



PEF 3307

Resistência dos Materiais

AULA 02

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Conteúdo da aula

- Introdução à resistência dos Materiais
- Cálculo de reações de apoio
- Cargas distribuídas
- Exemplos
- Conceito de tensão/esforço

Estruturas Estáticas

Estática dos sistemas rígidos

Equações de Equilíbrio:

$$R = m \cdot a = 0 \xrightarrow{a=0} R = 0; \quad \sum R = 0 \quad (\text{Forças}); \quad \sum M = 0 \quad (\text{Momento})$$

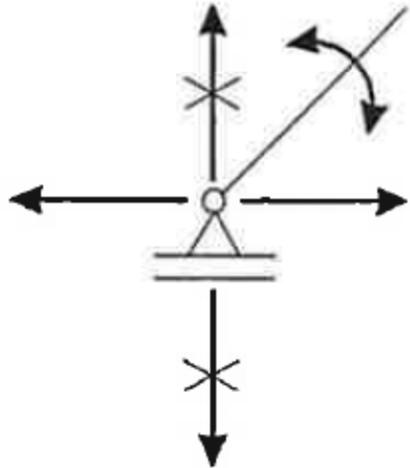
$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_A = 0$$

A é um ponto qualquer do plano da estrutura (pólo)

Restrições de movimento e reações associadas (plano)

Duas translações e uma rotação

a) 1º. Gênero ou articulação móvel ou apoio simples: impede uma translação no plano



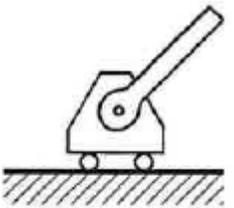
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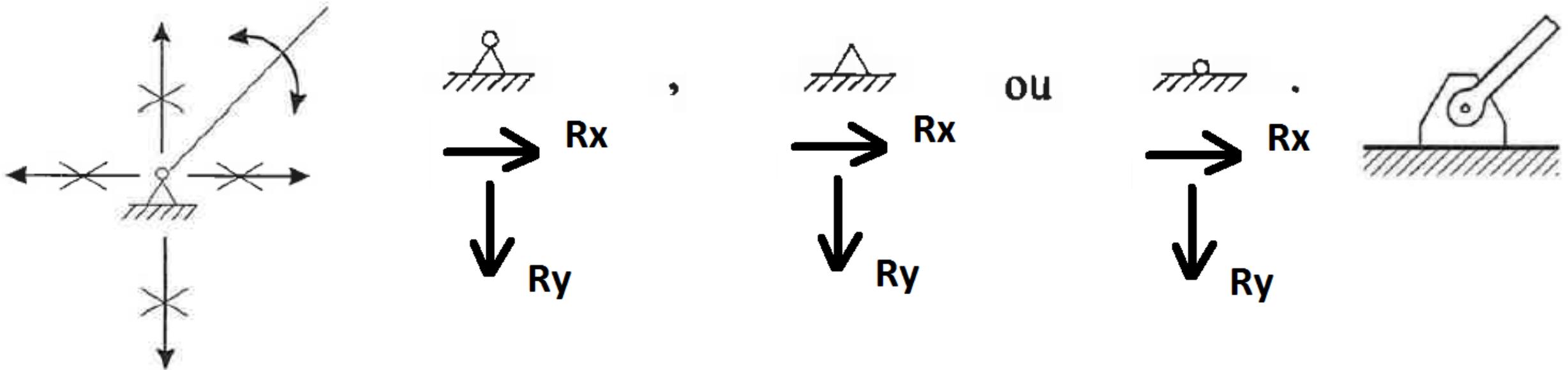


ou



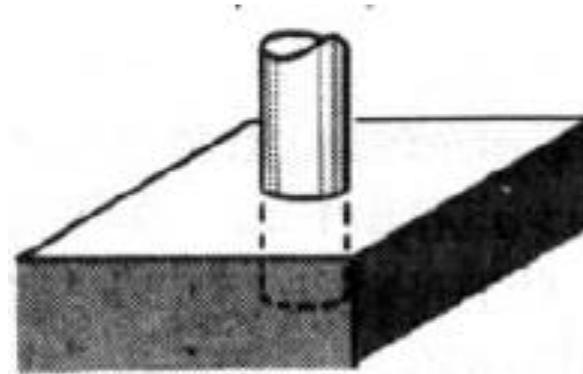
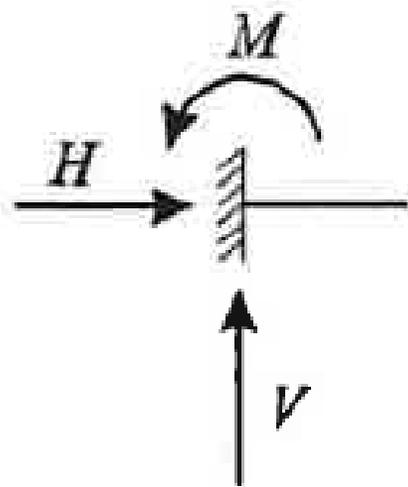
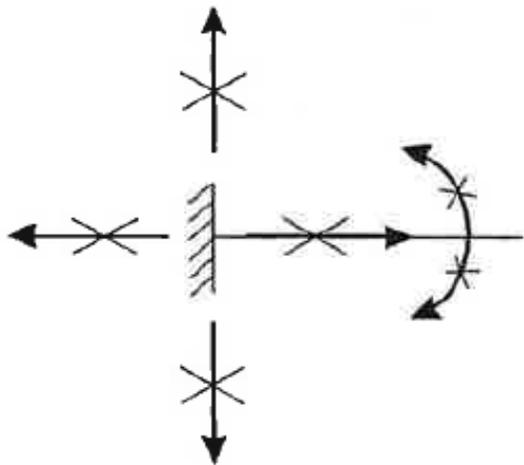
Graus de liberdade e apoios (plano)

b) 2º. Gênero ou articulação fixa/apoio fixo: impedem duas translações no plano

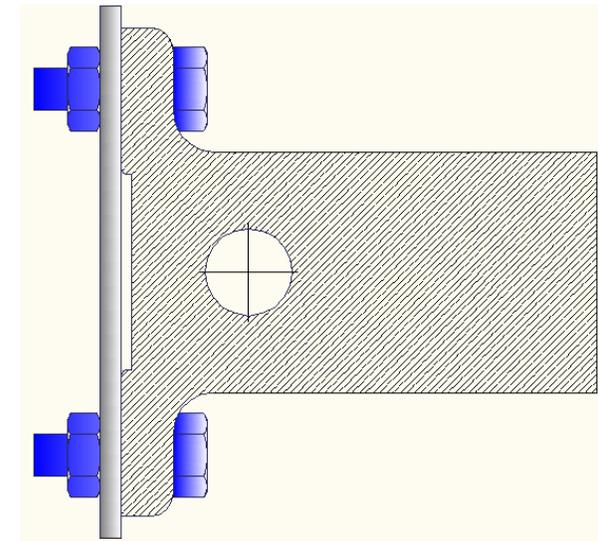


Graus de liberdade e apoios (plano)

c) Engaste: impedem duas translações e uma rotação no plano

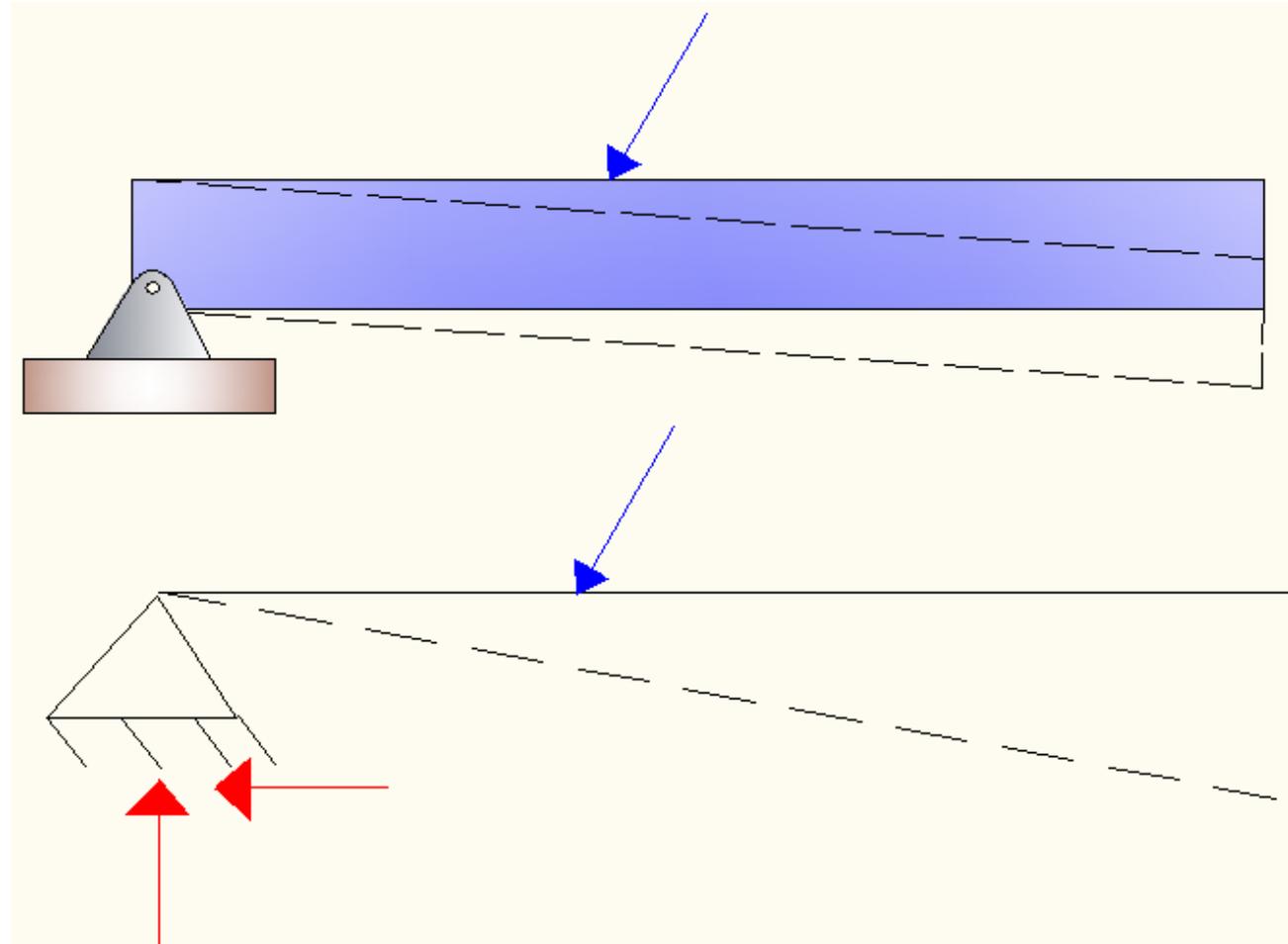


Engastamento



Classificação das estruturas quanto à estaticidade

a) Estrutura Hipostática: menos de 3 vínculos

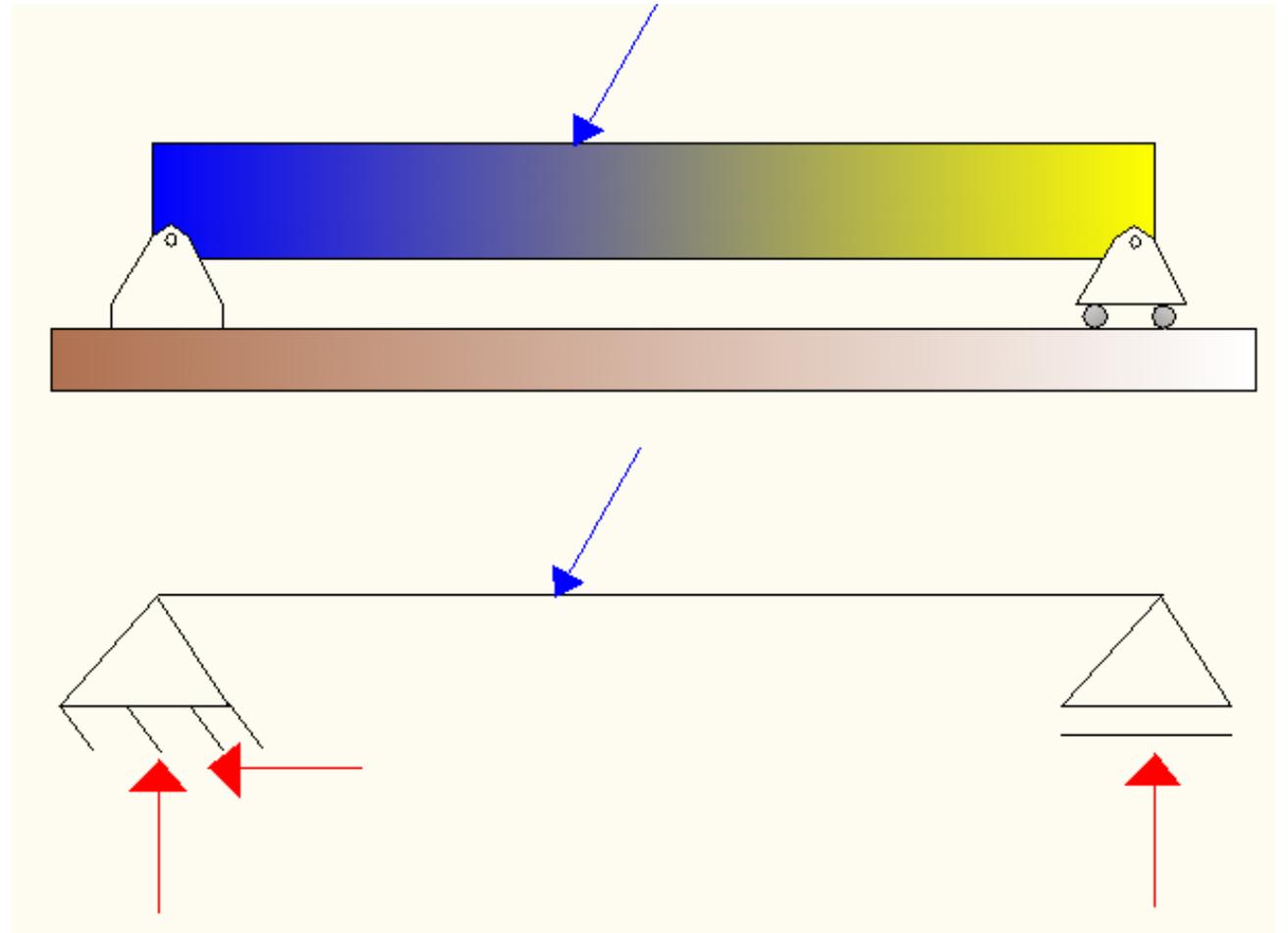


Algun movimento está livre

Classificação das estruturas quanto à estaticidade

b) Estrutura Isostática: exatamente 3 vínculos

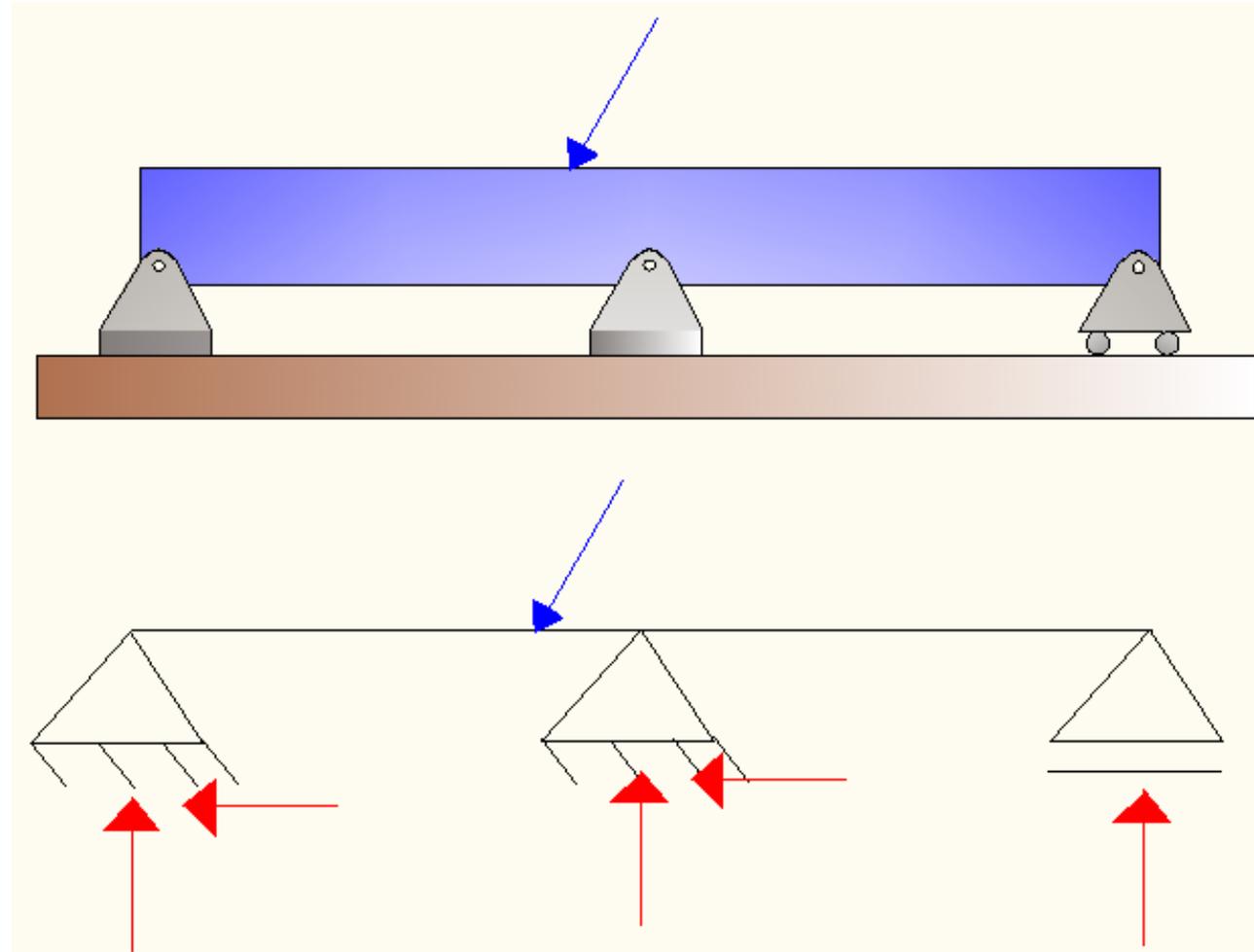
- **3 movimentos impedidos**



Classificação das estruturas quanto à estaticidade

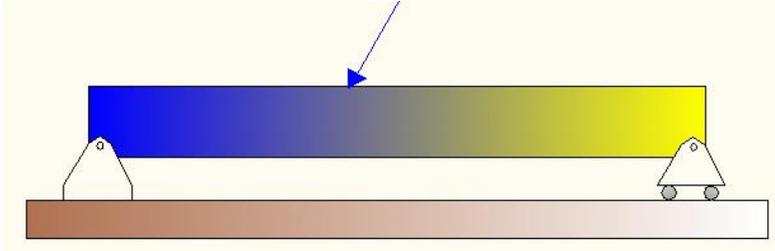
c) Estrutura Hiperestática: mais de 3 vínculos

- **Mais de 3 movimentos impedidos**

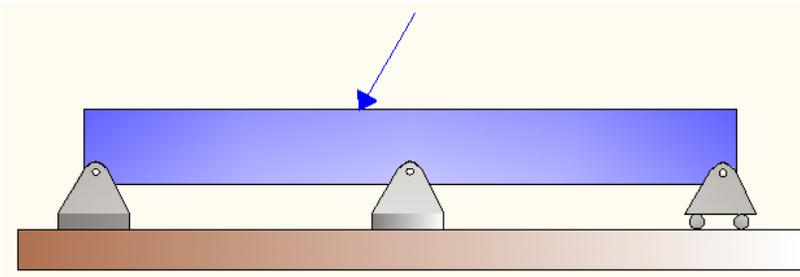


Estruturas isostáticas e hiperestáticas deformam

Serão estudados nesse curso esses tipos

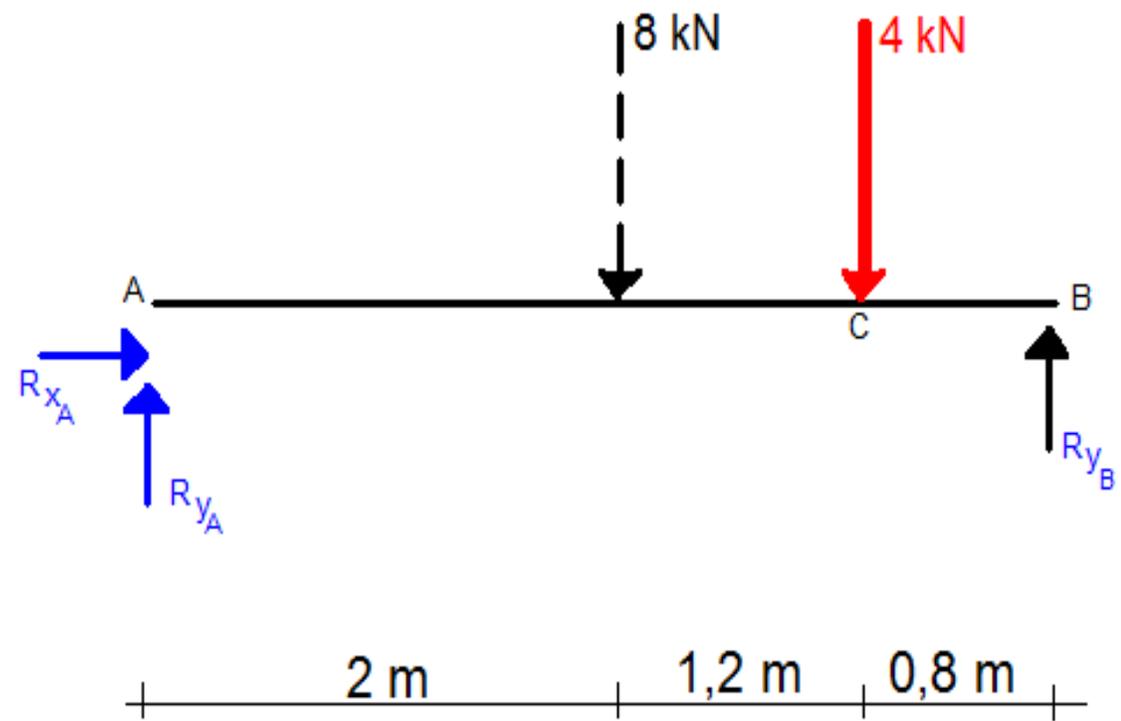
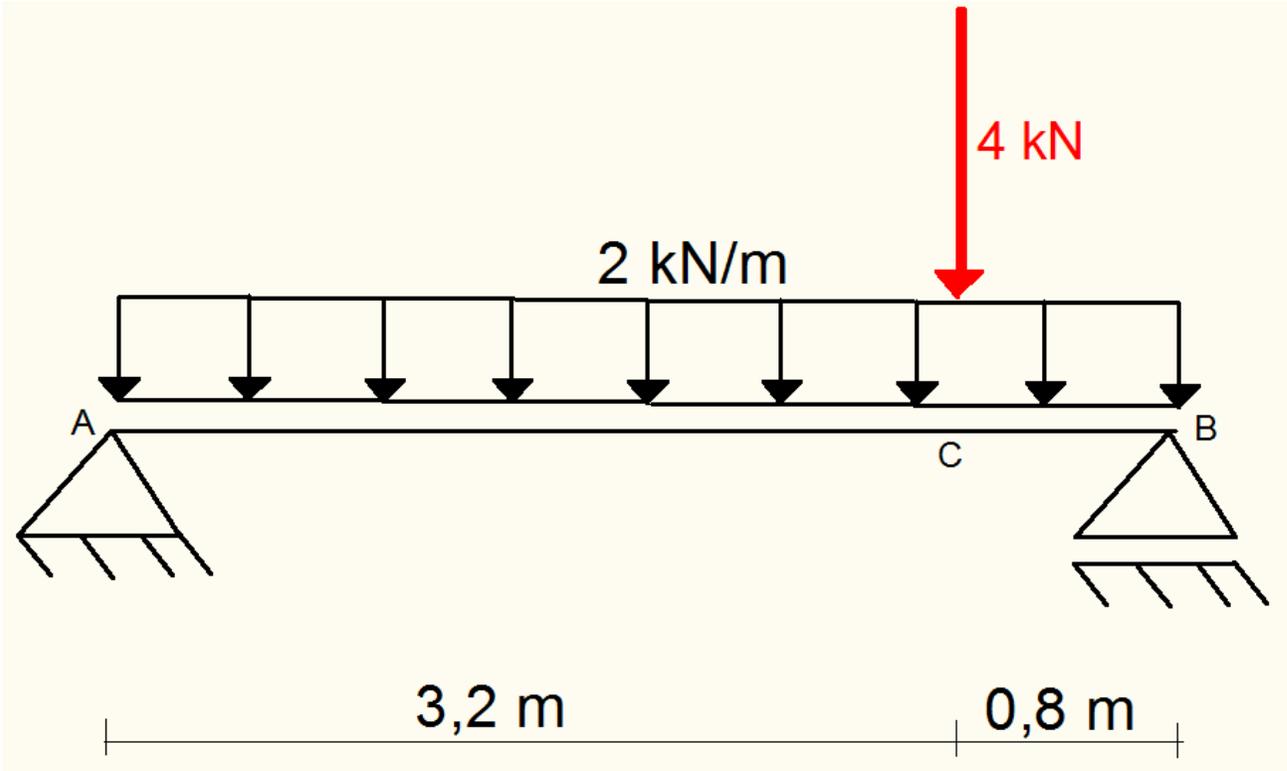


