimate, smooth polish surface

$\sigma_{erb} = 130 \text{ MPa} = m_e \sigma_u$, $m_t = 1$, $m_d = 0.8$

$m_s = 1.0$, $N_e = 5 \times 10^8$, $m' = 0.75$, $k_f' = k_f$

$$S_{er} = \frac{m\sigma_u}{k_f} = \frac{m_e m_t m_d m_s \sigma_u}{k_f} = 44.07 \text{ MPa}$$

$\sigma_u' = \sigma_u$, $S_{ar}' = m'\sigma_u / k_f' = 151.3$ MPa

Two points on S-N curve: $S_a = A N_f^B$

$(S_{er}, N_f) = (44.07, 5 \times 10^8)$, $(S_{ar}', N_f) = (151.3, 10^3)$

$$B = \frac{\log 151.3 - \log 44.07}{\log 10^3 - \log 5 \times 10^8} = -0.0940$$

$$A = \frac{S_a}{N_f^B} = \frac{151.3}{(10^3)^{-0.0940}} = 289.6 \text{ MPa}$$

(b) $N_f = ?$ for $S_a = 60$, $S_m = 30$ MPa

$k_f |S|_{max} = 2.36(60 + 30) = 212 \text{ MPa} < \sigma_o$

$$k_{fm} = k_f, \quad S_{ar} = \frac{S_a}{1 - \frac{k_{fm} S_m}{\sigma_u}} = 70.48 \text{ MPa}$$

$$N_f = \left(\frac{S_{ar}}{A}\right)^{1/B} = 3.38 \times 10^6 \text{ cycles}$$

(Estimate is OK as within 10^3 to 5×10^8 cycles range of equation.)

(10.29, p. 2)

(b) Alternate Solution: Use SWT.

$S_{ar} = \sqrt{S_{max} S_a} = \sqrt{(30+60)\,60} = 73.48$ MPa

$N_f = \left(\frac{S_{ar}}{A}\right)^{1/B} = 2.17 \times 10^6$ cycles ◀

(OK as within range 10^3 to 5×10^8)

10.30

A circular rod with a diameter change is axially loaded, Fig. A.12(a), and has a ground surface. (a) Estimate the *S-N* curve by Juvinall method, and find X_S for zero-to-tension loading for 30,000 cycles at $P_{max} = 70$ kN. (b) If needed, increase fillet radius for $X_S = 1.7$.

(a) First, list the various given values, determine k_t, and estimate k_f from Eqs. 10.7 and 10.9. Also calculate nominal stresses from applied load.

$$\hat{S}_a = \frac{4\hat{P}_a}{\pi d_1^2}, \qquad \hat{S}_m = \frac{4\hat{P}_m}{\pi d_1^2}$$

$$\hat{S}_{ar} = \sqrt{\hat{S}_{max}\hat{S}_a} = \sqrt{(\hat{S}_m + \hat{S}_a)\hat{S}_a}$$

Material: AISI 4142 (450 HB) steel Stresses in MPa

d_1, mm	d_2, mm	ρ, mm	d_2/d_1	ρ/d_1	Surface
15.0	18.0	1.0	1.20	0.0667	ground

σ_u, MPa	σ_u, ksi	log α	α, mm	k_t	k_f
1757	255	-1.4696	0.0339	1.95	1.919

$\hat{N}$	$\hat{P}_a$, N	$\hat{P}_m$, N	$\hat{S}_a$	$\hat{S}_m$	$\hat{S}_{ar}$
30,000	35,000	35,000	198.1	198.1	280.1

Next, establish the *S-N* curve point at N_e cycles based on Table 10.1.

$$m_e = (700\text{ MPa})/\sigma_u \qquad \text{(Fig. 9.24)}$$

$$m = m_e m_t m_d m_s, \quad \sigma_{er} = m\sigma_u, \qquad S_{er} = \frac{\sigma_{er}}{k_f}$$

(10.30, p.2)

N_e	m_e	m_t	m_d	m_s	m	σ_{er}	S_{er}
1,000,000	0.398	1.00	0.800	0.720	0.229	403.2	210.1

Then establish the *S-N* curve point at $N' = 1000$ cycles based on Table 10.2.

$$m' = 0.75\,, \qquad k'_f = k_f \qquad \text{(axial)}$$

$$S'_{ar} = \frac{m'\sigma'_u}{k'_f}\,, \qquad \sigma'_u = \sigma_u$$

N'	m'	k'_f	S'_{ar}
1000	0.750	1.919	686.7

Finally, use the above two points to obtain the constants for the estimated *S-N* curve. Estimate the life for the applied stresses, and from this X_N and then X_S.

$$B = \frac{\log S'_{ar} - \log S_{er}}{\log N' - \log N_e}\,, \qquad A = \frac{S'_{ar}}{(N')^B}$$

$$S_{ar} = AN_f^B\,, \qquad N_{f2} = \left(\frac{\hat{S}_{ar}}{A}\right)^{1/B} \qquad (N' \geq N_f \geq N_e)$$

$$X_N = \frac{N_{f2}}{\hat{N}}\,, \qquad X_S = X_N^{-B}$$

B	A	$\hat{S}_{ar}$	N_{f2}	X_N	X_S
-0.1714	2244	280.1	187,014	**6.23**	**1.37**

(10.30, p.3)

(b) Since X_S is not adequate, try a larger fillet radius and repeat the above calculations.

Material: AISI 4142 (450 HB) steel Stresses in MPa

d_1, mm	d_2, mm	ρ, mm	d_2/d_1	ρ/d_1	Surface
15.0	18.0	**2.5**	1.20	0.1667	ground

σ_u, MPa	σ_u, ksi	log α	α, mm	k_t	k_f
1757	255	-1.4696	0.0339	1.55	1.543

$\hat{N}$	$\hat{P}_a$, N	$\hat{P}_m$, N	$\hat{S}_a$	$\hat{S}_m$	$\hat{S}_{ar}$
30,000	35,000	35,000	198.1	198.1	280.1

N_e	m_e	m_t	m_d	m_s	m	σ_{er}	S_{er}
1,000,000	0.398	1.00	0.800	0.720	0.229	403.2	261.4

N'	m'	k'_f	S'_{ar}
1000	0.750	1.543	854.2

B	A	$\hat{S}_{ar}$	N_{f2}	X_N	X_S
-0.1714	2792	280.1	667,872	**22.26**	**1.70**

This X_S for the new $\rho = 2.5$ mm is adequate.

10.31

Double-edge-notched plates of AISI 4340 steel as in Fig. 10.11 have an *S-N* curve for axial loading at $R = -1$ given by points below, and a polished notch surface. Estimate the *S-N* curve by (a) Juvinall and (b) Shigley. Plot estimates with data and comment.

S_a, MPa	696	617	538	459	379
N_f, cycles	10^2	$10^{2.5}$	10^3	$10^{3.5}$	10^4

S_a, MPa	300	231	193	176	176
N_f, cycles	$10^{4.5}$	10^5	$10^{5.5}$	10^6	10^7

First, estimate k_f from given ρ, k_t, and σ_u using Eqs. 10.7 and 10.9.

ρ, mm	k_t	σ_u, MPa	$\log \alpha$	α, mm	k_f
2.54	2.43	786	-0.8539	0.1400	2.355

Next, establish the *S-N* curve point at N_e cycles based on Table 10.1.

Item	Juvinall	Shigley
N_e	1,000,000	1,000,000
m_e	0.50	0.504
m_t	1.00	0.85
m_d	0.90	1.00
m_s	1.00	1.00
m	0.450	0.428
σ_{er}	353.7	336.7
S_{er}	150.2	143.0

Stresses in MPa

$$m = m_e m_t m_d m_s$$

$$\sigma_{er} = m\sigma_u\,, \qquad S_{er} = \sigma_{er}/k_f$$

Then establish the *S-N* curve point at $N' = 1000$ cycles using Table 10.2. Special calculations are needed for each method.

Juvinall: $\quad m' = 0.75\,, \qquad k'_f = k_f$

(10.31, p.2)

Shigley:

$$\sigma'_f = \sigma_u + 345, \qquad b' = -\frac{\log(\sigma'_f/\sigma_{er})}{\log(2N_e)}, \qquad m' = \frac{\sigma'_f (2000)^{b'}}{\sigma_u}$$

$$k'_f = 1 + (k_f - 1)(-0.18 + 6.24\times10^{-4}\sigma_u - 9.47\times10^{-8}\sigma_u^2)$$

σ'_f	b'	m'	k'_f
1131	-0.0835	0.763	1.341

Item	Juvinall	Shigley
N'	1000	1000
m'	0.75	0.763
k'_f	2.355	1.341
S'_{ar}	250.3	446.9

$$S'_{ar} = \frac{m'\sigma'_u}{k'_f}, \qquad \sigma'_u = \sigma_u$$

Finally, for each method, use the above two points to obtain constants for the estimated *S-N* curve. Also, extend the estimates to short lives by a log-log straight line from the N' point to σ_u at $N_f = 1$ cycle.

