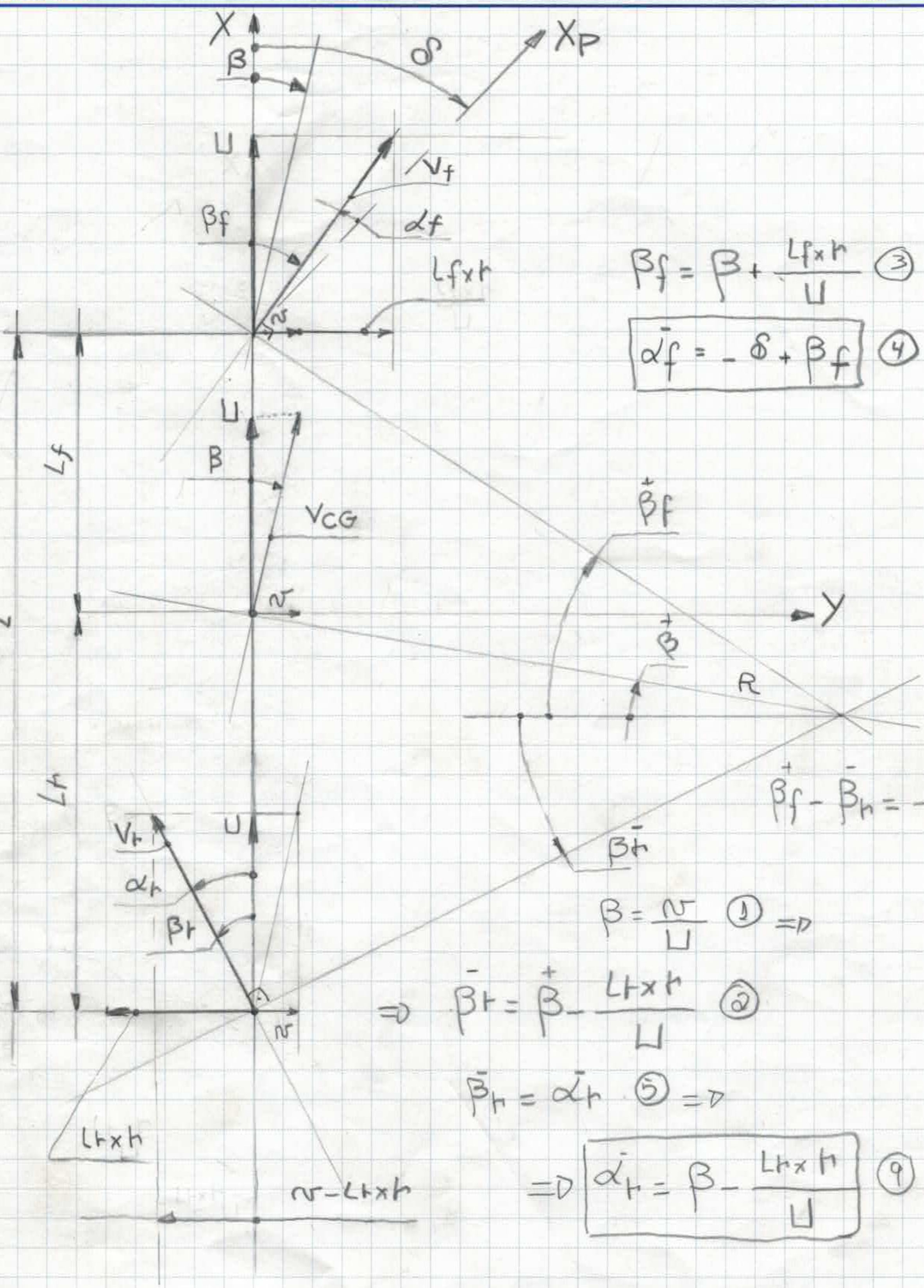
PROJETO DINAMICA VEICULAR – ESTABILIDADE DIRECIONAL FOLHA (2)



$$\beta_f = \beta + \frac{L_f \, r}{U} \quad (3)$$

$$\boxed{\overset{-}{\alpha}_f = -\delta + \beta_f} \quad (4)$$

$$\overset{+}{\beta}_f - \overset{-}{\beta}_r = \frac{L}{R} \quad (6)$$

$$\beta = \frac{v}{U} \quad (1) \Rightarrow$$

$$\Rightarrow \overset{-}{\beta}_r = \overset{+}{\beta} - \frac{L_r \, r}{U} \quad (2)$$

$$\overset{-}{\beta}_r = \overset{-}{\alpha}_r \quad (5) \Rightarrow$$

$$\Rightarrow \boxed{\overset{-}{\alpha}_r = \beta - \frac{L_r \, r}{U}} \quad (9)$$