Isostáticas, 95% do curso



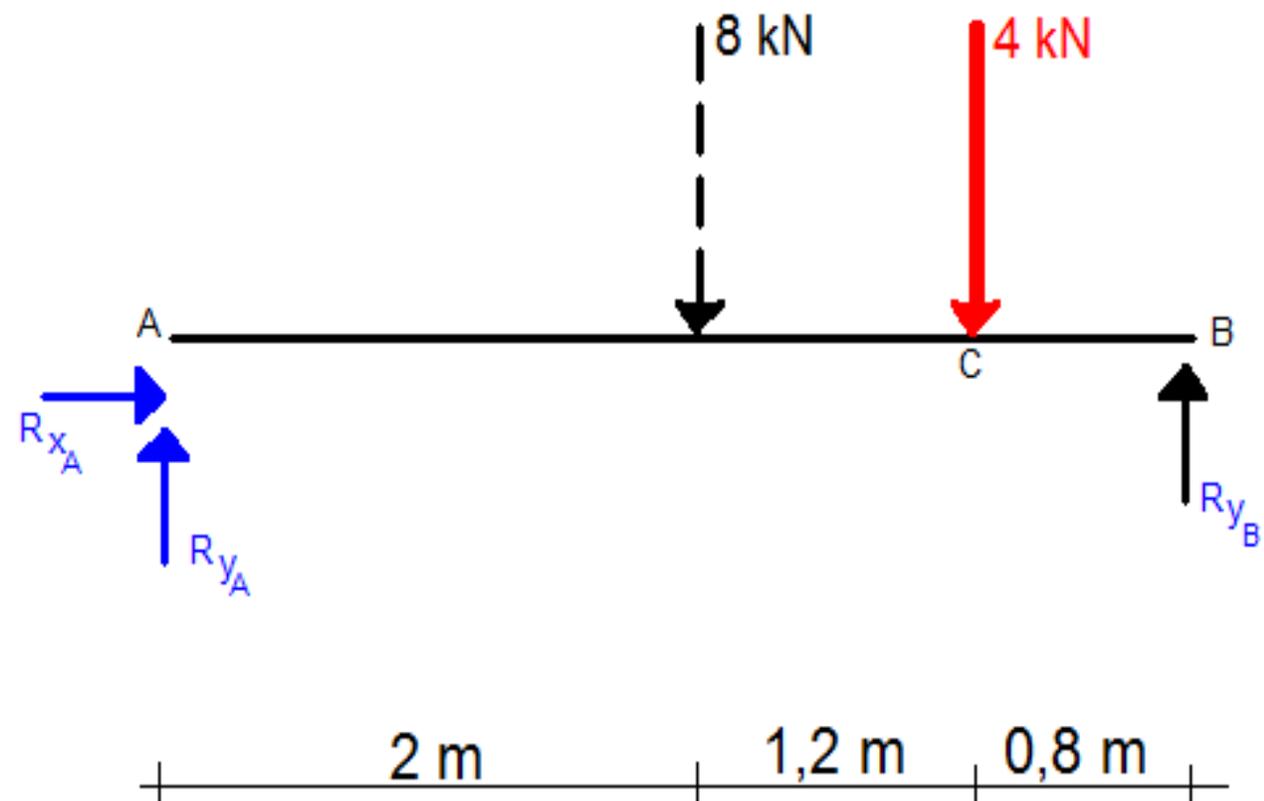
Hiperestáticas, 5% do curso

Exemplo 6: Calcule as reações da estrutura



$$\sum F_x = 0: \quad R_{xA} = 0$$

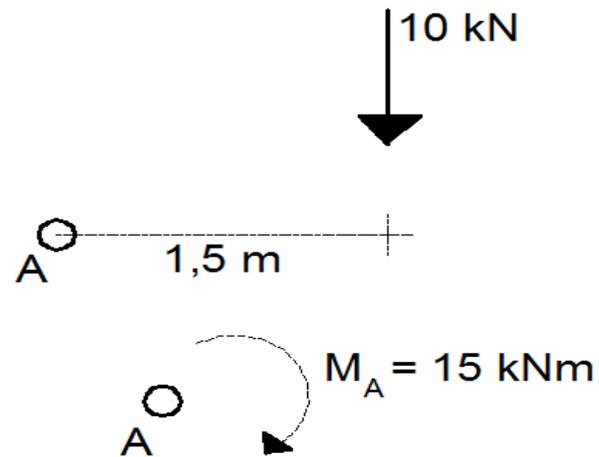
$$\sum F_y = 0: \quad R_{yA} + R_{yB} - 12 = 0$$



Lembrando que:

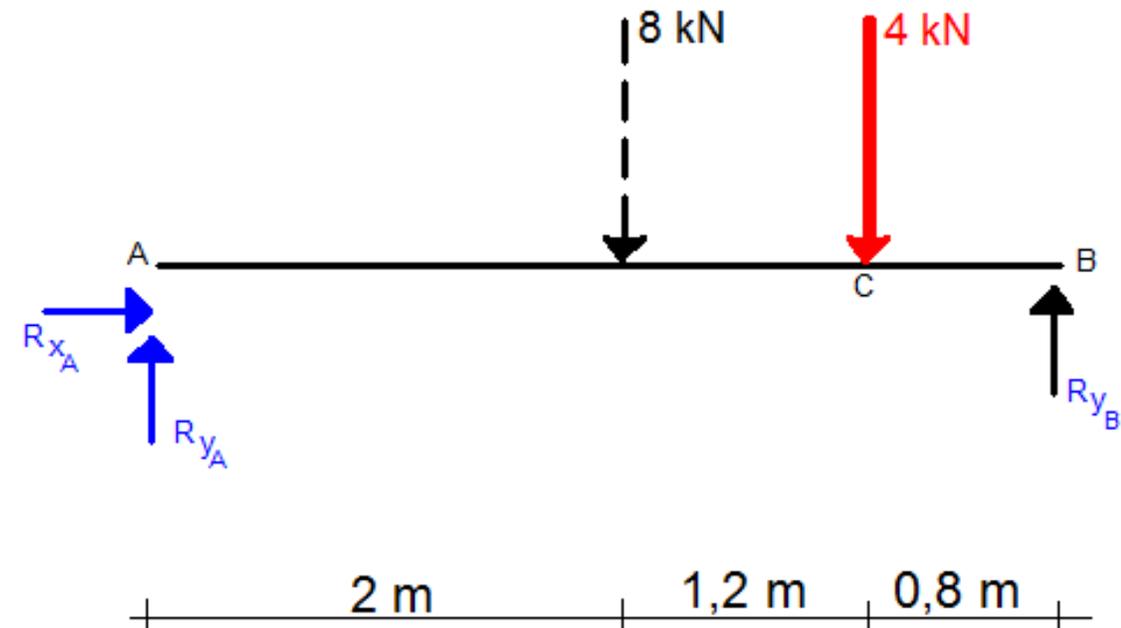
$$\vec{M}_O = \vec{OP} \wedge \vec{F}$$

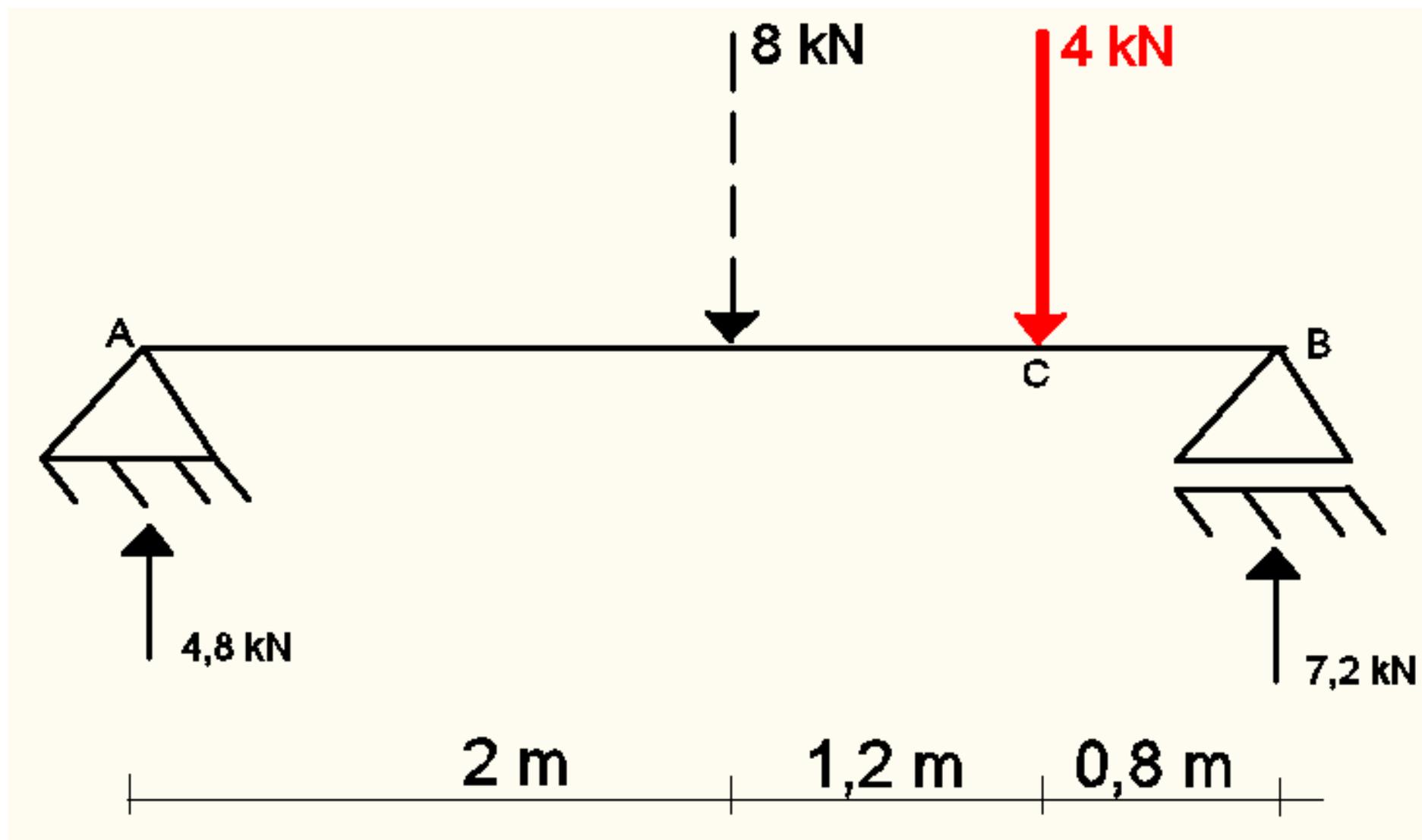
$$\|\vec{M}_O\| = \|\vec{OP}\| \cdot \|\vec{F}\| \sin \alpha$$



$$\sum M_A = 0: \quad + \curvearrowright \quad 4,0 \cdot R_{yB} - 8,0 \cdot 2,0 - 4,0 \cdot 3,2 = 0 \rightarrow R_{yB} = 7,2 \text{ kN}$$

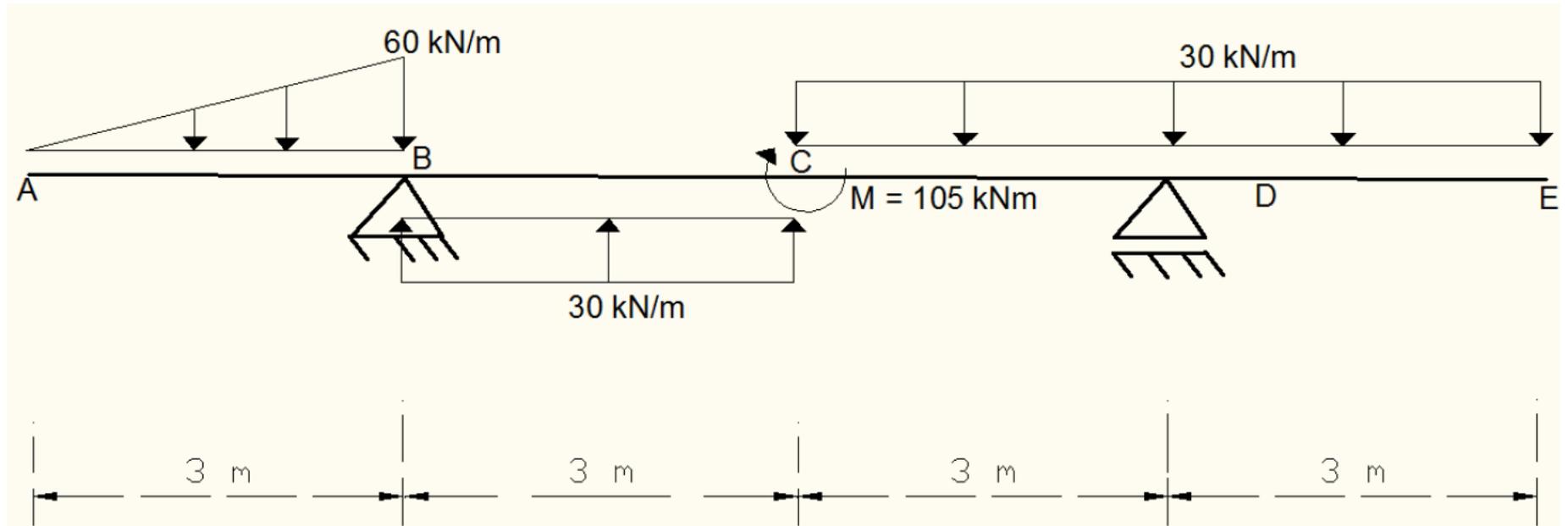
$$\therefore R_{yA} = 12 - 7,2 = 4,8 \text{ kN}$$



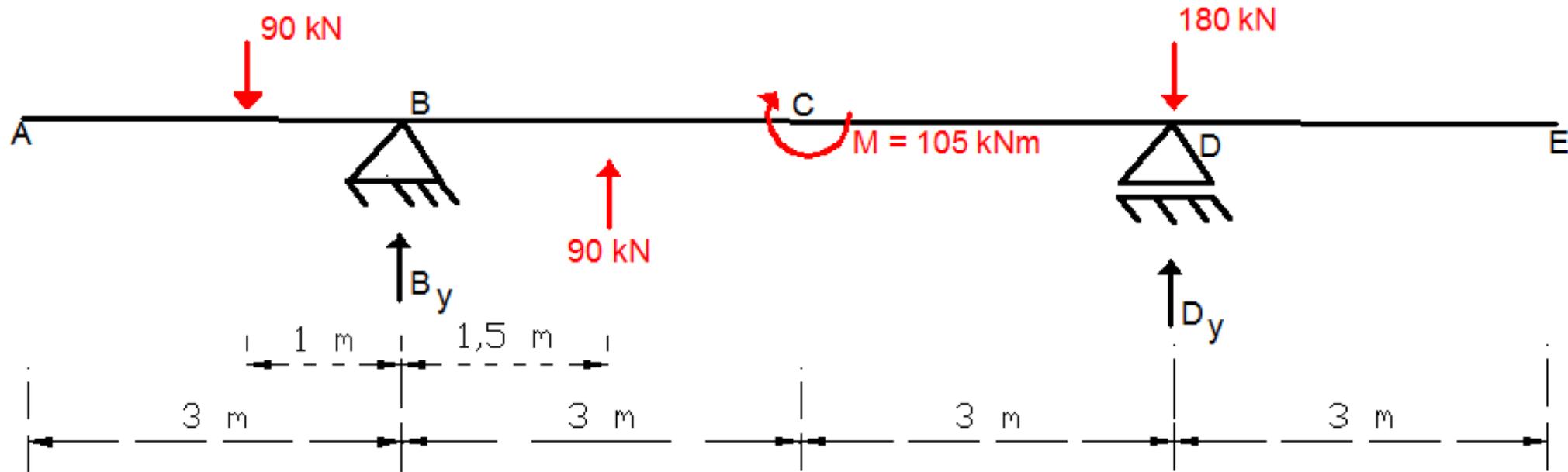


Exemplo 7*

R3) Determinar as reações da viga a seguir.



Exemplo 7 R3) Determinar as reações da viga a seguir.



$$\sum F_x = 0 \rightarrow B_x = 0$$

$$\sum F_y = 0 \rightarrow B_y + D_y = 180$$

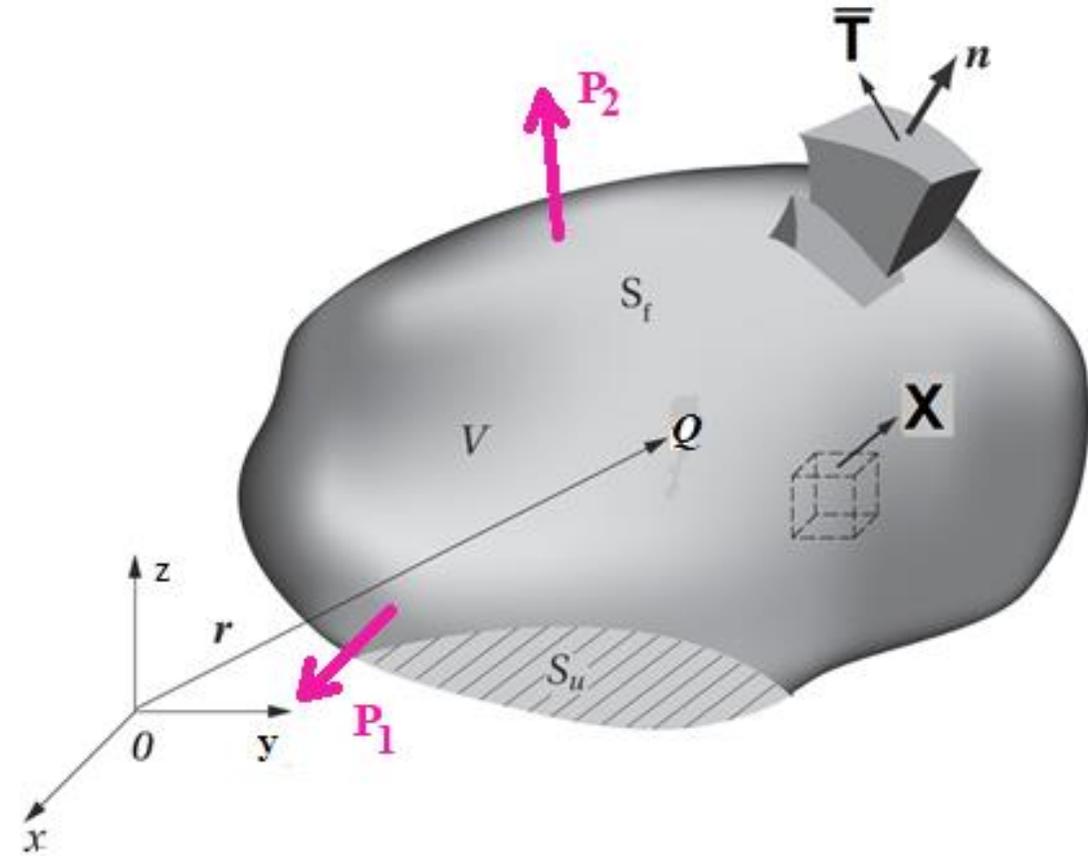
$$\sum M_B = 0 \rightarrow 6.D_y + 90.1 + 90.1,5 = 105 + 180.6 \rightarrow D_y = 160 \text{ kN}(\uparrow)$$

$$\therefore B_y = 20 \text{ kN}(\uparrow)$$

Tensão

Sólido deformável (V) em **equilíbrio estático**

Sujeito a forças de contato: P_1, P_2, \dots



Realize um corte imaginário que passe dentro do corpo

Tensão

Corte imaginário

Vetor tensão em **Q** no plano de normal n

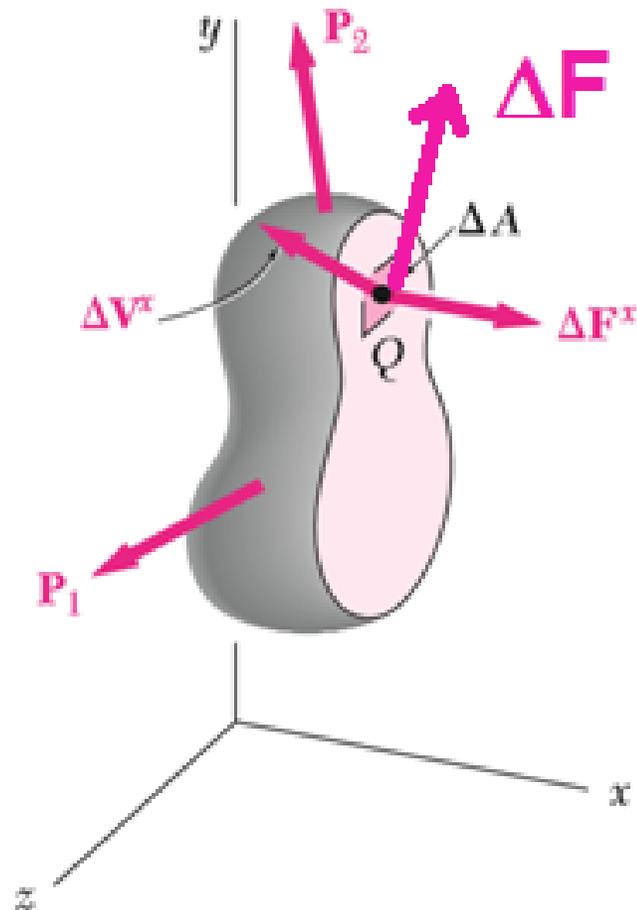
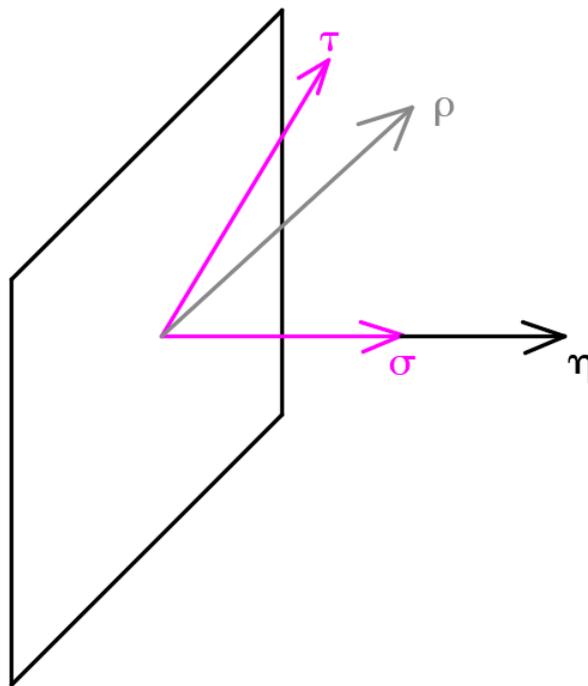
$$\rho_n = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}$$

$$\rho_n = \sigma + \tau$$

σ : tensão normal (perpendicular à ST)

τ : tensão cisalhante (paralelo à ST)

ΔA : área



Conhecer 3 tensões em Q : estado de tensão completamente determinado

ST: seção transversal 18

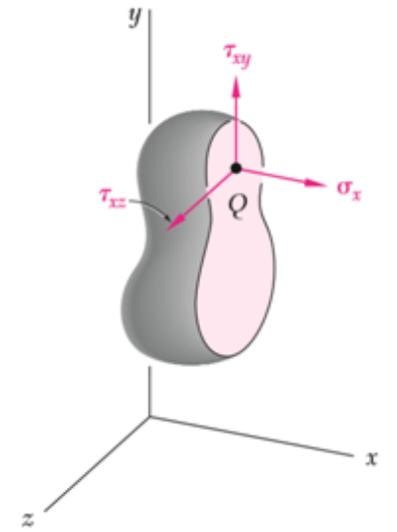
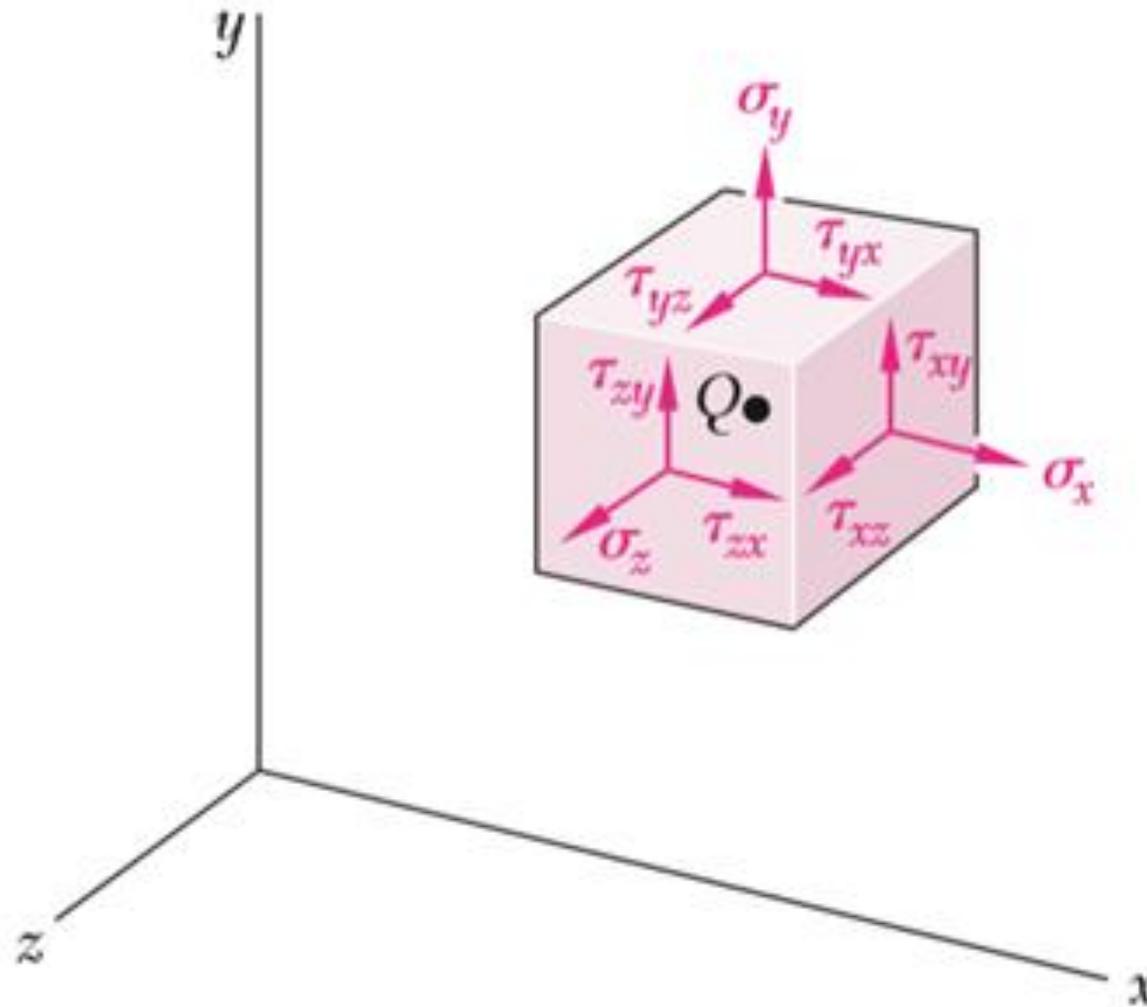
Tensão