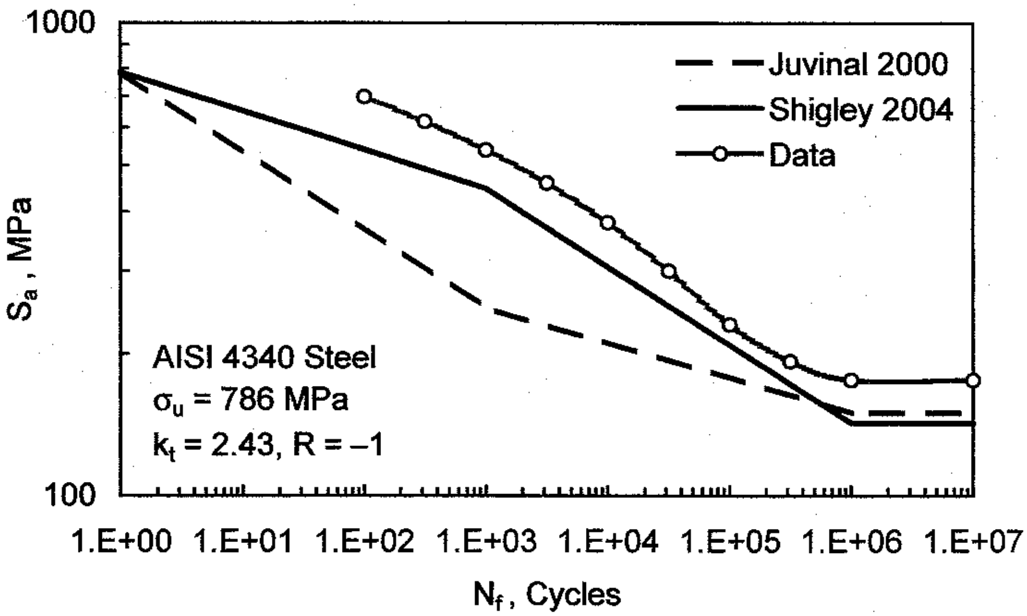
$$B = \frac{\log S'_{ar} - \log S_{er}}{\log N' - \log N_e}, \qquad A = \frac{S'_{ar}}{(N')^B}$$

$$S_{ar} = AN_f^B \qquad (N' \ge N_f \ge N_e)$$

Item	Juvinall	Shigley
B	-0.0739	-0.1650
A	417.1	1397.0

The plot that follows compares the estimates and the data. Shigley generally follows the trend of the data but is more conservative. Juvinall is grossly conservative at intermediate and short lives.

(10.31, p, 3)



1000
100
Juvinal 2000
Shigley 2004
Data
S_a , MPa
AISI 4340 Steel
σ_u = 786 MPa
k_t = 2.43, R = –1
1.E+00 1.E+01 1.E+02 1.E+03 1.E+04 1.E+05 1.E+06 1.E+07
N_f, Cycles

10.32

Axially loaded, double-edge-notched plates of AISI 4130 steel with $k_t = 4.00$ gave test data at $R = -1$ as below. Estimate the *S-N* curve by (a) Juvinall and (b) Shigley. Plot estimates with data and comment.

S_a, MPa	621	448	293	259	224
N_f, cycles	104	682	8,440	19,700	30,500
S_a, MPa	186	155	121	103	86.2
N_f, cycles	94,300	269,000	537,900	1,719,000	>10,325,000

First, estimate k_f from given ρ, k_t, and σ_u using Eqs. 10.7 and 10.9.

ρ, mm	k_t	σ_u, MPa	log α	α, mm	k_f
1.45	4.00	807	-0.8725	0.1341	3.746

Next, establish the *S-N* curve point at N_e cycles based on Table 10.1.

Item	Juvinall	Shigley
N_e	1,000,000	1,000,000
m_e	0.50	0.504
m_t	1.00	0.85
m_d	0.90	1.00
m_s	1.00	1.00
m	0.450	0.428
σ_{er}	363.2	345.7
S_{er}	96.9	92.3

Stresses in MPa
Assume polished surface

$$m = m_e m_t m_d m_s$$

$$\sigma_{er} = m\sigma_u \,, \qquad S_{er} = \sigma_{er}/k_f$$

Then establish the *S-N* curve point at $N' = 1000$ cycles using Table 10.2. Special calculations are needed for each method.

Juvinall: $\quad m' = 0.75 \,, \qquad k'_f = k_f$

(10.32, p.2)

Shigley:

$$\sigma'_f = \sigma_u + 345\,, \qquad b' = -\frac{\log(\sigma'_f/\sigma_{er})}{\log(2N_e)}\,, \qquad m' = \frac{\sigma'_f(2000)^{b'}}{\sigma_u}$$

$$k'_f = 1 + (k_f - 1)(-0.18 + 6.24\times10^{-4}\sigma_u - 9.47\times10^{-8}\sigma_u^2)$$

σ'_f	b'	m'	k'_f
1152	-0.0830	0.760	1.719

Item	Juvinall	Shigley
N'	1000	1000
m'	0.75	0.760
k'_f	3.746	1.719
S'_{ar}	161.6	356.7

$$S'_{ar} = \frac{m'\sigma'_u}{k'_f}\,, \qquad \sigma'_u = \sigma_u$$

Finally, for each method, use the above two points to obtain constants for the estimated *S-N* curve. Also, extend the estimates to short lives by a log-log straight line from the N' point to σ_u at $N_f = 1$ cycle.

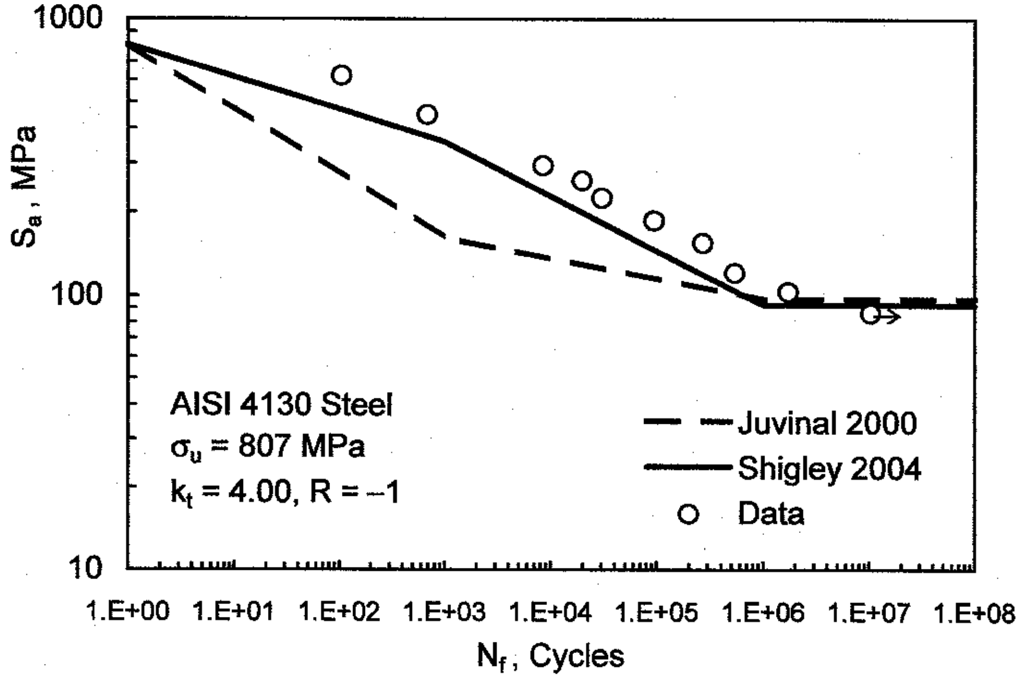
$$B = \frac{\log S'_{ar} - \log S_{er}}{\log N' - \log N_e}\,, \qquad A = \frac{S'_{ar}}{(N')^B}$$

$$S_{ar} = AN_f^B \qquad (N' \geq N_f \geq N_e)$$

Item	Juvinall	Shigley
B	-0.0739	-0.1957
A	269.3	1378.5

The plot that follows compares the estimates and the data. Shigley generally follows the trend of the data but is more conservative. Juvinall is grossly conservative at intermediate and short lives.

(10.32, p. 3)



1000
100
10
S_a, MPa
1.E+00
1.E+01
1.E+02
1.E+03
1.E+04
1.E+05
1.E+06
1.E+07
1.E+08
N_f, Cycles
AISI 4130 Steel
σ_u = 807 MPa
k_t = 4.00, R = −1
Juvinal 2000
Shigley 2004
Data

10.33

Axially loaded, circumferentially notched round bars of 2014-T6 Al with $k_t = 2.65$ gave test data at $R = -1$ as below. Estimate the *S-N* curve by Juvinall. Plot estimate with data and comment.

