8.1 AISI 4340 steel (Fig. 8.32), $\sigma_o = 800$ and 1600 MPa.

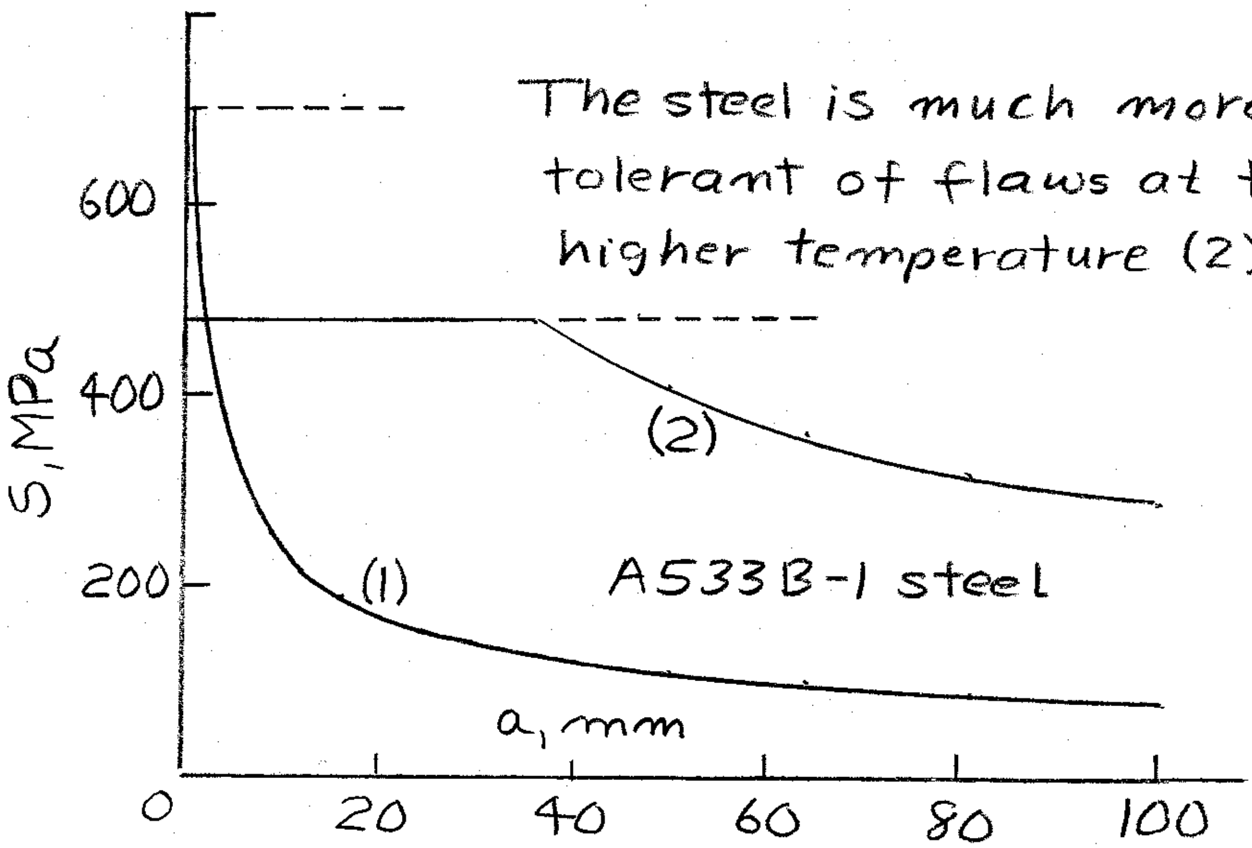
(a) $K_{Ic} = ?$ (b) $a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_o}\right)^2$

σ_o, MPa	K_{Ic}, MPa$\sqrt{m}$	a_t, mm
800	185	17.0
1600	40	0.20

The much smaller a_t for the higher σ_o indicates a greater sensitivity to flaws, so that brittle fracture would be an important design consideration.

8.2 $a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_o}\right)^2$, $K_{Ic} = S\sqrt{\pi a}$

	Temp. °C	K_{Ic} MPa$\sqrt{m}$	σ_o MPa	a_t m	a_t mm
(1)	−150	42	700	0.00115	1.15
(2)	+10	160	480	0.0354	35.4



The steel is much more tolerant of flaws at the higher temperature (2). ◀

(1) $S = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{42 \text{ MPa}\sqrt{m}}{\sqrt{\frac{\pi a \text{ mm}}{1000 \text{ mm/m}}}} = \frac{749}{\sqrt{a}}$ MPa

(2) $S = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{160}{\sqrt{\frac{\pi a}{1000}}} = \frac{2855}{\sqrt{a}}$ MPa

8.3 For each metal in Table 8.1, calculate a_t and plot on log scale vs. σ_o.

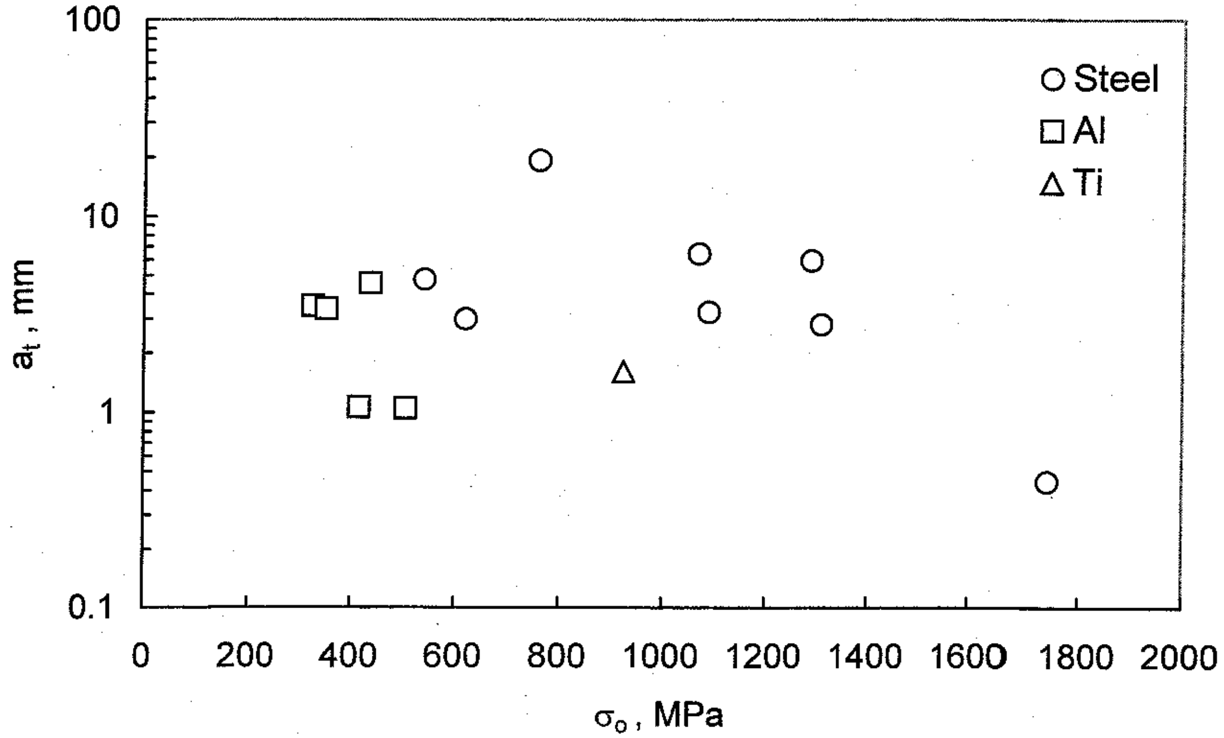
$$a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_o}\right)^2 = \frac{1}{\pi}\left(\frac{66\ MPa\sqrt{m}}{540\ MPa}\right)^2$$

$$a_t = 4.75 \times 10^{-3}\ m = 4.75\ mm$$

(for AISI 1144 steel, others similarly)

K_{Ic} MPa $m^{0.5}$	σ_o MPa	a_t mm
(a) Steels		
66	540	4.75
60	620	2.98
187	760	19.27
110	1090	3.24
123	1310	2.81
176	1290	5.93
152	1070	6.42
65	1740	0.44

K_{Ic} MPa $m^{0.5}$	σ_o MPa	a_t mm
(b) Aluminum Alloys		
24	415	1.06
34	325	3.48
36	350	3.37
29	505	1.05
52	435	4.55
(c) Titanium Alloy		
66	925	1.62



(8.3, p.2)

Although there is considerable scatter, it appears that a_t decreases with increasing σ_o for steels. Aluminum alloys show a similar trend, but with a_t smaller than for steels of similar strength.

8.4 Calculate a_t for various given materials and comment on trends.

$$a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_o}\right)^2 = \frac{1}{\pi}\left(\frac{66\ MPa\sqrt{m}}{540\ MPa}\right)^2$$

$a_t = 4.75 \times 10^{-3}\,m = 4.75\ mm$

(for AISI 1144 steel; others similarly)

Use σ_{ut} for brittle materials where there is no yield strength.

Material	K_{Ic} MPa $m^{0.5}$	σ_o MPa	a_t mm
1144 Steel	66	540	4.75
A517-F Steel	187	760	19.27
300-M, 650 Steel	152	1070	6.42
300-M, 300 Steel	65	1740	0.44
2219-T851 Al	36	350	3.37
7075-T651 Al	29	505	1.05
ABS Polymer	3.0	42[1]	1.62
Epoxy	0.60	59[1,2]	0.033
Glass	0.76	50[2]	0.074
Si_3N_4	5.6	450[2]	0.049

Notes: [1]Average strength from Table 4.3. [2]Ultimate strength, rather than yield, is used.

a_t is a fairly large (easily found) size for most of the steels, and is smaller for the Al alloys. The values are very small for epoxy, glass, and Si_3N_4, and in these cases may be similar to the natural flaws in the material. Hence, these three may be "internally flawed."

8.6 Center-cracked plate, AISI 1144 steel, $b = 40$, $t = 15$ mm. $K_{Ic} = 66\ MPa\sqrt{m}$ (Tbl. 8.1)

$P = ?$ for $X_K = K_{Ic}/K = 3$ if $a = 10, 24$ mm

(a) $a = 10$ mm, $\alpha = a/b = 10/40 = 0.25$

$K = F S_g \sqrt{\pi a}$, $F \approx 1$ (Fig. 8.12)

$$S_g = \frac{P}{2bt}, \quad K = F \frac{P}{2bt}\sqrt{\pi a}$$

$$K = \frac{K_{Ic}}{X_K} = \frac{66\ MPa\sqrt{m}}{3.0} = 22\ MPa\sqrt{m}$$

$$22\ MPa\sqrt{m} = \frac{P, N}{2(40)(15)\ mm^2}\sqrt{\pi(0.010\ m)}$$

$P = 148{,}900\ N = 148.9\ kN$ ◀

(b) $a = 24$ mm, $\alpha = a/b = 0.6$

$$F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} = 1.292 \quad \text{(Fig. 8.12)}$$

$$22 = 1.292\, \frac{P}{2(40)(15)}\sqrt{\pi(0.024)}$$

$P = 74{,}400\ N = 74.4\ kN$ ◀

8.7 F_1 = tabulated values , $\alpha = a/b$

(a) $F_2 = \dfrac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}}$ Fig. 8.12(a)

(b) $F_3 = \sqrt{\sec \dfrac{\pi\alpha}{2}}$ (c) $F_4 = \sqrt{\dfrac{2}{\pi\alpha} \tan \dfrac{\pi\alpha}{2}}$

($\pi a/2$ in radians)

For each $\alpha = 0, 0.1, 0.2, \ldots, 0.9$, calculate F_2, F_3, and F_4, and also the ratios F_2/F_1, F_3/F_1, and F_4/F_1.

