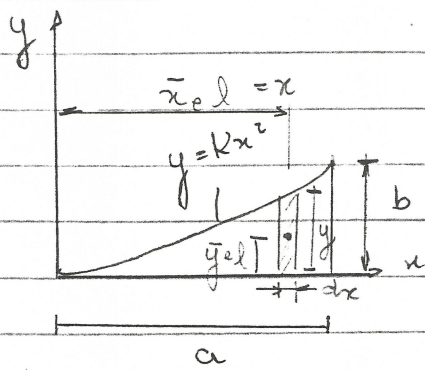


5.44 Prob. resolvido) - Determinar, por integração direta, o centróide da área parabólica ilustrada.



P/ $x=a$ temos $y=b$

$$y = kx^2 \\ b = ka^2 \Rightarrow k = \frac{b}{a^2}$$

$$y = \frac{b}{a^2} x^2$$

$$\bar{x}_{el} = x \\ \bar{y}_{el} = y/2$$

3ª Lem. dif. vertical

$$dA = y dx$$

$$A = \int dA = \int y dx = \int_0^a \frac{b}{a^2} x^2 dx = \frac{b}{a^2} \frac{x^3}{3} \Big|_0^a$$

$$A = \frac{ba^3}{3a^2} \Rightarrow A = \frac{ab}{3}$$

$$\bar{x} \cdot A = \int \bar{x}_{el} \cdot dA = \int x \cdot y dx = \int_0^a x \frac{b}{a^2} x^2 dx = \frac{b}{a^2} \frac{x^4}{4} \Big|_0^a$$

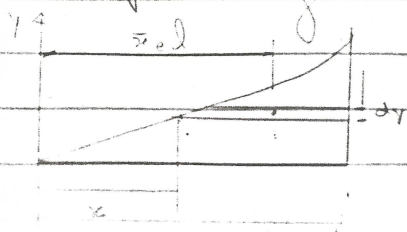
$$\bar{x} \cdot A = \frac{a^4 b}{4} \Rightarrow \bar{x} = \frac{a^4 b}{4} \cdot \frac{3}{ab}$$

$$\boxed{\bar{x} = \frac{3a}{4}}$$

$$\bar{y} \cdot A = \int \bar{y}_{el} \cdot dA = \int \frac{y}{2} \cdot y dx = \frac{1}{2} \int y^2 dx = \frac{1}{2} \int_0^a \frac{b^2}{a^4} x^4 dx \\ = \frac{1}{2} \frac{b^2}{a^4} \frac{a^5}{5} = \frac{b^2 a}{10}$$

$$\bar{y} = \frac{b^2 a}{10} \cdot \frac{3}{ab} \Rightarrow \boxed{\bar{y} = \frac{3b}{10}}$$

3ª Lem. dif. horizontal



$$dA = (a-x) dy$$

$$\bar{x}_{el} = \frac{a-x}{2} + x$$

$$\bar{x}_{el} = \frac{a+x}{2}$$