S_a, MPa	262	200	138	124	100
N_f, cycles	2,100	13,300	137,000	661,000	2,420,000

S_a, MPa	82.7	79.3
N_f, cycles	17,200,000	27,600,000

First, estimate k_f from given ρ and k_t using Eqs. 10.6 and 10.9.

σ_u, MPa	ρ, mm	k_t	α, mm	k_f
494	0.813	2.65	0.51	2.014

Next, establish the *S-N* curve point at N_e cycles based on Table 10.1.

Item	Juvinall
N_e	5.00E+08
m_e	0.263
m_t	1.00
m_d	0.90
m_s	1.00
m	0.237
σ_{er}	117.0
S_{er}	58.1

Stresses in MPa

Assume polished surface

$$m_e = (130\text{ MPa})/\sigma_u \qquad \text{(Fig. 9.25)}$$

$$m = m_e m_t m_d m_s$$

$$\sigma_{er} = m\sigma_u\,, \qquad S_{er} = \sigma_{er}/k_f$$

Then establish the *S-N* curve point at $N' = 1000$ cycles using Table 10.2.

Juvinall: $\quad m' = 0.75\,, \qquad k'_f = k_f$

(10.33, p. 2)

Item	Juvinall
N'	1000
m'	0.75
k'f	2.014
S'ar	184.0

$$S'_{ar} = \frac{m'\sigma'_u}{k'_f}, \qquad \sigma'_u = \sigma_u$$

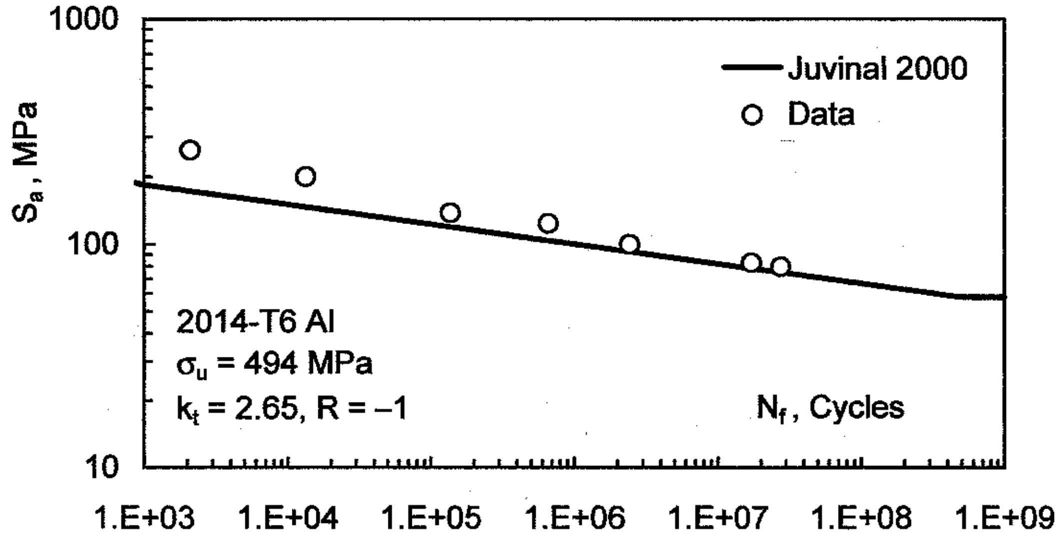
Finally, use the above two points to obtain constants for the estimated *S-N* curve.

$$B = \frac{\log S'_{ar} - \log S_{er}}{\log N' - \log N_e}, \qquad A = \frac{S'_{ar}}{(N')^B}$$

$$S_{ar} = AN_f^B \qquad (N' \geq N_f \geq N_e)$$

Item	Juvinall
B	-0.0878
A	337.5

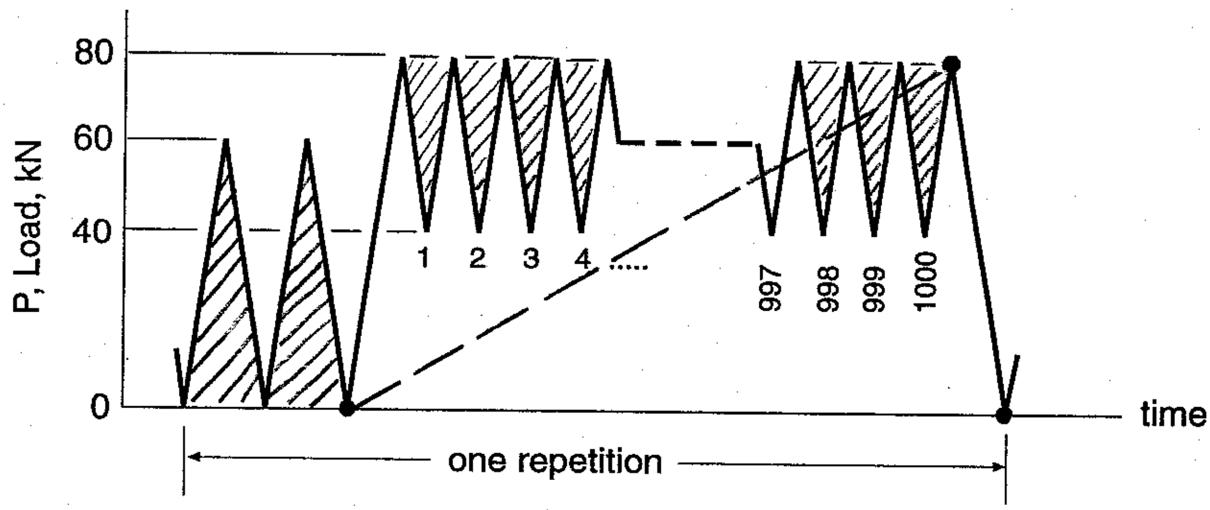
The plot that follows compares the estimate and the data. The Juvinall estimate is reasonably accurate at intermediate and long lives, but overly conservative at short lives.



10.34

The given repeating load history is applied to the Prob. 10.30 axially loaded rod with a diameter change, Fig. A.12(a). An estimated *S-N* curve is given, and 100 repetions are expected in service. Estimate (a) repetitions to failure, (b) safety factors in life and stress.

First perform cycle counting on the load history. Then for each level of cycling, calculate nominal stresses and S_{ar} using the SWT equation, and from this the life N_f. Next, apply the P-M rule, and from the resulting number of repetitions to failure, obtain the safety factors.



$$P_a = \frac{P_{max} - P_{min}}{2}, \qquad S_{max} = \frac{4P_{max}}{\pi d_1^2}, \qquad S_a = \frac{4P_a}{\pi d_1^2}$$

$$S_{ar} = \sqrt{S_{max} S_a}, \qquad S_{ar} = A(N_f)^B$$

$$N_f = \left(\frac{S_{ar}}{A}\right)^{1/B}, \qquad B_f\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep}} = 1$$

(10.34, p.2)

AISI 4142 steel (450 HB), $k_f = 1.92$ Stresses in MPa

σ_u	A	B	d_1, mm	$\hat{B}$, reps
1757	2250	-0.172	15	100

j	N_j	P_{min}, N	P_{max}, N	P_a, N	S_{max}	S_a
1	2	0	60,000	30,000	339.5	169.8
2	1000	40,000	80,000	20,000	452.7	113.2
3	1	0	80,000	40,000	452.7	226.4

j	S_{ar}	N_{fj}	N_j/N_{fj}
1	240.1	4.47E+05	4.48E-06
2	226.4	6.29E+05	1.59E-03
3	320.1	8.39E+04	1.19E-05
		$\Sigma =$	1.61E-03
		$B_f = 1/\Sigma =$	**623**

$$X_N = \frac{B_f}{\hat{B}}, \qquad X_S = X_N^{-B}$$

$X_N =$ **6.23**

$X_S =$ **1.370**

10.35 The shaft illustrated failed after 2×10^7 rotations and was made of AISI 1040 steel with $\sigma_u = 620$, $\sigma_o = 320$ MPa, 25 % RA. The fillet radius at the failure location was smaller than on the design drawings. Estimate X_S for both cases. Does the manufacturing error explain the failure? Fig. A.12 (b) applies.

At the failure cross section:

$$M_a = (190{,}500\ \mathrm{N})(156 - 70/2)\ \mathrm{mm} = 2.305\times10^7\ \mathrm{N\cdot mm}$$

$$S_a = \frac{32 M_a}{\pi d_1^3} = \frac{32(2.305\times10^7\ \mathrm{N\cdot mm})}{\pi(149\ \mathrm{mm})^3} = 71.0\ \mathrm{MPa}$$

For both the design and actual service cases, list the various given values, determine k_t, and estimate k_f from Eqs. 10.10 and 10.11.

