LISTA 3

① $Y = X\beta + \varepsilon$ $\qquad E(\varepsilon|X) = 0$

$\varepsilon|X \sim N(0; \Sigma)$

$\to \Sigma \neq \sigma^2 I$

a)

$$Y = \begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1T} \\ y_{21} \\ \vdots \\ y_{NT} \end{bmatrix} \quad X = \begin{bmatrix} 1 & X1_{11} & X2_{11} & X3_{11} \\ 1 & X1_{12} & X2_{12} & X3_{12} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & X1_{1T} & X2_{1T} & X3_{1T} \\ 1 & X1_{21} & X2_{21} & X3_{21} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & X1_{NT} & X2_{NT} & X3_{NT} \end{bmatrix} \quad \beta = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{1T} \\ \varepsilon_{21} \\ \vdots \\ \varepsilon_{NT} \end{bmatrix}$$

$$\hat{\beta}^{MQOE} = (X'X)^{-1}X'Y$$

$$= (X'X)^{-1}X'(X\beta + \varepsilon)$$

$$= \underbrace{(X'X)^{-1}X'X}_{I}\beta + (X'X)^{-1}X'\varepsilon$$

$$\boxed{\hat{\beta}^{MQOE} = \beta + (X'X)^{-1}X'\varepsilon}$$

• Achando a distribuição de $\hat{\beta}^{MQOE}$:

$$E(\hat{\beta}^{MQOE}|X) = E(\beta|X) + E((X'X)^{-1}X'\varepsilon|X)$$

$$= \beta + (X'X)^{-1}X' \underbrace{E(\varepsilon|X)}_{=0}$$

$$\boxed{E(\hat{\beta}^{MQOE}|X) = \beta}$$

$$Var(\hat{\beta}^{MQOE}|X) = \underbrace{Var(\beta|X)}_{0} + Var((X'X)^{-1}X'\varepsilon|X)$$

$$(X'X)^{-1}X' \underbrace{Var(\varepsilon|X)}_{=\Sigma} X (X'X)^{-1}$$

$$\boxed{Var(\hat{\beta}^{MQOE}|X) = (X'X)^{-1}X'\Sigma X(X'X)^{-1}}$$

$$\boxed{\hat{\beta}^{MQOE} \overset{A}{\sim} N\left(\beta,\ (X'X)^{-1}X'\Sigma X(X'X)^{-1}\right)}$$

② $\ln y_{it} = \beta_0 + \beta_1 x_{it} + \varepsilon_{it}$

$\varepsilon_{it} = a_i + u_{it}$

N indivíduos

$T = 1, 2$

a) $Var(a|S,X,G) = \sigma_a^2$

$Var(u|S,X,G) = \sigma_u^2$

Também suponho que vale

(I) $u_{it} \perp\!\!\!\perp u_{js} \;\; \forall i \neq j \;\; e \;\; \forall t,s$

(II) Amostra aleatória

(III) $u_{it} \perp\!\!\!\perp a_i, x_{is} \;\; \forall t,s$

$$\Sigma = \begin{bmatrix} Var(\varepsilon_{11}) & Cov(\varepsilon_{11},\varepsilon_{12}) & 0 & 0 \\ Cov(\varepsilon_{12},\varepsilon_{11}) & Var(\varepsilon_{12}) & 0 & 0 \\ 0 & 0 & Var(\varepsilon_{21}) & Cov(\varepsilon_{21},\varepsilon_{22}) \\ 0 & 0 & Cov(\varepsilon_{22},\varepsilon_{21}) & Var(\varepsilon_{22}) \end{bmatrix}$$

$$Var(\varepsilon_{11}) = Var(a_1 + u_{11}) = \underbrace{Var(a_1)}_{\sigma_a^2} + \underbrace{Var(u_{11})}_{\sigma_u^2} + 2\underbrace{cov(a_1, u_{11})}_{=0\text{, por causa de (III)}}$$

$$\boxed{Var(\varepsilon_{11}) = \sigma_a^2 + \sigma_u^2 = Var(\varepsilon_{12}) = Var(\varepsilon_{21}) = Var(\varepsilon_{22})}$$

$$Cov(\varepsilon_{11},\varepsilon_{12}) = Cov(a_1 + u_{11}, a_1 + u_{12}) = \underbrace{cov(a_1,a_1)}_{Var(a_1) \,=\, \sigma_a^2} + \underbrace{cov(a_1,u_{12})}_{=0\text{, por causa de (III)}} + \underbrace{cov(u_{11},a_1)}_{=0\text{, por causa de (III)}} + \underbrace{cov(u_{11},u_{12})}_{=0\text{, por causa de (I)}}$$