$\alpha = a/b$	Tada F_1	Fig. 8.12a F_2	Secant F_3	Tangent F_4	Fig. 8.12a F_2/F_1	Secant F_3/F_1	Tangent F_4/F_1
0	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.006	1.0048	1.0062	1.0041	0.9988	1.0002	0.9982
0.2	1.025	1.0208	1.0254	1.0170	0.9959	1.0004	0.9922
0.3	1.058	1.0510	1.0594	1.0398	0.9934	1.0013	0.9828
0.4	1.109	1.1001	1.1118	1.0753	0.9920	1.0025	0.9696
0.5	1.187	1.1759	1.1892	1.1284	0.9907	1.0019	0.9506
0.6	1.303	1.2924	1.3043	1.2085	0.9918	1.0010	0.9274
0.7	1.488	1.4784	1.4841	1.3360	0.9935	0.9974	0.8979
0.8	1.816	1.8082	1.7989	1.5650	0.9957	0.9906	0.8618
0.9	2.578	2.5743	2.5283	2.1133	0.9986	0.9807	0.8197

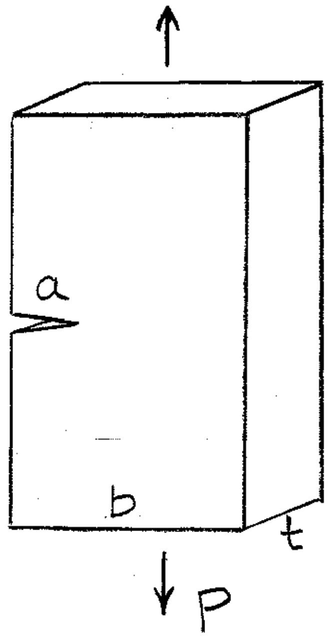
(a) F_2 is within 1% for $\alpha \leq 0.9$

(b) F_3 is within 2% for $\alpha \leq 0.9$

(c) F_4 is within 2% for $\alpha \leq 0.3$, 5% for $\alpha \leq 0.5$, and 18% off at $\alpha = 0.9$. ◀

8.8 AISI 1045 steel, HB = 400, σ_o = 1300 MPa,

K_{Ic} = 80 MPa$\sqrt{m}$ (Fig. 8.7)



$b = 120,\ t = 12\ mm$

$$\frac{K_{Ic}}{K} = X_K = 3.0$$

$$K = \frac{80}{3.0} = 26.7\ MPa\sqrt{m}$$

(a) $P = ?$ for $a = 18\ mm$

$$\frac{a}{b} = \frac{18}{120} = 0.15 = \alpha$$

$$K = F S_g \sqrt{\pi a}, \quad S_g = \frac{P}{bt}$$

$$F = 0.265(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}} = 1.283$$

$$26.7\ MPa\sqrt{m} = 1.283 \frac{P,\ N}{(120)(12)\ mm^2} \sqrt{\pi(0.018m)}$$

$$P = 126{,}000\ N = 126\ kN$$ ◀

(b) $X_o' = P_o / P = ?$, P from (a)

$P_o = bt\sigma_o\left[-\alpha + \sqrt{2\alpha^2 - 2\alpha + 1}\right]$ (Fig. A.16 (d))

$P_o = (120\ mm)(12\ mm)(1300\ MPa) \times$

$$\left[-0.15 + \sqrt{2(0.15)^2 - 2(0.15) + 1}\right]$$

(8.8, p.2)

$P_o = 1.335 \times 10^6 \text{ N} = 1335 \text{ kN}$

$X_o' = P_o/P = 1335/126 = 10.6$ ◀

The $P = 126$ kN force is far below fully plastic yielding, with $X_K = 3.0$ controlling. ◀

(c) $a = ?$ for $P = 100$ kN, $X_K = 3.0$

$K = F S_g \sqrt{\pi a}$, $F = F(\alpha = a/b)$ as above

$$26.7 \text{ MPa}\sqrt{m} = F \frac{100{,}000 \text{ N}}{120(12) \text{ mm}^2} \sqrt{\pi \frac{a, \text{mm}}{1000 \text{ mm/m}}}$$

Solve iteratively: Pick "a". Calculate $\alpha = a/b$, F, and K. Vary "a" until $K = 26.7$. Result is $a = 24.4$ mm ◀

Also check $X_o' > 3.0$ as in (b) for this "a".

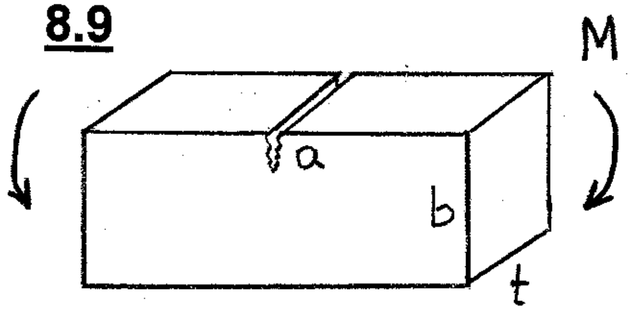
$P_o = 1159$ MPa, $X_o' = 11.66$ OK ◀

b, mm	t, mm	σ_o, MPa	K_{Ic}, MPa$\sqrt{m}$
120	12	1300	80.0

	P, N	S, MPa	a, mm	a, m	α	F
(a,b)	**125,897**	87.43	18.00	0.01800	0.1500	1.283
(c)	100,000	69.44	**24.38**	0.02438	0.2032	1.387

	K, MPa$\sqrt{m}$	X_K	P_o, N	X_o
(a,b)	26.67	3.00	1.335E+06	**10.60**
(c)	26.67	3.00	1.159E+06	11.59

8.9



$X_K = 2.5$

$b = 20$ mm

$t = 10$ mm

ABS plastic

$K_{Ic} = 3.0\ \text{MPa}\sqrt{m}$, $\sigma_o = 42$ MPa (average)

(Tables 8.2, 4.3)

(a) $a = ?$ for $M = 10$ N·m

$$S_g = \frac{6M}{b^2 t} = \frac{6(10{,}000\ \text{N}\cdot\text{mm})}{(20\ \text{mm})^2(10\ \text{mm})} = 15.0\ \text{MPa}$$

$K = 1.12\, S_g \sqrt{\pi a}$ $\quad (a/b \le 0.4)$ $\quad$ (Fig. 8.13)

$K = 3.0/2.5 = 1.2\ \text{MPa}\sqrt{m}$

$$a = \frac{1}{\pi}\left(\frac{K}{1.12\, S_g}\right)^2 = \frac{1}{\pi}\left(\frac{1.2\ \text{MPa}\sqrt{m}}{1.12 \times 15\ \text{MPa}}\right)^2$$

$a = 0.00162$ m $= 1.62$ mm ◀

$a/b = 1.62/20 = 0.081$, $F = 1.12$ is O.K.

(b) $a = ?$ for $M = 3.0$ N·m $= 3000$ N·mm

$$S_g = \frac{6M}{b^2 t} = \frac{6(3000)}{20^2\, 10} = 4.5\ \text{MPa}$$

Calculation with $F = 1.12$ as above gives:

$a = 18.0$ mm, $\alpha = a/b = 0.90$, not ≤ 0.4

(8.9, p.2)

Hence, the equation for varying F is needed.

$$F = \sqrt{\frac{1}{x}\tan x}\left[\frac{0.923 + 0.199(1-\sin x)^4}{\cos x}\right]$$

where $x = \pi\alpha/2$, $\alpha = a/b$

$K = FS\sqrt{\pi a}$, $1.2\,\text{MPa}\sqrt{m} = F\,(4.5\,\text{MPa})\sqrt{\pi(a, m)}$

Pick an "a", then calculate, α, x, F, and K.
Vary "a" iteratively until $K = 1.2\,\text{MPa}\sqrt{m}$.
$a = 10.1$ mm ◀

b, mm	t, mm	X	K_{Ic}, MPa $\sqrt{m}$	σ_o, MPa
20	10	2.5	3.0	42

	M, N-mm	S, MPa	a, mm	a, m	$\alpha = a/b$	$\pi\alpha/2$
(a)	10,000	15.00	**1.624**	0.001624	0.08120	---
(a)	10,000	15.00	**1.870**	0.001870	0.09352	0.14690
(b)	3,000	4.50	**10.125**	0.010125	0.50627	0.79525

	F	Comment	K, MPa $\sqrt{m}$	X_K	M_o, N-m	X'_o	
(a)	1.120	Approx F	1.200	2.50	35,456	**3.55**	(c)
(a)	1.044	Exact F	1.200	2.50	34,512	**3.45**	(c)
(b)	1.495	Exact F	1.200	2.50	10,238	**3.41**	(c)

(c) Consider fully plastic yielding.

$M_o = \frac{b^2 t \sigma_o}{4}(1-\alpha)^2$ (Fig. A.16), $X_o' = M_o/M$

See results in table above: (a) $X_o' = 3.55$,
(b) $X_o' = 3.41$. So $X_K = 2.5$ controls for (a), (b) ◀

8.10 Single edge cracked plate.

$b = 30$, $t = 4$, $a = 6$ mm, $P = 7.5$ kN

(a) For $X_K = 2.8$, what K_{Ic} required?

$$K = FS\sqrt{\pi a}, \quad K = \frac{K_{Ic}}{X_K}$$

$$K_{Ic} = X_K F \frac{P}{bt}\sqrt{\pi a} \qquad \text{(Fig. 8.12(c))}$$

$$F = 0.265(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{1.5}}$$

$$\alpha = a/b = 6/30 = 0.20, \quad F = 1.380$$

$$K_{Ic} = 2.8(1.38)\frac{7500\text{ N}}{30(4)\text{ mm}^2}\sqrt{\pi(0.006\text{ m})}$$

$$K_{Ic} = 33.2\text{ MPa}\sqrt{m}$$ ◀

(b) $X_o' = 2.0$ for fully plastic yielding

$\sigma_o = ?$, $X_o' = P_o/P$ Use Fig. A.16 (d).

$$P_o = bt\sigma_o\left[-\alpha + \sqrt{2\alpha^2 - 2\alpha + 1}\right] = X_o P$$

$$2(7500\text{ N}) = (30\text{ mm})(4\text{ mm})(\sigma_o\text{ MPa}) \times \left[-0.2 + \sqrt{2(0.2)^2 - 2(0.2) + 1}\right]$$

$$\sigma_o = 200\text{ MPa} \quad \text{(minimum)}$$ ◀

(c) 2024-T351 or 2219-T851 meet above.

8.11 Bending members, Fig. 8.13(a), $b = 60$, $t = 20$ mm, 18 Ni maraging steel (vac. melt)
$M = 10$ kN·m, $a = 1$ mm.
$K_{Ic} = 176 \text{ MPa}\sqrt{m}$, $\sigma_o = 1290$ MPa (Tbl. 8.1)

(a) Failure $M = ?$, $X = ?$ Figs. 8.13, A.13(b).

Brittle fracture: $K = FS_c\sqrt{\pi a} = K_{Ic}$

$$S_c = \frac{6M_c}{b^2 t}, \quad F = 1.12 \quad (a/b \leq 0.4)$$

$$F\,\frac{6M_c}{b^2 t}\sqrt{\pi a} = K_{Ic}, \qquad M_c = \frac{b^2 t\, K_{Ic}}{6F\sqrt{\pi a}}$$

$$M_c = \frac{(60\text{ mm})^2(20\text{ mm})(176\text{ MPa}\sqrt{m})}{6(1.12)\sqrt{\pi(0.001\text{ m})}}$$

$$M_c = 33.64 \times 10^6 \text{ N·mm} = 33.64 \text{ kN·m} \qquad \blacktriangleleft$$

$$X_c = \frac{M_c}{M} = \frac{33.64}{10} = 3.36 \qquad \blacktriangleleft$$

($F = 1.12$ is OK as $a/b = 1/60 = 0.0167$)

Fully plastic yielding: $M_o = \dfrac{b^2 t\,\sigma_o}{4}(1 - a/b)^2$

$$M_o = \frac{(60\text{ mm})^2(20\text{ mm})(1290\text{ MPa})}{4}\left(1 - \frac{1\text{ mm}}{60\text{ mm}}\right)^2$$

$$M_o = 22.45 \times 10^6 \text{ N·mm} = 22.45 \text{ kN·m} \qquad \blacktriangleleft$$

$$X_o' = \frac{M_o}{M} = \frac{22.45}{10} = 2.25 \quad \text{(controls)} \qquad \blacktriangleleft$$

(8.11, p. 2)

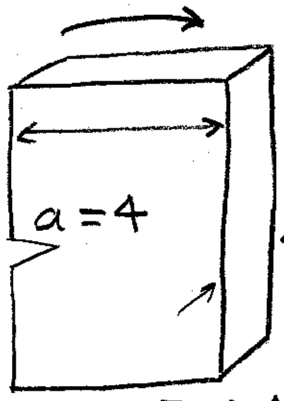
(b) $a = 5\ mm$, $\alpha = a/b = 0.0833$

Repeat above with new "a".