Testando 2 e 3 com 6, tem-se:

$$\bar{\beta}_f - \bar{\beta}_r = \frac{L}{R} \Rightarrow$$

$$\Rightarrow \cancel{\beta} + \frac{L_f \times r}{U} - \cancel{\beta} + \frac{L_r \times r}{U} = \frac{L}{R} \Rightarrow$$

$$\frac{(L_f + L_r) \times r}{U} = \frac{L}{R}. \quad \text{Como } \frac{r}{U} = \frac{1}{R}, \text{ vem:}$$

$$L \times \frac{1}{R} = \frac{L}{R} \quad \text{c.q.d.}$$

De 4 $\Rightarrow$

$$\Rightarrow \delta = \beta_f - \bar{\alpha}_f \quad (7). \quad \text{Substituindo 6 em 7} \Rightarrow$$

$$\Rightarrow \boxed{\delta = \frac{L}{R} + \bar{\alpha}_r - \bar{\alpha}_f} \quad (8)$$

Calculo dos forças laterais:

$\bar{F}y_f = \bar{C}\alpha_f \cdot \bar{\alpha}_f$ (9) - equação válida na região linear de $Fy \times \alpha$.

Substituindo 4 em 9 $\Rightarrow$

$$\Rightarrow Fy_f = C\alpha_f \times \left(\beta + \frac{L_f \times r}{U} - \delta\right) \Rightarrow$$

$$\Rightarrow \boxed{Fy_f = C\alpha_f \cdot \beta + C\alpha_f \cdot \frac{L_f \times r}{U} - C\alpha_f \cdot \delta} \quad (10)$$

$$Fy_r = C\alpha_r \cdot \alpha_r \quad (11)$$

Substituindo 9 em 11 tem-se:

$$Fy_r = C\alpha_r \cdot \left(\beta - \frac{L_r \times r}{U}\right) \Rightarrow$$

$$\Rightarrow \boxed{Fy_r = C\alpha_r \cdot \beta - C\alpha_r \cdot \frac{L_r \times r}{U}} \quad (12)$$

Somando 10 e 12 tem-se:

$$\boxed{Fy = C\alpha_f \cdot \beta + C\alpha_f \cdot \frac{L_f \times r}{U} - C\alpha_f \cdot \delta + C\alpha_r \cdot \beta - C\alpha_r \cdot \frac{L_r \times r}{U}} \quad (13)$$

Escrevendo:

$$F_y = \frac{\partial F_y}{\partial \beta}\cdot\beta + \frac{\partial F_y}{\partial r}\cdot r + \frac{\partial F_y}{\partial \delta}\cdot\delta \Rightarrow \text{na região linear } F_y \times \alpha$$

$$\Rightarrow F_y = Y_\beta\cdot\beta + Y_r\cdot r + Y_\delta\cdot\delta \quad (14)$$

Substituindo (13) em (14) $\Rightarrow$

$$\Rightarrow \boxed{Y_\beta = C\alpha_f + C\alpha_r} \quad (15)$$

$$\Rightarrow \boxed{Y_r = \frac{1}{U}\left(C\alpha_f\cdot L_f - C\alpha_r\cdot L_r\right)} \quad (16)$$

$$\Rightarrow \boxed{Y_\delta = -C\alpha_f} \quad (17)$$

Cálculo dos torques laterais:

$N_f = C\alpha_f\cdot\alpha_f\, L_f$ (18) na região linear $F_y \times \alpha$

$N_r = C\alpha_r\cdot\alpha_r\cdot L_r$ (19)

Somando os torques:

$$N = C\alpha_f\cdot\left(\beta + \frac{L_f\times r}{U} - \delta\right)\cdot L_f - C\alpha_r\left(\beta - \frac{L_r\times r}{U}\right)L_r \Rightarrow$$

$$\boxed{N = C\alpha_f\cdot\beta\cdot L_f + C\alpha_f\cdot\frac{L_f\times r}{U}\cdot L_f - C\alpha_f\cdot\delta\cdot L_f - C\alpha_r\,\beta\cdot L_r + C\alpha_r\cdot\frac{L_r\times r}{U}\cdot L_r} \quad (20)$$