Case	d_1, mm	d_2, mm	ρ, mm	d_2/d_1	ρ/d_1	Surface
design	149.0	171.5	6.35	1.151	0.0426	machined
actual	149.0	171.5	2.00	1.151	0.0134	machined

Case	σ_u, MPa	σ_u, ksi	log β	β, mm	k_t	k_f
design	620	90	-0.88230	0.1311	2.0	1.87
actual	620	90	-0.88230	0.1311	2.7	2.35

For each case, based on Table 10.1 and the Juvinall method, establish the fatigue limit on the *S-N* curve at N_e = 1,000,000 cycles.

(10.35, p.2)

$$m = m_e m_t m_d m_s$$

$$\sigma_{er} = m\sigma_u \,, \qquad S_{er} = \sigma_{er}/k_f$$

Case	m_e	m_t	m_d	m_s	m	σ_{er}	S_{er}
design	0.500	1.00	0.700	0.770	0.270	167.1	89.1
actual	0.500	1.00	0.700	0.770	0.270	167.1	71.0

$X_S = S_{er} / S_a$

$X_S = 89.1/71.0 = 1.26$ (design)

$X_S = 71.0/71.0 = 1.00$ (actual)

Hence, for the actual case, there is no safety margin, explaining the failure. Note that the design safety factor is already too low. (X = 1.00 for the actual case is a coincidence. Using Peterson for k_f and/or Shigley for σ_{er} would give a similar value, but not exactly 1.00.)

10.36 Bailey Bridge panels cycled at $S_{min} = 7.9$ MPa, with given load pattern, and two peak $S_{max} = 240$ and 209 MPa.

(a) Estimate life from Fig. 10.17 data.

$$S_{max} = A N_f^{B} = 2350\, N_f^{-0.247} \text{ MPa}$$

For each level of cycling, calculate:

$$S_{max} = (\text{Peak } S_{max})(\%\text{ of peak})/100$$

$$N_f = \left(\frac{S_{max}}{A}\right)^{1/B}$$

Then apply:

$$B_f \left[\Sigma \frac{N_j}{N_{fj}} \right]_{per\ rep.} = 1$$

B_f = repetitions to failure (calculated)

(b) Compare B_f from data:

$$\text{Ratio} = \frac{\text{Data } B_f}{\text{Calc } B_f}$$

Details are on the next page. The test data lives are 3 to 5 times longer than the calculated ones. Local yielding effects may explain the difference. See Sections 10.10 and 14.6 for discussion. ◀

(10.36, p, 2)

A, MPa = 2350 B = -0.247

Peak S_{max}, MPa = 240

% of Peak S_{max}	Cycles per Block, N_j	S_{max} MPa	N_{fj}	N_j/N_{fj}
20	11,700	48	6.94E+06	1.69E-03
40	8,400	96	4.19E+05	2.00E-02
60	3,140	144	8.12E+04	3.87E-02
80	700	192	2.53E+04	2.76E-02
90	57	216	1.57E+04	3.62E-03
100	3	240	1.03E+04	2.92E-04
Σ =	24,000		Σ =	9.19E-02
			Calc $B_f = 1/\Sigma$ =	**10.88**

Data B_f = 33
Ratio Data/Calc = 3.03

Peak S_{max}, MPa = 209

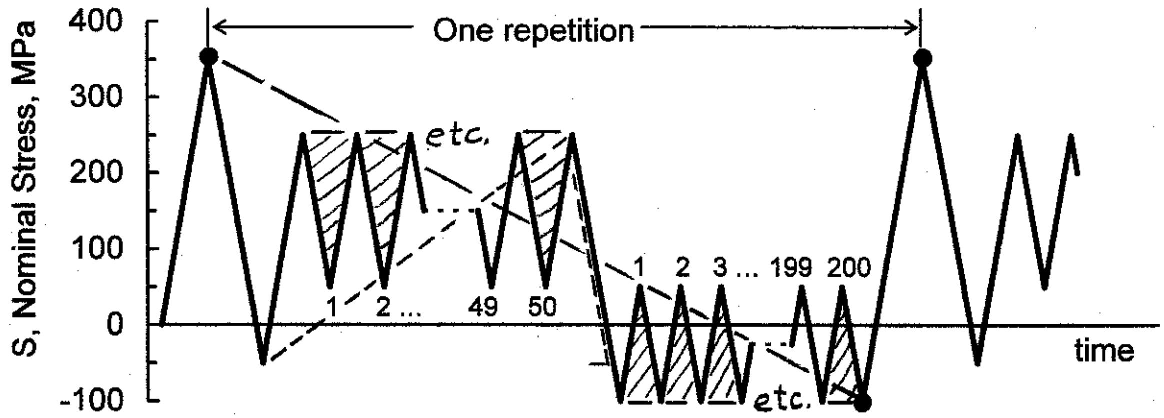
% of Peak S_{max}	Cycles per Block, N_j	S_{max} MPa	N_{fj}	N_j/N_{fj}
20	11,700	41.8	1.22E+07	9.63E-04
40	8,400	83.6	7.34E+05	1.14E-02
60	3,140	125.4	1.42E+05	2.21E-02
80	700	167.2	4.44E+04	1.58E-02
90	57	188.1	2.75E+04	2.07E-03
100	3	209	1.80E+04	1.67E-04
Σ =	24,000		Σ =	5.25E-02
			Calc $B_f = 1/\Sigma$ =	**19.05**

Data B_f = 100
Ratio Data/Calc = 5.25

10.37

The given repeating nominal stress history is applied to 2024-T351 Al double-edge-notched plates ($k_t = 2.15$), for which constants for a Walker fit are given. In service, 200 repetions are expected. Estimate (a) repetitions to failure, (b) safety factors in life and stress.

First perform cycle counting on the nominal stress history. Then for each level of cycling, calculate S_{ar} using the Walker equation, and from this the life N_f. Next, apply the P-M rule, and from the resulting number of repetitions to failure, obtain the safety factors.



A	B	γ	$\hat{B}$, reps
1531	-0.2175	0.7326	200

Stresses in MPa

$$R = S_{\min}/S_{\max}, \qquad S_{ar} = S_{\max}\left(\frac{1-R}{2}\right)^{\gamma}, \qquad N_f = \left(\frac{S_{ar}}{A}\right)^{1/B}$$

$$B_f = 1 \Big/ \left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep.}}$$

(10.37, p.2)

j	N_j	S_{min}	S_{max}	R	S_{ar}	N_{fj}	N_j/N_{fj}
1	50	50	250	0.200	127.8	9.10E+04	5.50E-04
2	1	-50	250	-0.200	172.0	2.32E+04	4.31E-05
3	200	-100	50	-2.000	67.3	1.73E+06	1.15E-04
4	1	-100	350	-0.286	253.2	3.92E+03	2.55E-04

$\Sigma =$ 9.63E-04

$1/\Sigma = B_f =$ **1038**

$$X_N = \frac{B_f}{\hat{B}}, \qquad X_S = X_N^{-B}$$

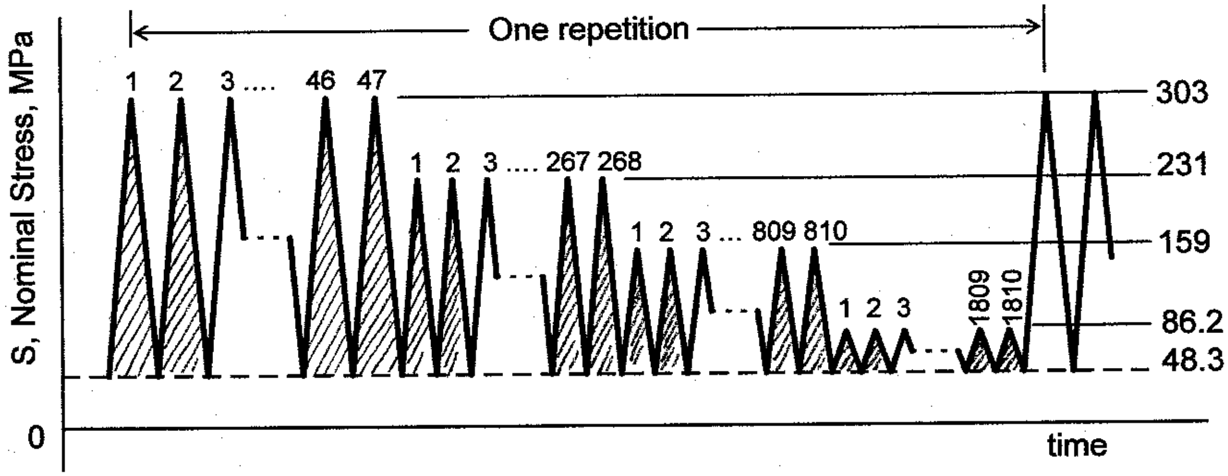
$X_N =$ **5.19**

$X_S =$ **1.431**

10.38

The given repeating nominal stress history is applied to 7075-T6 Al double-edge-notched plates ($k_t = 4.00$), for which constants for a Walker fit are given. (a) Estimate the number of repetitions to failure. (b) Compare the calculated life to test data and comment.