$$\boxed{Cov(\varepsilon_{11},\varepsilon_{12}) = \sigma_a^2 = Cov(\varepsilon_{12},\varepsilon_{11}) = Cov(\varepsilon_{21},\varepsilon_{22}) = Cov(\varepsilon_{22},\varepsilon_{21})}$$

$$\Sigma = \begin{bmatrix} \sigma_a^2+\sigma_u^2 & \sigma_a^2 & 0 & 0 \\ \sigma_a^2 & \sigma_a^2+\sigma_u^2 & 0 & 0 \\ 0 & 0 & \sigma_a^2+\sigma_u^2 & \sigma_a^2 \\ 0 & 0 & \sigma_a^2 & \sigma_a^2+\sigma_u^2 \end{bmatrix}$$

② b) $E(\varepsilon | X) = 0$

$$\hat{\beta}^{MQOE} = (X'X)X'Y$$

$$\hat{\beta}^{MQOE} \overset{A}{\sim} N\left(\beta, (X'X)^{-1}X'\Sigma X(X'X)^{-1}\right)$$

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③ $y_{it} = A . K_{it}^{b_1} . T_i^{b_2} . e^{\varepsilon_{it}}$, $\varepsilon_{it} = u_i + v_{it}$

aplica ln

$$\ln y_{it} = \ln A + b_1 \ln K_{it} + b_2 \ln T_i + \varepsilon_{it}$$

hipóteses
$$\begin{cases} v_{it} \perp\!\!\!\perp u_i, K_{it}, T_i, v_{js} & \forall i \neq j , \forall t \neq s \\ E(u) = E(v) = 0 \\ Var(u | K, T) = \sigma_u^2 \\ Var(v | K, T, u) = \sigma_v^2 \end{cases}$$

a) $Var(\varepsilon_{it}) = Var(u_i + v_{it})$

$$= \underbrace{Var(u_i)}_{\sigma_u^2} + \underbrace{Var(v_{it})}_{\sigma_v^2} + 2 \underbrace{Cov(u_i, v_{it})}_{= 0; \text{ por causa de } v_{it} \perp\!\!\!\perp u_i}$$

$$\boxed{Var(\varepsilon_{it}) = \sigma_u^2 + \sigma_v^2}$$

$$Cov(\varepsilon_{it}, \varepsilon_{is}) = Cov(u_i + v_{it}, u_i + v_{is})$$

$$= \underbrace{Cov(u_i, u_i)}_{Var(u_i) = \sigma_u^2} + \underbrace{cov(u_i, v_{is})}_{= 0, \; u_i \perp\!\!\!\perp v_{is}} + \underbrace{cov(v_{it}, u_i)}_{= 0, \; v_{it} \perp\!\!\!\perp u_i} + \underbrace{cov(v_{it}, v_{is})}_{= 0, \text{ supondo } v_{it} \perp\!\!\!\perp v_{is}}$$

$$\boxed{Cov(\varepsilon_{it}, \varepsilon_{is}) = \sigma_u^2}$$

$\boxed{Cov(\varepsilon_{it}, \varepsilon_{js}) = 0}$ → lembrar que os indivíduos não estão relacionados entre si

⇓

ε do indivíduo i $\perp\!\!\!\perp$ ε do indivíduo j

④ $$\ln y_{it} = \beta_0 - r S_{it} + \beta_1 X_{it} - \beta_0 X_{it}^2 + \gamma g_i + \varepsilon_{it}$$

$\ln y_{it}$	$r S_{it}$	$\beta_1 X_{it}$	$\beta_0 X_{it}^2$	γg_i
salário	educação	experiência no mercado de trabalho	experiência ao quadrado	sexo do indivíduo

$$\varepsilon_{it} = a_i + u_{it}$$

a) Um possível fator fixo (a_i) no erro seria a "habilidade inata" dos indivíduos, ou seja, a habilidade/aptidão com que os indivíduos nascem e que ajudam eles a obter maiores níveis de educação ou terem melhor desempenho no mercado de trabalho.

Assim, teríamos que essa "habilidade inata", que está no erro (a_i), estaria correlacionada com os nossos regressores (S, X, G)

$\Downarrow$

Portanto, $E(a \mid S, X, G) \neq 0$

Como $E(\varepsilon \mid S, X, G) = \underbrace{E(a \mid S, X, G)}_{\neq 0} + \underbrace{E(u \mid S, X, G)}_{\text{supondo } = 0}$

Temos que $E(\varepsilon \mid S, X, G) \neq 0$.

b) Igual questão ② a)