$X_c = 1.50$, $X_o' = 1.95$ ◁

Brittle fracture now controls, vs. yielding for (a). X_c is low noting typical statistical scatter in K_{Ic} (Table B.4). Replace. ◀

8.12 Rectangular beam, 7475-T7351 Al



$b = 40$ mm $X'_o = 2.0$

$t = 20$ $X_K = 3.5$

$a/b = 0.1$ (a) X'_o, X_K OK ?

$M = 1.0$ kN·m (b) $b = ?$ if not

Fig. 8.13: $F = 1.12$ $(a/b < 0.4)$

$$S = \frac{6M}{b^2 t} = \frac{6(1.0 \times 10^6 \text{ N·mm})}{(40)^2(20) \text{ mm}^3} = 187.5 \text{ MPa}$$

Table 8.1: $K_{Ic} = 52 \text{ MPa}\sqrt{m}$, $\sigma_o = 435$ MPa

Consider fully plastic yielding, with crack

$$M_o = \frac{b^2 t \sigma_o}{4}\left(1 - \frac{a}{b}\right)^2 \quad \text{Fig. A.16(b)}$$

$$M_o = \frac{(40\text{mm})^2(20\text{mm})(435 \text{ MPa})}{4}\left(1 - \frac{4\text{mm}}{40\text{ mm}}\right)^2$$

$M_o = 2.82 \times 10^6$ N·mm $= 2.82$ kN·m

$X'_o = M_o/M = 2.82 > 2.0$ OK ◁

Consider brittle fracture

$K_{Ic}/X_K = K = FS\sqrt{\pi a}$

$K = 1.12(187.5 \text{ MPa})\sqrt{\pi(0.004 \text{ m})}$

(8.12, p. 2)

$K = 23.54\ \text{MPa}\sqrt{m}, \quad X_K = \dfrac{52\ \text{MPa}\sqrt{m}}{23.54\ \text{MPa}\sqrt{m}}$

$X_K = 2.21, \quad < 3.5 \quad$ not OK

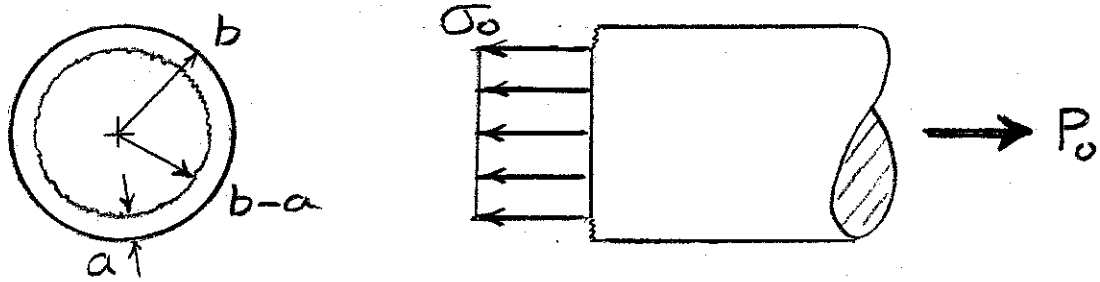
(b) $\dfrac{K_{Ic}}{X_K} = K = F S \sqrt{\pi a} = F \dfrac{6M}{b^2 t} \sqrt{\pi a}$

$$\frac{52\ \text{MPa}\sqrt{m}}{3.5} = 1.12\ \frac{6\,(1.0 \times 10^6\ \text{N}\cdot\text{mm})}{(b,\ \text{mm})^2\,(20\ \text{mm})}\sqrt{\pi\,(0.004\ \text{m})}$$

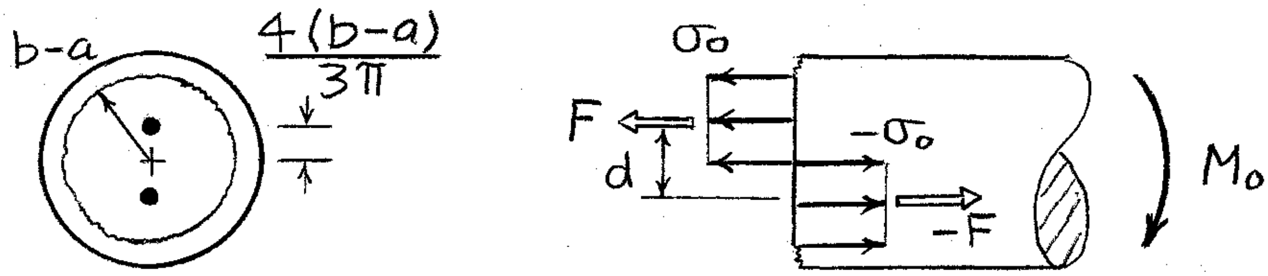
$b = 50.35\ \text{mm}$ ◀

$\alpha = a/b = 4/50.35 = 0.079, \quad F = 1.12$ is OK

8.14 Find P_o, M_o for a circumferential crack in shaft case, as $f(a, b, \sigma_o)$



$$P_o = \pi (b-a)^2 \sigma_o = \pi b^2 \sigma_o (1 - a/b)^2 \quad ◀$$



Couple:

$$M_o = Fd = \left(\frac{\pi (b-a)^2 \sigma_o}{2}\right)\left(2\,\frac{4(b-a)}{3\pi}\right)$$

$$M_o = \frac{4}{3}(b-a)^3 \sigma_o = \frac{4}{3} b^3 \sigma_o (1 - a/b)^3 \quad ◀$$

8.15 Tube with longitudinal crack.

$p = 20$ MPa, $r_{avg} = 25$, $t = 2$, $a = 5$ mm.

Find X_K, X_o (without crack). Ti-6Al-4V.
(Table 8.1)

$$\lambda = \frac{a}{\sqrt{r_{avg}t}} = \frac{5}{\sqrt{25(2)}} = 0.707$$

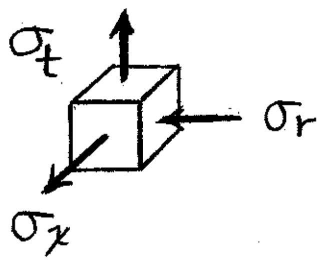
$$F = \sqrt{1 + 1.25\lambda^2} = 1.275$$

$$K = \frac{F p r_{avg}}{t}\sqrt{\pi a} = \frac{1.275\,(20\,\text{MPa})(25\,\text{mm})}{2\,\text{mm}}$$

$$K = \frac{1.275\,(20\,\text{MPa})(25\,\text{mm})}{2\,\text{mm}}\sqrt{\pi\,(0.005\,\text{m})}$$

$$K = 39.94\ \text{MPa}\sqrt{\text{m}}$$

$$X_K = \frac{K_{Ic}}{K} = \frac{66\ \text{MPa}\sqrt{\text{m}}}{39.94\ \text{MPa}\sqrt{\text{m}}} = 1.65 \quad ◀$$



$$\sigma_x = \frac{p r_1}{2t}, \quad \sigma_t = \frac{p r_1}{t}, \quad \sigma_r = -p$$

$$\sigma_x = \frac{(20\,\text{MPa})(25-1)\,\text{mm}}{2\,(2\,\text{mm})} = 120\ \text{MPa}$$

$\sigma_t = 240$ MPa, $\sigma_r = -20$ MPa

$\sigma_t, \sigma_x, \sigma_r = \sigma_1, \sigma_2, \sigma_3$ (all $\tau = 0$)

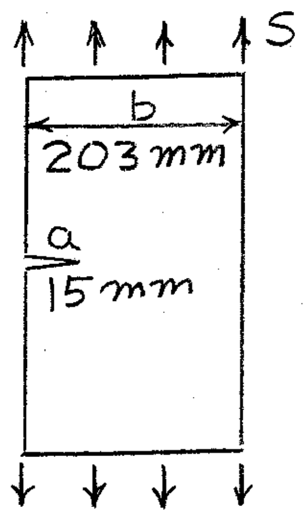
$$\bar{\sigma}_S = \text{MAX}\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right) = \sigma_o / X_o$$

$$\bar{\sigma}_S = 260\ \text{MPa} = 925\ \text{MPa} / X_o, \quad X_o = 3.56 \quad ◀$$

The X_K is too low! ◀

8.16 Structural member, A572 steel, Fig. 8.37. Dynamic loading at $-30°C$, $a = 15$ mm. (a) $M_c = ?$ (b) Compare to design $M_x = 176$ kN·m from $X_o = 1.67$. $K_{Ic} = 40 \text{ MPa}\sqrt{m}$.

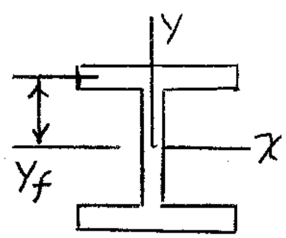
(a) $K_{Ic} = F S_c \sqrt{\pi a}$ Model flange as Fig. 8.12 (c), S at mid-thickness of flange.



$a/b = 15/203 = 0.074$

$F \approx 1.12$

Also note web tends to limit asymmetry effect on F.



$$S_c = \frac{K_{Ic}}{F\sqrt{\pi a}} = \frac{M_c y_f}{I_x}$$

$$\frac{40 \text{ MPa}\sqrt{m}}{1.12\sqrt{\pi(0.015 \text{ m})}} = \frac{(M_c, \text{ N·mm})(303/2 - 13.1/2) \text{ mm}}{(1.29 \times 10^{-4} \text{ m}^4)(1000 \text{ mm/m})^4}$$

$M_c = 146.4 \times 10^6$ N·mm $= 146.4$ kN·m ◀

(b) M_c to fracture < allowable $M_x = 176$ kN·m, If service loads reach design M_x, fracture is likely. ◀

Comments: M_i to yield edge of beam, with $\sigma_o = 345$ MPa from Fig. 8.37, is:

(8.16, p2)

$$\sigma_o = \frac{M_i\,y}{I_x}, \quad 345\ \text{MPa} = \frac{(M_i,\ \text{N}\cdot\text{mm})\,(303/2\ \text{mm})}{1.29 \times 10^8\ \text{mm}^4}$$

$$M_i = 293.8 \times 10^6\ \text{N}\cdot\text{mm} = 293.8\ \text{kN}\cdot\text{m}$$

$X_o = 1.67$ then gives an allowable moment:

$$M_x = \frac{M_i}{X_o} = \frac{293.8}{1.67} = 175.9\ \text{kN}\cdot\text{m} \quad \triangleleft$$

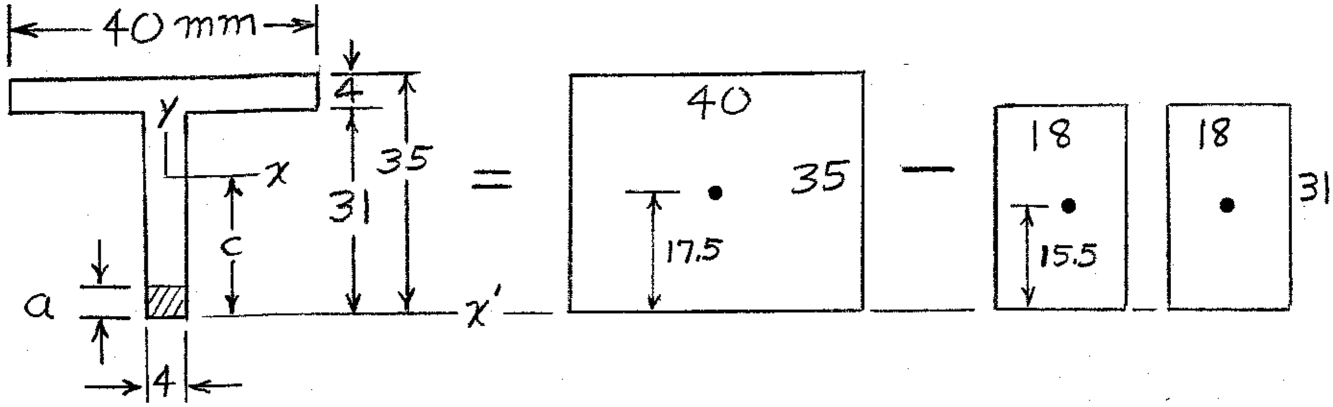
which is close to the given allowable $M_x = 176\ \text{kN}\cdot\text{m}$.

8.17 T-section, 7075-T651 Al, crack "a," $M_x = 180$ N·m. (a) Verify $c, \bar{I}_x$, (b) $X_K = ?$ for $a = 1.5$ mm. (c) $a = ?$ for $X_K = 3.0$. (d) Use 7475-T7351?