First perform cycle counting on the nominal stress history. Then for each level of cycling, calculate S_{ar} using the Walker equation, and from this the life N_f. Next, apply the P-M rule, and compare the resulting number of repetitions to failure with the test data.



A	B	γ
779	-0.197	0.486

Stresses in MPa

$$R = S_{\min}/S_{\max}\,, \qquad S_{ar} = S_{\max}\left(\frac{1-R}{2}\right)^{\gamma}, \qquad N_f = \left(\frac{S_{ar}}{A}\right)^{1/B}$$

$$B_f = 1\Big/\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep.}}$$

(10.38, p.2)

j	N_j	S_{min}	S_{max}	R	S_{ar}	N_{fj}	N_j/N_{fj}
1	47	48.3	303	0.159	198.8	1.02E+03	4.59E-02
2	268	48.3	231	0.209	147.2	4.72E+03	5.68E-02
3	810	48.3	159	0.304	95.2	4.30E+04	1.88E-02
4	1810	48.3	86.2	0.560	41.3	2.99E+06	6.05E-04
						$\Sigma =$	1.22E-01
						$1/\Sigma = B_f =$	**8.19** ◀

Test Data, Repetitions to Failure

No.	1	2	3	4	5	6	Geometric Mean
B_f	18.7	18	18	18	16	15	**17.23**

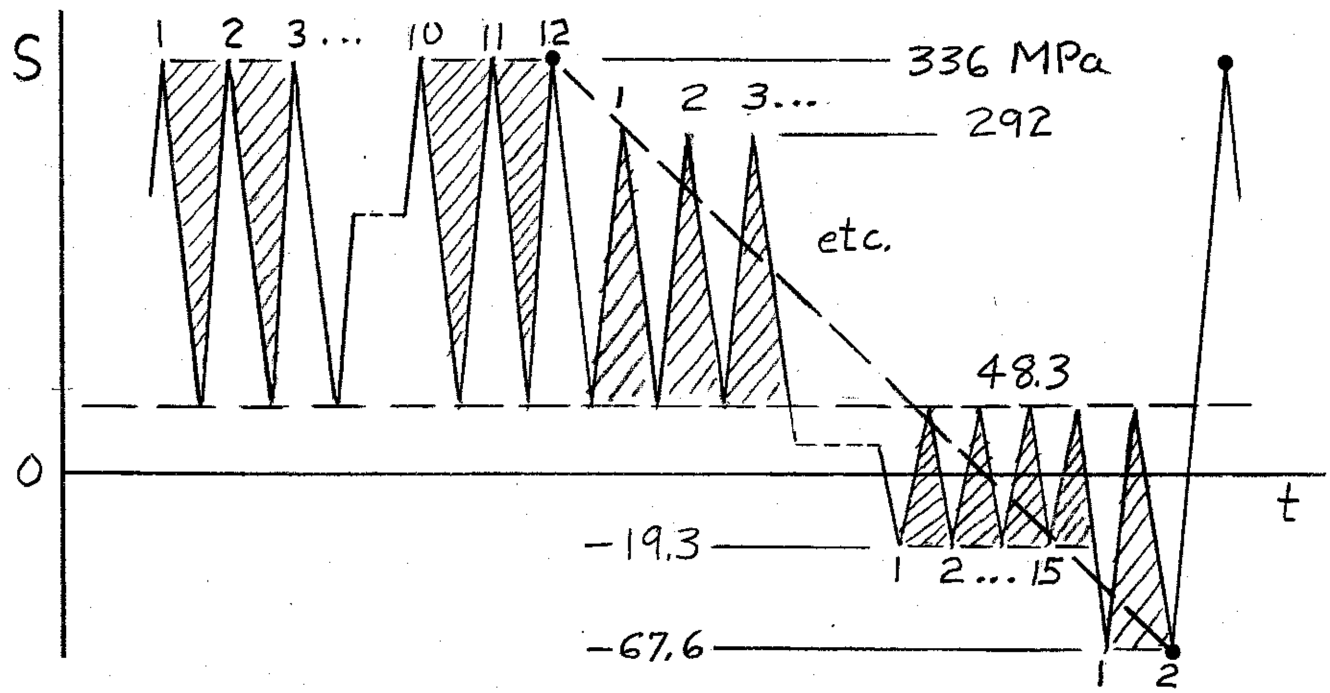
Ratio Data/Calc = **2.10** ◀

The experimental fatigue lives are approximately twice the estimate, which is reasonable agreement. ◀

10.39

The given repeating nominal stress history is applied to 7075-T6 Al double-edge-notched plates ($k_t = 4.00$), for which constants for a Walker fit are given. (a) Estimate the number of repetitions to failure. (b) Compare the calculated life to test data and comment.

First perform cycle counting on the nominal stress history. Note that the major cycle is formed by one peak at the highest level, $S_{max} = 336$ MPa, and one valley at the lowest level, $S_{min} = -67.6$ MPa. Also, there are 11 cycles between $S_{min} = 48.3$ and $S_{max} = 336$ MPa, and one between $S_{min} = -67.6$ and $S_{max} = 48.3$ MPa. Otherwise, each adjacent valley and peak form a cycle. Then for each level of cycling, calculate S_{ar} using the Walker equation, and from this the life N_f. Next, apply the P-M rule, and compare the resulting number of repetitions to failure with the test data.



(10.39, p.2)

A	B	γ
779	-0.197	0.486

Stresses in MPa

$$R = S_{\min}/S_{\max}\,, \qquad S_{ar} = S_{\max}\left(\frac{1-R}{2}\right)^{\gamma}, \qquad N_f = \left(\frac{S_{ar}}{A}\right)^{1/B}$$

$$B_f = 1\Big/\left[\sum \frac{N_j}{N_{fj}}\right]_{\text{one rep.}}$$

j	N_j	S_{min}	S_{max}	R	S_{ar}	N_{fj}	N_j/N_{fj}
1	11	48.3	336	0.144	222.5	5.79E+02	1.90E-02
2	35	48.3	292	0.165	190.9	1.26E+03	2.78E-02
3	88	48.3	255	0.189	164.4	2.69E+03	3.27E-02
4	180	48.3	219	0.221	138.5	6.41E+03	2.81E-02
5	300	48.3	181	0.267	111.1	1.96E+04	1.53E-02
6	510	48.3	143	0.338	83.6	8.34E+04	6.11E-03
7	780	48.3	105	0.460	55.6	6.62E+05	1.18E-03
8	1030	48.3	67.6	0.714	26.2	2.98E+07	3.45E-05
9	15	-19.3	48.3	-0.400	40.6	3.25E+06	4.61E-06
10	1	-67.6	48.3	-1.400	52.8	8.60E+05	1.16E-06
11	1	-67.6	336	-0.201	262.3	2.51E+02	3.98E-03
						Σ =	1.34E-01
						$1/\Sigma = B_f$ =	**7.45**

Test Data, Repetitions to Failure, and Geometric Mean

No.	1	2	3	4	5	6	Mean
B_f	13.7	12.1	12.1	11	11	10	**11.59**

Ratio Data/Calc = **1.56**

The experimental fatigue lives are approximately 50% above the estimate, which is reasonable agreement.

10.40 Man-Ten notched member under the load history of Fig. 9.48. Constant amplitude data give:

$$\Delta P = 379\,(2N_f)^{-0.223} \quad kN \qquad (R=-1)$$

$$\frac{\Delta P}{2} = P_a = A\,(2N_f)^B, \quad A = 189.5\,kN, \quad B = -0.223$$

(a) Estimate life for peak loads of 71.17, 35.58, and 15.57 kN. To account for various mean loads, P_m, use:

$$P_{ar} = \sqrt{P_{max} P_a} = \sqrt{(P_m + P_a) P_a} = A\,(2N_f)^B$$

Loads in Fig. 9.48 in % of peak load. Hence, for each Range (ΔP) and Mean (P_m), tabulate the N_j value from the matrix and calculate:

$$P_a,\,kN = \frac{\Delta P,\,\%}{2} \times \frac{\text{Peak } P,\,kN}{100}$$

$$P_m,\,kN = (P_m,\,\%) \times \frac{\text{Peak } P,\,kN}{100}$$

$$P_{ar} = \sqrt{(P_m + P_a) P_a}$$

$$N_f = \frac{1}{2}\left(\frac{P_{ar}}{A}\right)^{1/B}$$

However, if $P_m + P_a = P_{max}$ is ≤ 0, use $P_{ar} = 0$, $N_{fj} = \infty$, and $N_j / N_{fj} = 0$

(10.40, p.2)

Then apply $B_f \left[\sum \frac{N_j}{N_{fj}} \right]_{per\,rep.} = 1$

Detailed calculations for peak P = 71.17 kN are on four pages that follow. Do peak P = 35.58 and 15.57 kN similarly.