Escrevendo:

$$N = N_\beta\cdot\beta + N_r\cdot r + N_\delta\cdot\delta \quad (21)$$

De (20) e (21) $\Rightarrow$

$$\Rightarrow N_\beta = \left(C\alpha_f\cdot L_f - C\alpha_r\cdot L_r\right) \quad (22)$$

$$\Rightarrow N_r = \frac{1}{U}\left(C\alpha_f\cdot L_f^2 + C\alpha_r\cdot L_r^2\right) \quad (23)$$

$$\Rightarrow N_\delta = -C\alpha_f\cdot L_f \quad (24)$$

Resumindo as derivadas de controle e estabilidade:

$$\boxed{Y_\beta = C_{\alpha_f} + C_{\alpha_r} \quad e \quad N_r = \frac{1}{U}\left(C_{\alpha_f} \cdot L_f^2 + C_{\alpha_r} \cdot L_r^2\right)}$$

derivadas de amortecimento

$$\boxed{Y_\delta = -C_{\alpha_f} \quad e \quad N_\delta = -C_{\alpha_f} \cdot L_f}$$

derivadas de controle

$$\boxed{Y_r = \frac{1}{U}\left(C_{\alpha_f} \cdot L_f - C_{\alpha_r} \cdot L_r\right) \quad e \quad N_\beta = \left(C_{\alpha_f} \cdot L_f - C_{\alpha_r} \cdot L_r\right)}$$

derivadas de união

Métrica de estabilidade direcional estática

De ⑧ $\longrightarrow \overset{+}{\delta} = \frac{L}{R} - \bar{\alpha_f} + \overset{+}{\alpha_r} \Rightarrow$

$F_{y_f} = m_f \cdot \frac{V_{CG}^2}{R} \Rightarrow \frac{W_f}{g} \cdot \frac{V_{CG}^2}{R}$ e $F_{y_f} = C_{\alpha_f} \alpha_f$ na região linear

$\Rightarrow \bar{\alpha_f} = \frac{W_f}{C_\alpha} \cdot \frac{V_{CG}^2}{Rg}$ e

$\Rightarrow \bar{\alpha_r} = \frac{W_r}{C_{\alpha_r}} \cdot \frac{V_{CG}^2}{Rg}$

$\overset{+}{\delta} = \frac{L}{R}\left(-\frac{W_f}{C_{\alpha_f}} + \frac{W_r}{C_{\alpha_r}}\right) \cdot \bar{a_y} \Rightarrow$

$\delta = \frac{L}{R} + K \cdot \bar{a_y}$ onde $\boxed{K = \left(\frac{W_r}{C_{\alpha_r}} - \frac{W_f}{C_{\alpha_f}}\right) \quad (25)}$

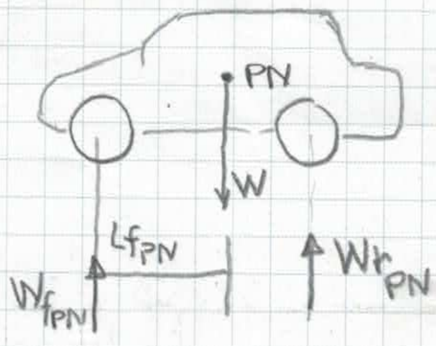
Quando $K > 0 \Rightarrow \uparrow a_y, \uparrow V_{CG} \Rightarrow \delta \uparrow$

K é chamado de gradiente de esterçamento. É a métrica de estabilidade direcional estática.

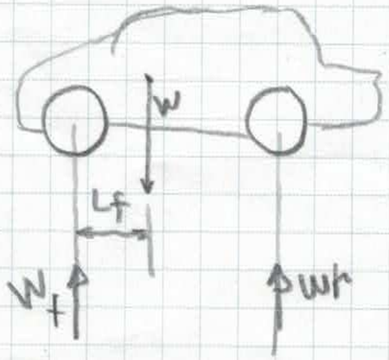
Esta métrica pode ser vista de uma forma diferente, a margem de estabilidade direcional estática SM.