Decomposição do vetor tensão

$$\sigma_x = \lim_{\Delta A \rightarrow 0} \frac{\Delta F^x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \rightarrow 0} \frac{\Delta V_y^x}{\Delta A}$$

$$\tau_{xz} = \lim_{\Delta A \rightarrow 0} \frac{\Delta V_z^x}{\Delta A}$$



Estado tensional definidos nas **facetas paralelas** aos planos coordenados

Simetria de tensões

A combinação de forças geradas pela tensão devem satisfazer as condições para o equilíbrio:

$$\sum F_x = \sum F_y = \sum F_z = 0$$

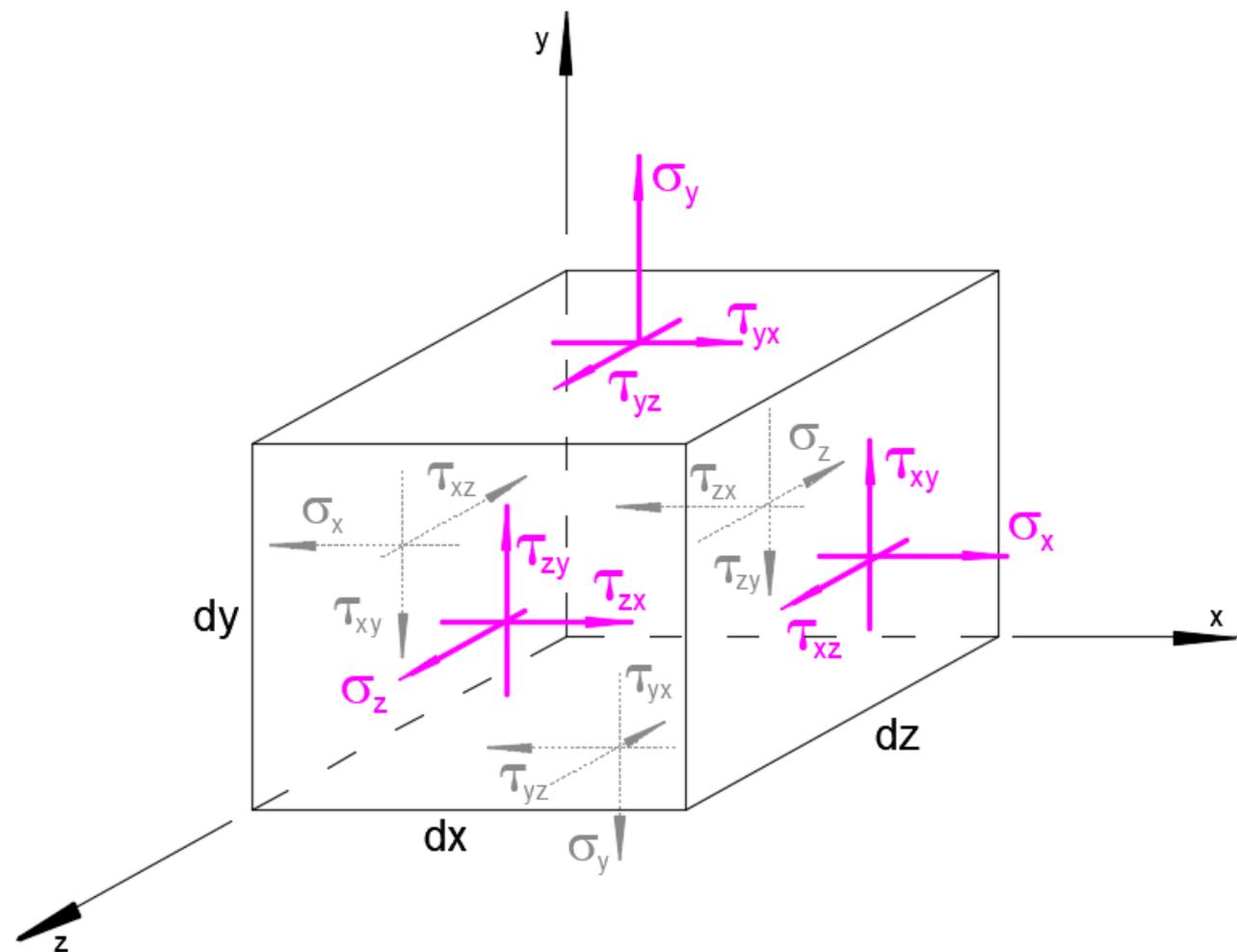
$$\sum M_x = \sum M_y = \sum M_z = 0$$

Considere os momentos em torno do eixo z, pólo no centro do volume:

$$\sum M_Q = 0 = (\tau_{xy} dy dz) dx - (\tau_{yx} dx dz) dy$$

$$\tau_{xy} = \tau_{yx}$$

Similarmente, $\tau_{yz} = \tau_{zy}$ e $\tau_{zx} = \tau_{xz}$



$$\tau_{yz} = \tau_{zy}$$

$$\tau_{zx} = \tau_{xz}$$

**Simetria de tensões
de cisalhamento**

Notação das 6 tensões

Notação:

$$\sigma = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = [\sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{xy} \quad \tau_{xz} \quad \tau_{yz}]^T$$

Às vezes é conveniente escrever na forma:

$$\sigma = [\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \sigma_{xy} \quad \sigma_{xz} \quad \sigma_{yz}]^T$$

Ou

$$\sigma = [\sigma_{11} \quad \sigma_{22} \quad \sigma_{33} \quad \sigma_{12} \quad \sigma_{13} \quad \sigma_{23}]^T$$

com $x = 1$, $y = 2$ e $z = 3$

**Tensor das tensões de Cauchy
(simétrico)**

$$\mathbf{T} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

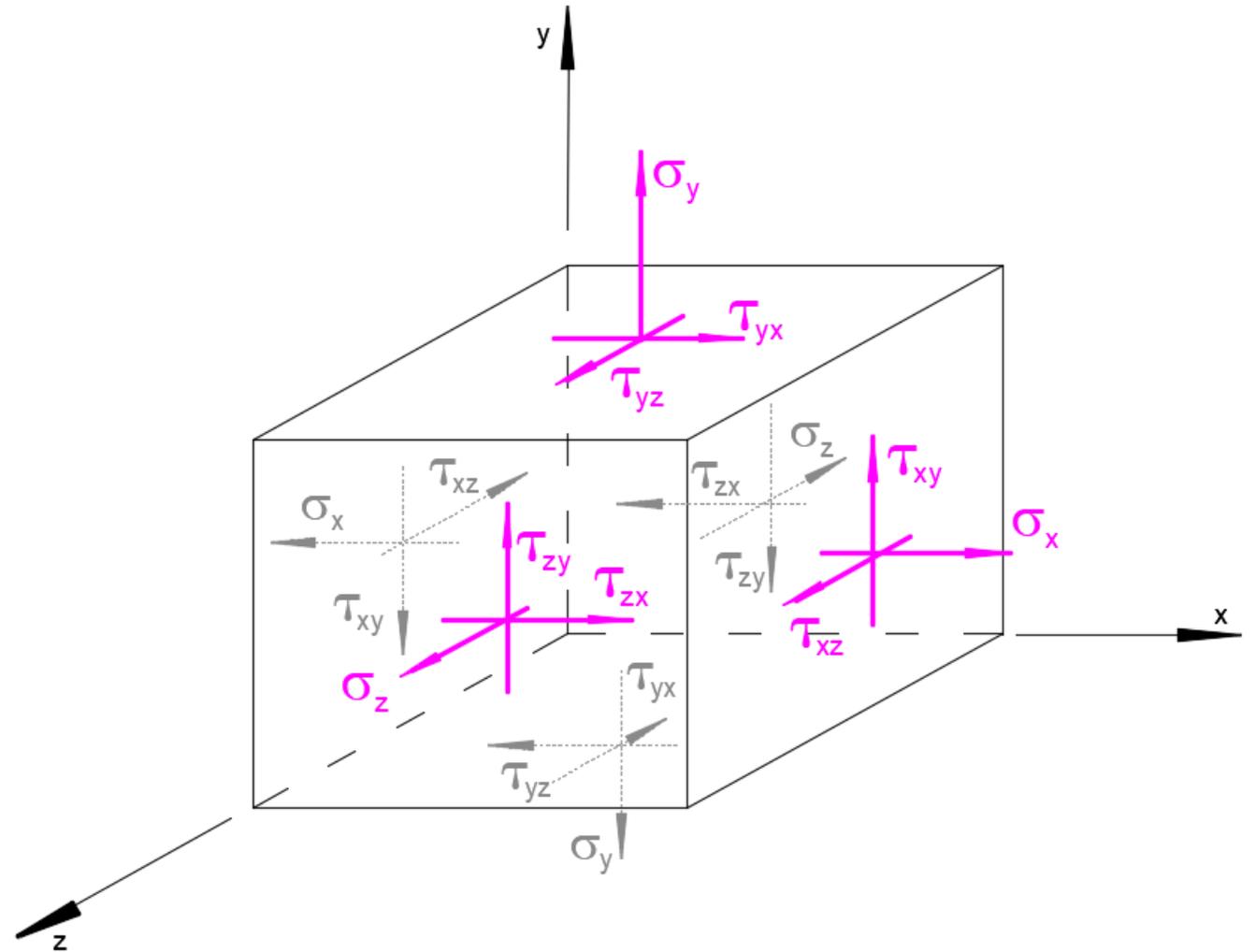
**Estado triplo
de tensões**

Sinal das componentes de tensão

A σ é positiva no sentido da normal à faceta

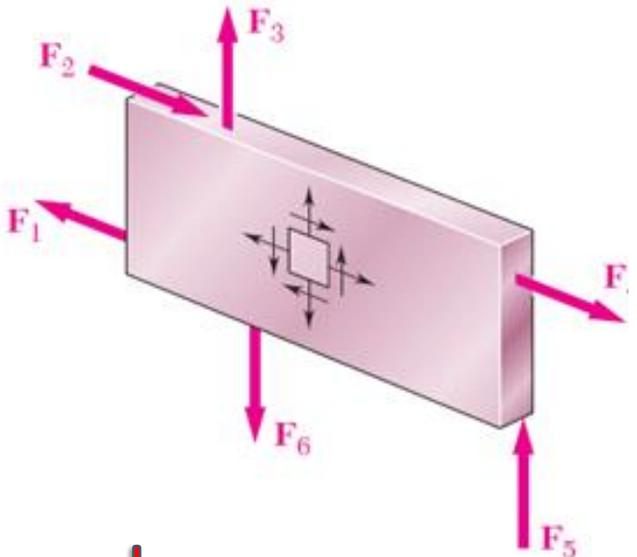
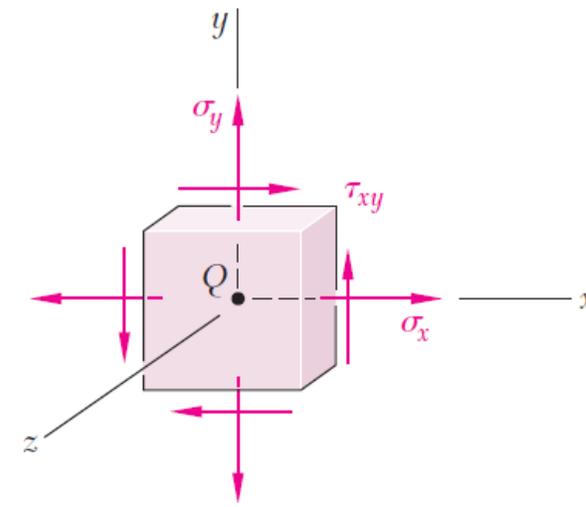
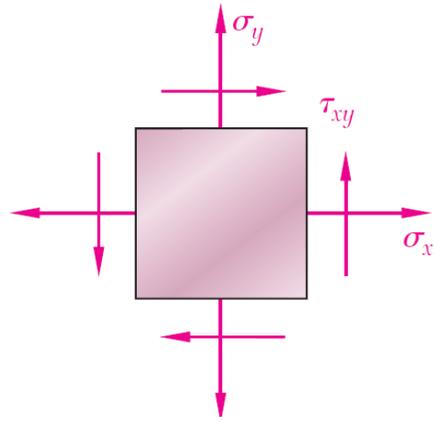
Se normal à faceta está no sentido do eixo, τ acompanha o seu sentido, caso contrário, sentido oposto.

$$T = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

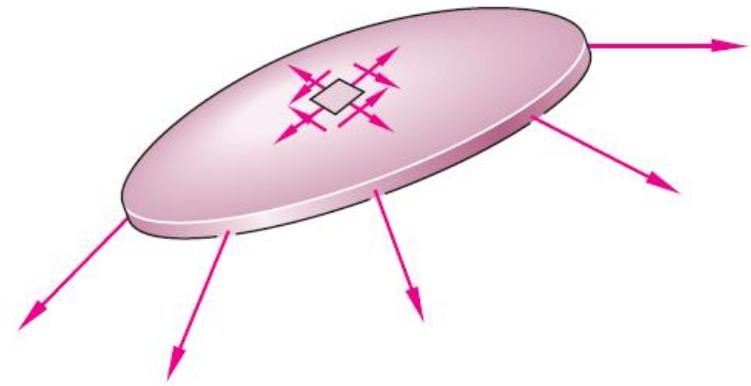


Estado plano de tensões

$$\mathbf{T} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$



chapa



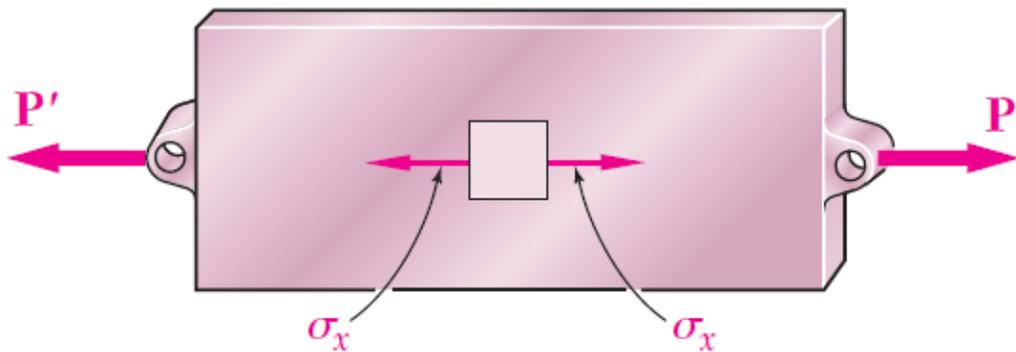
membrana



viga

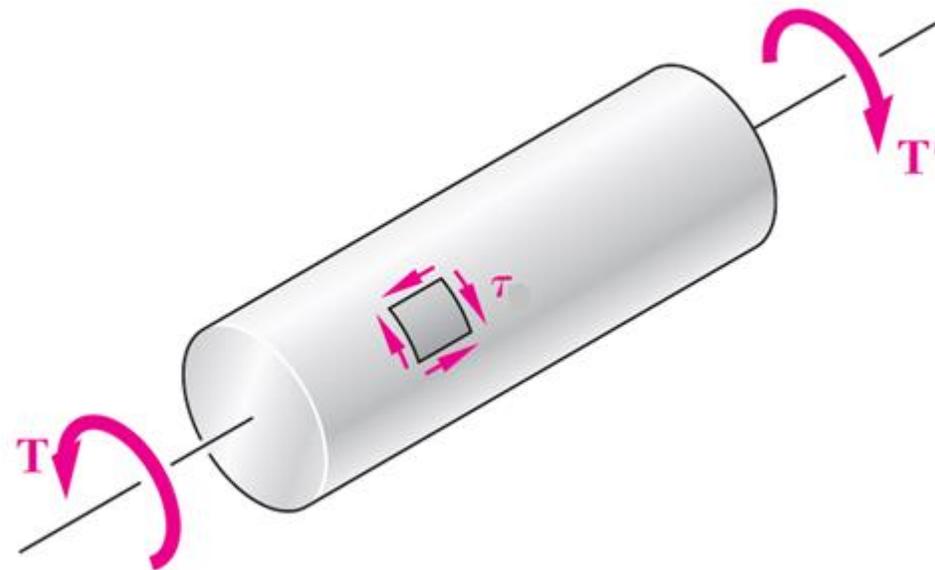
Estado uniaxial de tensões

$$\mathbf{T} = [\sigma_x]$$



Barra (corda) tracionada

$$\mathbf{T} = [\tau_{xy}]$$

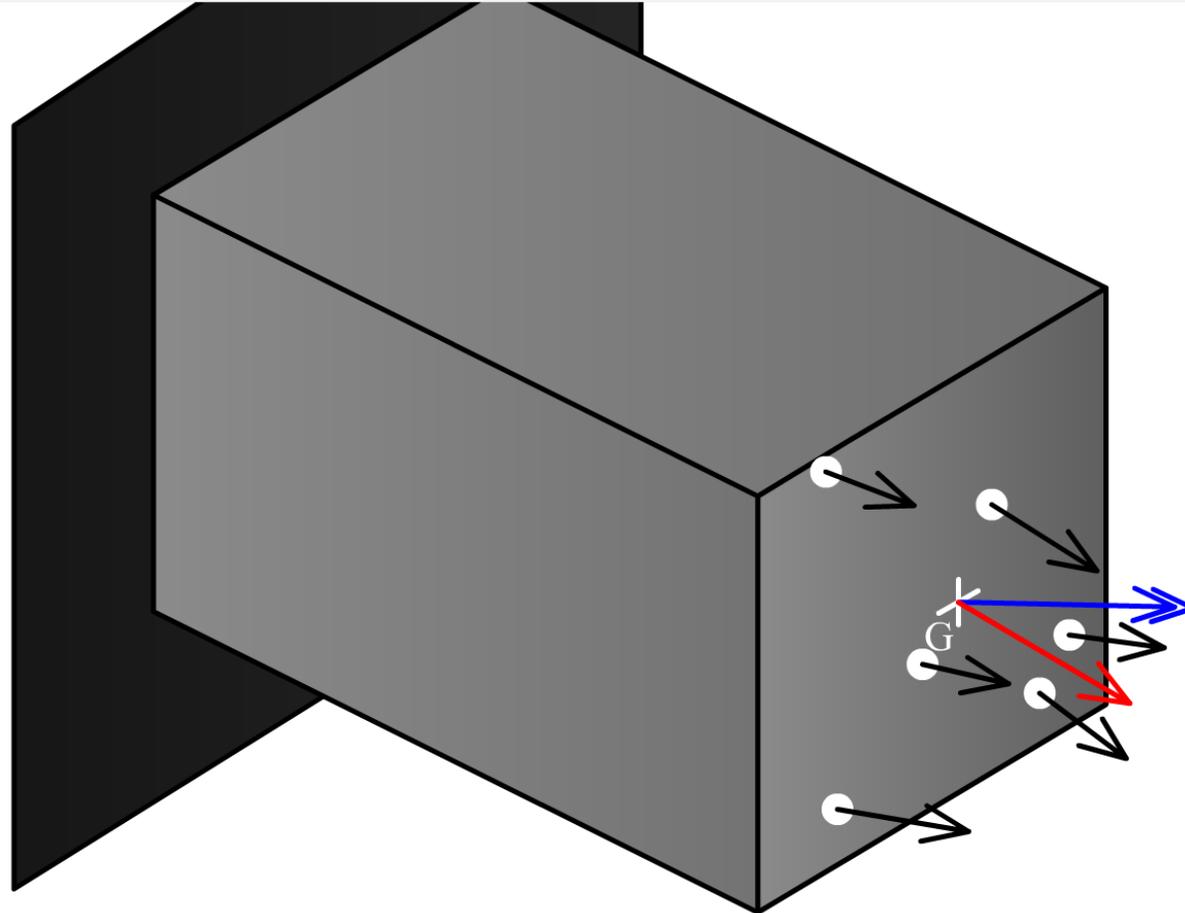


Torção (torque) em eixo

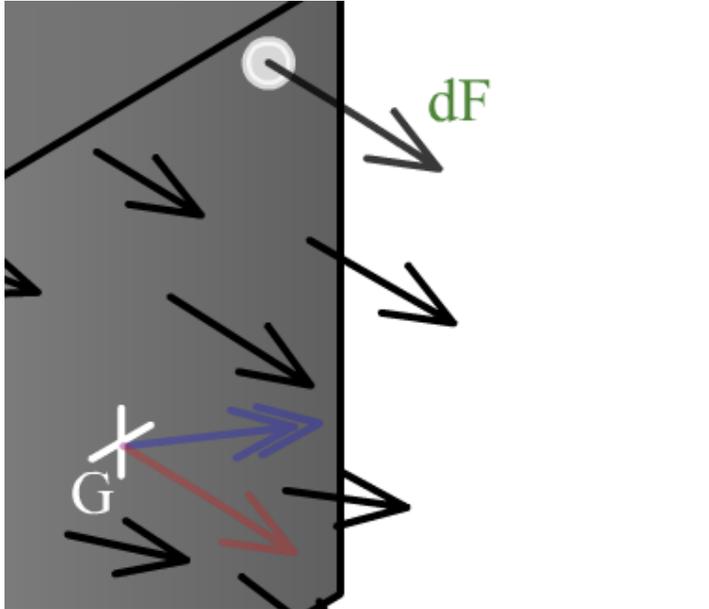
TENSÕES E ESFORÇOS SOLICITANTES: ELEMENTOS LINEARES

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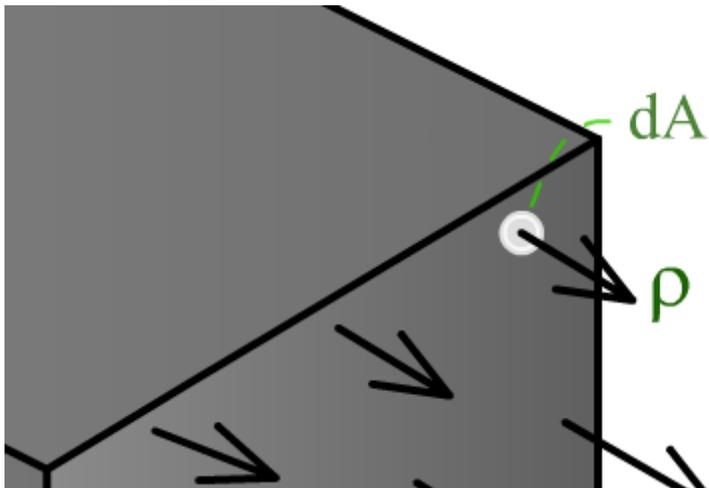
Tensões no plano



Tensão: $\vec{\rho} = \vec{\sigma} + \vec{\tau}$

Tensão normal a seção: $\vec{\sigma}$

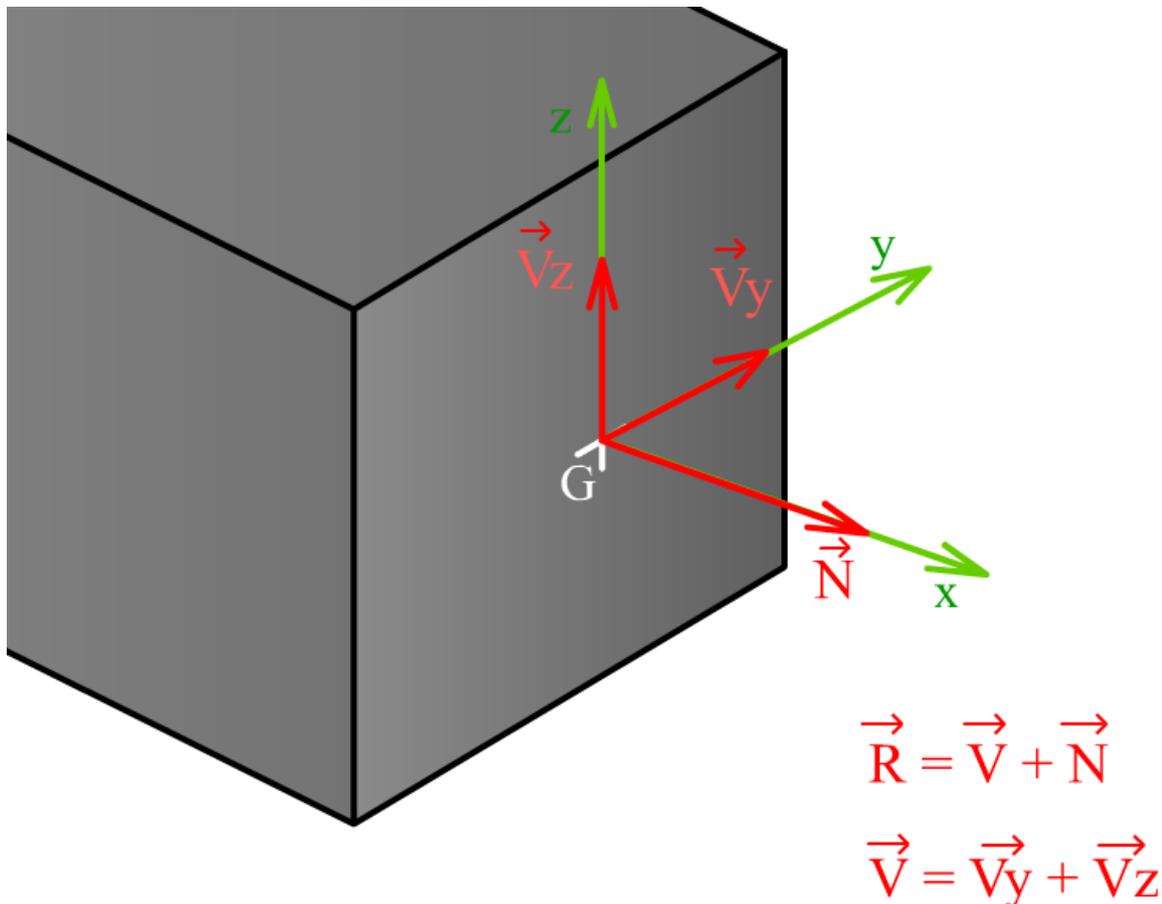
Tensão paralela a seção: $\vec{\tau}$



$$\vec{\rho}_{m\acute{e}dia} = \lim_{\Delta A \rightarrow 0} \frac{\vec{\Delta F}}{\Delta A}$$