Peak P, kN	Calc B_f	Data B_f	Ratio
71.17	3.56	11.04	3.10
35.58	79.7	168.5	2.12
15.57	3241	4531	1.40

(b) Compare to test data. First compare the geometric mean of the three data points with the calculated B_f. For peak P = 71.17 kN:

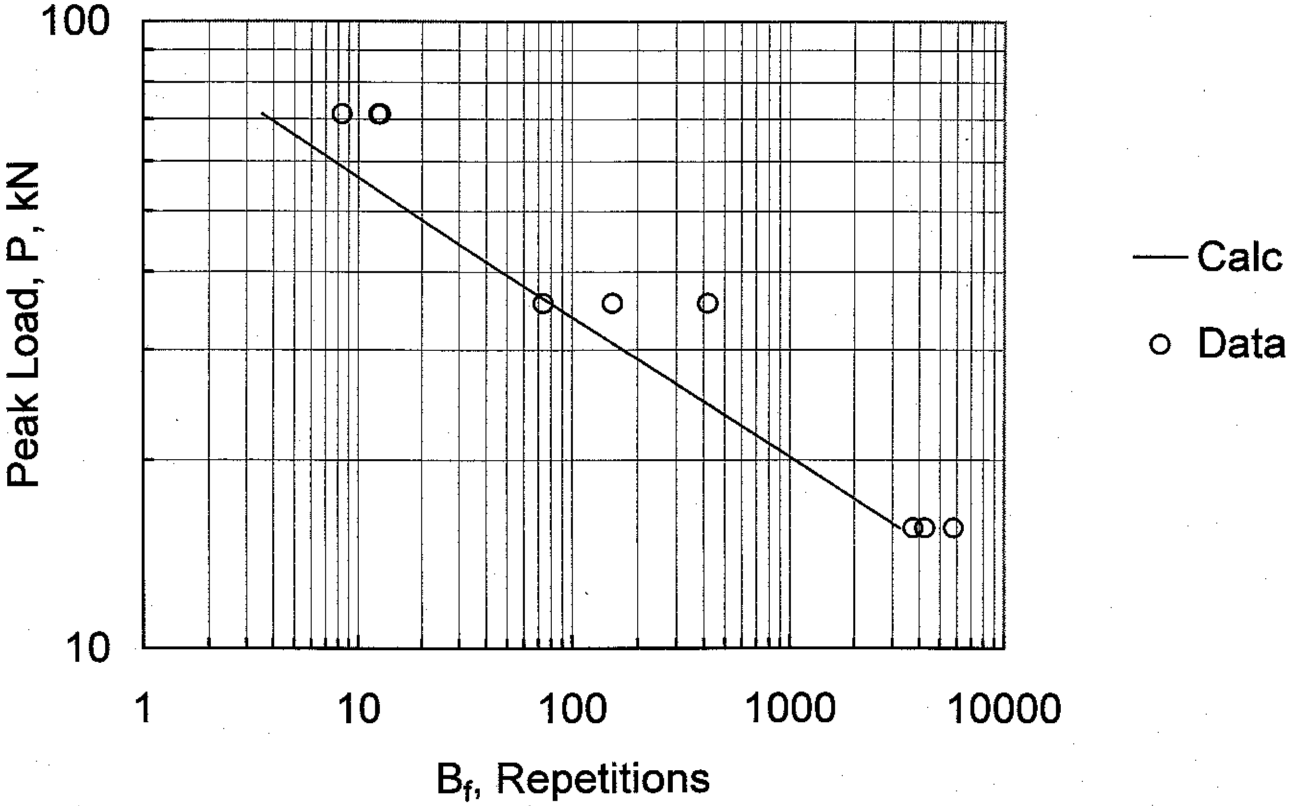
Data $B_f = (8.4 \times 12.8 \times 12.5)^{1/3} = 11.04$ kN

$$\text{Ratio} = \frac{\text{Data } B_f}{\text{Calc } B_f} = \frac{11.04}{3.56} = 3.10$$

Results for all three load levels are in the table above.

The calculated values form a straight line on a log-log plot. This line is plotted with the data on the next page.

(10.40, p.3)



The calculated lives are in reasonable agreement with the data considering its scatter, with the calculated line falling near the bottom of the scatter

(10.40, p.4)

A	B	Peak P
kN		kN
189.5	-0.223	71.17

Data B_f			
Test 1	Test 1	Test 1	Geo Mean
8.4	12.8	12.5	11.04

N_j	ΔP, %	P_m, %	P_a, kN	P_m, kN	P_{ar}, kN	N_{fj}	N_j/N_{fj}
4	20	-15	7.117	-10.676	0		0
1	20	-10	7.117	-7.117	0		0
5	20	-5	7.117	-3.559	5.032	5.83E+06	8.58E-07
2	20	0	7.117	0.000	7.117	1.23E+06	1.62E-06
2	20	5	7.117	3.559	8.717	4.96E+05	4.03E-06
5	20	10	7.117	7.117	10.065	2.60E+05	1.92E-05
3	20	25	7.117	17.793	13.315	7.42E+04	4.04E-05
6	20	30	7.117	21.351	14.234	5.50E+04	1.09E-04
15	20	35	7.117	24.910	15.097	4.23E+04	3.55E-04
27	20	40	7.117	28.468	15.914	3.34E+04	8.09E-04
29	20	45	7.117	32.027	16.691	2.69E+04	1.08E-03
32	20	50	7.117	35.585	17.433	2.22E+04	1.44E-03
22	20	55	7.117	39.144	18.145	1.85E+04	1.19E-03
12	20	60	7.117	42.702	18.830	1.57E+04	7.65E-04
6	20	65	7.117	46.261	19.491	1.34E+04	4.46E-04
2	20	70	7.117	49.819	20.130	1.16E+04	1.72E-04
2	25	-15	8.896	-10.676	0		0
4	25	-10	8.896	-7.117	3.979	1.67E+07	2.39E-07
3	25	-5	8.896	-3.559	6.891	1.42E+06	2.11E-06
9	25	0	8.896	0.000	8.896	4.53E+05	1.99E-05
8	25	5	8.896	3.559	10.526	2.13E+05	3.76E-05
10	25	10	8.896	7.117	11.936	1.21E+05	8.25E-05
4	25	15	8.896	10.676	13.195	7.73E+04	5.17E-05
6	25	20	8.896	14.234	14.345	5.31E+04	1.13E-04
2	25	25	8.896	17.793	15.409	3.86E+04	5.19E-05
7	25	30	8.896	21.351	16.404	2.91E+04	2.40E-04
17	25	35	8.896	24.910	17.342	2.27E+04	7.49E-04
37	25	40	8.896	28.468	18.232	1.81E+04	2.04E-03
36	25	45	8.896	32.027	19.080	1.48E+04	2.43E-03
43	25	50	8.896	35.585	19.893	1.23E+04	3.51E-03
33	25	55	8.896	39.144	20.673	1.03E+04	3.20E-03
13	25	60	8.896	42.702	21.425	8.79E+03	1.48E-03
7	25	65	8.896	46.261	22.151	7.57E+03	9.24E-04
1	25	70	8.896	49.819	22.855	6.58E+03	1.52E-04

(10.40, p. 5)