From Table 8.1, K_{Ic} in MPa$\sqrt{m}$, σ_o in MPa:

7075-T651: $K_{Ic} = 29$, $\sigma_o = 505$

7475-T7351: $K_{Ic} = 52$, $\sigma_o = 435$



$$(a)\ c = \bar{y} = \frac{\Sigma A y_c}{\Sigma A} = \frac{(40 \times 35)(17.5) - 2(18 \times 31)(15.5)}{(40 \times 35) - 2(18 \times 31)}$$

$$c = \frac{7204\ \text{mm}^3}{284\ \text{mm}^2} = 25.359\ \text{mm}$$

$$I_{x'} = \Sigma \frac{bh^3}{3} = \frac{40(35)^3}{3} - 2\frac{18(31)^3}{3} = 214{,}175\ \text{mm}^4$$

$$I_{x'} = \bar{I}_x + (\Sigma A)c^2, \quad \bar{I}_x = I_{x'} - (\Sigma A)c^2$$

$$\bar{I}_x = 214{,}175 - (284)(25.359)^2 = 31{,}538\ \text{mm}^4$$

(b) $K = FS\sqrt{\pi a} = K_{Ic}/X_K$, $F \approx 1.12$

$S = My/\bar{I}_x$, use $y = c$, bottom edge

(8.17, p.2)

$$S = \frac{(180{,}000\ N\cdot mm)(25.36\ mm)}{31{,}538\ mm^4} = 144.73\ MPa$$

$$K = 1.12\,(144.73\ MPa)\sqrt{\pi(0.0015\ m)} = 11.13\ MPa\sqrt{m}$$

$$X_K = K_{Ic}/K = 29/11.13 = 2.61$$

(c) $K_{Ic}/X_K = FS\sqrt{\pi a}$

$$(29\ MPa\sqrt{m})/3.0 = 1.12\,(144.73\ MPa)\sqrt{\pi(a,\ m)}$$

$$a = 1.132 \times 10^{-3}\ m = 1.13\ mm$$

(d) Redo (b) and (c) with $K_{Ic} = 52\ MPa\sqrt{m}$

For $a = 1.5\ mm$, $X_K = 52/11.13 = 4.67$

For $X_K = 3.0$:

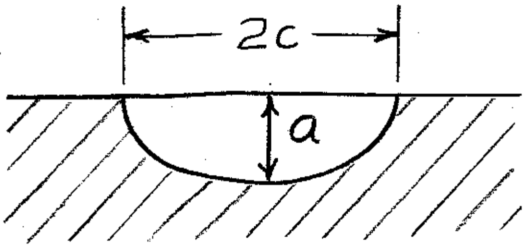
$52/3 = 1.12\,(144.73)\sqrt{\pi a}$, $a = 3.64\ mm$

Check yielding since σ_o lower.

$$X_o = \frac{\sigma_o}{S} = \frac{435\ MPa}{144.73\ MPa} = 3.01 \quad OK$$

7475-T7351 has larger X_K for $a = 1.5\ mm$, and for $X_K = 3.0$, permissible "a" is more than 3X larger, so easier to find by inspection. Change to new alloy if fracture would have severe consequences.

8.18 ASTM A470-8 steel, $S = 250$ MPa



$2c = 50$ mm

$a = 15$ mm

$K_{Ic} = 60 \text{ MPa}\sqrt{m}$

(Table 8.1)

Safe?

Fig. 8.19 applies

$$K = FS\sqrt{\frac{\pi a}{Q}}, \quad F = 1.12 \quad \left(\frac{a}{t}, \frac{c}{b} \text{ small}\right)$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} = 1 + 1.464\left(\frac{15 \text{ mm}}{25 \text{ mm}}\right)^{1.65}$$

$$Q = 1.630$$

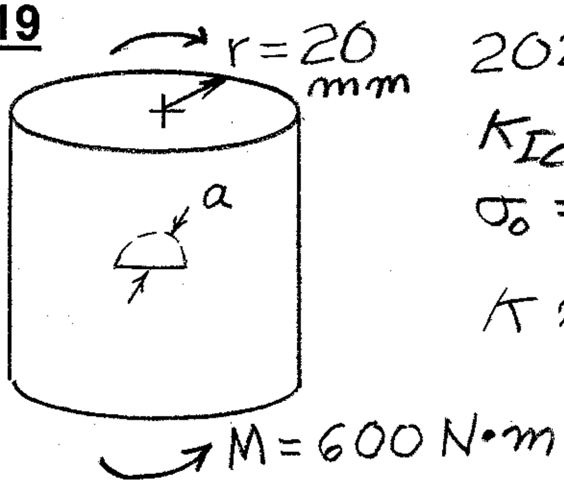
$$K = 1.12(250 \text{ MPa})\sqrt{\frac{\pi(0.015 \text{ m})}{1.630}}$$

$$K = 47.6 \text{ MPa}\sqrt{m}$$

$$X_K = \frac{K_{Ic}}{K} = \frac{60 \text{ MPa}\sqrt{m}}{47.6 \text{ MPa}\sqrt{m}} = 1.26 \quad \text{No!}$$

X_K is inadequate to safely operate.

8.19



2024-T351 Al, Table 8.1

$K_{Ic} = 34\ \text{MPa}\sqrt{m}$

$\sigma_o = 325\ \text{MPa}$

$K \approx 0.728\, S\sqrt{\pi a}$ (Fig. 8.17)

$$X_K = \frac{K_{Ic}}{K} = 3$$

$$S = \frac{Mr}{I} = \frac{Mr}{\pi r^4/4} = \frac{4M}{\pi r^3} = \frac{4(6 \times 10^5\ \text{N}\cdot\text{mm})}{\pi (20\ \text{mm})^3} = 95.5\ \text{MPa}$$

$$K = \frac{K_{Ic}}{3} = \frac{34}{3} = 11.33\ \text{MPa}\sqrt{m}$$

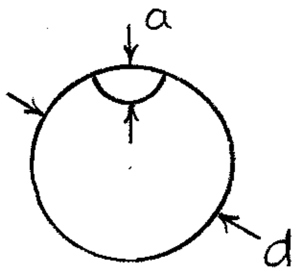
$$a = \frac{1}{\pi}\left(\frac{K}{0.728S}\right)^2 = \frac{1}{\pi}\left(\frac{11.33\ \text{MPa}\sqrt{m}}{0.728(95.5\ \text{MPa})}\right)^2$$

$$a = 8.45 \times 10^{-3}\ \text{m} = 8.45\ \text{mm}$$ ◀

$$\frac{a}{d} = \frac{8.45}{40} = 0.21, \quad 0.21 < 0.35$$

So $F = 0.728$ is OK (Fig. 8.17)

8.20 Solid circular shaft, $d = 50\ mm$, $M = 8.0\ kN \cdot m$, 300-M (300°C) steel. Table 8.1:



$K_{Ic} = 65\ MPa\sqrt{m}$, $\sigma_o = 1740\ MPa$

(a) $a_c = ?$ (b) $a = ?$ for $X_K = 3.5$

(c) Compare a_c and a.

(a) $K = FS\sqrt{\pi a}$, $S = \dfrac{32M}{\pi d^3}$, $F = 0.728$

for $a/d < 0.35$ (Fig. 8.17(d))

$$K = K_{Ic} = F\frac{32M}{\pi d^3}\sqrt{\pi a_c}$$

$$65\ MPa\sqrt{m} = 0.728\frac{32(8.0\times10^6\ N\cdot mm)}{\pi(50\ mm)^3}\sqrt{\pi a_c}$$

$a_c = 5.97\times10^{-3}\ m = 5.97\ mm$ ($F = 0.728$ is OK) ◀

(b) $K = K_{Ic}/X_K = FS\sqrt{\pi a}$

$$\frac{65\ MPa\sqrt{m}}{3.5} = 0.728\frac{32(8.0\times10^6\ N\cdot mm)}{\pi(50\ mm)^3}\sqrt{\pi a}$$

$a = 4.87\times10^{-4}\ m = 0.487\ mm$ ($F = 0.728$ is OK) ◀

(c) $X_a = a_c/a = 5.97/0.487 = 12.25$ ◀

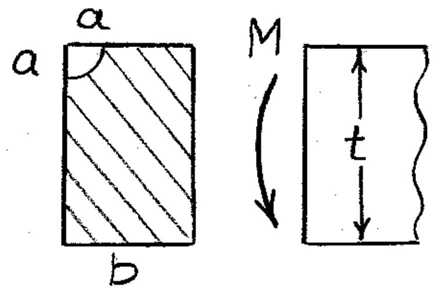
A large safety factor in "a" is needed to achieve an adequate X_K. From Eq 8.17:

$$X_a = \frac{a_c}{a} = \left(\frac{F}{F_c}X_K\right)^2,\quad F = F_c = 0.728,\quad X_a = X_K^2$$ ◀

8.21 Rectangular beam, 2219-T851 Al

$M = 160\ \text{N}\cdot\text{m}$, $b = 10\ \text{mm}$, corner $a = 2\ \text{mm}$

(a) $t = ?$ for $X_K = \dfrac{K_{Ic}}{K} = 3.0$



$F \approx 0.722$, $\dfrac{a}{t} < 0.35$, $\dfrac{a}{b} < 0.2$

From Fig. 8.17(c), with dimensions b, t, and "a" defined there.

$K_{Ic} = 36\ \text{MPa}\sqrt{\text{m}}$, $\sigma_o = 350\ \text{MPa}$ (Table 8.1)

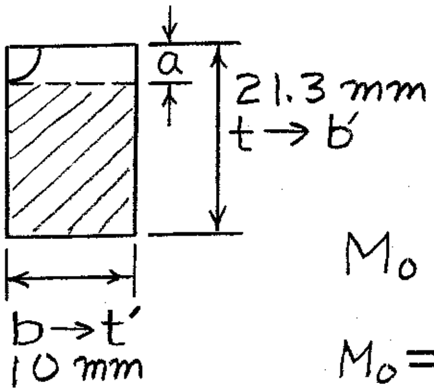
$$K = \frac{K_{Ic}}{X_K} = FS\sqrt{\pi a}\,,\quad S = \frac{6M}{bt^2}\qquad \text{(Fig. 8.17)}$$

$$\frac{36\ \text{MPa}\sqrt{\text{m}}}{3.0} = 0.722\,\frac{6(160{,}000\ \text{N}\cdot\text{mm})}{(10\ \text{mm})(t,\ \text{mm})^2}\sqrt{\pi(0.002\ \text{m})}$$

$t = 21.3\ \text{mm}$ ◀

$$\frac{a}{t} = \frac{2}{21.3} = 0.094,\quad \frac{a}{b} = \frac{2}{10} = 0.20,\ F \text{ is OK}$$

(b) Check fully plastic yielding. Use Fig. A.16(b).



$$M_o = \frac{(b')^2 t' \sigma_o}{4}\left(1 - \frac{a}{b'}\right)^2$$

Use units mm, $\text{MPa} = \text{N}/\text{mm}^2$

$$M_o = \frac{21.3^2\, 10\,(350)}{4}\left(1 - \frac{2}{21.3}\right)^2\ \text{N}\cdot\text{mm}$$

$M_o = 326\ \text{N}\cdot\text{m}$, $X_o' = M_o/M = 2.04$, OK ◀

8.22 Annealed Ti-6Al-4V, Table 8.1

$\sigma_o = 925\ MPa$, $K_{Ic} = 66\ MPa\sqrt{m}$

Solid circular shaft, $M = 5.0\ kN\cdot m$

$a = 3.0\ mm$, half-circular surface crk.