ESFORÇOS SOLICITANTES

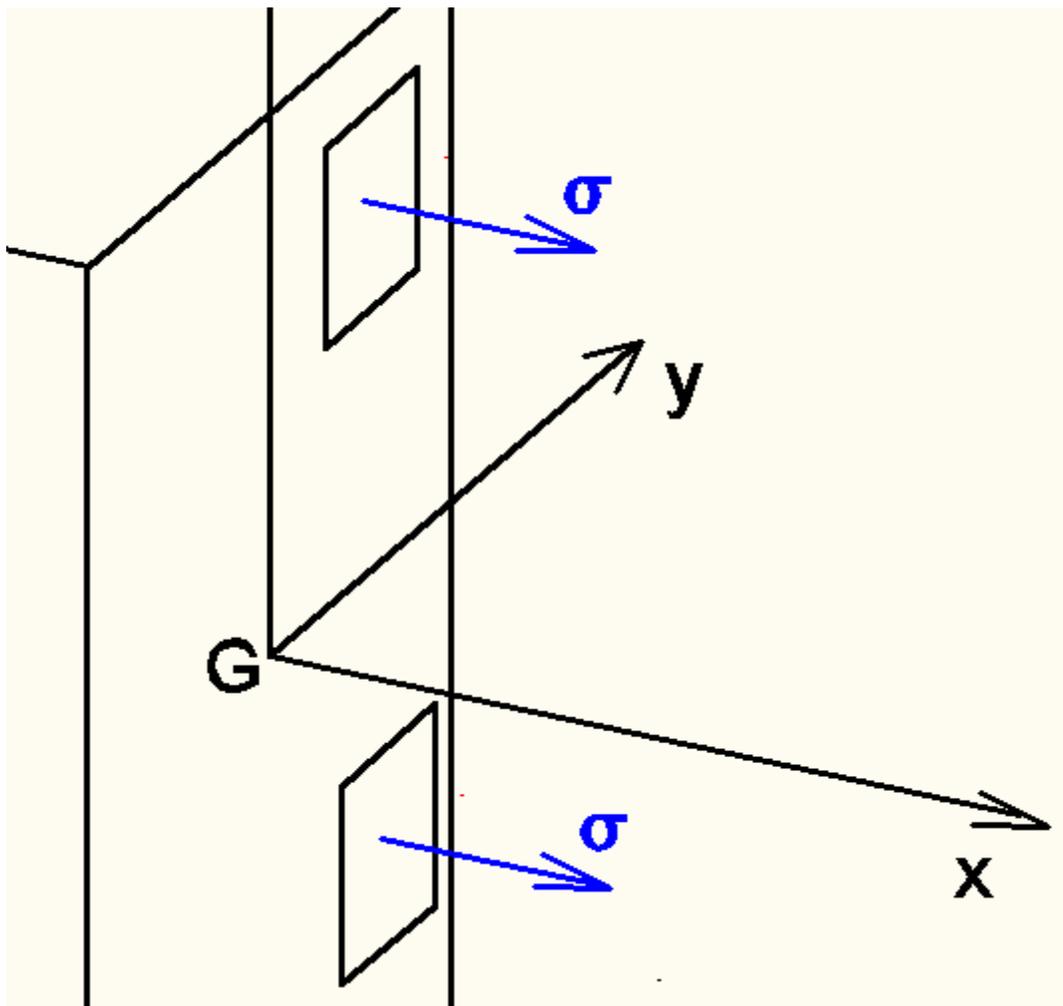
Esforços solicitantes: força/momento resultante das tensões transferidos para o centroide de cada seção transversal



N: Esforço Normal

**V: Esforço Cisalhante
ou Cortante**

ESFORÇO NORMAL

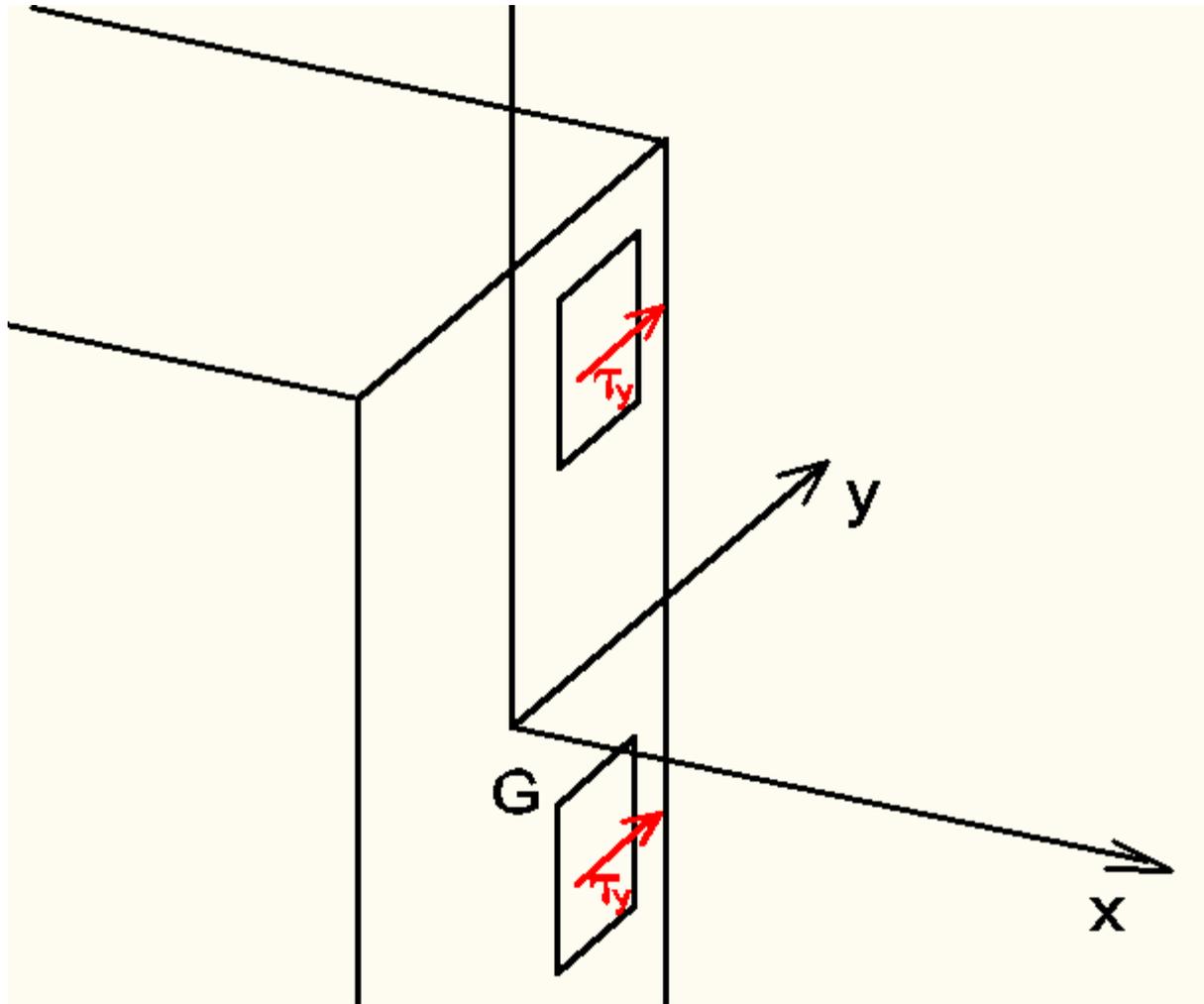


$$N = \int_A \sigma dA$$

N: ESFORÇO NORMAL

σ : TENSÃO NORMAL

ESFORÇO CISALHANTE (Cortante)

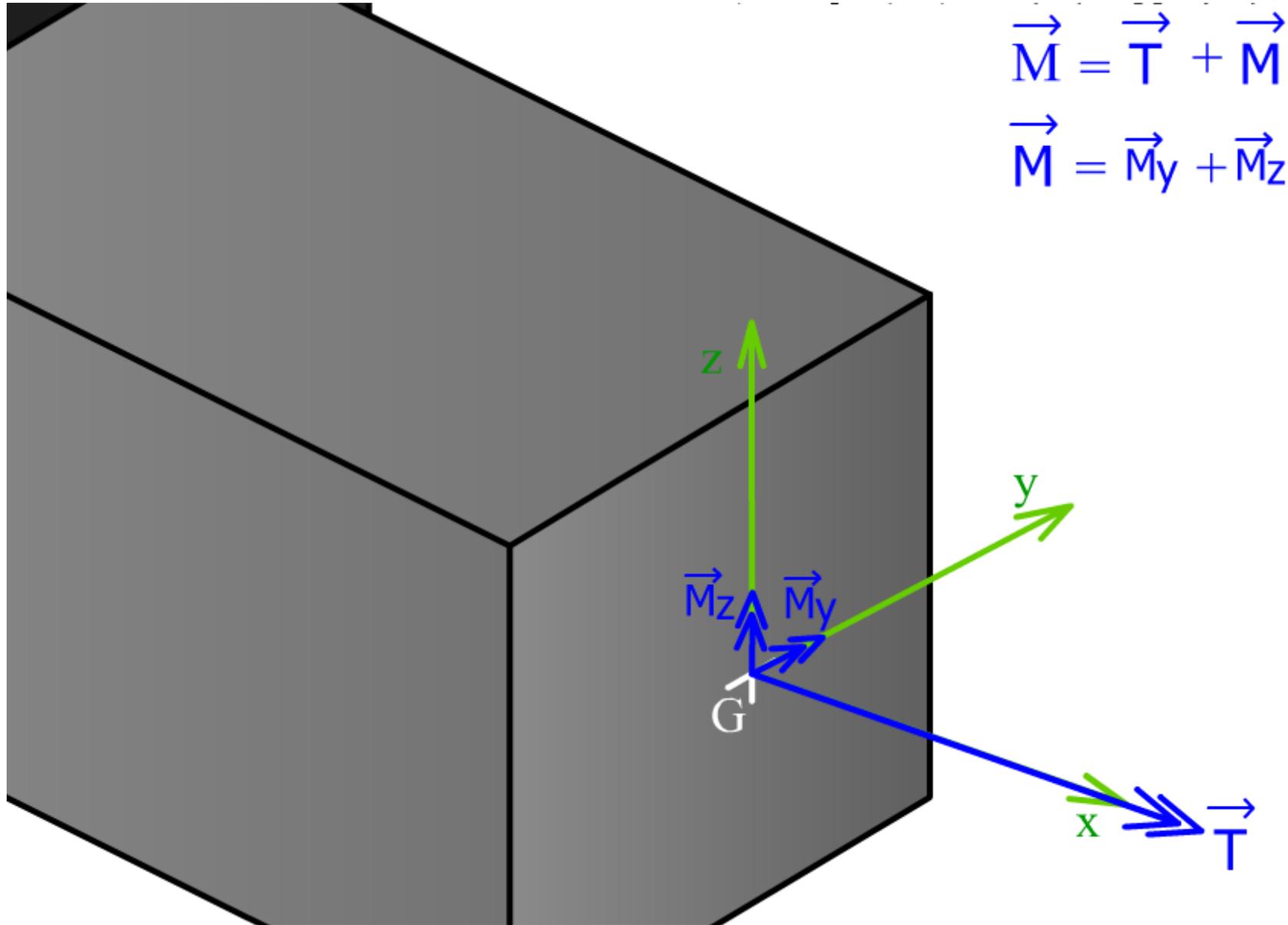


$$V_y = \int_A \tau_y dA$$

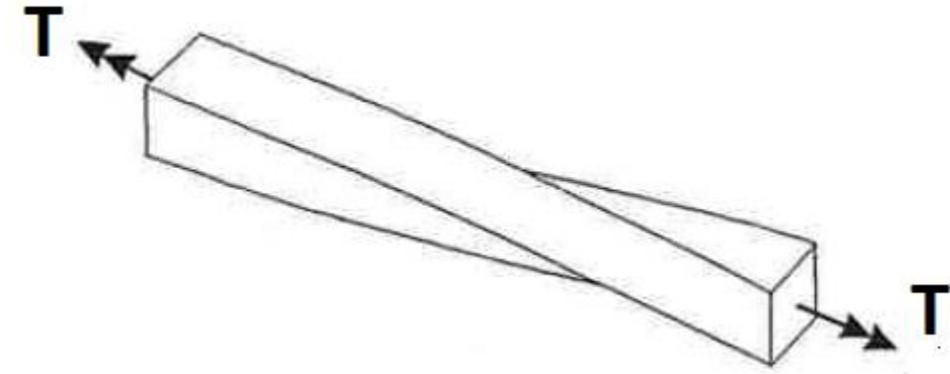
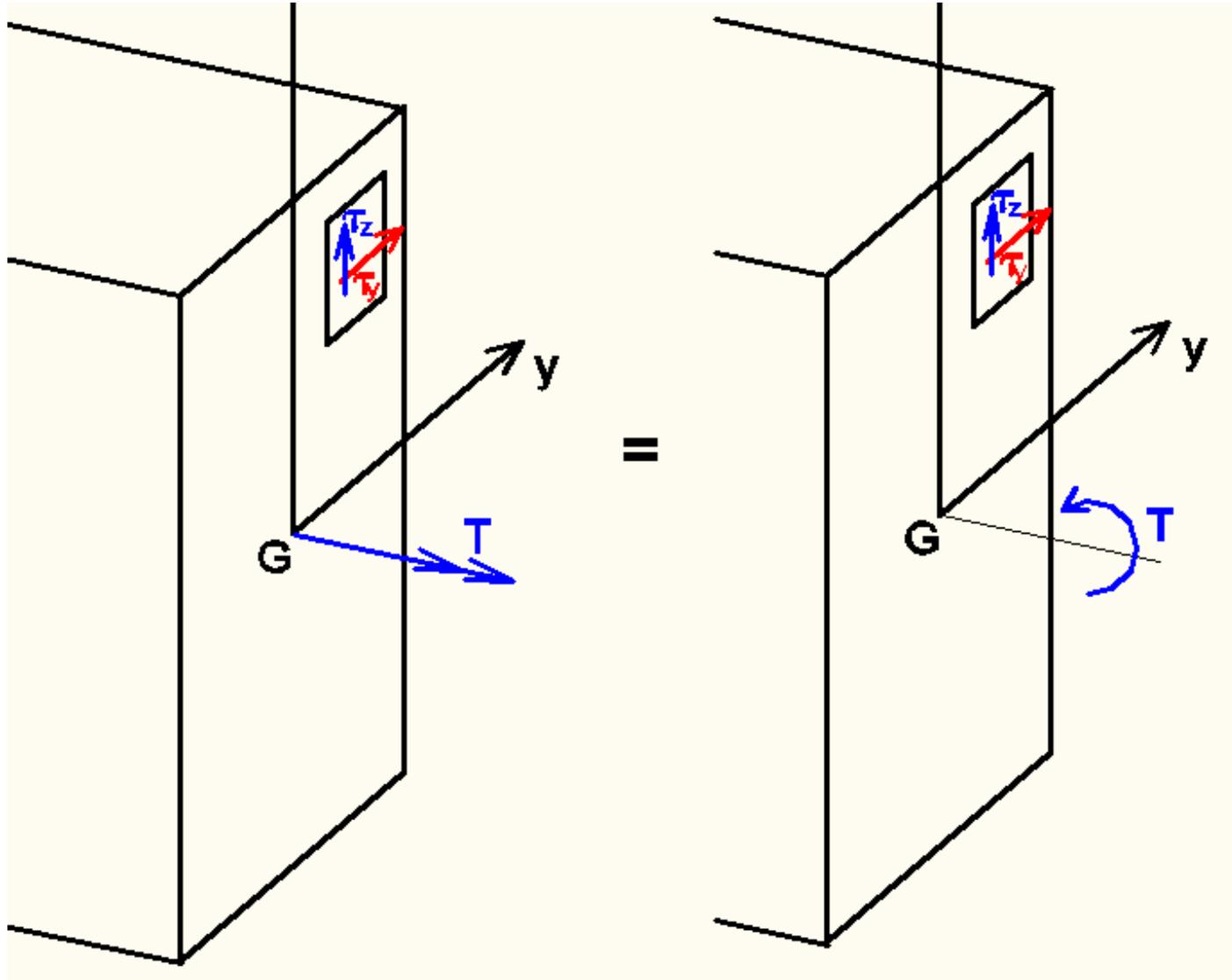
$$V_z = \int_A \tau_z dA$$

Tensão Cisalhante: τ

ESFORÇOS DE MOMENTO

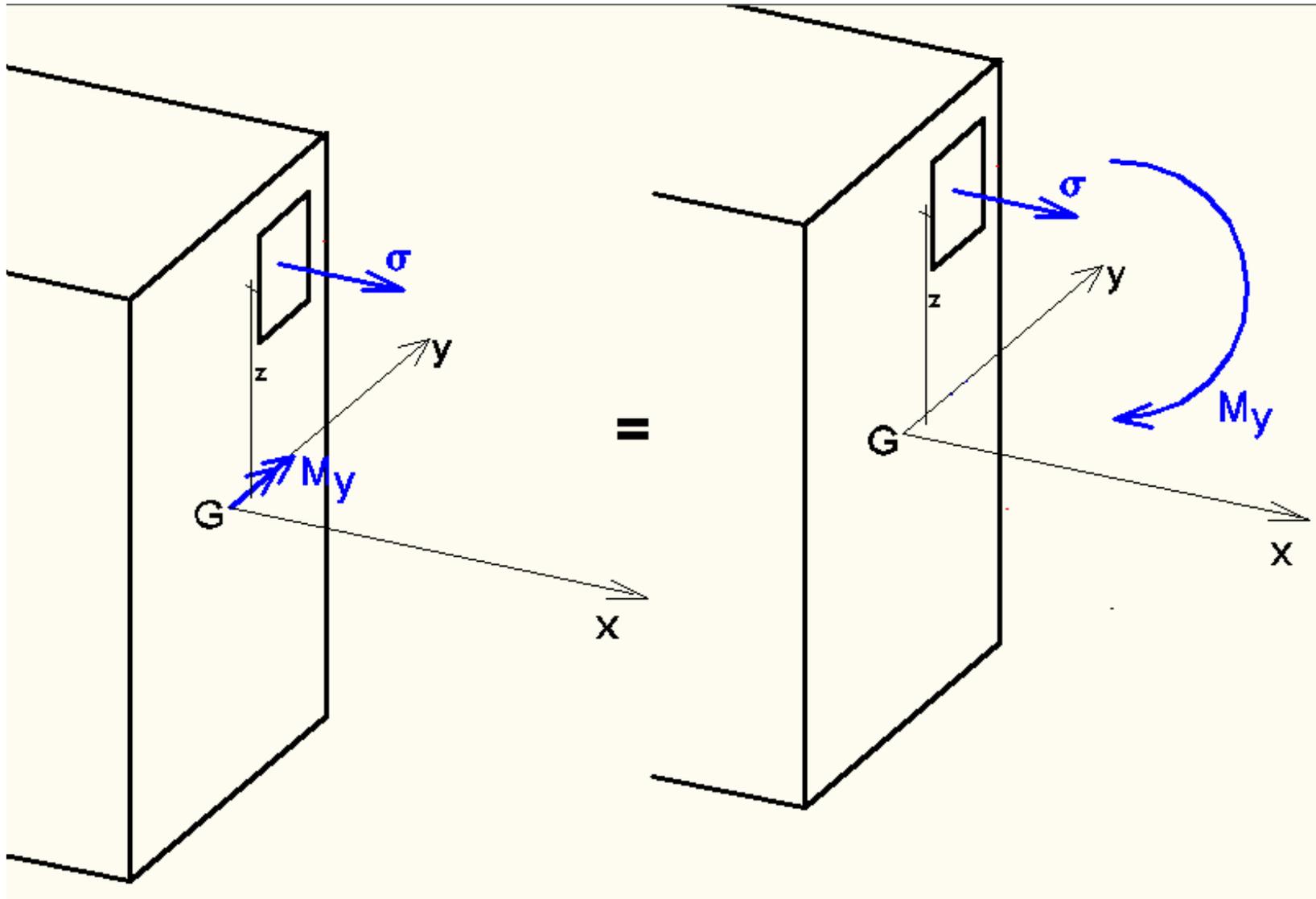


MOMENTO TORÇOR (T)



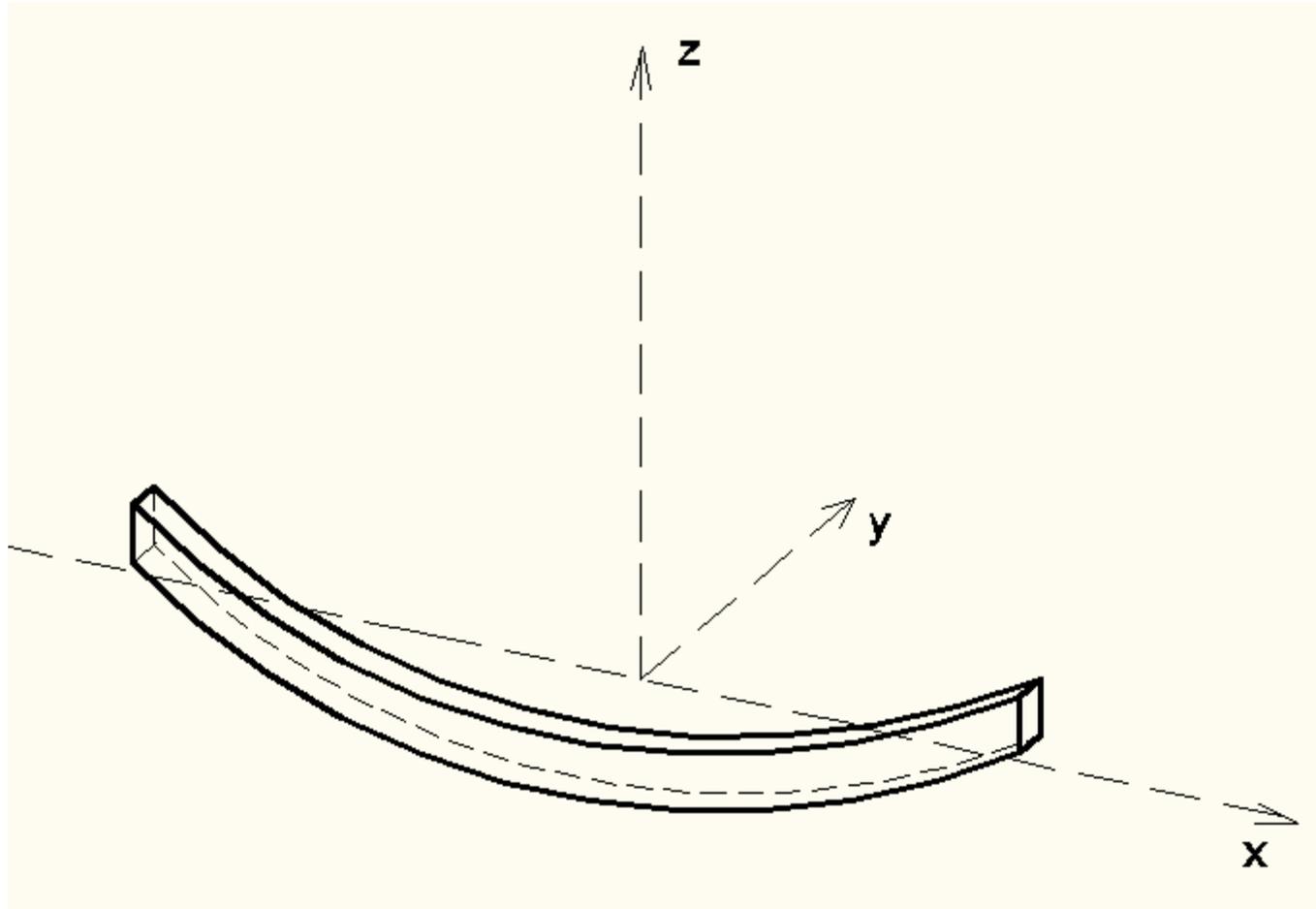
$$T = \int_A (\tau_z y - \tau_y z) dA$$

MOMENTO FLETOR (M_y)