2	25	75	8.896	53.378	23.537	5.77E+03	3.47E-04
1	30	-15	10.676	-10.676	0		0
1	30	-10	10.676	-7.117	6.164	2.35E+06	4.26E-07
5	30	-5	10.676	-3.559	8.717	4.96E+05	1.01E-05
3	30	0	10.676	0.000	10.676	2.00E+05	1.50E-05
1	30	5	10.676	3.559	12.327	1.05E+05	9.53E-06
1	30	10	10.676	7.117	13.782	6.36E+04	1.57E-05
4	30	15	10.676	10.676	15.097	4.23E+04	9.47E-05
3	30	20	10.676	14.234	16.307	2.99E+04	1.00E-04
4	30	30	10.676	21.351	18.491	1.70E+04	2.35E-04
13	30	35	10.676	24.910	19.491	1.34E+04	9.67E-04
20	30	40	10.676	28.468	20.442	1.09E+04	1.84E-03
20	30	45	10.676	32.027	21.351	8.93E+03	2.24E-03
23	30	50	10.676	35.585	22.223	7.46E+03	3.08E-03
20	30	55	10.676	39.144	23.062	6.32E+03	3.16E-03
8	30	60	10.676	42.702	23.871	5.42E+03	1.48E-03
6	30	65	10.676	46.261	24.654	4.69E+03	1.28E-03
1	30	70	10.676	49.819	25.413	4.09E+03	2.44E-04
1	35	-15	12.455	-10.676	4.707	7.86E+06	1.27E-07
1	35	-10	12.455	-7.117	8.154	6.69E+05	1.49E-06
4	35	-5	12.455	-3.559	10.526	2.13E+05	1.88E-05
2	35	0	12.455	0.000	12.455	1.00E+05	2.00E-05
3	35	5	12.455	3.559	14.122	5.70E+04	5.26E-05
2	35	10	12.455	7.117	15.613	3.64E+04	5.50E-05
1	35	20	12.455	14.234	18.232	1.81E+04	5.51E-05
3	35	25	12.455	17.793	19.409	1.37E+04	2.19E-04
2	35	30	12.455	21.351	20.519	1.07E+04	1.87E-04
8	35	35	12.455	24.910	21.572	8.53E+03	9.38E-04
17	35	40	12.455	28.468	22.576	6.95E+03	2.44E-03
16	35	45	12.455	32.027	23.537	5.77E+03	2.77E-03
11	35	50	12.455	35.585	24.461	4.85E+03	2.27E-03
11	35	55	12.455	39.144	25.350	4.14E+03	2.66E-03
7	35	60	12.455	42.702	26.210	3.56E+03	1.97E-03
2	35	65	12.455	46.261	27.042	3.10E+03	6.46E-04
1	40	-10	14.234	-7.117	10.065	2.60E+05	3.84E-06
1	40	-5	14.234	-3.559	12.327	1.05E+05	9.53E-06
1	40	0	14.234	0.000	14.234	5.50E+04	1.82E-05
2	40	5	14.234	3.559	15.914	3.34E+04	5.99E-05
1	40	10	14.234	7.117	17.433	2.22E+04	4.51E-05
1	40	15	14.234	10.676	18.830	1.57E+04	6.37E-05
4	40	30	14.234	21.351	22.506	7.05E+03	5.67E-04

(10.40, p.6)

7	40	35	14.234	24.910	23.604	5.70E+03	1.23E-03
15	40	40	14.234	28.468	24.654	4.69E+03	3.20E-03
16	40	45	14.234	32.027	25.661	3.92E+03	4.09E-03
9	40	50	14.234	35.585	26.629	3.32E+03	2.71E-03
8	40	55	14.234	39.144	27.564	2.84E+03	2.82E-03
2	40	60	14.234	42.702	28.468	2.46E+03	8.13E-04
1	45	-10	16.013	-7.117	11.936	1.21E+05	8.25E-06
4	45	0	16.013	0.000	16.013	3.24E+04	1.23E-04
3	45	5	16.013	3.559	17.703	2.07E+04	1.45E-04
2	45	30	16.013	21.351	24.461	4.85E+03	4.12E-04
1	45	35	16.013	24.910	25.599	3.96E+03	2.53E-04
9	45	40	16.013	28.468	26.689	3.28E+03	2.74E-03
7	45	45	16.013	32.027	27.736	2.76E+03	2.53E-03
2	45	50	16.013	35.585	28.745	2.35E+03	8.50E-04
3	45	55	16.013	39.144	29.719	2.03E+03	1.48E-03
1	45	60	16.013	42.702	30.663	1.76E+03	5.68E-04
2	50	-5	17.793	-3.559	15.914	3.34E+04	5.99E-05
2	50	0	17.793	0.000	17.793	2.02E+04	9.89E-05
2	50	5	17.793	3.559	19.491	1.34E+04	1.49E-04
1	50	10	17.793	7.117	21.052	9.51E+03	1.05E-04
2	50	30	17.793	21.351	26.391	3.45E+03	5.79E-04
2	50	35	17.793	24.910	27.564	2.84E+03	7.04E-04
3	50	40	17.793	28.468	28.690	2.37E+03	1.26E-03
3	50	45	17.793	32.027	29.773	2.01E+03	1.49E-03
1	50	50	17.793	35.585	30.818	1.72E+03	5.80E-04
1	50	55	17.793	39.144	31.828	1.49E+03	6.71E-04
1	50	60	17.793	42.702	32.808	1.30E+03	7.68E-04
1	50	65	17.793	46.261	33.759	1.14E+03	8.74E-04
1	55	-5	19.572	-3.559	17.703	2.07E+04	4.83E-05
1	55	0	19.572	0.000	19.572	1.32E+04	7.58E-05
2	55	30	19.572	21.351	28.301	2.52E+03	7.92E-04
2	55	35	19.572	24.910	29.506	2.09E+03	9.55E-04
4	55	40	19.572	28.468	30.663	1.76E+03	2.27E-03
4	55	45	19.572	32.027	31.778	1.50E+03	2.66E-03
2	55	50	19.572	35.585	32.856	1.29E+03	1.55E-03
1	55	60	19.572	42.702	34.911	9.85E+02	1.02E-03
1	55	70	19.572	49.819	36.852	7.73E+02	1.29E-03
1	60	-10	21.351	-7.117	17.433	2.22E+04	4.51E-05
1	60	-5	21.351	-3.559	19.491	1.34E+04	7.44E-05
1	60	30	21.351	21.351	30.195	1.89E+03	5.30E-04
1	60	35	21.351	24.910	31.428	1.58E+03	6.34E-04

(10.40, p. 7)

3	60	40	21.351	28.468	32.614	1.34E+03	2.25E-03
2	60	45	21.351	32.027	33.759	1.14E+03	1.75E-03
1	60	50	21.351	35.585	34.866	9.91E+02	1.01E-03
2	65	35	23.130	24.910	33.334	1.21E+03	1.65E-03
1	65	40	23.130	28.468	34.547	1.03E+03	9.69E-04
2	70	35	24.910	24.910	35.227	9.46E+02	2.11E-03
1	70	45	24.910	32.027	37.660	7.01E+02	1.43E-03
1	75	15	26.689	10.676	31.579	1.54E+03	6.48E-04
1	75	35	26.689	24.910	37.109	7.49E+02	1.34E-03
2	75	40	26.689	28.468	38.367	6.45E+02	3.10E-03
1	85	5	30.247	3.559	31.977	1.46E+03	6.85E-04
1	85	15	30.247	10.676	35.182	9.51E+02	1.05E-03
3	85	20	30.247	14.234	36.680	7.89E+02	3.80E-03
3	85	25	30.247	17.793	38.119	6.64E+02	4.52E-03
4	90	25	32.027	17.793	39.944	5.38E+02	7.43E-03
1	95	10	33.806	7.117	37.194	7.41E+02	1.35E-03
1	95	20	33.806	14.234	40.299	5.17E+02	1.93E-03
4	95	25	33.806	17.793	41.765	4.41E+02	9.07E-03
1	95	30	33.806	21.351	43.181	3.80E+02	2.63E-03
5	100	20	35.585	14.234	42.105	4.25E+02	1.18E-02
3	100	25	35.585	17.793	43.583	3.64E+02	8.24E-03
1	100	30	35.585	21.351	45.012	3.15E+02	3.17E-03
3	105	20	37.364	14.234	43.908	3.52E+02	8.52E-03
3	105	25	37.364	17.793	45.397	3.03E+02	9.89E-03
3	105	30	37.364	21.351	46.839	2.64E+02	1.14E-02
2	110	25	39.144	17.793	47.209	2.54E+02	7.86E-03
3	110	30	39.144	21.351	48.662	2.22E+02	1.35E-02
3	115	25	40.923	17.793	49.018	2.15E+02	1.40E-02
1	120	15	42.702	10.676	47.742	2.42E+02	4.13E-03
1	120	25	42.702	17.793	50.826	1.83E+02	5.47E-03
1	120	30	42.702	21.351	52.299	1.61E+02	6.22E-03
2	125	25	44.481	17.793	52.631	1.56E+02	1.28E-02
1	135	20	48.040	14.234	54.696	1.32E+02	7.60E-03
1	150	25	53.378	17.793	61.635	7.70E+01	1.30E-02
854						$\Sigma =$	0.28119
						Calc $B_f = 1/\Sigma =$	3.56

Peak load, kN = 71.17

Data $B_f =$ 11.04

Ratio = 3.10

10.41 Four notched members, with (a) assumed to be an engineering component, and k_{tn}, k_{tg}, dimensions, and S_g as given.