(a) $X_o = 2$, $d_o = ?$, $S = \frac{Mc}{I}$, $c = d/2$

$$I = \frac{\pi d^4}{64}, \quad S = \frac{32M}{\pi d^3} = \frac{\sigma_o}{X_o}$$

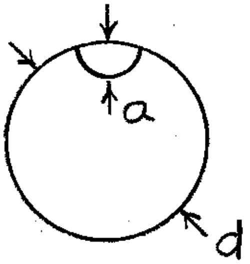
$$d = \left(\frac{32 M X_o}{\pi \sigma_o}\right)^{1/3} = \left(\frac{32\,(0.005\ MN\cdot m)\,2}{\pi\ 925\ MN/m^2}\right)^{1/3}$$

$d = d_o = 0.0479\ m = 47.9\ mm$ ◀

(b) $X_K = 3$, $d_c = ?$, $F = 0.728$ $(a/d < 0.35)$

$$K = F S \sqrt{\pi a} = \frac{K_{Ic}}{X_K}, \quad S = \frac{32M}{\pi d^3}$$ Fig. 8.17(d)

$$\frac{K_{Ic}}{X_K} = F\left(\frac{32M}{\pi d^3}\right)\sqrt{\pi a}$$



$$d = \left(\frac{32 M F X_K \sqrt{a}}{K_{Ic}\sqrt{\pi}}\right)^{1/3}$$

$$d = \left(\frac{32\,(0.005\ MN\cdot m)(0.728)(3)\sqrt{0.003\ m}}{66\ MPa\sqrt{m}\quad \sqrt{\pi}}\right)^{1/3}$$

$d = d_c = 0.0547\ m = 54.7\ mm$ (F is OK) ◀

(c) Use the larger, $d = 54.7\ mm$ ◀

8.23 Shaft, $d = 50$ mm, with circumferential crack, $a = 5$ mm. 18 Ni maraging steel $K_{Ic} = 123\ \text{MPa}\sqrt{m}$ (air melted, Table 8.1)

(a) $X = ?$ if $M = 1.5$ kN·m

Fig. 8.14 (b) applies: $S_g = 4M/(\pi b^3)$

$K = F S_g \sqrt{\pi a}$, $\alpha = a/b$, $\beta = 1 - \alpha$

$b = d/2 = 25$ mm, $\alpha = 5/25 = 0.2$, $\beta = 0.8$

$$F = \frac{3}{8\beta^{2.5}}\left[1 + \frac{\beta}{2} + \frac{3}{8}\beta^2 + \frac{5}{16}\beta^3 + \frac{35}{128}\beta^4 + 0.537\beta^5\right]$$

$F = 1.368$

$$K = F S_g \sqrt{\pi a} = 1.368\,\frac{4\,(1.5\times 10^6\ \text{N}\cdot\text{mm})}{\pi\,(25\ \text{mm})^3}\sqrt{\pi(0.005\ \text{m})}$$

$K = 20.95\ \text{MPa}\sqrt{m}$, $X = K_{Ic}/K = 5.87$ ◀

(b) Add 120 kN tension (Fig. 8.14(a))

$$F_2 = \frac{1}{2\beta^{1.5}}\left[1 + \frac{\beta}{2} + \frac{3}{8}\beta^2 - 0.363\beta^3 + 0.731\beta^4\right] = 1.225$$

$K_2 = F_2 S_{g2}\sqrt{\pi a}$, $S_{g2} = P/(\pi b^2)$

$$K_2 = 1.225\,\frac{120{,}000\ \text{N}}{\pi\,(25\ \text{mm})^2}\sqrt{\pi\,(0.005\ \text{m})} = 9.39\ \text{MPa}\sqrt{m}$$

$K = K_1 + K_2 = 20.95 + 9.39 = 30.33\ \text{MPa}\sqrt{m}$

where K_1 is K due to bending from (a).

$X = K_{Ic}/K = 123/30.33 = 4.05$ ◀

8.24 Circular shaft, $d = 60$ mm, with half-circular surface crack, $a = 10$ mm.
Ti-6Al-4V, annealed: $K_{Ic} = 66$ MPa$\sqrt{m}$ (Table 8.1). $M = 1.2$ kN·m, $P = ?$, $X_K = 4.0$.

Fig. 8.17(d): $F = 0.728$, $a/d < 0.2$ or 0.35.
$a/d = 10/60 = 0.167$, this $F = F_t = F_b$.

$$K_{Ic} / X_K = K = F_t S_t \sqrt{\pi a} + F_b S_b \sqrt{\pi a}$$

$$66/4 \text{ MPa}\sqrt{m} = 0.728\sqrt{\pi(0.010\text{ m})}\,(S_t + S_b)$$

$$S_t = \frac{4P}{\pi d^2} = \frac{4\,(P, N)}{\pi\,(60\text{ mm})^2} = (3.537 \times 10^{-4})\,P,\ \text{MPa}$$

$$S_b = \frac{32M}{\pi d^3} = \frac{32(1.2 \times 10^6\text{ N·mm})}{\pi\,(60\text{ mm})^3} = 56.59\text{ MPa}$$

$$16.5 = 0.728\sqrt{0.010\pi}\,(3.537 \times 10^{-4} P + 56.59)$$

$P = 201{,}500$ N $= 202$ kN ◀

8.25

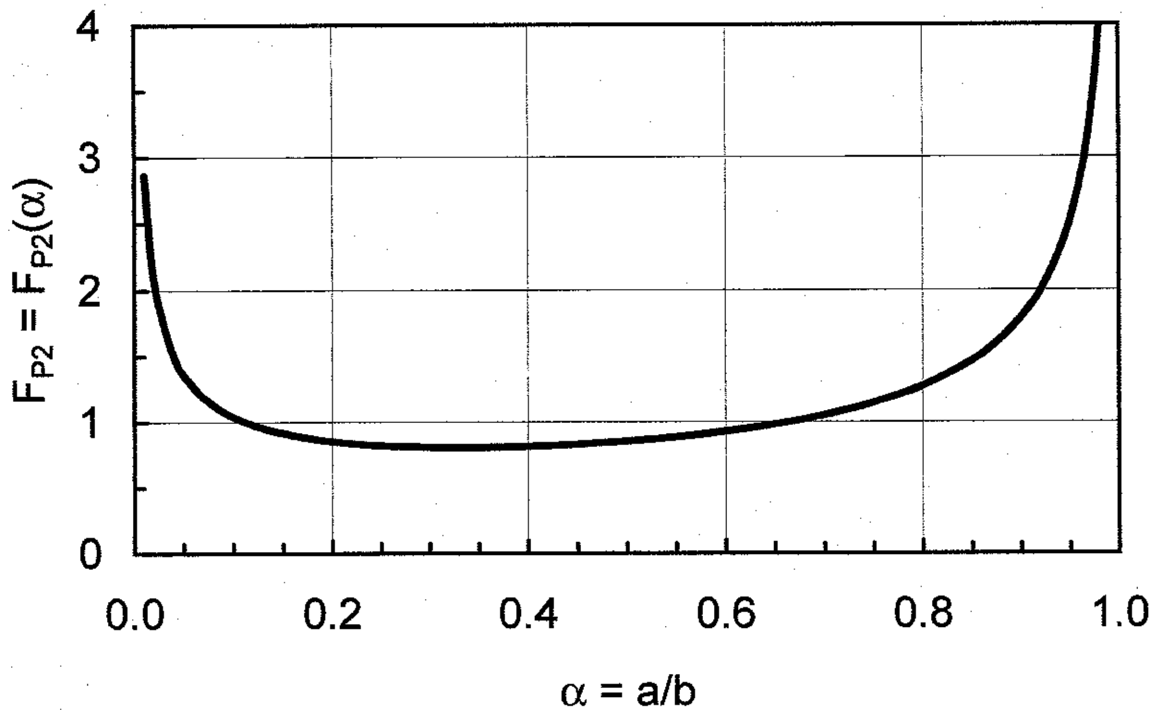
Infinite collinear array of cracks loaded on one side, K_2 in Fig. 8.23(b). Calculate and plot F_{P2} for various $\alpha = a/b$. Compare to approximations.

$$K_2 = F_{P2}\frac{P}{t\sqrt{b}}, \qquad F_{P2} = \frac{1}{2}\left(\frac{1}{\sqrt{\sin\pi\alpha}} + \sqrt{\frac{1}{2}\tan\frac{\pi\alpha}{2}}\right)$$

$$K_2' = F_{P2}'\frac{P}{t\sqrt{b}}, \qquad F_{P2}' = \frac{1}{2}\left(\frac{1}{\sqrt{\pi\alpha}} + \frac{\sqrt{\pi\alpha}}{2}\right) \qquad (10\%,\ \alpha \le 0.38)$$

$$K_2'' = F_{P2}''\frac{P}{t\sqrt{b}}, \qquad F_{P2}'' = 0.89 \qquad (10\%,\ 0.12 \le \alpha \le 0.65)$$

Calculation results are on the next page. For α approacing zero or unity, F_{P2} and hence K_2 approaches infinity. Ratios F'_{P2}/F_{P2} and F''_{P2}/F_{P2} show that the approximations are within 10% over their respective ranges, with the minor exception that F''_{P2} does exceed the exact F_{P2} by 10.4% around $\alpha = 0.34$.



(8.25, p.2)

Infinite collinear array of cracks loaded on one side, K_2 in Fig. 8.23(b)

$\alpha = a/b$	F_{P2}	F'_{P2}	F'_{P2}/F_{P2}	F''_{P2}	F''_{P2}/F_{P2}
0.010	2.865	2.865	1.000	---	---
0.020	2.058	2.057	1.000	---	---
0.040	1.501	1.499	0.999	---	---
0.060	1.264	1.260	0.997	---	---
0.080	1.128	1.123	0.995	---	---
0.100	1.040	1.032	0.992	---	---
0.120	0.979	0.968	0.989	0.89	0.910
0.140	0.933	0.920	0.985	0.89	0.953
0.180	0.874	0.853	0.976	0.89	1.019
0.220	0.838	0.809	0.965	0.89	1.062
0.260	0.818	0.779	0.952	0.89	1.088
0.300	0.808	0.758	0.937	0.89	1.101
0.340	0.806	0.742	0.921	0.89	1.104
0.380	0.810	0.731	0.902	0.89	1.099
0.400	0.814	---	---	0.89	1.093
0.450	0.830	---	---	0.89	1.072
0.500	0.854	---	---	0.89	1.043
0.550	0.886	---	---	0.89	1.005
0.600	0.927	---	---	0.89	0.960
0.650	0.981	---	---	0.89	0.907
0.700	1.051	---	---	---	---
0.750	1.144	---	---	---	---
0.800	1.272	---	---	---	---
0.850	1.464	---	---	---	---
0.870	1.570	---	---	---	---
0.890	1.705	---	---	---	---
0.910	1.884	---	---	---	---
0.920	1.997	---	---	---	---
0.930	2.135	---	---	---	---
0.940	2.305	---	---	---	---
0.950	2.524	---	---	---	---
0.960	2.822	---	---	---	---
0.970	3.258	---	---	---	---
0.980	3.990	---	---	---	---
0.990	5.642	---	---	---	---

8.26 Row of holes loaded on one side.

$S = 60$ MPa, $2b = 24$, $d = 4$, $\ell = 3$ mm

$X_K, X_o = ?$ 2024-T351 AL (Table 8.1)

$a = d/2 + \ell = 5$ mm, $\alpha = a/b = 5/12 = 0.417$

$$K = \frac{0.89P}{t\sqrt{b}}, \quad S = \frac{P}{2bt} \qquad \text{(Fig. 8.23(b))}$$

$$K = \frac{0.89(2btS)}{t\sqrt{b}} = 1.78\, S\sqrt{b}$$

$$K = 1.78\,(60\text{ MPa})\sqrt{0.012\text{ m}} = 11.70\text{ MPa}\sqrt{\text{m}}$$

$$X_K = K_{Ic}/K = 34/11.70 = 2.91 \quad \triangleleft$$

Fully plastic yielding

$$\sigma_o = \frac{P_o}{(2b-2a)t} = \frac{2btS_o}{2bt(1-\alpha)} = \frac{S_o}{1-\alpha}$$

$$S_o = \sigma_o(1-\alpha) = 325\,(1-0.417) = 189.6\text{ MPa}$$

$$X_o' = S_o/S = 189.6/60 = 3.16 \quad \triangleleft$$

$X_K = 2.91$ controls ◀

8.27 Cylindrical pressure vessel

$d_1 = 150$, $t = 5$ mm, $p = 20$ MPa, $X_o = 2$

Leak-before-break, $X_a = 9$, i.e., $c_c = X_a t$

(a) Safe if 300-M steel (300°C)?

$K_{Ic} = 65\ \text{MPa}\sqrt{m}$, $\sigma_o = 1740$ MPa (Table 8.1)

$$\sigma_t = \frac{p r_1}{t} = \frac{(20\ \text{MPa})(75\ \text{mm})}{5\ \text{mm}} = 300\ \text{MPa}$$

$$\sigma_x = \frac{p r_1}{2t} = 150\ \text{MPa}, \quad \sigma_r = -p = -20\ \text{MPa}$$

$$\sigma_t, \sigma_x, \sigma_r = \sigma_1, \sigma_2, \sigma_3$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} = 277\ \text{MPa}$$

$$X_o = \sigma_o / \bar{\sigma}_H = 6.27 \quad \text{OK} \quad \triangleleft$$

$$c_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 = \frac{1}{\pi}\left(\frac{65\ \text{MPa}\sqrt{m}}{300\ \text{MPa}}\right)^2 = 0.01494\ m$$

$$X_a = \frac{c_c}{t} = \frac{14.9\ \text{mm}}{5\ \text{mm}} = 2.99,\ 2.99 < 9 \quad \text{No!} \blacktriangleleft$$

(b) Safe if A517-F steel?