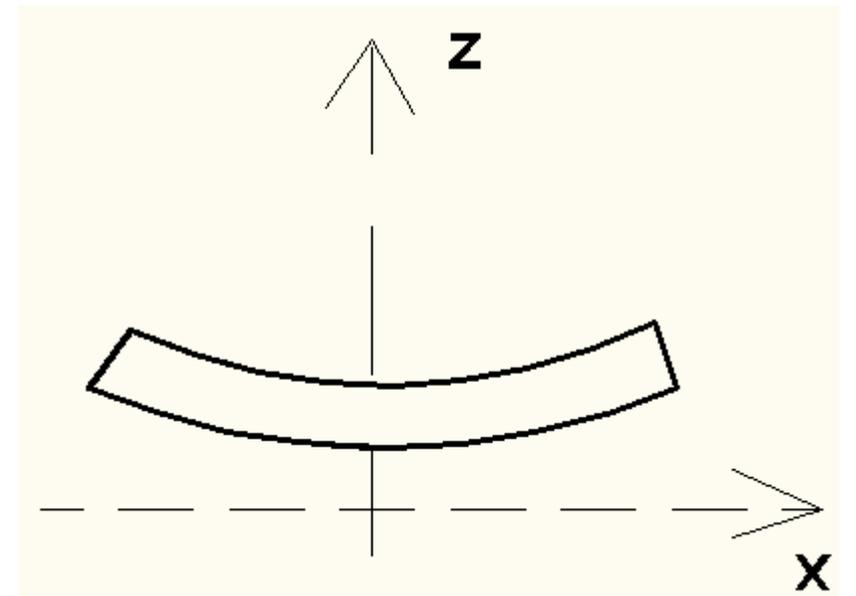


$$M_y = \int_A \sigma \cdot z \, dA$$

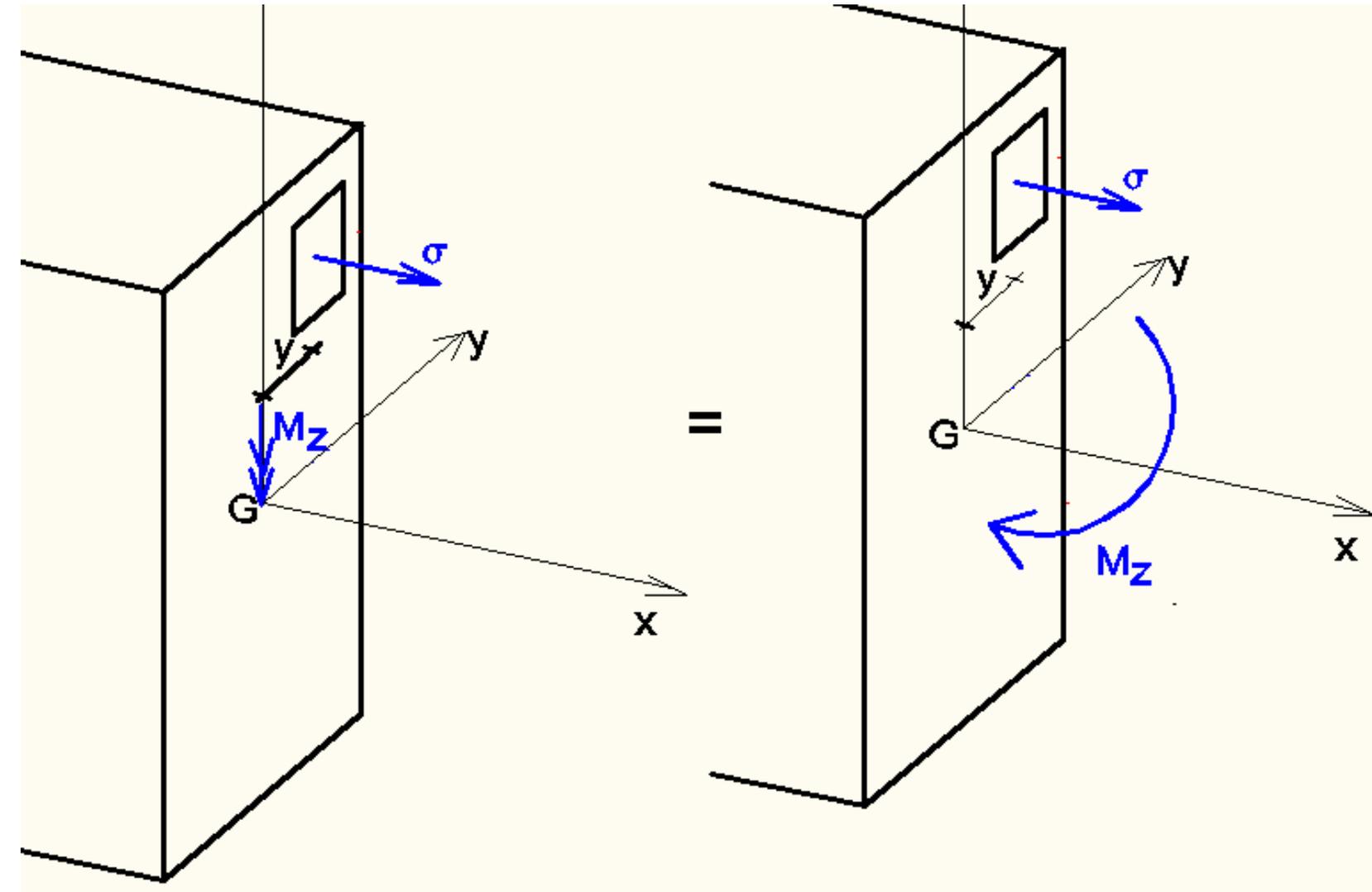
MOMENTO FLETOR (M_y)



Curvatura em torno de y

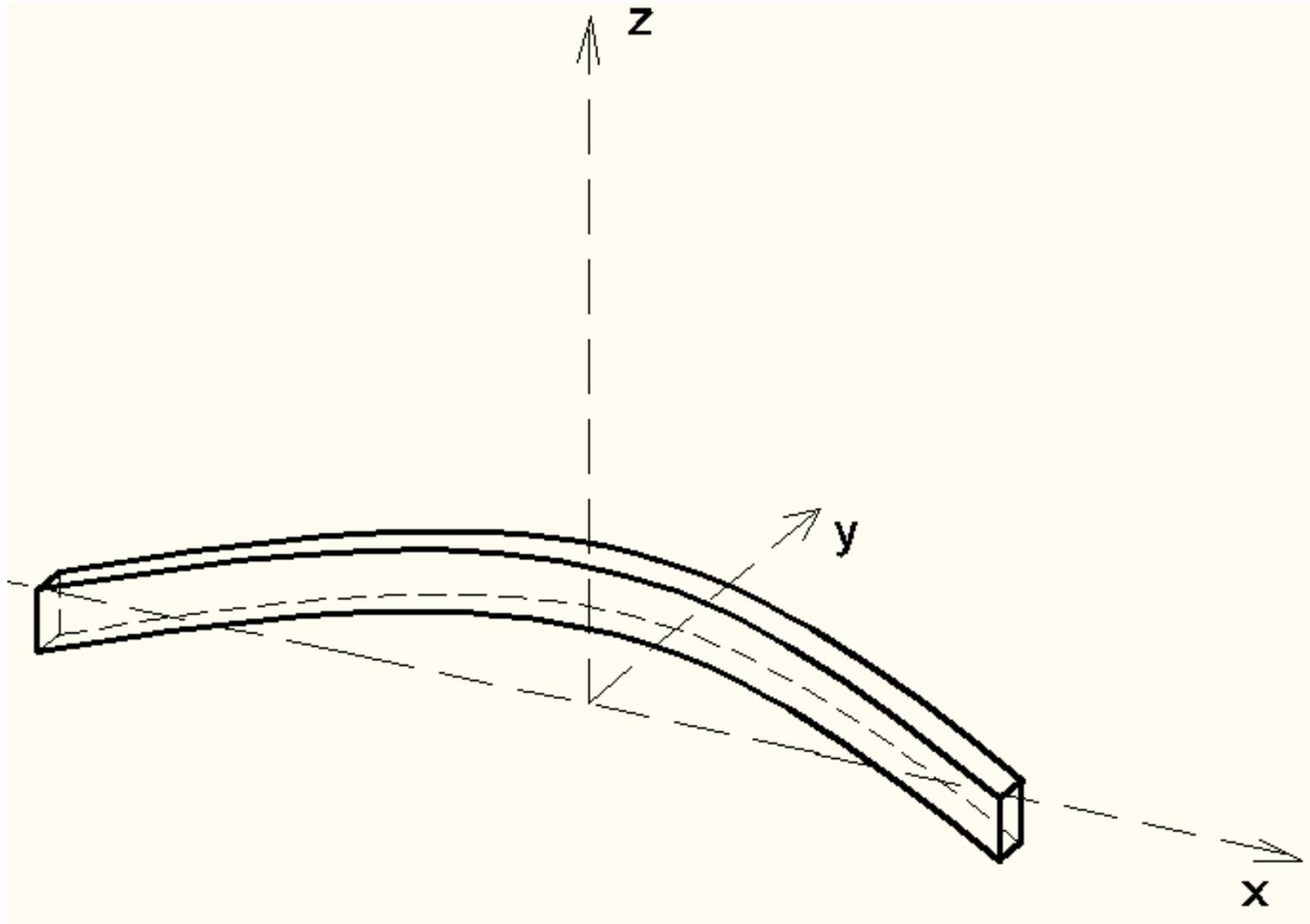


MOMENTO FLETOR (M_z)

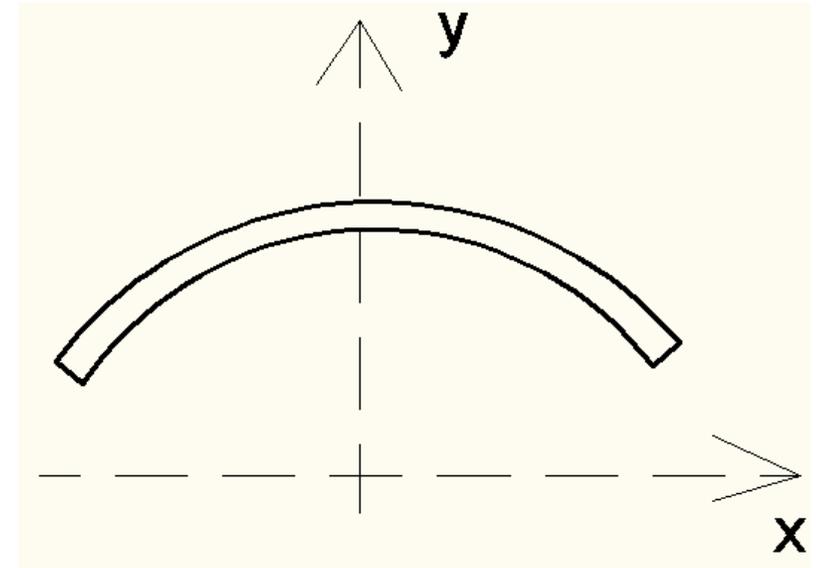


$$M_z = \int_A \sigma \cdot y \, dA$$

MOMENTO FLETOR (M_z)



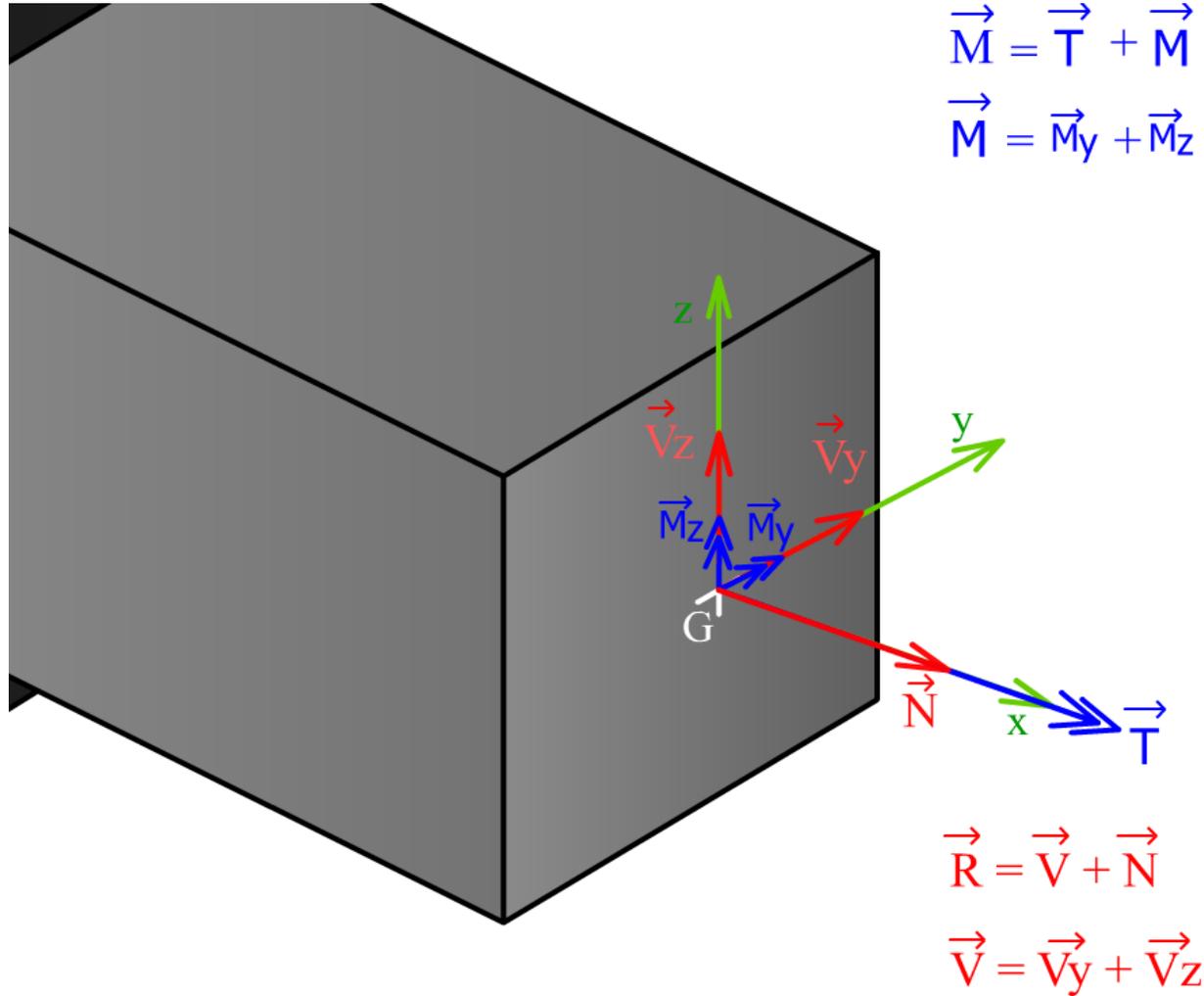
Curvatura em torno de z



TOTAL DE ESFORÇOS (6)

$$\vec{M} = \vec{T} + \vec{M}$$

$$\vec{M} = \vec{M}_y + \vec{M}_z$$



$$N = \int_A \sigma dA$$

$$V_y = \int_A \tau_y dA$$

$$V_z = \int_A \tau_z dA$$

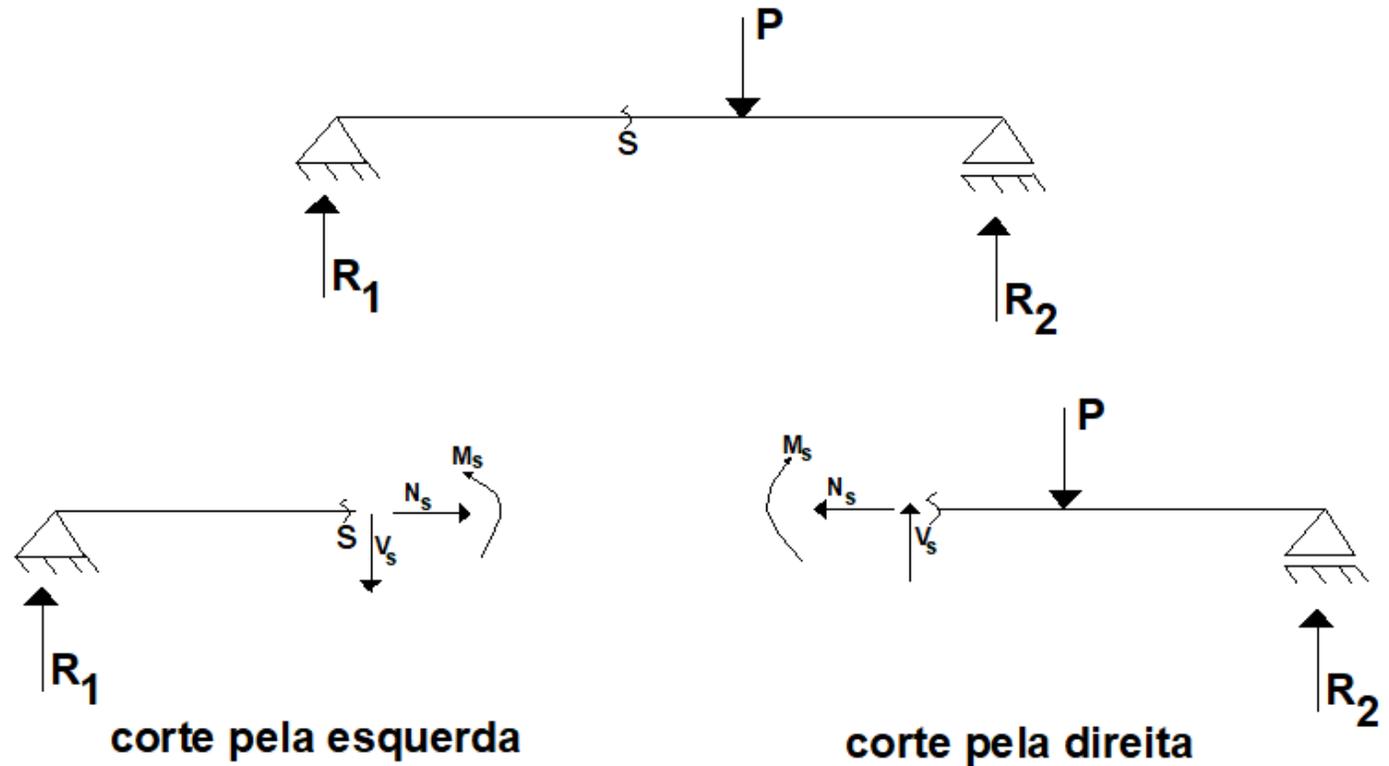
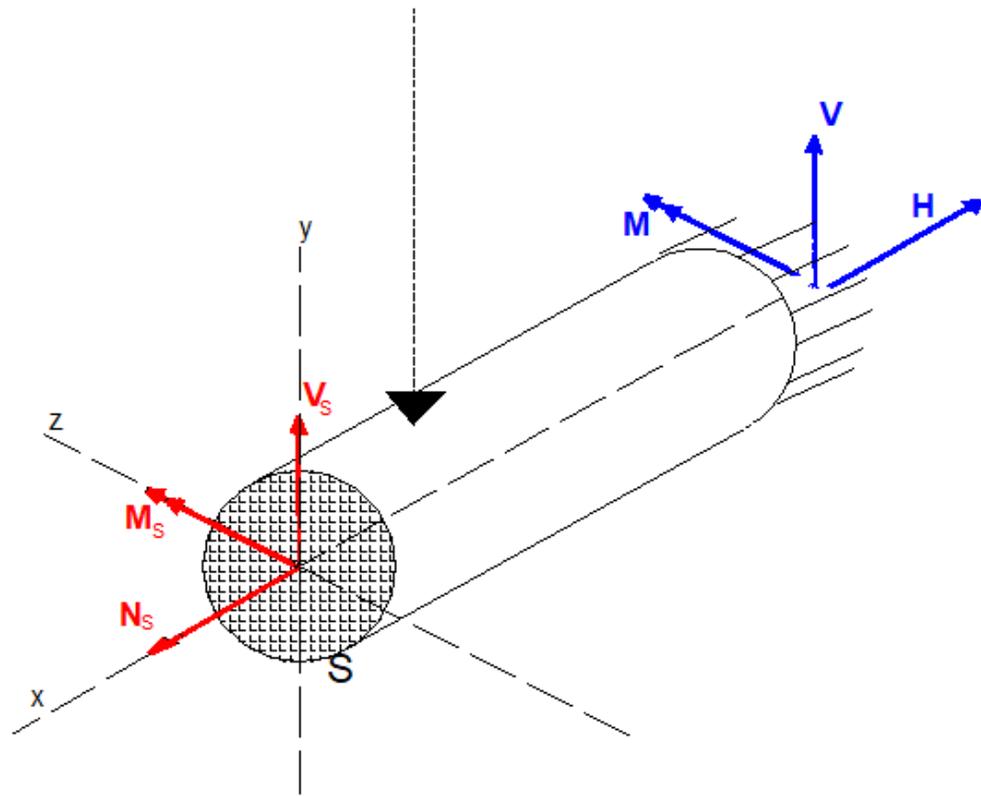
$$M_z = \int_A \sigma \cdot y dA$$

$$M_y = \int_A \sigma \cdot z dA$$

$$T = \int_A (\tau_z y - \tau_y z) dA$$

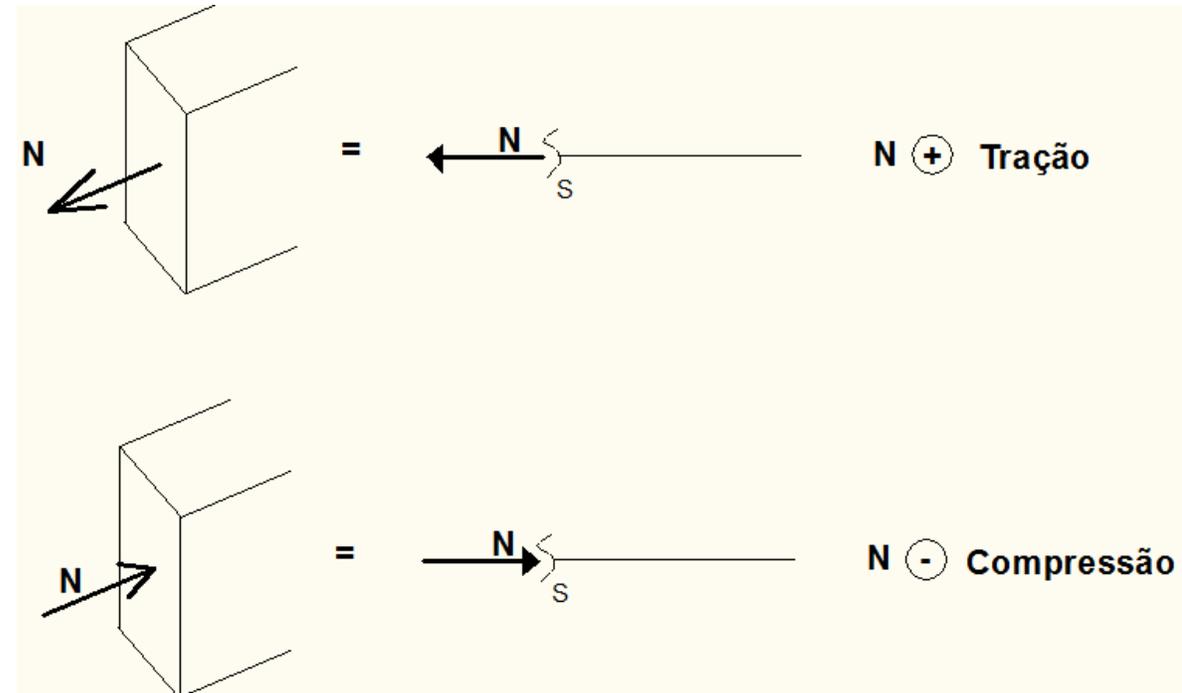
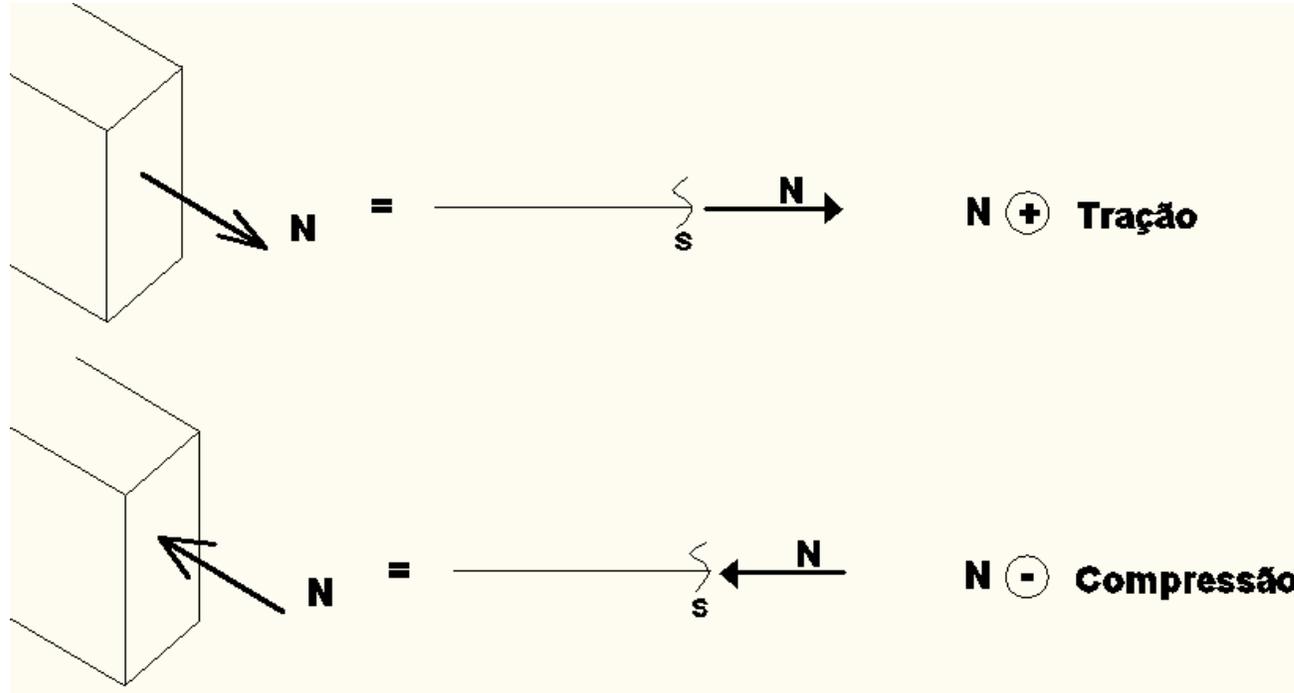
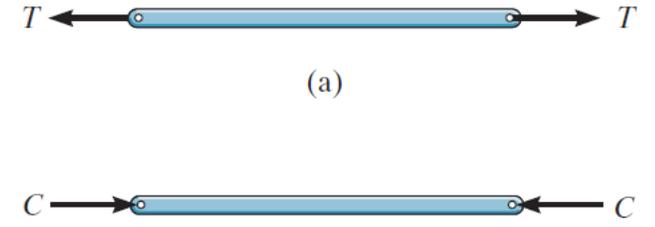
ESFORÇOS SOLICITANTES – SISTEMAS PLANOS

Para o caso de cargas/geometria contidas no plano, tem-se somente 3 componentes: **M**, **V** e **N**, dispensando os índices.