(a) Determine l' for each; $F = F(\alpha)$, $a = c + l$.

$$l' = \frac{c}{(1.12 k_{tg}/F)^2 - 1}$$

Approximate values of F corresponding to small $\alpha = a/b$, obtained from Figs. 8.13(a), 8.14(a), 8.12(b), and 8.12(a), respectively, may be used to approximate l' as follows:

Case	k_{tg}	w_2 or d_2 mm	c mm	b mm	~ F	~ l' mm
(a)	3.77	55.2	7.2	55.2	1.12	0.545
(b)	4.55	12.0	1.2	6.0	1.12	0.061
(c)	3.24	54.0	9.0	27.0	1.12	0.948
(d)	3.43	44.0	7.0	22.0	1.00	0.509

◁

Accurate values require a trial and error solution with variable $F = F(\alpha)$, $\alpha = a/b$, from the same Figures. Using $\theta = \pi\alpha/2$ and $\beta = 1-\alpha$ as convenient, these are:

$$\text{Case (a): } F = \sqrt{\frac{\tan\theta}{\theta}}\left[\frac{0.923 + 0.199(1-\sin\theta)^4}{\cos\theta}\right]$$

(10.41, p.2)

Case (b): $F = \frac{1}{2\beta^{1.5}}\left[1 + \frac{\beta}{2} + \frac{3}{8}\beta^2 - 0.363\beta^3 + 0.731\beta^4\right]$

Case (c): $F = (1 + 0.122\cos^4\theta)\sqrt{\frac{\tan\theta}{\theta}}$

Case (d): $F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}}$

To proceed, note that l' occurs where $K_A = K_B$ from Eq. 8.24 and 8.25.

$K_A = 1.12\, k_{tg} S_g \sqrt{\pi l}, \quad K_B = F S_g \sqrt{\pi a}$

Use these in the form

$$\frac{K_A}{k_{tg} S_g} = 1.12\sqrt{\pi l}, \quad \frac{K_B}{k_{tg} S_g} = \frac{F}{k_{tg}}\sqrt{\pi(c + l)}$$

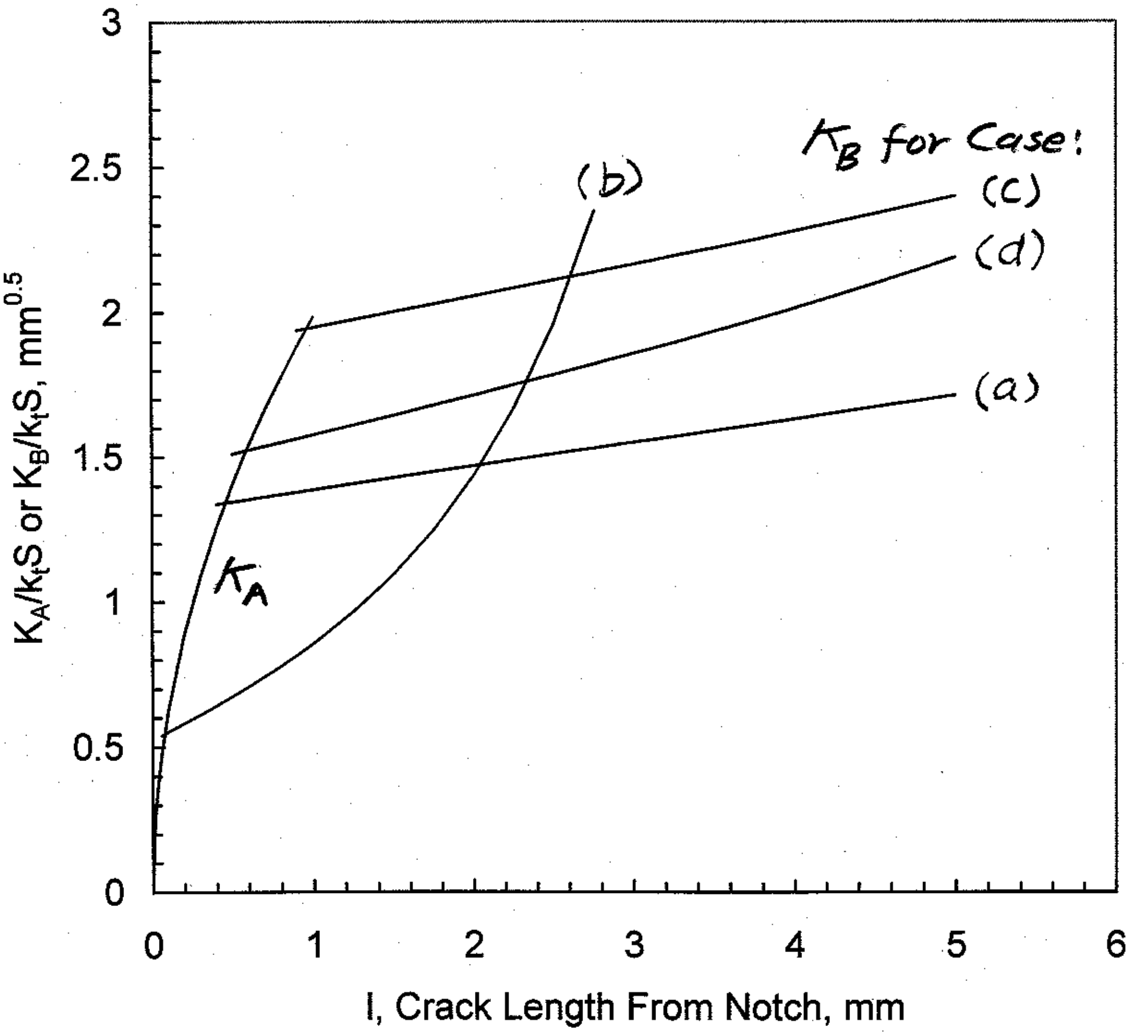
With varying F as above, adjust l until $K_A/(k_{tg} S_g) - K_B/(k_{tg} S_g) = 0$ is obtained; the final $l = l'$. The result follows:

Case	l'	a	$K_A/(k_{tg}S)$	$\alpha = a/b$	θ or β	F(a)	$K_B/(k_{tg}S)$	$K_A - K_B$
	mm	mm	$mm^{0.5}$				$mm^{0.5}$	
(a)	0.456	7.7	1.341	0.139	0.218	1.031	1.341	0.000
(b)	0.075	1.3	0.545	0.213	0.787	1.238	0.545	0.000
(c)	0.959	10.0	1.944	0.369	0.579	1.126	1.944	0.000
(d)	0.589	7.6	1.524	0.345	~~~	1.070	1.524	0.000

▲

(10.41, p.3)

(b) For various l, plot $K_A/(k_{tg} S_g)$, and also plot $K_B/(k_{tg} S_g)$ for each case, using k_{tg}, c, and varying F as appropriate.



(c) For the component, Case (a), the l' value, and also the approximate variation of K vs. l in the plot, is most closely matched by Case (d), the plate with a central hole.

10.42 AWS category E' applies to a cover-plated beam. Daily occurrences of ΔS for typical loading are given. Find (a) life in years, (b) X_N, X_S for 75 year service life.

$\Delta S = A' N_f^B$, with A', B from Table 10.3.

Apply P-M rule and calculate X_N, X_S.

$N_f = \left(\frac{\Delta S}{A'}\right)^{1/B}$, $B_f = 1 \Big/ \left[\sum \frac{N_j}{N_{fj}}\right]_{\text{per day}}$

AWS Category	A', MPa	B
E'	5001	-0.3330

ΔS, MPa	N_j	N_{fj}	N_j/N_{fj}
1.8	121	2.20E+10	5.51E-09
5.4	335	8.11E+08	4.13E-07
9.0	255	1.75E+08	1.46E-06
12.6	136	6.37E+07	2.14E-06
16.2	76	2.99E+07	2.54E-06
19.8	48	1.64E+07	2.93E-06
23.4	16	9.92E+06	1.61E-06
27.0	9	6.45E+06	1.39E-06
30.6	3	4.43E+06	6.77E-07
34.2	1	3.17E+06	3.15E-07
		Σ =	1.35E-05
		B_f, days = $1/\Sigma$ =	74,175
		B_f, years =	**203.1**

(10.42, p.2)

$X_N = B_f / \hat{B}$, $\hat{B} = 75$ years

$X_N = 203.1/75 = 2.71$ ◀

$X_S = X_N^{-B} = 2.71^{-(-0.333)} = 1.39$ ◀

Comments: If the AWS value $\Delta S_{TH} = 18$ MPa is employed in the life estimate, using $N_j / N_{fj} = 0$ below this level, $B_f = 395$ yrs. results. This is expected to be nonconservative, as cycles below the fatigue limit cause fatigue damage if combined with cycles above. If the same is done using $\Delta S_{TH}/2 = 9$ MPa per AASHTO, $B_f = 210$ yrs. results. Also X_N and X_S above are relative to the design curve, which is about $\times 2$ in life below the mean of the original data.