$K_{Ic} = 187\ \text{MPa}\sqrt{m}$, $\sigma_o = 760$ MPa (Table 8.1)

$$X_o = \frac{\sigma_o}{\bar{\sigma}_H} = \frac{760\ \text{MPa}}{277\ \text{MPa}} = 2.74 \quad \text{OK} \quad \triangleleft$$

$$c_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 = \frac{1}{\pi}\left(\frac{187\ \text{MPa}\sqrt{m}}{300\ \text{MPa}}\right)^2 = 0.1237\ m$$

$$X_a = c_c / t = 123.7\ \text{mm} / 5\ \text{mm} = 24.7 \quad \text{Yes!} \quad \blacktriangleleft$$

(8.27, p.2)

(c) What minimum K_{Ic} required?

$$c_c = X_a t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2, \quad K_{Ic} = \sigma_t \sqrt{\pi X_a t}$$

$$K_{Ic} = 300\,\text{MPa}\sqrt{\pi\,(9)(0.005\,\text{m})}$$

$$K_{Ic} = 113\ \text{MPa}\sqrt{\text{m}}$$ ◀

(d) What $X_K = K_{Ic}/K$ achieved?

$$K_{Ic} = \sigma_t \sqrt{\pi X_a t} \quad \text{(from above)}$$

$$\frac{K_{Ic}}{\sqrt{X_a}} = \sigma_t \sqrt{\pi t} = \hat{K} = K \text{ at leak}$$

$$\frac{K_{Ic}}{\hat{K}} = \sqrt{X_a}, \quad X_K = \sqrt{X_a} = \sqrt{9} = 3$$ ◀

8.28 Fig. 8.35 steels except A217 used at 22°C. Design with $S = \sigma_o/2$ and $K = K_{Ic}/2$. What is largest permissible half circular surface crack for each?

$$K = 0.728\, S\sqrt{\pi a} \qquad \text{(Fig. 8.17(b))}$$

$$\frac{K_{Ic}}{2} = 0.728\,\frac{\sigma_o}{2}\sqrt{\pi a}$$

$$a = \frac{1}{\pi}\left(\frac{K_{Ic}}{0.728\,\sigma_o}\right)^2$$

Read K_{Ic} from Fig. 8.35 at 22°C, and tabulate with σ_o also given.

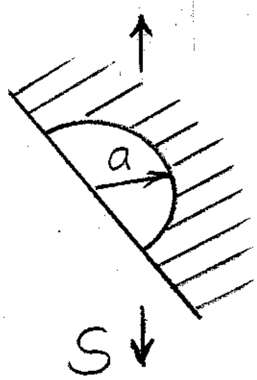
$$a = \frac{1}{\pi}\left(\frac{170\ \text{MPa}\sqrt{\text{m}}}{0.728\,(682\ \text{MPa})}\right)^2 = 0.03732\ \text{m}$$

(for 403 SS; others similarly)

Material	K_{Ic} MPa·m$^{0.5}$	σ_o MPa	a mm
403 SS	170	682	37.32
A471	130	931	11.71
A469	75	590	9.71
A470	45	626	3.10

In inspection for cracks, larger size is easier to find, so that fracture is most easily avoided in 403 SS, and least in A470.

8.29 2.25Cr-1Mo steel, Fig. 8.38, at +50°C and −50°C, S = 400 MPa. For half-circular surface crack, find a for $X_K = 3.0$.



$$K = FS\sqrt{\pi a}, \quad F = 0.728$$

(Fig. 8.17(b))

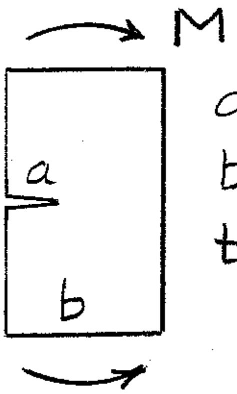
$$X_K = \frac{K_{Ic}}{K} = 3.0$$

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic},\ MPa\sqrt{m}}{3 \times 0.728 \times 400\ MPa}\right)^2$$

Temp., °C	K_{Ic}, MPa$\sqrt{m}$	a_c, mm
+50	140	8.17
−50	50	1.04

The steel is clearly more likely to have a problem with brittle fracture at the lower temperature, as a_c is in that case too small to be easily found by inspection. Since the transition to low K_{Ic} at low temperature occurs for BCC metals, a better choice of material for −50°C might be an austenitic stainless steel, such as AISI 304 or 316, which has an FCC structure, and no transition to low K_{Ic} at low temperature.

8.30 Consider AISI 4340 steel, Fig. 8.32. (a) $\sigma_o = 800$, (b) $\sigma_o = 1600$ MPa. Calculate M_c and M_o for both. (c) Use high σ_o ?



$a = 10$ mm
$b = 50$ mm
$t = 20$ mm

$K = FS\sqrt{\pi a}$
$a/b = 0.20$
$F = 1.12$ (Fig. 8.13)
$S = \dfrac{6M}{b^2 t}$

$$K_{Ic} = 1.12 \frac{6M_c}{b^2 t}\sqrt{\pi a}$$

$$M_c = \frac{b^2 t K_{Ic}}{6.72\sqrt{\pi a}} = \frac{(50\,mm)^2(20\,mm)(K_{Ic}, MPa\sqrt{m})}{6.72\sqrt{\pi(0.010\,m)}}$$

$$M_o = \frac{b^2 t \sigma_o}{4}\left(1 - \frac{a}{b}\right)^2 \quad \text{(Fig. A.16 (b))}$$

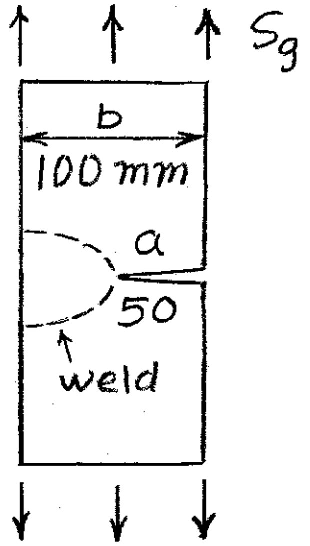
$$M_o = \frac{(50\,mm)^2(20\,mm)(\sigma_o, MPa)}{4}(0.80)^2$$

M_c, M_o above in N·mm; $(N \cdot mm) \times 10^{-6} = kN \cdot m$

	σ_o MPa	K_{Ic} $MPa\sqrt{m}$	M_c kN·m	M_o kN·m
(a)	800	187	7.85	6.40 ◄
(b)	1600	41	1.72 ◄	12.80

(c) The higher σ_o causes M at failure to be much lower due to low K_{Ic}. Do not use. ◄

8.31 Welded A533B-1 steel, Fig. 8.33. Find failure S_g at (a) $-75°C$, (b) $200°C$. Then (c) comment on using this steel vs. °C.



Assume geometry acts as crack.

$$K_{Ic} = F S_{gc} \sqrt{\pi a}$$

$\alpha = a/b = 0.50$, $F = 2.815$

F from $F(\alpha)$ in Fig. 8.12(c).

$$S_{gc} = \frac{K_{Ic}}{F\sqrt{\pi a}}$$

$$S_{gc}, \text{MPa} = \frac{K_{Ic}, \text{MPa}\sqrt{m}}{2.815\sqrt{\pi(0.05\,m)}}$$

$$S_{go} = \frac{P_o}{bt} = \sigma_o\left[-\alpha + \sqrt{2\alpha^2 - 2\alpha + 1}\right] \quad \text{(Fig. A.16(d))}$$

$$S_{go}, \text{MPa} = (\sigma_o, \text{MPa})[0.2071] \quad (\alpha = 0.5)$$

b, mm	a, mm	a, m	α = a/b	F
100	50	0.050	0.50	2.815

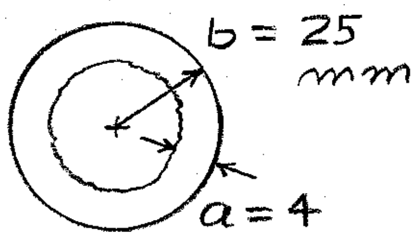
	T, °C	K_{Ic}, MPa $\sqrt{m}$	σ_o, MPa	S_{gc}, MPa	S_{go}, MPa
(a)	-75	52	550	**46.60** ◀	113.91
(b)	200	200	400	179.24	**82.84** ◀

(c) If flaws are present, sudden fracture may occur at $-75°C$. Avoid this steel at low temp. ◀

8.32 Shaft, 50 mm dia, moment, M. Circumferential crack, $a = 4$ mm. 300 M steel, 650 or 300 °C temper, Table 8.1 properties. M_o or M_c = ?

$$K = FS\sqrt{\pi a}, \quad S = \frac{4M}{\pi b^3}$$

(Fig. 8.14)



$$\alpha = \frac{a}{b} = 0.16$$

$$F = \frac{3}{8\beta^{2.5}}\left[1 + \frac{1}{2}\beta + \frac{3}{8}\beta^2 + \frac{5}{16}\beta^3 + \frac{35}{128}\beta^4 + 0.537\beta^5\right]$$

$\beta = 1 - \alpha = 0.84$, $F = 1.293$

$$K = K_{Ic} = F\,\frac{4M_c}{\pi b^3}\sqrt{\pi a}$$

$$M_c = \frac{\pi b^3 K_{Ic}}{4F\sqrt{\pi a}}, \quad M_o = \frac{4\sigma_o b^3}{3}(1-\alpha)^3$$

(Pr. 8.14)

Calculate M_c and M_o for each temper.

300-M	K_{Ic}	σ_o	b	a	α	β	F	M_c	M_o
°C temper	MPa$\sqrt{\text{m}}$	MPa	mm	mm	a/b	1 - a/b		kN-m	kN-m
650	152	1070	25	4	0.16	0.84	1.293	12.86	**13.21**
300	65	1740	25	4	0.16	0.84	1.293	**5.50**	21.49

The lower of M_c, M_o controls for each,

$M_{fail} = M_o = 13.21$ kN·m for 650 °C ◀

$M_{fail} = M_c = 5.50$ kN·m for 300 °C ◀

Temper 650 °C clearly better; yields first. ◀

8.33 (a) Determine ΔT for fracture due to thermal shock of each material, Assume $a = 1$ mm half-circular surface crack.

$$K_{Ic} = FS\sqrt{\pi a}, \quad F = 0.728 \quad \text{(Fig. 8.17(b))}$$

$$\sigma_x = \sigma_y = S = -\frac{E\alpha(\Delta T)}{1-\nu}, \quad S = \frac{K_{Ic}}{F\sqrt{\pi a}}$$

$$\Delta T = -\frac{K_{Ic}}{F\sqrt{\pi a}}\,\frac{1-\nu}{E\alpha}$$

α, E, ν in Table P8.33

K_{Ic} im Table 8.2

For soda-lime glass:

$$\Delta T = \frac{-(0.76\ \text{MPa}\sqrt{m})(1-0.2)}{0.728\sqrt{\pi(0.001\ m)}\,(69{,}000\ \text{MPa})(9.1\times10^{-6}\ {}^1/{}_{^\circ C})}$$

$\Delta T = -23.7\ ^\circ C$, Others similarly

Material	α	E	ν	K_{Ic}	ΔT	Rank
	$10^{-6}/^\circ C$	GPa		MPa $\sqrt{m}$	°C	
S-L glass	9.1	69	0.20	0.76	-23.7	4
MgO	13.5	300	0.18	2.9	-14.4	5
Al_2O_3	8.0	400	0.22	4.0	-23.9	3
SiC	4.5	396	0.22	3.7	-39.7	2
Si_3N_4	2.9	310	0.27	5.6	-111.4	1

Si_3N_4 has the best resistance, MgO the worst, as ranked above.