Existe um ponto na direção x do veículo que se o centro de massa do veículo estiver sobre

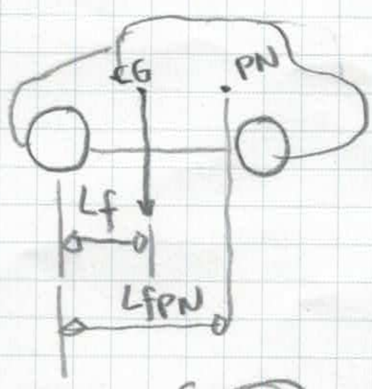
Se, $K = 0$. Este ponto recebe o nome de "ponto neutro de manobra". Sua posição será determinada tendo o eixo dianteiro como referência:



Com o centro de gravidade do veículo em outra posição tem-se:



A margem de estabilidade direcional estática é definida como:



$$\boxed{SM = \frac{Lf_{PN} - Lf}{L} \times 100} \quad (26)$$

Da equação (25),

$$K = \left(\frac{Wr_{PN}}{C\alpha r} - \frac{Wf_{PN}}{C\alpha f} \right) = 0 \quad (27)$$

→ definição de ponto neutro de manobra.

⇒

$$\frac{W_{fPN}}{-C_{\alpha f}} + \frac{W_{rPN}}{C_{\alpha r}} = 0 \quad (28)$$

sendo $Y_\delta = -C_{\alpha f}$ e

$Y_\beta = C_{\alpha f} + C_{\alpha r} \Rightarrow C_{\alpha r} = Y_\beta - C_{\alpha f} \Rightarrow$

$$C_{\alpha r} = Y_\beta + Y_\delta \quad (29)$$

De (28) e (29) tem-se:

$$\frac{W_{fPN}}{Y_\delta} + \frac{W_{rPN}}{Y_\delta + Y_\beta} = 0 \quad (30)$$

, sendo $W_{f_{PN}} = \frac{W}{L} \cdot L_{r_{PN}}$ e

$W_{r_{PN}} = \frac{W}{L} L_{f_{PN}}$

$$\Rightarrow \frac{W}{L}\left(\frac{L_{rPN}}{Y_\delta} + \frac{L_{fPN}}{Y_\delta + Y_\beta}\right) = 0 \Rightarrow$$

$$\frac{L_{fPN}}{Y_\delta + Y_\beta} + \frac{L - L_{fPN}}{Y_\delta} = 0 \Rightarrow$$

$$\Rightarrow \frac{L_{fPN}}{Y_\delta + Y_\beta} + \frac{L}{Y_\delta} - \frac{L_{fPN}}{Y_\delta} = 0 \Rightarrow$$

$$\Rightarrow L_{f_{PN}}\left(\frac{1}{Y_\delta + Y_\beta} - \frac{1}{Y_\delta}\right) + \frac{L}{Y_\delta} = 0$$

$$L_{f_{PN}}\left(\frac{Y_\delta - (Y_\delta + Y_\beta)}{Y_\delta (Y_\delta + Y_\beta)}\right) + \frac{L}{Y_\delta} = 0 \Rightarrow$$

$$\Rightarrow L_{f_{PN}}\left(\frac{-Y_\beta}{Y_\delta (Y_\delta + Y_\beta)}\right) = -\frac{L}{Y_\delta} \Rightarrow$$

*

$$L_{f_{PN}} = \left(-\frac{L}{Y_\delta}\right)\frac{Y_\delta (Y_\delta + Y_\beta)}{(-Y_\beta)} = \boxed{L_{f_{PN}} = L \times \left(\frac{Y_\delta + Y_\beta}{Y_\beta}\right)} \quad (31) \Rightarrow$$

$$\Rightarrow SM = \frac{-L_f + L_{f_{PN}}}{L} \times 100 \Rightarrow \boxed{SM = \frac{-L_f + L\left(\frac{Y_\delta + Y_\beta}{Y_\beta}\right)}{L} \times 100} \Rightarrow \quad (32)$$