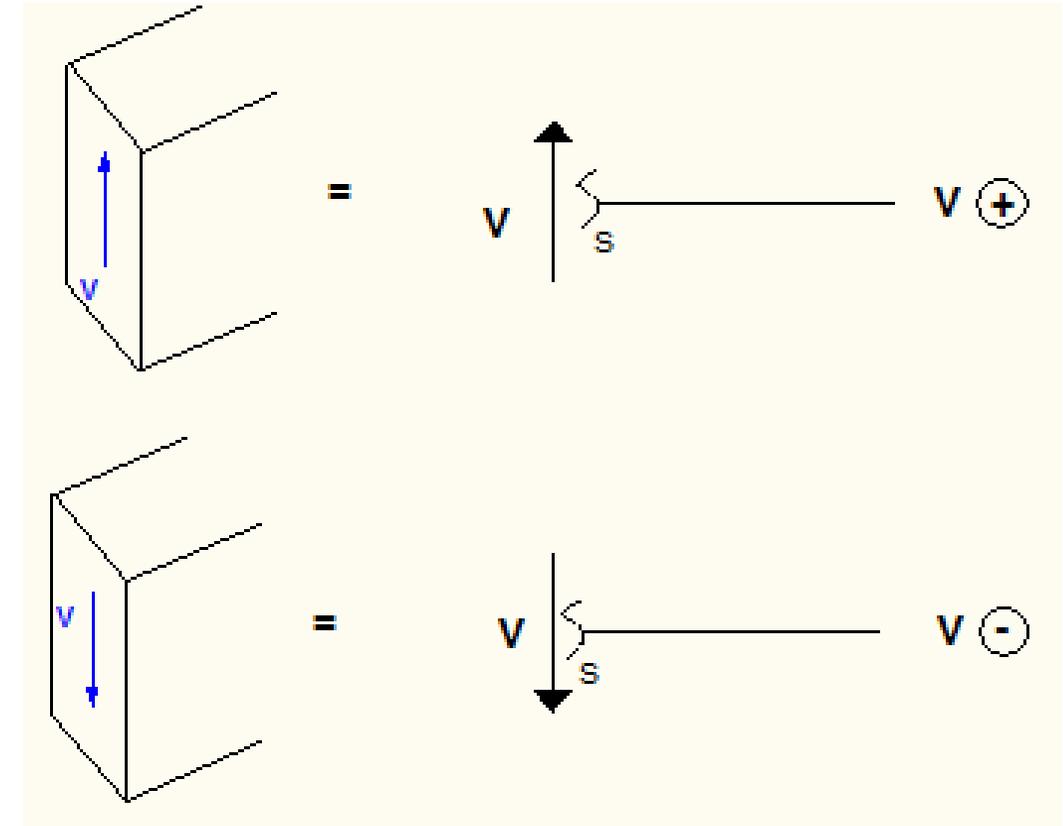
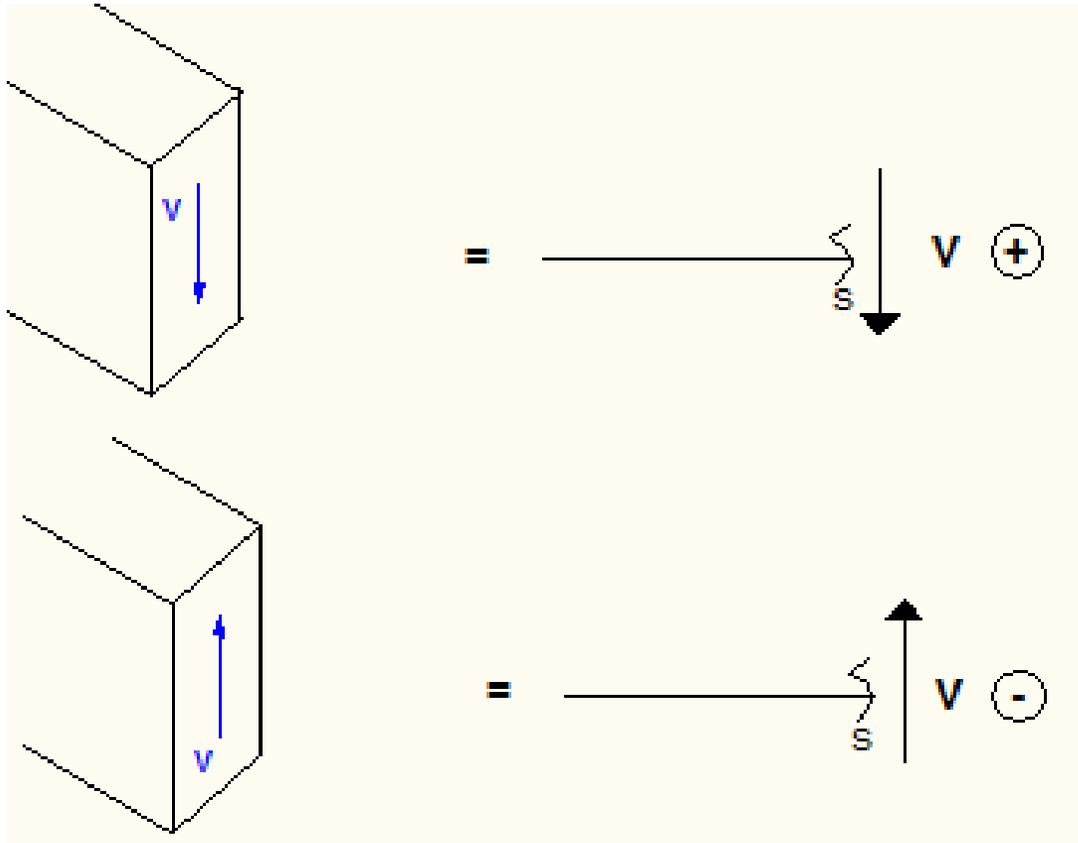
ESFORÇO NORMAL (N)

Convenção de sinal:



Diagramas de esforços - Convenção de sinais

Esforço Cortante (V)



Força cortante

Gira o trecho de barra em que atua no sentido horário

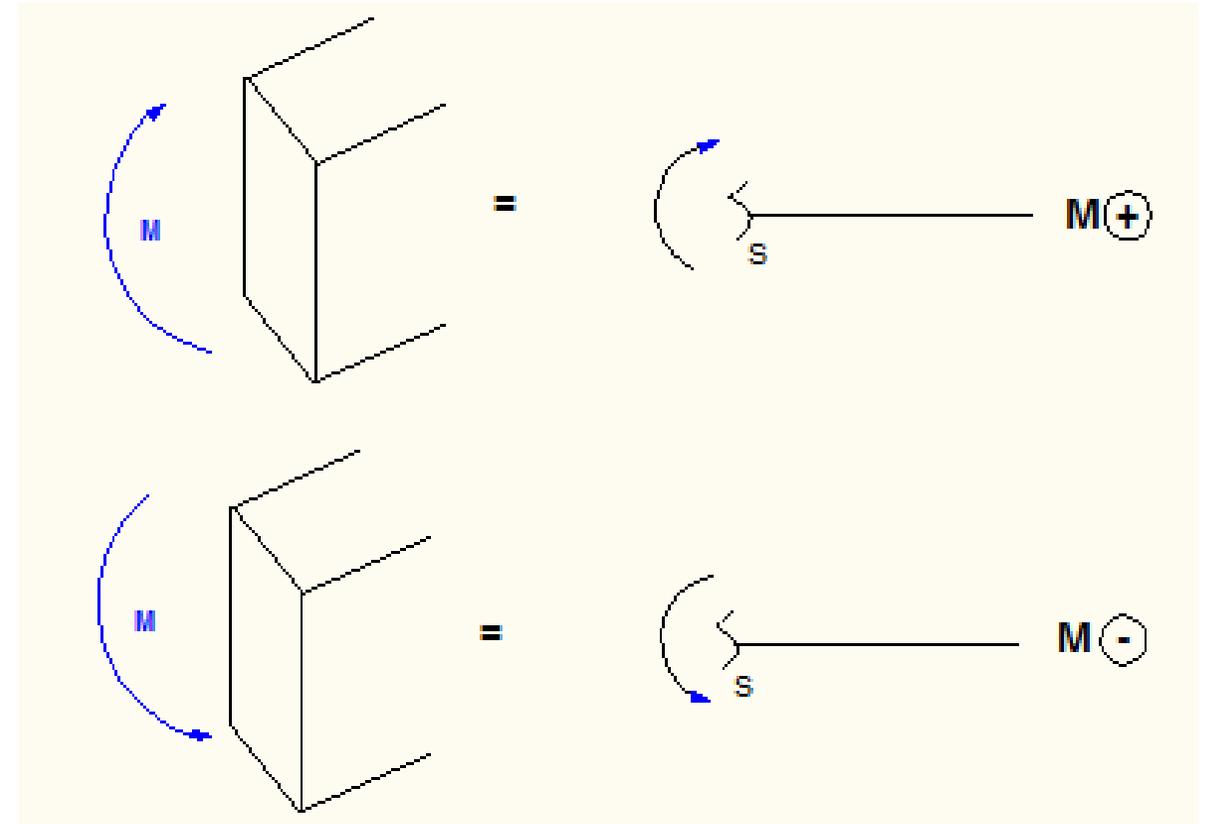
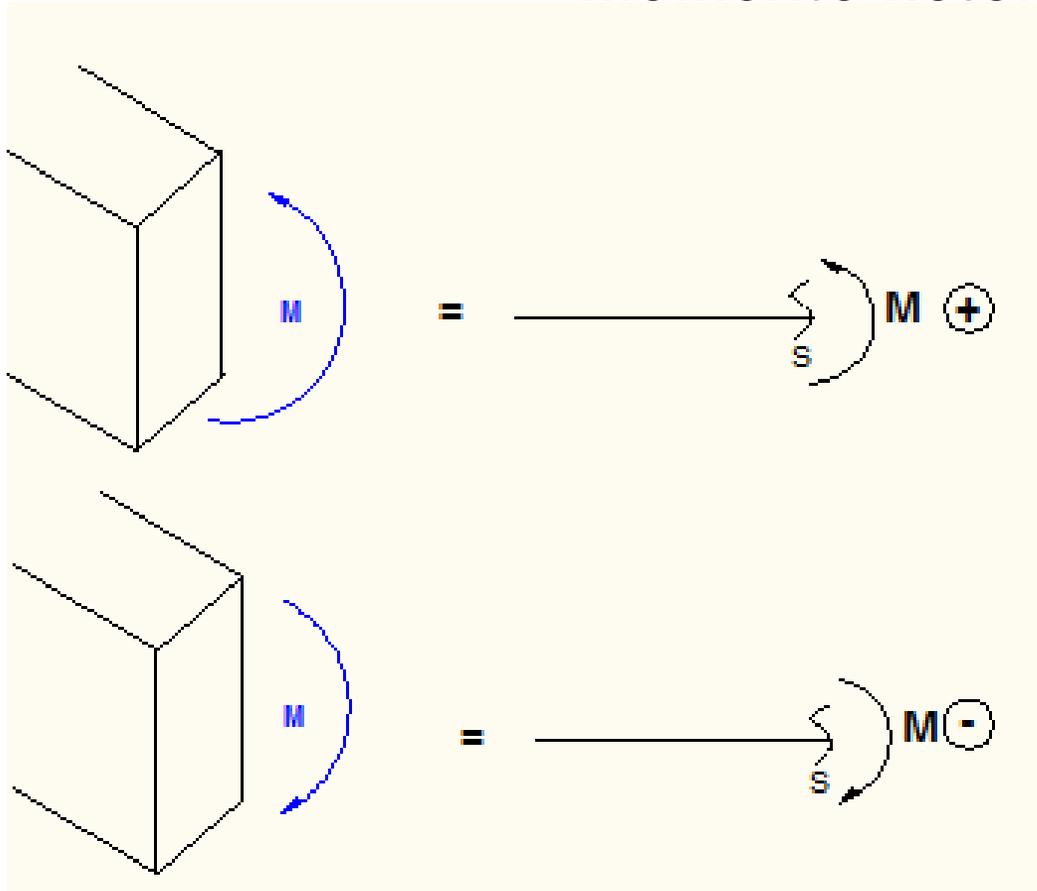
$$V > 0$$

Gira o trecho de barra em que atua no sentido anti-horário

$$V < 0$$

Diagramas de esforços - Convenção de sinais

Momento fletor (M)



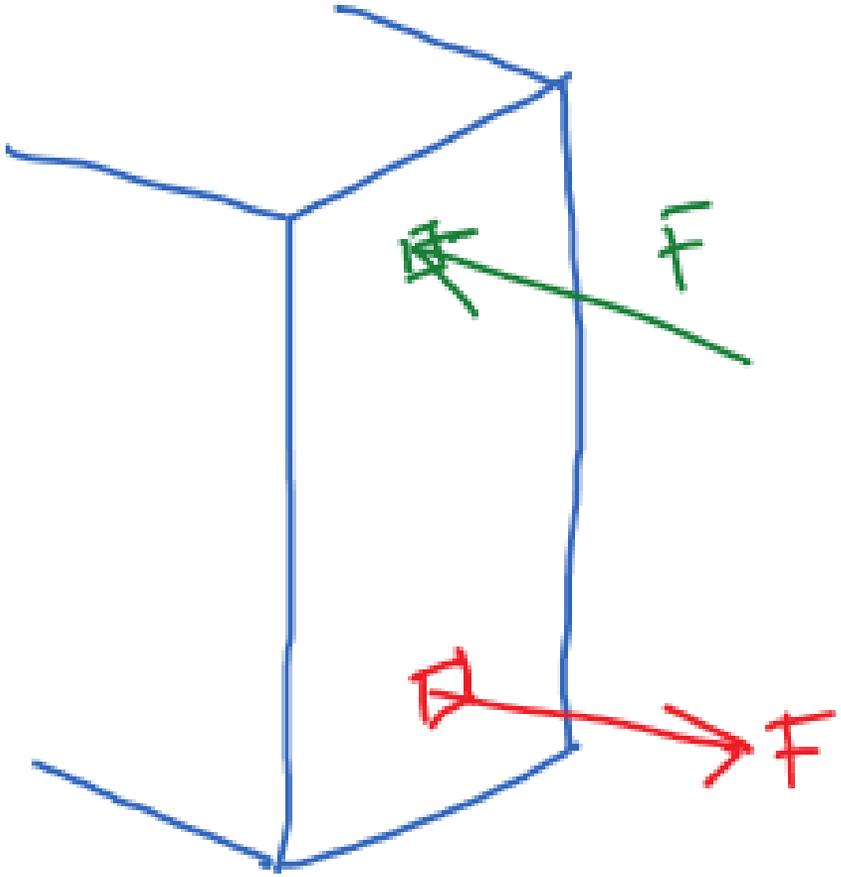
Momento fletor

Traciona as fibras inferiores da barra

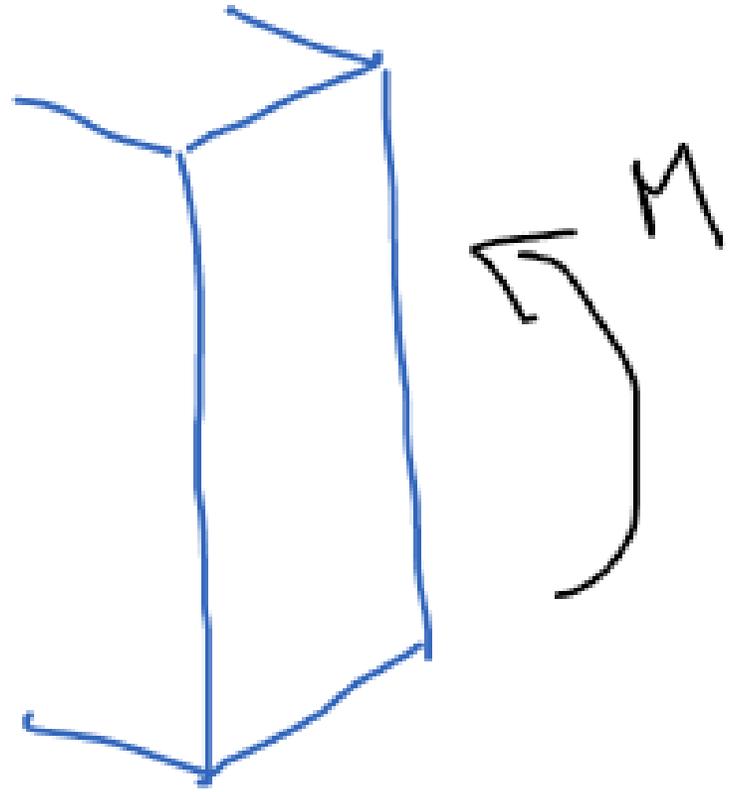
$M > 0$

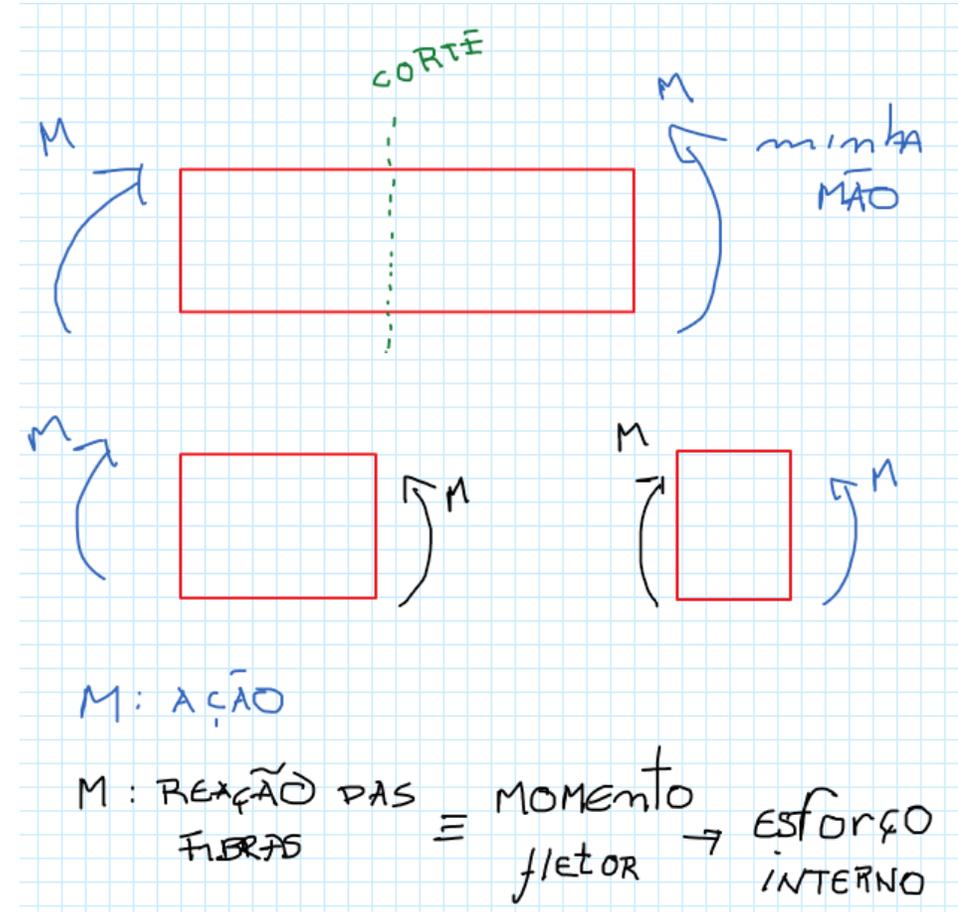
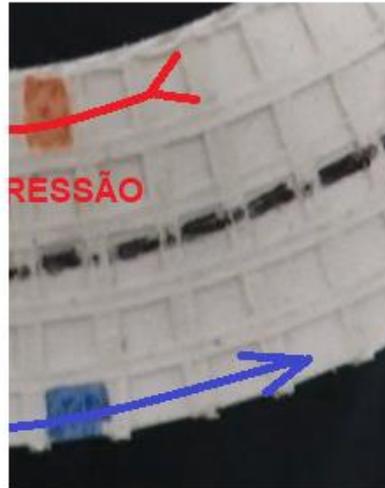
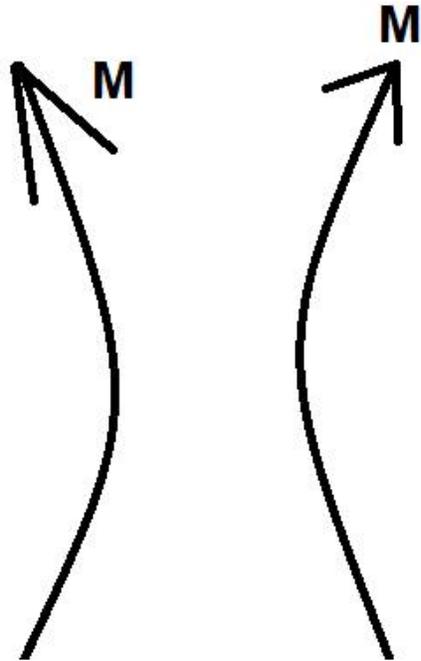
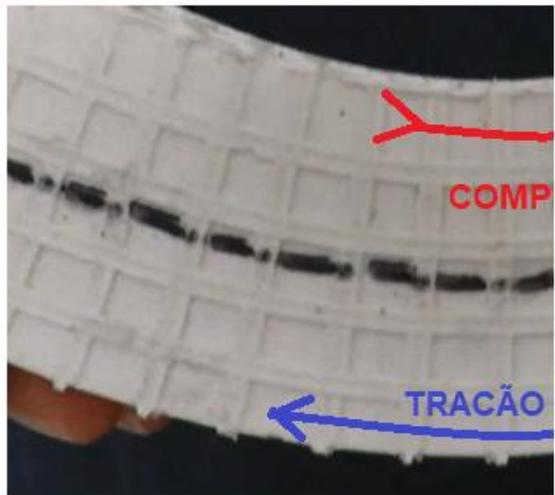
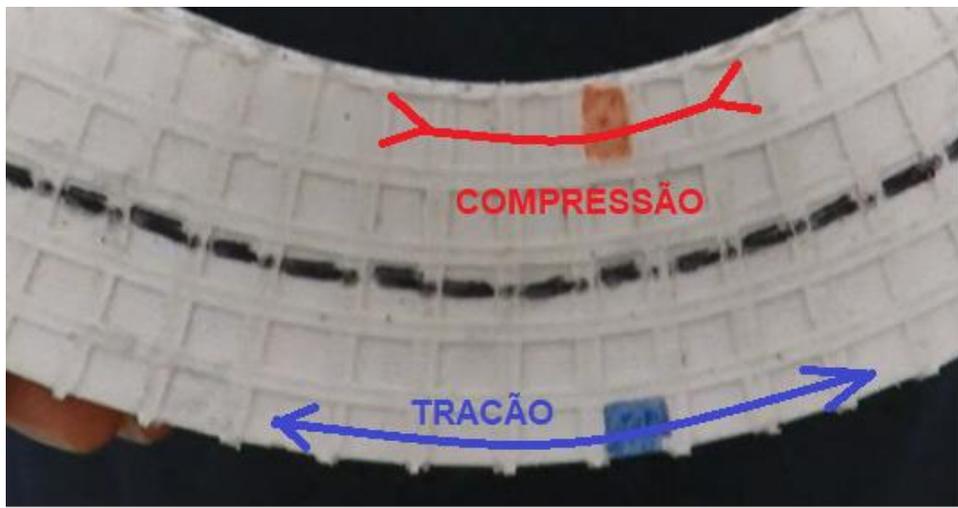
Traciona as fibras superiores da barra