(8.33) (b) Thermal shock of listed mat'ls.

$$S = -\frac{E \alpha \Delta T}{1-\nu} \qquad (Eq. 5.41)$$

Use Sec. 3.8 method to rank materials as to resistance to ΔT with a small half-circular surface crack present.

$$K_{Ic} = F S \sqrt{\pi a}, \quad F = 0.728 \quad (Fig. 8.17(b))$$

$$-\Delta T = \frac{1-\nu}{E\alpha} S = \frac{1-\nu}{E\alpha} \frac{K_{Ic}}{F\sqrt{\pi a}}$$

$$-\Delta T = \left[\frac{1}{F\sqrt{\pi a}}\right]\left[\frac{K_{Ic}(1-\nu)}{E\alpha}\right] = f_1 f_2$$

Maximize $f_2 = K_{Ic}(1-\nu)/(E\alpha)$

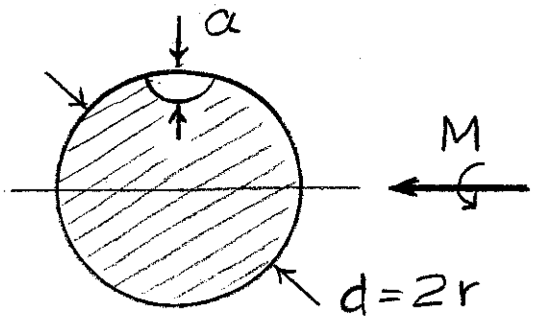
Material	α	E	ν	K_{Ic}	f_2 for ΔT	Rank
	$10^{-6}/°C$	GPa		$MPa\sqrt{m}$	$°C\ m^{0.5}$	
S-L glass	9.1	69	0.20	0.76	0.968	4
MgO	13.5	300	0.18	2.9	0.587	5
Al_2O_3	8.0	400	0.22	4.0	0.975	3
SiC	4.5	396	0.22	3.7	1.620	2
Si_3N_4	2.9	310	0.27	5.6	4.547	1

Higher K_{Ic} gives better ΔT resistance. But lower ν, E, and α are beneficial. Lower α gives less thermal effect. Lower E and ν give less stress for a given ΔT.

8.34 Solid round shaft, $M = 3.8$ kN·m

AISI 4340 steel, $X_o = 2.0$, $X_k = 3.0$

K_{Ic} vs. σ_o in Fig. 8.32, $a = 1.0$ mm.



(a) What d_o, d_c needed for $\sigma_o = 800$ MPa?

(b) Which controls?

(c) Repeat for other σ_o.

(d) Find optimum σ_o, d.

(e) Look at $a = 0.5, 2.0$ mm.

(a, b) Yielding:

$$\sigma = \frac{Mc}{I} = \frac{\sigma_o}{X_o}, \quad I = \frac{\pi d^4}{64}, \quad c = d/2$$

$$\frac{M(d/2)}{\pi d^4/64} = \frac{\sigma_o}{X_o}, \quad d_o = \left(\frac{32 M X_o}{\pi \sigma_o}\right)^{1/3}$$

For $\sigma_o = 800$ MPa

$$d_o = \left[\frac{32(2)(3.8\times10^6\ \text{N·mm})}{\pi\ 800\,\text{MPa}}\right]^{1/3} = 45.9\ \text{mm} \quad \blacktriangleleft$$

Fracture: Fig. 8.17(d), $S = 32M/(\pi d^3)$

$$K = FS\sqrt{\pi a}, \quad F = 0.728 \ (a/b < 0.35)$$

$$K = 0.728\,\frac{32M}{\pi d^3}\sqrt{\pi a} = \frac{K_{Ic}}{X_K}$$

$$d_c = \left[\frac{0.728 \cdot 32 M X_K \sqrt{\pi a}}{\pi K_{Ic}}\right]^{1/3}$$

(8.34, p.2)

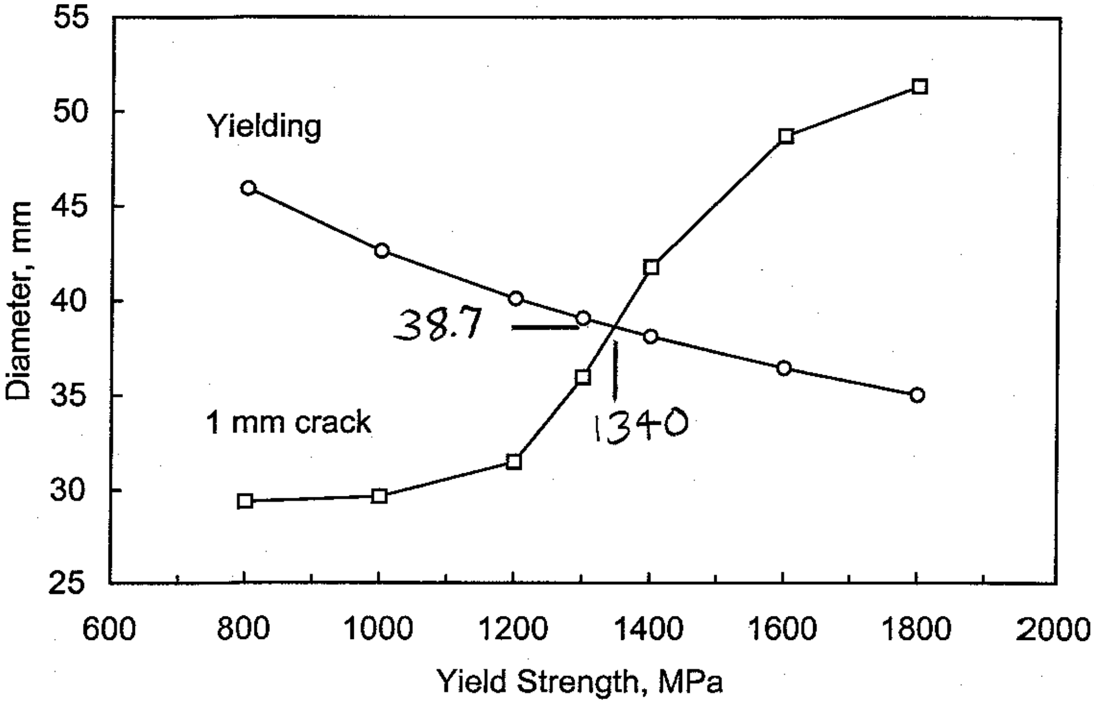
For $\sigma_o = 800$ MPa, with $K_{Ic} = 187$ MPa$\sqrt{m}$

$$d_c = \left[\frac{0.728 \cdot 32\,(3.8 \times 10^6\ N\cdot mm)(3)\sqrt{\pi\,(0.001 m)}}{\pi\ 187\ MPa\sqrt{m}}\right]^{\frac{1}{3}}$$

$d_c = 29.4$ mm ($d_o = 45.9$ mm controls) ◀

Similarly calculate d_o and d_c for the other σ_o, K_{Ic} combinations. (C)

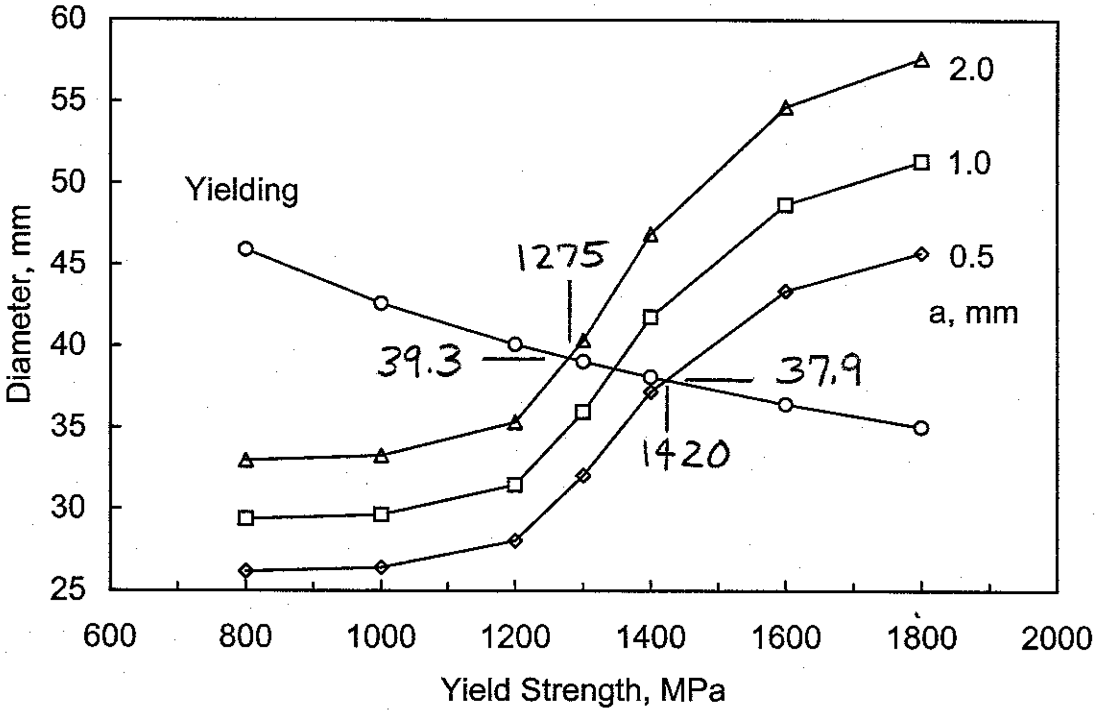
σ_o	K_{Ic}	Yielding	Fracture, d_c, mm, for a, mm =		
MPa	MPa-m$^{0.5}$	d_o, mm	1.00	0.50	2.00
800	187	45.91	29.37	26.17	32.97
1000	182	42.62	29.64	26.40	33.27
1200	152	40.11	31.47	28.04	35.33 ◀
1300	102	39.05	35.95	32.03	40.35
1400	65	38.10	41.77	37.22	46.89
1600	41	36.44	48.71	43.39	54.67
1800	35	35.04	51.35	45.74	57.63



(8.34, p.3)

At any yield strength, the larger of d_o or d_c controls, The minimum d occurs at $\sigma_o = 1340$ MPa, $d = 38.7$ mm (d) ◀

(e) Also consider $a = 0.5$ and $a = 2.0$ mm. Repeat calculations for these a; see table above.



a mm	σ_o MPa	$d_o = d_c$ mm
0.50	1420	37.9
1.00	1340	38.7
2.00	1275	39.3

For larger crack size, a, lower σ_o and larger d are required for min. weight. ◀

8.39 Shaft with $d = 20$ mm and circumferential crack, $a = 1.5$ mm. Combined loads $M = 150$ N·m, $T = 300$ N·m. What is X_K?
AISI 4130 steel: $K_{Ic} = 110$ MPa$\sqrt{m}$ (Tbl. 8.1)
Assume $K_{IIIc} = K_{Ic}/2$ in:

$$\left(\frac{K_I}{K_{Ic}}\right)^2 + \left(\frac{K_{III}}{K_{IIIc}}\right)^2 = 1, \quad \left(\frac{K_I}{K_{Ic}}\right)^2 + \left(\frac{2K_{III}}{K_{Ic}}\right)^2 = 1$$

$$K_{Ic} = \sqrt{(K_I)^2 + (2K_{III})^2} \quad \text{(at fracture)}$$

For the design situation, define an effective K_I.

$$\overline{K}_I = \sqrt{K_I^2 + 4K_{III}^2}, \quad X_K = K_{Ic} / \overline{K}_I$$

From 8.14(b), $\alpha = a/b = 1.5/10 = 0.15$
$\beta = 1 - \alpha = 0.85$, $F = 1.277$

$$K_I = F S_g \sqrt{\pi a}, \quad S_g = \frac{4M}{\pi b^3}, \quad b = d/2$$

$$K_I = \frac{4FM}{\pi b^3}\sqrt{\pi a} = \frac{4(1.277)(150{,}000 \text{ N·mm})}{\pi (10 \text{ mm})^3}\sqrt{\pi (0.0015 \text{ m})}$$

$K_I = 16.75$ MPa$\sqrt{m}$

Similarly, from Fig. 8.14(c), $\alpha = 0.15$
$\beta = 0.85$, $F = 1.195$

(8.39, p.2)

$$K_{III} = FS_g\sqrt{\pi a}\,, \quad S_g = \frac{2T}{\pi b^3}\,, \quad K_{III} = \frac{2FT}{\pi b^3}\sqrt{\pi a}$$

$$K_{III} = \frac{2(1.195)(300{,}000\ N\cdot mm)}{\pi(10\ mm)^3}\sqrt{\pi(0.0015\ m)}$$

$$K_{III} = 15.67\ MPa\sqrt{m}$$

$$\bar{K} = \sqrt{16.75^2 + 4(15.67)^2} = 35.5\ MPa\sqrt{m}$$

$$X_K = \frac{K_{Ic}}{\bar{K}} = \frac{110}{35.5} = 3.10$$

8.40 For Ex. 8.4: (a) Does pl. strain apply, and is LEFM applicable? (b) $r_{o\sigma}$ or $r_{o\varepsilon} = ?$

A517-F steel; $K_{Ic} = 187$ MPa$\sqrt{m}$, $\sigma_o = 760$ MPa

$t = 50$, $2c = 40$, $a = 10$ mm (Fig. 8.19(b))

$K = 49.2$ MPa$\sqrt{m}$, as calc. for given stresses.