*

$$SM = \frac{1}{L}\left(-Lf + L\left(\frac{y\delta + y\beta}{y\beta}\right)\right) \times 100$$

$$SM = \frac{1}{L}\left(\frac{-y\beta \cdot Lf + Ly\delta + Ly\beta}{y\beta}\right) \times 100 \Rightarrow$$

$$\Rightarrow SM = \frac{1}{L}\left(\frac{\cancel{-y\beta \cdot Lf} + \cancel{Lf\, y\beta} + Lr\, y\beta + Ly\delta}{y\beta}\right) \times 100 \Rightarrow$$

$$\Rightarrow SM = \frac{1}{L}\left(\frac{Lr\, y\beta + Ly\delta}{y\beta}\right) \times 100 \Rightarrow$$

$$\Rightarrow SM = \frac{1}{L}\left(\frac{Lr\, y\beta + Lf\, y\delta + Lr \cdot y\delta}{y\beta}\right) \times 100 \Rightarrow$$

$$\Rightarrow SM = \frac{1}{L}\left(\frac{Lr(y\beta + y\delta) + Lf \cdot y\delta}{y\beta}\right) \times 100 \Rightarrow$$

$$\Rightarrow \boxed{SM = \frac{1}{L}\left(\frac{Lf \cdot y\delta + Lr(y\beta + y\delta)}{y\beta}\right) \times 100} \rightarrow$$ testado no simulador

Resumindo:

$$K = \left(\frac{Wr}{C\alpha r} + \frac{Wf}{y\delta}\right)$$

$$C\alpha r = y\beta - C\alpha_r = y\beta + y\delta$$

$$\Rightarrow K = \frac{Wr}{y\beta + y\delta} + \frac{Wf}{y\delta} \quad e$$

$$SM = \frac{1}{L}\left(\frac{Lf\, y\delta + Lr(y\beta + y\delta)}{y\beta}\right) \times 100$$

$$mV(r+\dot{\beta}) - Y_\beta \cdot \beta - Y_r \cdot r = Y_\delta \cdot \delta \quad (1)$$

$$I_{zz}\dot{r} - N_r r - N_\beta \beta = N_\delta \delta \quad (2)$$

De (1) $\Rightarrow mVr + mV\dot{\beta} - Y_\beta \beta - Y_r \cdot r = Y_\delta \cdot \delta \Rightarrow$

$\Rightarrow mV\dot{\beta} - Y_\beta \cdot \beta + mVr - Y_r \cdot r = Y_\delta \cdot \delta \Rightarrow$ Aplicamo Laplace

$\Rightarrow mVs\beta - Y_\beta \cdot \beta + mVr - Y_r \cdot r = Y_\delta \cdot \delta \Rightarrow$

$\Rightarrow (mVs - Y_\beta) \cdot \beta + (mV - Y_r) r = Y_\delta \cdot \delta \quad \div\, mV \Rightarrow$

$$\Rightarrow \left(s - \frac{Y_\beta}{mV}\right) \cdot \beta + \left(1 - \frac{Y_r}{mV}\right) r = \frac{Y_\delta}{mV} \cdot \delta \quad (3)$$

De (2) $\rightarrow sr - \dfrac{N_r}{I_{zz}} \cdot r - \dfrac{N_\beta}{I_{zz}} \cdot \beta = \dfrac{N_\delta}{I_{zz}} \cdot \delta$

Na forma matricial

$$\begin{bmatrix} \left(s - \frac{Y_\beta}{mV}\right) & \left(1 - \frac{Y_r}{mV}\right) \\ \frac{-N_\beta}{I_{zz}} & \left(s - \frac{N_r}{I_{zz}}\right) \end{bmatrix} \begin{Bmatrix} \beta(s) \\ r(s) \end{Bmatrix} = \begin{bmatrix} \frac{Y_\delta}{mV} \cdot \delta \\ \frac{N_\delta}{I_{zz}} \cdot \delta \end{bmatrix}$$

a matriz inversa de

$$\begin{bmatrix} \left(s - \frac{Y_\beta}{mV}\right) & \left(1 - \frac{Y_r}{mV}\right) \\ \frac{-N_\beta}{I_{zz}} & \left(s - \frac{N_r}{I_{zz}}\right) \end{bmatrix}^{-1} = \frac{\begin{bmatrix} \left(s - \frac{N_r}{I_{zz}}\right) & -\left(1 - \frac{Y_r}{mV}\right) \\ \frac{N_\beta}{I_{zz}} & \left(s - \frac{Y_\beta}{mV}\right) \end{bmatrix}}{\left(s - \frac{Y_\beta}{mV}\right)\left(s - \frac{N_r}{I_{zz}}\right) - \left(1 - \frac{Y_r}{mV}\right)\left(\frac{-N_\beta}{I_{zz}}\right)}$$

onde o polinomio característico é:

$$D(s) = \left(s - \frac{Y_\beta}{mV}\right)\left(s - \frac{N_r}{I_{zz}}\right) - \left(1 - \frac{Y_r}{mV}\right)\left(-\frac{N_\beta}{I_{zz}}\right) \quad (4)$$

$$\beta(s) = \frac{\left(s - \frac{N_r}{I_{zz}}\right) \times \frac{Y_\delta}{mV} - \left(1 - \frac{Y_r}{mV}\right) \cdot \frac{N_\delta}{I_{zz}}}{D(s)} \cdot \delta(s) \quad \therefore$$

$$\frac{\beta(s)}{\delta(s)} = \frac{\frac{Y_\delta}{mV} \cdot s + \left(\frac{Y_r \cdot N_\delta - N_r \cdot Y_\delta - mV \cdot N_\delta}{mV\, I_{zz}}\right)}{D(s)}$$

$$\frac{r(s)}{\delta(s)} = \frac{\left(s - \frac{Y\beta}{mV}\right) \cdot \frac{N\delta}{Izz} - \frac{N\beta}{Izz} \cdot \frac{Y\delta}{mV}}{D(s)} \Rightarrow$$

$$\frac{r}{\delta}(s) = \frac{\frac{N\delta}{Izz} \cdot s - \left(\frac{Y\beta . N\delta + N\beta . Y\delta}{mV Izz}\right)}{D(s)}$$

Calculo do fator de amortecimento, frequencia natural ωn e amortecida ωa

$$D(s) = 0$$

$$\left(s - \frac{Y\beta}{mV}\right)\left(s - \frac{Nr}{Izz}\right) - \left(1 - \frac{Yr}{mV}\right)\left(\frac{-N\beta}{Izz}\right) = 0$$

$$s^2 - \frac{Nr}{Izz} s - \frac{Y\beta}{mV} \cdot s + \frac{Y\beta}{mV} \cdot \frac{Nr}{Izz} + \frac{N\beta}{Izz} - \frac{Yr}{mV} \cdot \frac{N\beta}{Izz} = 0 \Rightarrow$$

$$\Rightarrow s^2 + \frac{-(mV . Nr + Y\beta . Izz)\, s}{mV Izz} + \frac{Y\beta . Nr + N\beta . mV - Yr . N\beta}{mV Izz} = 0$$

fazendo analogia

$$s^2 + 2\xi \omega n\, s + \omega n^2 = 0 \Rightarrow$$

$$2\xi \omega n = -\frac{mVNr + Izz . Y\beta}{mV Izz} \Rightarrow$$

$$\boxed{\xi = -\frac{mV\bar{N}r + Izz\, \bar{Y}\beta}{2 . \omega n . mV Izz}} \quad *$$

$$\omega n^2 = \frac{Y\beta . Nr + N\beta . mV - Yr . N\beta}{mV Izz} \Rightarrow$$

$$\boxed{\omega n^2 = \frac{\bar{Y}\beta . \bar{N}r - \overset{+}{N}\beta . \overset{+}{Y}r}{mV Izz} + \frac{\overset{+}{N}\beta}{Izz}} \quad *$$

$N\beta$ (estab. estat)

$$\boxed{\omega a = \omega n . \sqrt{1 - \xi^2}}$$

Escrevendo SM de outra forma:

$$SM = \frac{1}{L}\left(\frac{L_f \cdot Y_\delta + L_r (Y_\beta + Y_\delta)}{Y_\beta}\right) \times 100$$

$$L_f = \frac{W_r}{W} \cdot L \; ; \; L_r = \frac{W_f}{W} \cdot L$$

$$SM = \frac{1}{L}\left(\frac{\frac{W_r}{W} \cdot L \cdot Y_\delta + \frac{W_f}{W} \cdot L \cdot (Y_\beta + Y_\delta)}{Y_\beta}\right) \times 100$$

$$SM = \frac{L}{L} \cdot \frac{1}{W}\left(\frac{W_r \cdot Y_\delta + W_f \cdot (Y_\beta + Y_\delta)}{Y_\beta}\right) \times 100$$

$$SM = \frac{1}{W}\left(\frac{W_r \cdot Y_\delta + W_f \cdot Y_\beta + W_f \cdot Y_\delta}{Y_\beta}\right) \times 100$$

$$SM = \frac{1}{W}\left(\frac{W \cdot Y_\delta + W_f \cdot Y_\beta}{Y_\beta}\right) \times 100$$

$$SM = \left(\frac{W Y_\delta}{W \cdot Y_\beta} + \frac{W_f}{W} \cdot \frac{Y_\beta}{Y_\beta}\right) \times 100 \Rightarrow$$

$$\boxed{SM = \left(\frac{\overset{+}{Y_\delta}}{\overset{-}{Y_\beta}} + \frac{W_f}{W}\right) \times 100}$$ - O.K testado no simulador