Adapting Eq. 8.40 to this case:

$$2b,\ a,\ (t-a),\ h \ \geq 2.5\left(\frac{K}{\sigma_o}\right)^2 = 0.0105 \text{ m}$$

large, 10, 40 mm, large $\geq$ 10.5 mm

No, not plane strain, although close. ◀

$$a,\ (t-a),\ h \geq \frac{4}{\pi}\left(\frac{K}{\sigma_o}\right)^2 = 0.00533 \qquad \text{(Eq. 8.39)}$$

10, 40 mm, large $\geq$ 5.33 mm

Yes, LEFM is applicable. ◀

$$2r_{o\sigma} = \frac{1}{\pi}\left(\frac{K}{\sigma_o}\right)^2 = 0.00133 \text{ m} = 1.33 \text{ mm} \quad ◀$$

8.41 Double-edge-cracked plate (Fig 8.12b)
$b = 15.9$, $t = 6.35$, $a = 5.7$ mm, large h.
7075-T651 Al: $K_{Ic} = 29$ MPa$\sqrt{m}$,
$\sigma_o = 505$ MPa (Table 8.1)

In test: $P_{max} = 55.6$, $P_Q = 50.3$ kN, Type I

(a) $K_Q = F S_Q \sqrt{\pi a}$, $S_Q = \dfrac{P_Q}{2bt} = \dfrac{50{,}300\,N}{2(15.9)(6.35)}$ mm mm

$F = F(\alpha)$, $\alpha = a/b = 0.3585$

$F \approx 1.12$, or more accurately, $F = 1.125$

$K_Q = 1.125\,(249.1\text{ MPa})\sqrt{\pi\,(0.0057\text{ m})}$

$K_Q = 37.5$ MPa$\sqrt{m}$ ◀

(b) $t,\ a,\ (b-a),\ h \geq 2.5(K_Q/\sigma_o)^2 = 0.0138$ m

6.35, 5.7, 10.2, large ≥ 13.8 mm

No, not plane strain. ◀

$a,\ (b-a),\ h \geq \frac{4}{\pi}(K_Q/\sigma_o)^2 = 0.00702$ m

5.7, 10.2, large ≥ 7.02 mm

No, LEFM not applicable. ◀

(c) $K_Q \geq K_{Ic}$ is expected due to pl. stress.
Value possibly affected by plasticity. ◀

8.42 AISI 4340 steel, $\sigma_o = 1380$ MPa

Compact spec., $b = 50.8$, $t = 12.95$, $a = 25.4$ mm

$P_Q = P_{max} = 15.03$ kN, type 3

(a) $K_Q = F_P \dfrac{P}{t\sqrt{b}}$, $a/b = \alpha = 0.5$, $F_P = 9.659$ (Fig. 8.15c)

$$K_Q = \frac{9.659\,(0.01503\ \text{MN})}{(0.01295\ \text{m})\sqrt{0.0508\ \text{m}}} = 49.74\ \text{MPa}\sqrt{\text{m}}$$ ◀

(b) Valid K_{Ic}? $t, a, (b-a), h \geq 2.5(K_Q/\sigma_o)^2$

$12.95,\ 25.4,\ 25.4,\ 0.6 \times 50.8 \geq 3.25$ mm

Yes, valid $K_{Ic} = 49.7$ MPa$\sqrt{\text{m}}$ ◀

(c) $2r_{o\varepsilon} = \dfrac{1}{3\pi}\left(\dfrac{K_{Ic}}{\sigma_o}\right)^2 = 0.138$ mm ◀

8.43 For given results of fracture toughness tests, determine: (a) K_Q and whether valid K_{Ic}, (b) $2r_{o\sigma}$ or $2r_{o\varepsilon}$, (c) LEFM applicable?, (d) Plot K_Q vs. t, comment, (e) P_o, comment.

$$K_Q = F_P \frac{P_Q}{t\sqrt{b}}, \quad F_P = F_P(\alpha), \quad \alpha = a/b \quad \text{(Fig. 8.16)}$$

$$F_P = \frac{2+\alpha}{(1-\alpha)^{1.5}}(0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4)$$

Plane strain: $t, a, (b-a), h \geq 2.5\left(\frac{K_Q}{\sigma_o}\right)^2$

and $P_{max}/P_Q \leq 1.1$

$$2r_{o\sigma} = \frac{1}{\pi}\left(\frac{K_Q}{\sigma_o}\right)^2, \quad 2r_{o\varepsilon} = \frac{1}{3\pi}\left(\frac{K_Q}{\sigma_o}\right)^2$$

LEFM: $a, (b-a), h \geq \frac{4}{\pi}\left(\frac{K}{\sigma_o}\right)^2$

(a, b, c)

b, mm	h, mm	h/b	σ_o, MPa
50.8	30.5	0.600	505

7075-T651 Al

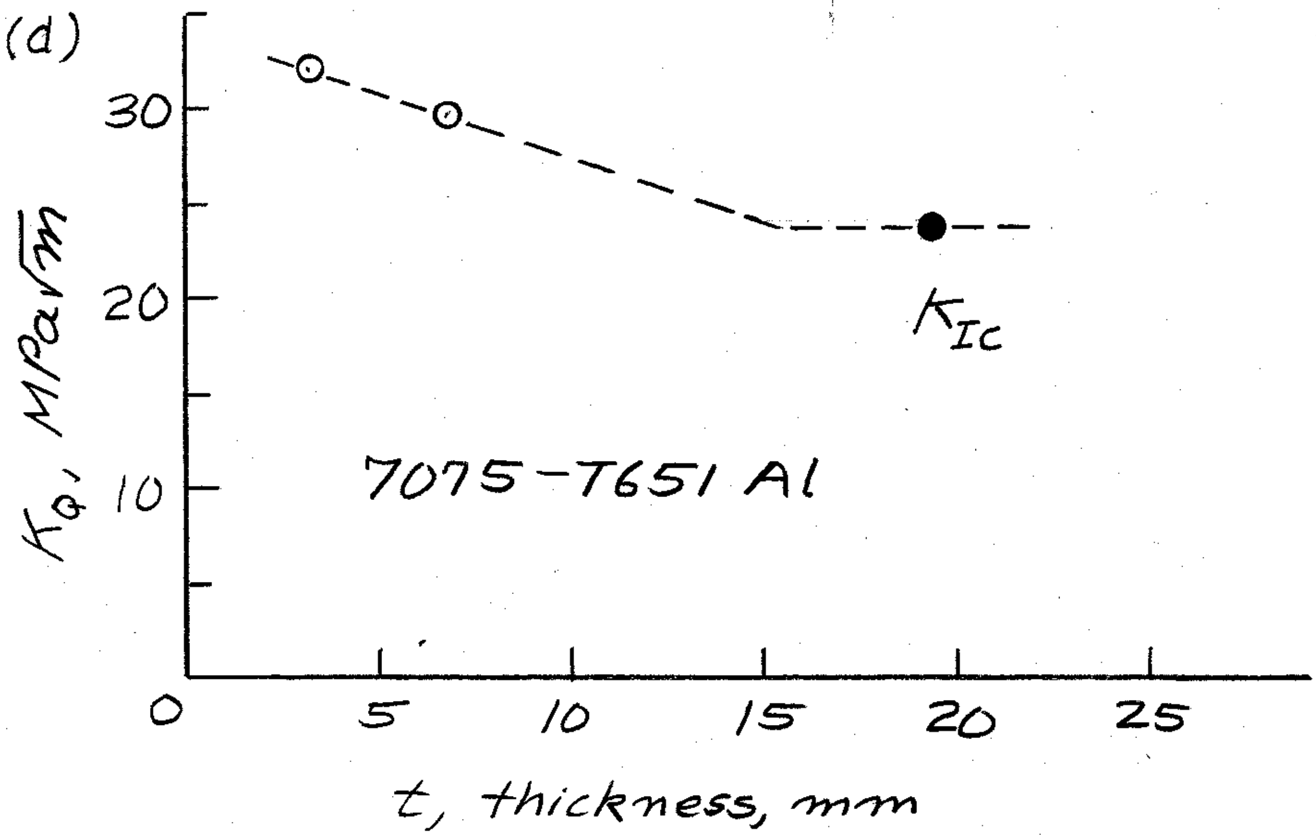
(Table 8.1)

Test No.	a_i mm	t mm	P_Q kN	P_{max} kN	α a/b	F_P	K_Q MPa$\sqrt{m}$
1	24.1	3.18	2.56	3.96	0.4744	8.945	31.95
2	24.7	6.86	4.96	6.16	0.4862	9.263	29.72
3	23.3	19.35	12.00	12.00	0.4587	8.547	23.52

(8.43, p.2)

Test No.	$2.5(K/\sigma_o)^2$ mm	P_{max}/P_Q	Plane strain?	K_{Ic} $MPa\sqrt{m}$	$(4/\pi)(K/\sigma_o)^2$ mm	LEFM OK?
1	10.01	1.55	no	---	5.10	yes
2	8.66	1.24	no	---	4.41	yes
3	5.42	1.00	yes	23.52	2.76	yes

(d)



K_Q decreases with t, but is expected to level off such that no further decrease occurs prior to the final value. The fracture surface changes from angular (shear) type to flat over this range.

(8.43, p. 3)

(e) $P_o = bt\sigma_o\left[-\alpha - 1 + \sqrt{2(1+\alpha^2)}\right]$

b, mm	σ_o, MPa
50.8	505

Test No.	t mm	α a/b	P_o kN	P_{max}/P_o	Fully plastic ?
1	3.18	0.4744	7.41	0.534	no
2	6.86	0.4862	15.19	0.406	no
3	19.35	0.4587	48.26	0.249	no

Since P_{max} is not close to P_o in any case, all three failures were controlled by fracture, rather than yielding. ◀

8.44 $K = FS\sqrt{\pi a}$, $a, (b-a), h \geq \frac{4}{\pi}\left(\frac{K}{\sigma_o}\right)^2$

(a) Largest S/σ_o for LEFM?
Assume $a < (b-a)$ and $a < h$, so that a controls inequality, so at the limit

$$a = \frac{4}{\pi}\left(\frac{K}{\sigma_o}\right)^2 = \frac{1}{\pi}\left(\frac{K}{FS}\right)^2, \quad \frac{S}{\sigma_o} = \frac{1}{2F}$$ ◀

(b)

F	S/σ_o limit
1.00	0.50
1.12	0.45
$2/\pi$	0.79

◀

(c) In Fig. 8.5, the $S/\sigma_o = 0.50$ limit applies, so that deviation from the LEFM line might be expected above $S = 0.5\,\sigma_o = 259$ MPa. One data point a little above this S seems to agree with the LEFM curve (solid), but the three at higher S clearly deviate from LEFM. Hence, there is reasonable agreement with the limit.

8.45 $K_c = 66\ MPa\sqrt{m}$, $\sigma_o = 518\ MPa$

Center-cracked plate: Fig. 8.12(a)

$K = S_g\sqrt{\pi a}$, $F \approx 1$ up to $\alpha = 0.4 = \frac{a}{b}$,

or $a = 0.4b = 0.4(152\ mm) = 60.8\ mm$

Plot: (a) $66 = S_g\sqrt{\pi a}$ $MPa\sqrt{m}$

(b) $K_e = 66 = S_g\sqrt{\pi(a + r_{o\sigma})}$ $MPa\sqrt{m}$

$$r_{o\sigma} = \frac{1}{2\pi}\left(\frac{K_c}{\sigma_o}\right)^2 = \frac{1}{2\pi}\left(\frac{66}{518}\right)^2 = 0.00258\ m$$

a mm	a m	(a) $S_g = \frac{66}{\sqrt{\pi a}}$	(b) $S_g = \frac{66}{\sqrt{\pi(a+0.00258)}}$
1	0.001	1178 MPa	622 MPa
2	0.002	833	550
4	0.004	589	459
10	0.010	372	332
20	0.020	263	248
30	0.030	215	206
40	0.040	186	180
50	0.050	167	162
60	0.060	152	149
7	0.007	445	380

(8.45, p.2)

(c) The $r_{o\sigma}$ adjustment gives some improvement around $a = 3$ to 10 mm; it cannot be correct above σ_o.