Qdo $Y_\delta \uparrow$ $SM \downarrow$

Qdo $\overset{-}{Y_\beta} \uparrow$ $SM \uparrow$

Qdo $SM = 0 \Rightarrow \frac{Y_\delta}{Y_\beta} = -\frac{W_f}{W}$ ou

$$\boxed{\frac{W_f}{W} = -\frac{\overset{+}{Y_\delta}}{\overset{-}{Y_\beta}}} = \frac{\bar{C}_{\alpha f}}{\bar{C}_{\alpha f} + \bar{C}_{\alpha r}}$$

$$K = 0 \Rightarrow \frac{W_r}{Y_\beta + Y_\delta} = -\frac{W_f}{Y_\delta}$$

Para a solução das equações de movimento, usou-se:
na forma matricial:

$$\begin{bmatrix} (s+a_{11}) & a_{12} \\ a_{21} & (s+a_{22}) \end{bmatrix} \begin{Bmatrix} Y_1(s) \\ Y_2(s) \end{Bmatrix} = \begin{bmatrix} b_{11}.U_1(s) + b_{12}.U_2(s) \\ b_{21}.U_1(s) + b_{22}.U_2(s) \end{bmatrix}$$

$$\begin{Bmatrix} Y_1(s) \\ Y_2(s) \end{Bmatrix} = \begin{bmatrix} (s+a_{11}) & a_{12} \\ a_{21} & (s+a_{22}) \end{bmatrix}^{-1} \begin{bmatrix} b_{11}U_1(s) + b_{12}.U_2(s) \\ b_{21}U_1(s) + b_{22}.U_2(s) \end{bmatrix}$$

A matriz inversa é:

$$\begin{bmatrix} (s+a_{11}) & a_{12} \\ a_{21} & (s+a_{22}) \end{bmatrix}^{-1} = \frac{\begin{bmatrix} (s+a_{22}) & -a_{12} \\ -a_{21} & (s+a_{11}) \end{bmatrix}}{(s+a_{11}).(s+a_{22}) - a_{12}.a_{21}} \therefore$$

$$\begin{Bmatrix} Y_1(s) \\ Y_2(s) \end{Bmatrix} = \frac{\begin{bmatrix} (s+a_{22}) & -a_{12} \\ -a_{21} & (s+a_{11}) \end{bmatrix}}{(s+a_{11})(s+a_{22}) - a_{12}.a_{21}} \begin{bmatrix} b_{11}U_1(s) + b_{12}U_2(s) \\ b_{21}U_1(s) + b_{22}U_2(s) \end{bmatrix}$$

onde $D(s) = (s+a_{11}).(s+a_{22}) - a_{12}.a_{21}$ é comum a $Y_1(s)$ e $Y_2(s)$, sendo chamado de polinômio característico.

Assim,

$$Y_1(s) = \frac{(s+a_{22}).b_{11} - a_{12}.b_{21}}{D(s)} U_1(s) + \frac{(s+a_{22})b_{12} - a_{12}.b_{22}}{D(s)}.U_2$$

$$Y_2(s) = \frac{(s+a_{11})b_{21} - a_{21}.b_{11}}{D(s)}.U_1(s) + \frac{(s+a_{11})b_{22} - a_{21}.b_{12}}{D(s)}.U_2(s)$$

Da equacoẽ de movimento,

$$a_{11} = \frac{-Y\beta}{mV} \quad ; \quad a_{12} = \left(1 - \frac{Yr}{mV}\right) \quad ; \quad b_{11} = \frac{Y\delta}{mV} \quad ; \quad b_{12} = 0$$

$$a_{21} = \frac{-N\beta}{Izz} \quad ; \quad a_{22} = \frac{-Nr}{Jzz} \quad ; \quad b_{21} = \frac{N\delta}{Izz} \quad ; \quad b_{22} = 0$$

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