$M < 0$



|||

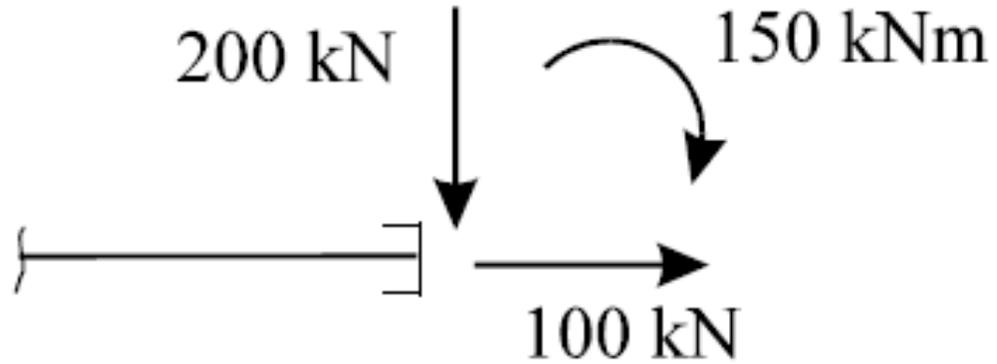




Momento traciona as fibras inferiores, em ambos os cortes

Exemplo 8

Corte à direita

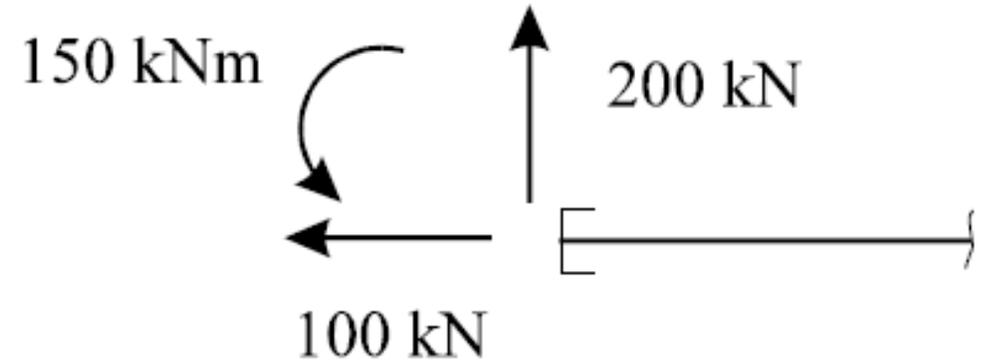


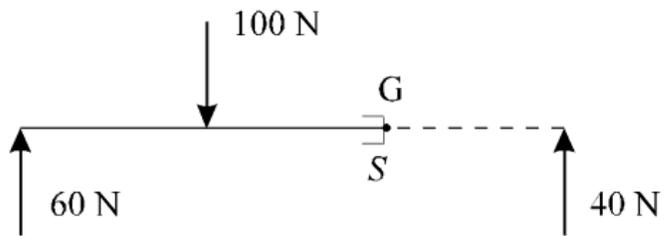
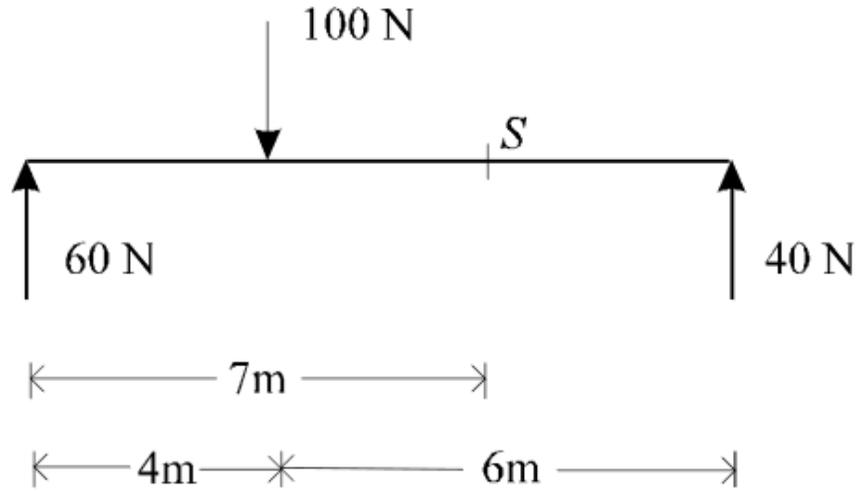
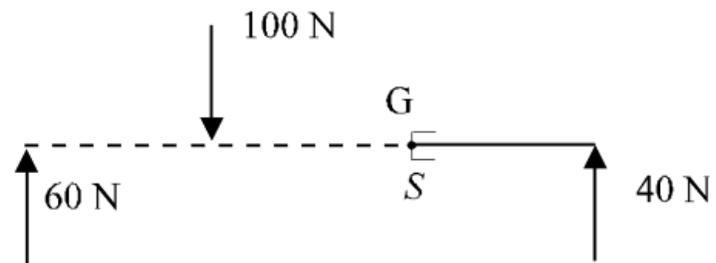
$$N = 100 \text{ kN}$$

$$V = 200 \text{ kN}$$

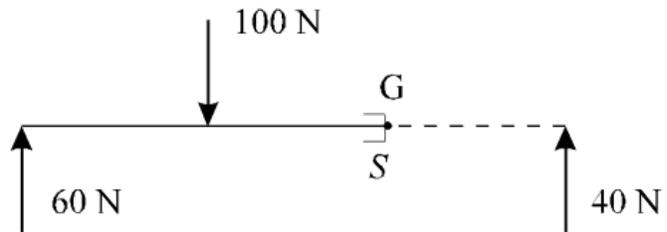
$$M = -150 \text{ kNm}$$

Corte à esquerda

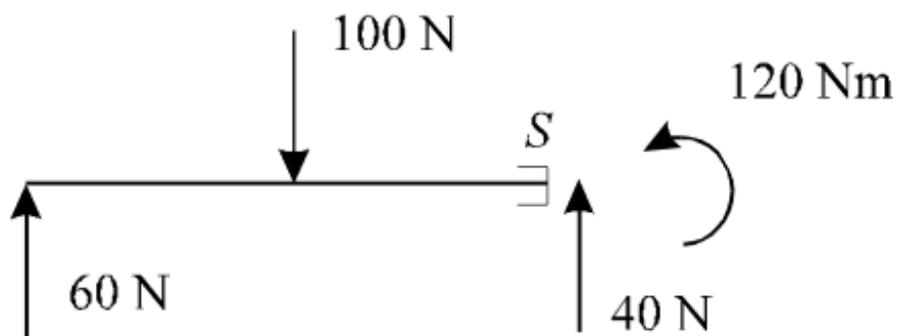


Exemplo 9:**Obter esforços na seção S****Corte à esquerda****Corte à direita**

Exemplo 9:

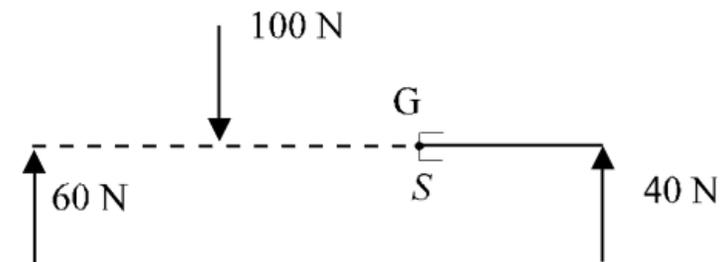


Corte à esquerda

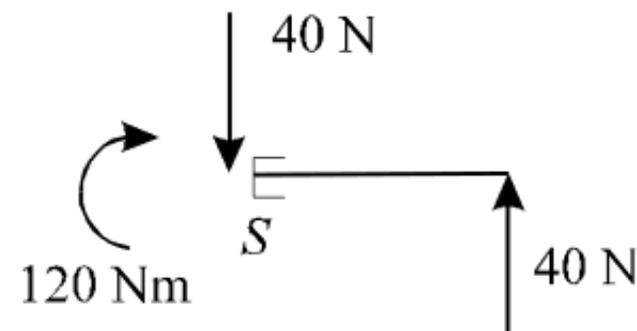


Handwritten free body diagram of the left part of the beam. It shows a horizontal line with an upward arrow of 60 at the left end, a downward arrow of 100 at 3m from the right end, and an upward arrow of 40 at the right end. A counter-clockwise curved arrow labeled 'M' is at the right end. Below the line, a dimension line shows a total length of 7m, divided into 4m and 3m segments.

$$\sum M_S = 0 \quad (+)$$
$$M + 100 \cdot 3 - 60 \cdot 7 = 0$$
$$M = 120 \text{ Nm}$$



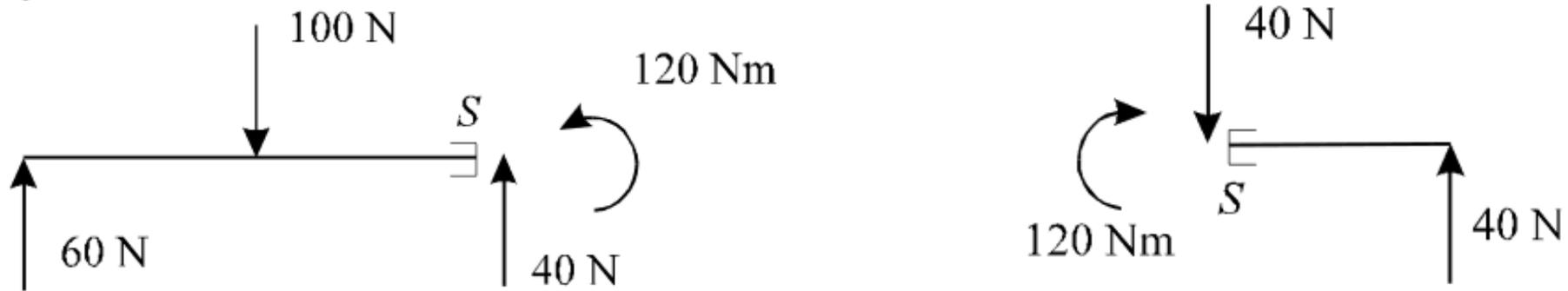
Corte à direita



Handwritten free body diagram of the right part of the beam. It shows a horizontal line with a downward arrow of 40 at the left end, an upward arrow of 40 at the right end, and a clockwise curved arrow labeled 'M' at the left end. Below the line, a dimension line shows a length of 3m.

$$\sum M_S = 0 \quad (+)$$
$$40 \cdot 3 - M = 0$$
$$M = 120 \text{ Nm}$$

Exemplo 9:



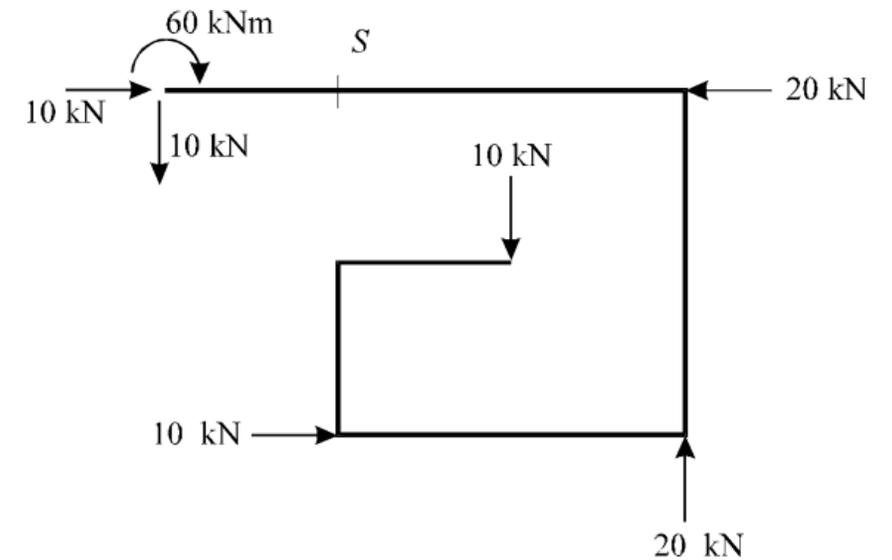
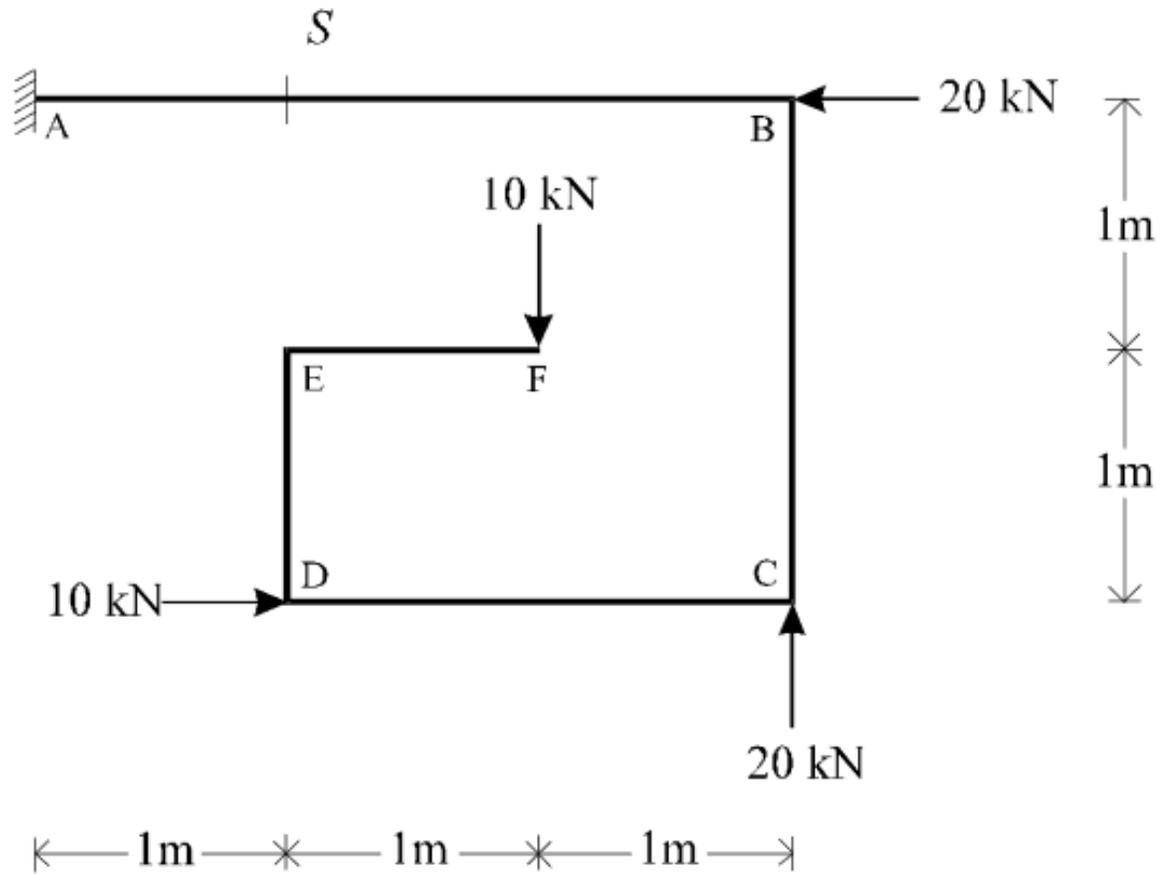
Em ambos os cortes:

Momento traciona as fibras inferiores

Cortante gira a seção no sentido anti-horário

Exemplo 10:

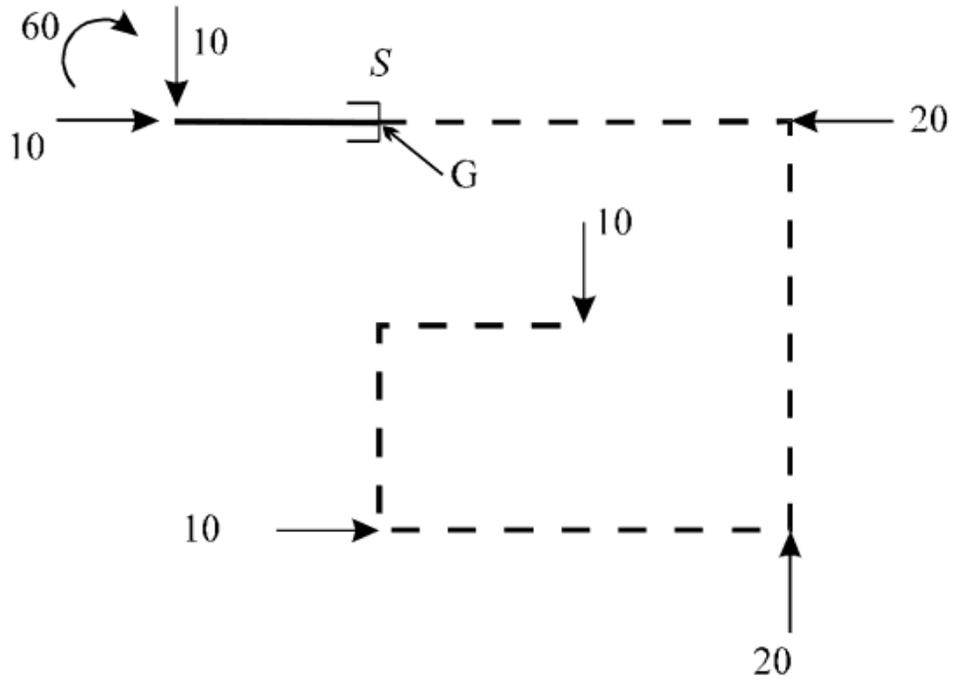
Obter os esforços na seção S



a) Cálculo das reações:

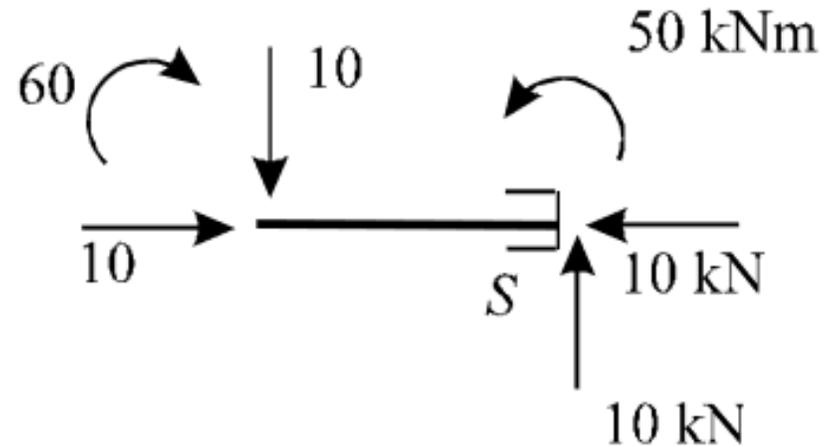
Exemplo 10:

b) Corte à esquerda:

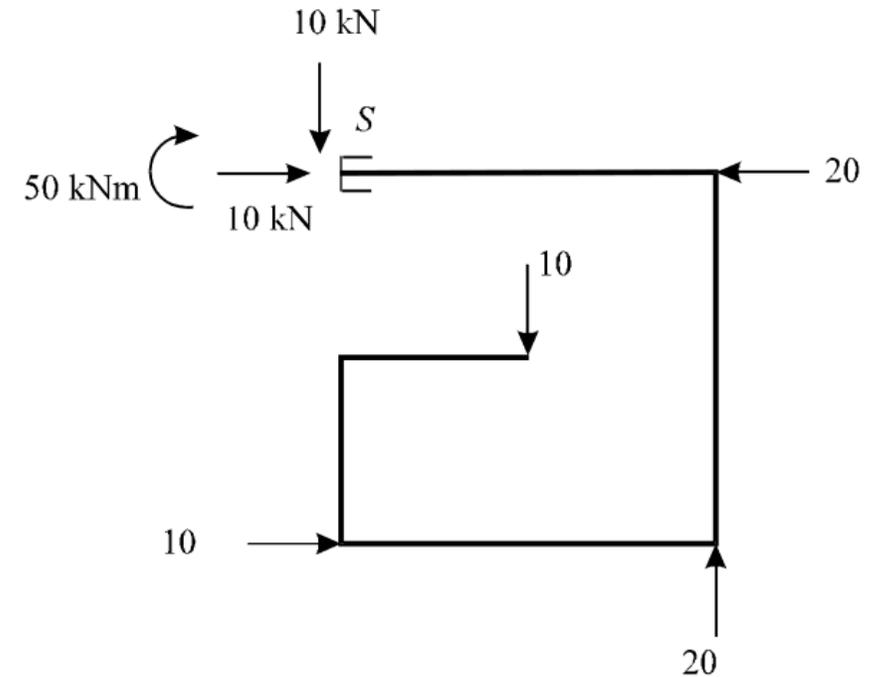
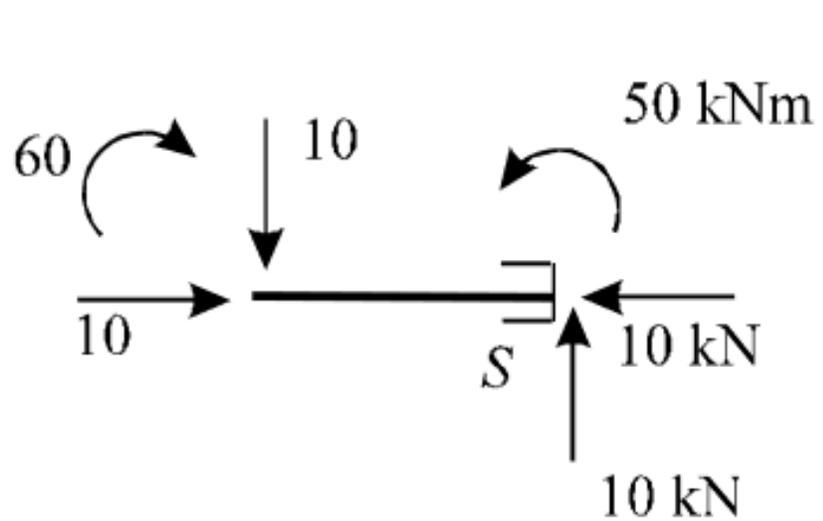


Realizando equilíbrio estático no corpo cortado:

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_s = 0$$



Exemplo 10: A seção S está com os seguintes esforços solicitantes:

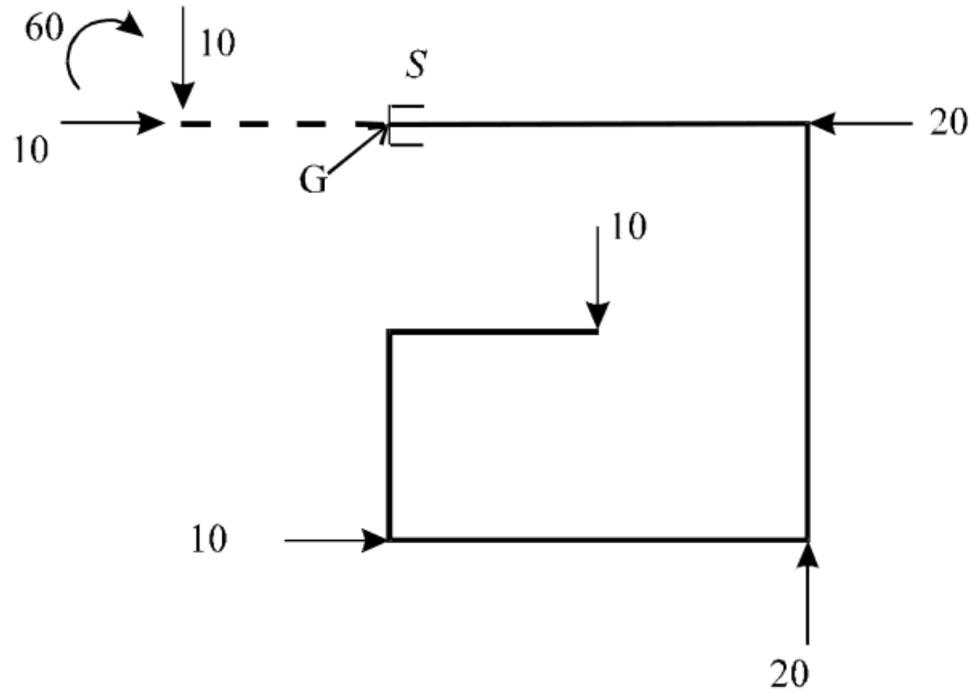


$$\begin{aligned} N_s &= -10 \text{ kN} \\ V_s &= -10 \text{ kN} \\ M_s &= 50 \text{ kNm} \end{aligned}$$

(sinal conforme convenção)

Exemplo 10:

c) Corte à direita:

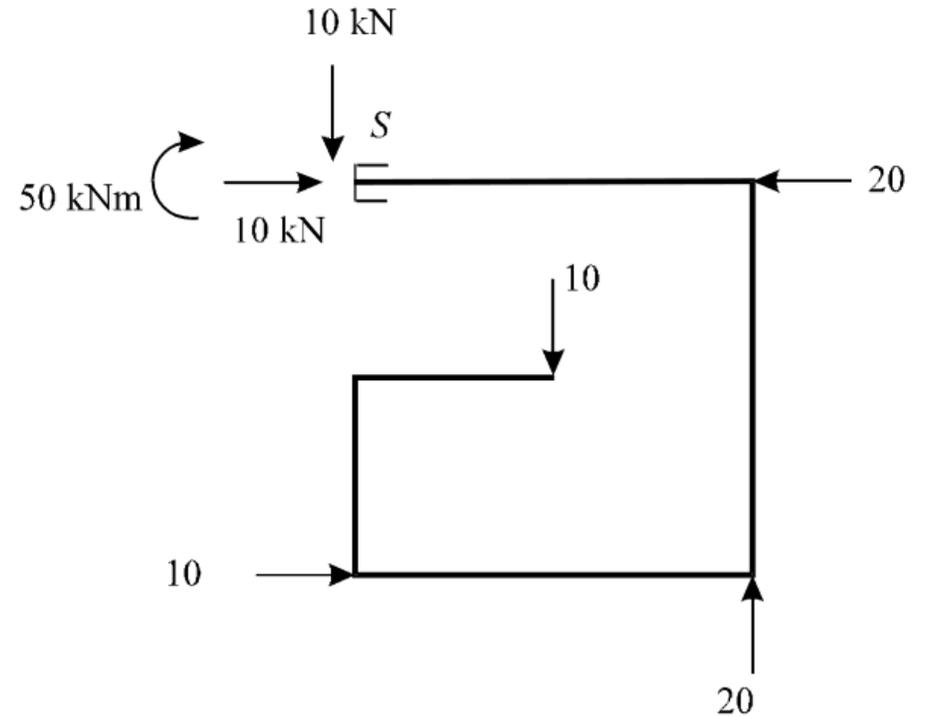


$$\begin{aligned} N_s &= -10 \text{ kN} \\ V_s &= -10 \text{ kN} \\ M_s &= 50 \text{ kNm} \end{aligned}$$

(sinal conforme convenção)

Realizando equilíbrio estático no corpo cortado:

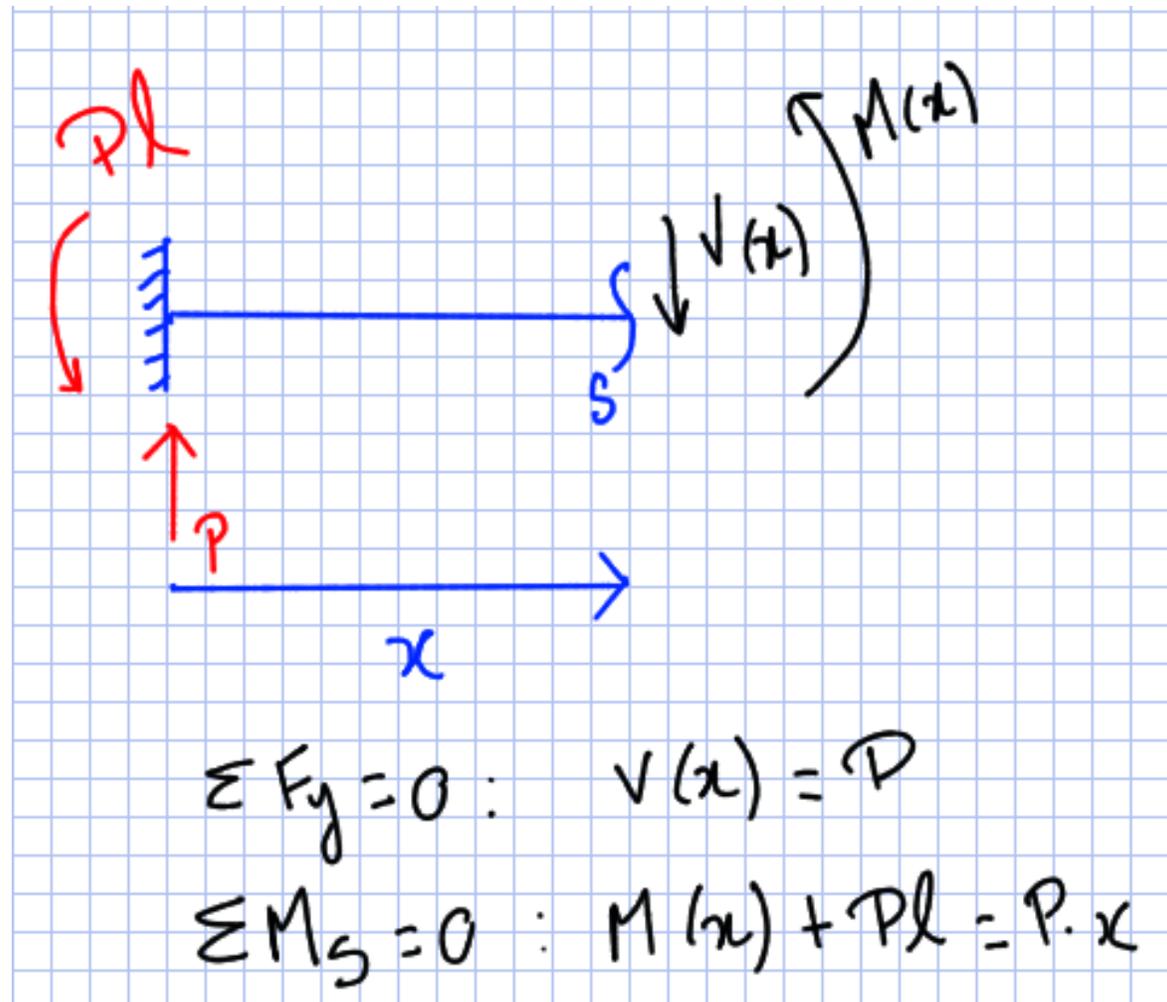
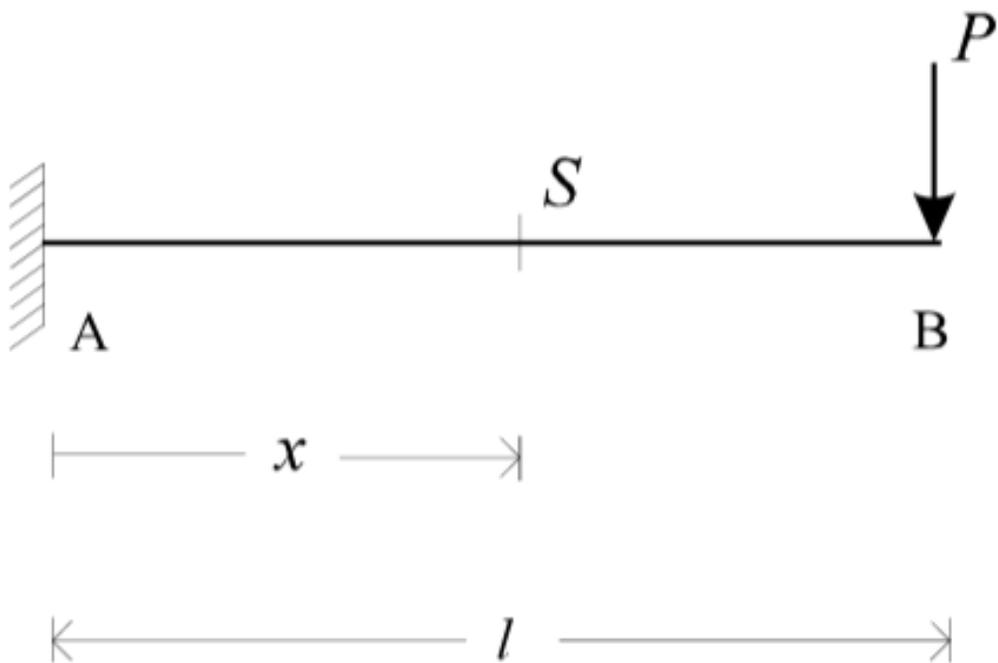
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_s = 0$$



Diagramas de esforços

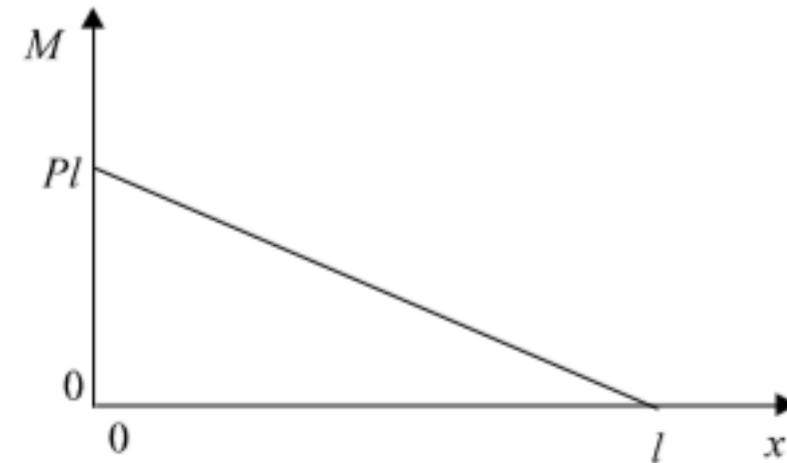
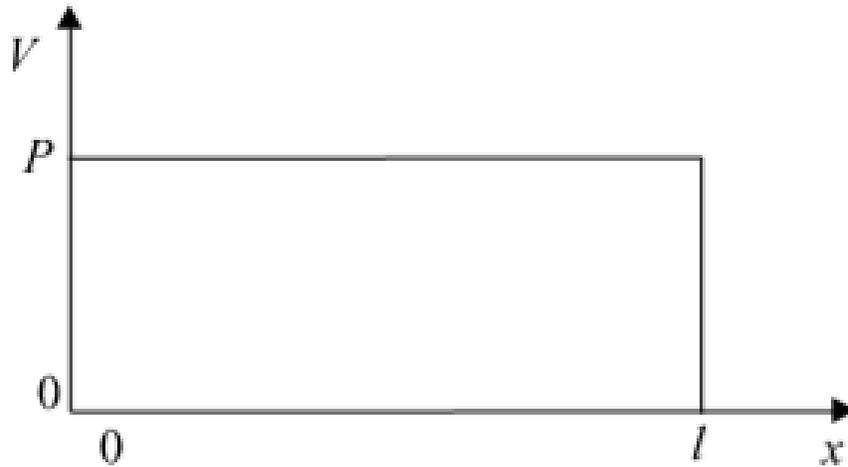
Seja a viga em balanço:

Corte S numa seção x :



Diagramas de esforços

Gráficos que mostram a variação da força cortante e do momento fletor ao longo da estrutura



$$0 \leq x \leq L$$

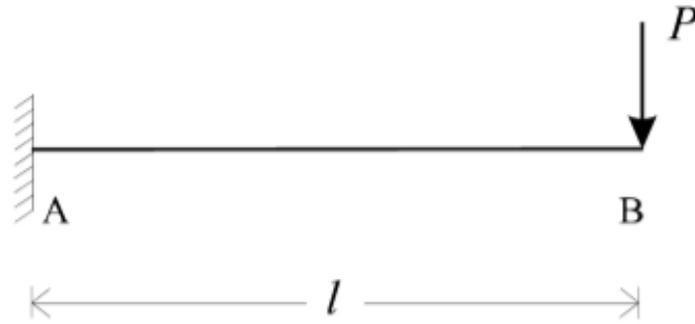
$$\sum F_y = 0 : V(x) = P$$

$$\sum M_S = 0 : M(x) + Pl = P \cdot x$$

Seção junto ao engaste é a mais solicitada

Diagramas de esforços

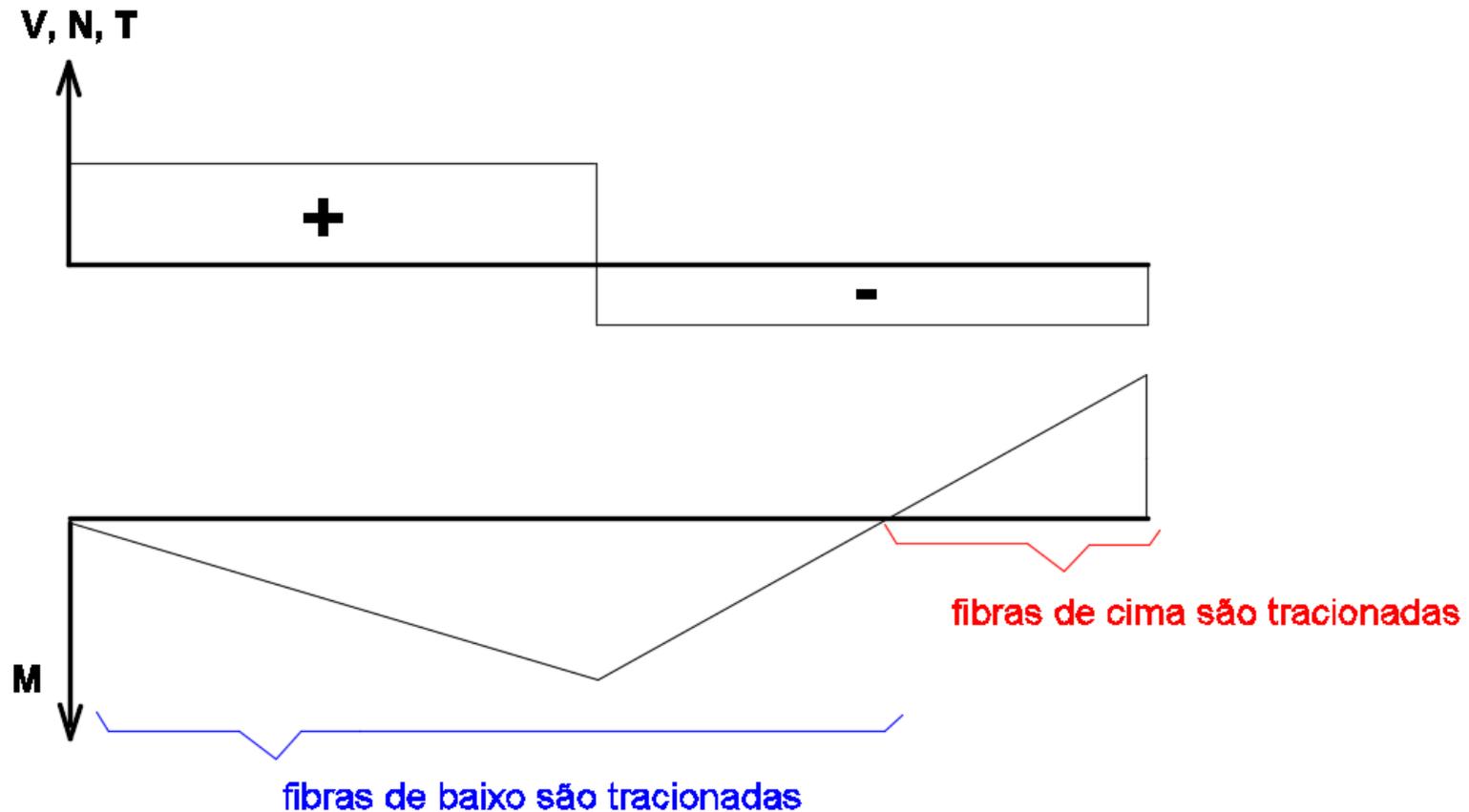
Valores plotados são na direção perpendicular, ou paralelo ao eixo, conforme definição do esforço



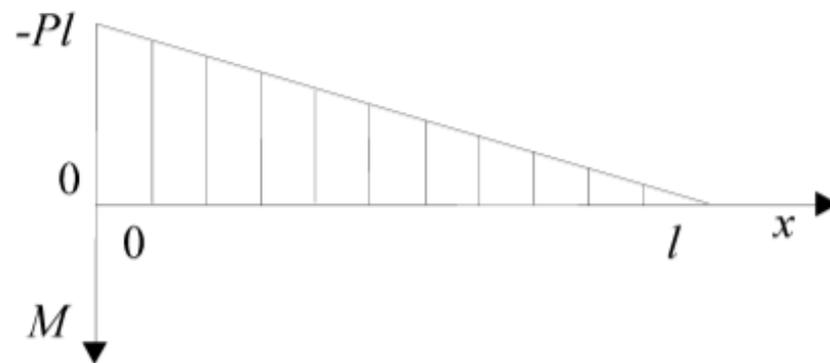
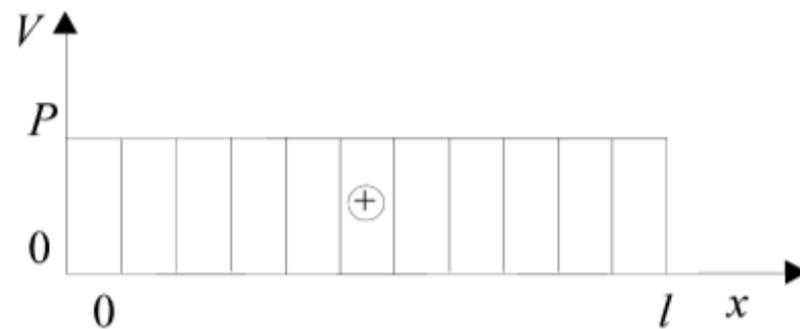
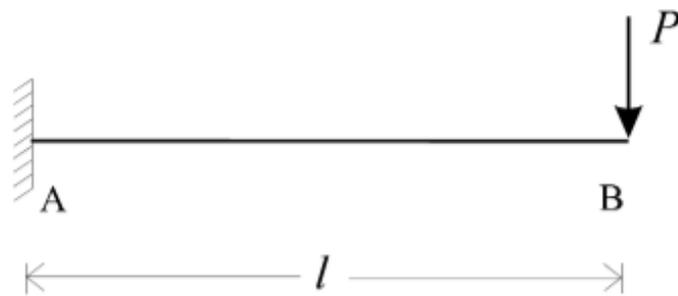
Diagramas de esforços - Desenhos

$N, V > 0$  Desenha acima do eixo

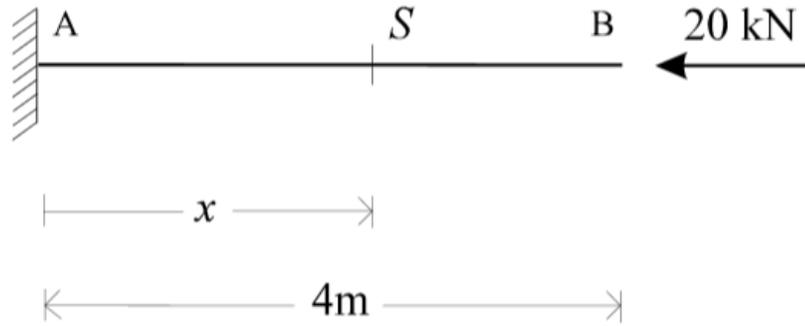
M :  Desenha o lado que traciona



Diagramas de esforços – Exemplo de diagrama



Exemplo 11: Força horizontal



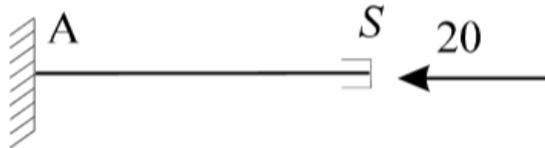
(a)

Por equilíbrio estático
no corpo cortado:

$$N(x) = -20$$

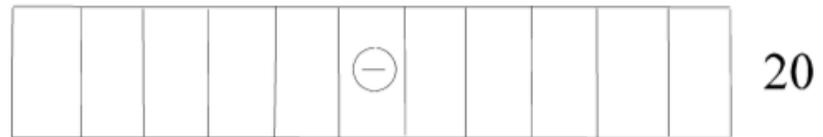
$$V(x) = 0$$

$$M(x) = 0$$



Diagramas

N (kN)



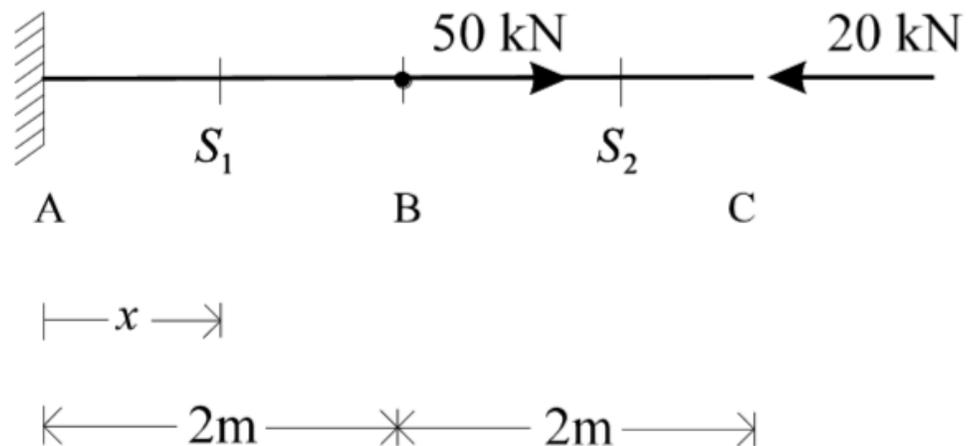
V (kN)



M (kNm)

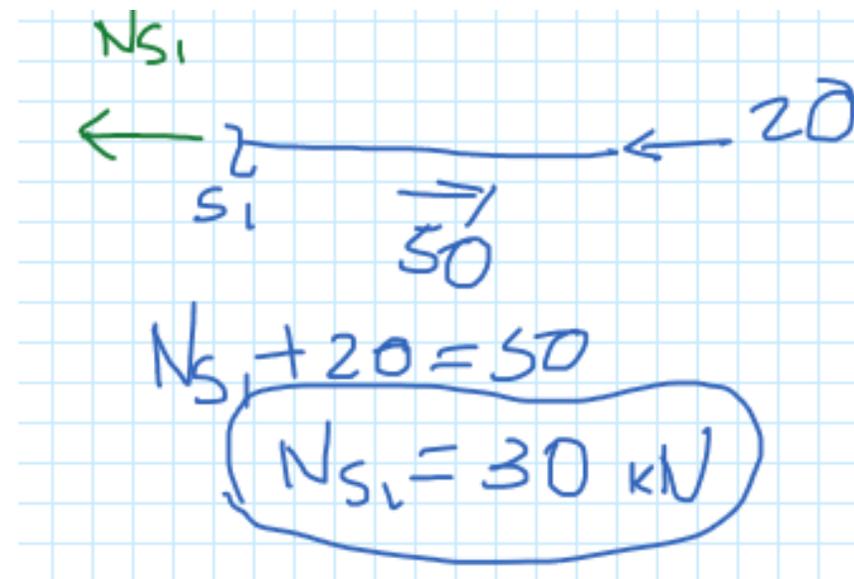
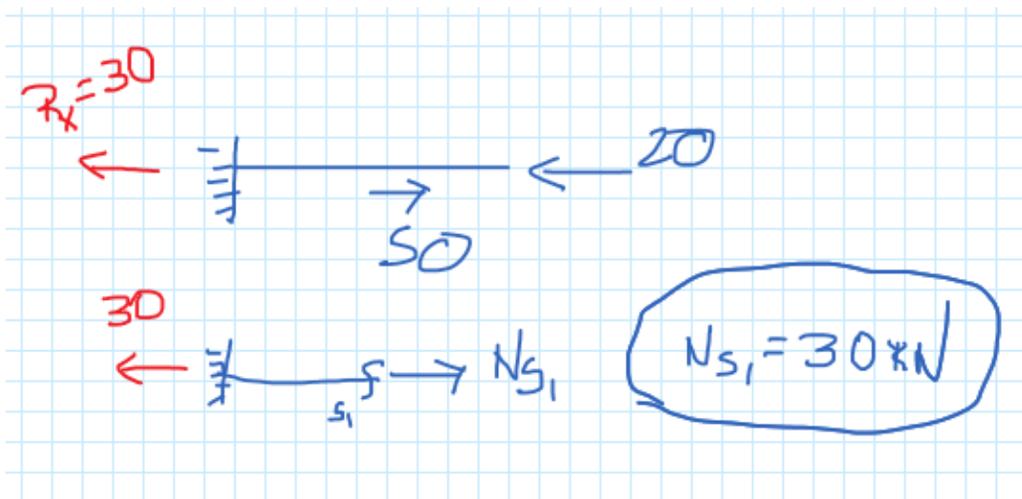


Exemplo 12: Duas forças horizontais



Note que entre AB e BC há uma força horizontal concentrada.

Trecho S_1 :



Exemplo 12: Duas forças horizontais

Trecho S_2 :

$$R_{TL} = 30$$



$$\sum F_x = 0: N_{S_2} + 50 = 30$$

$$N_{S_2} = -20 \text{ kN}$$



$$\sum F_x = 0: N_{S_2} = -20 \text{ kN}$$

Exemplo 12: Duas forças horizontais

Por equilíbrio estático
no corpo cortado:

- trecho AB: $0 \leq x < 2\text{m}$

$$N(x) = -20 + 50 = 30$$

$$V(x) = 0$$

$$M(x) = 0$$

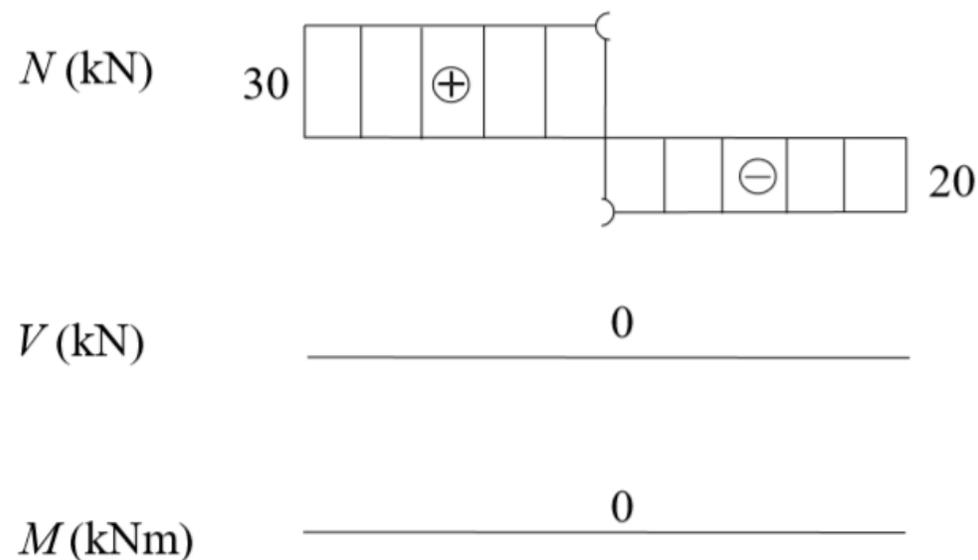
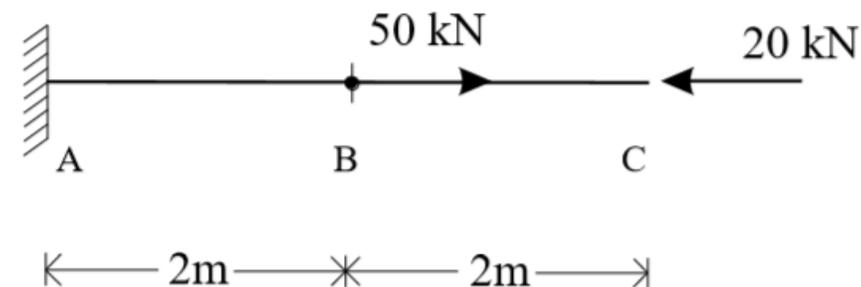
- trecho BC: $2\text{m} < x \leq 4\text{m}$

$$N(x) = -20$$

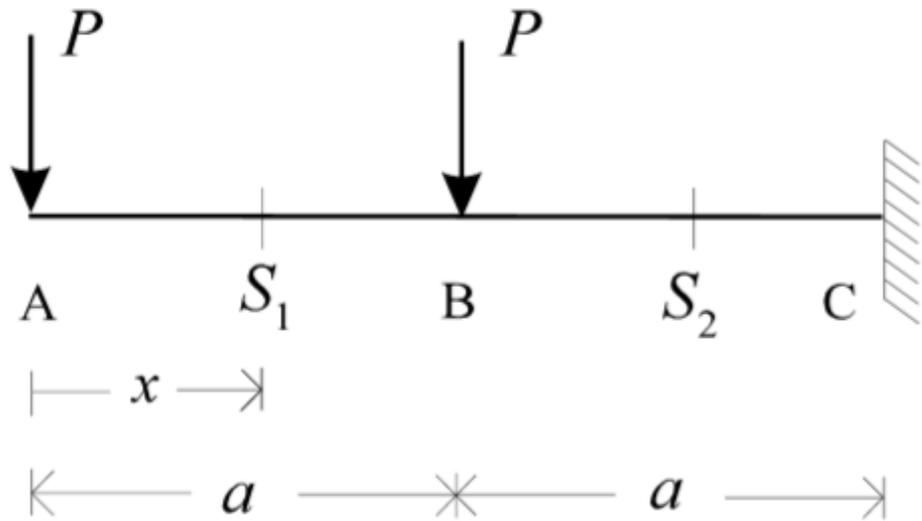
$$V(x) = 0$$

$$M(x) = 0$$

Diagramas

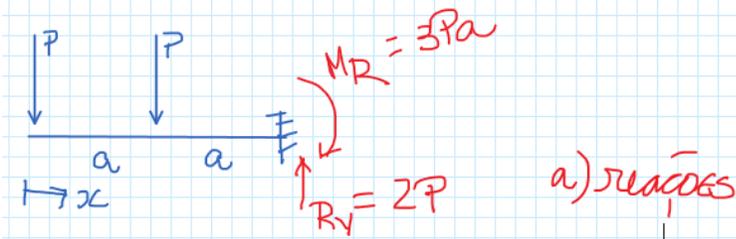


Exemplo 13: Duas forças verticais



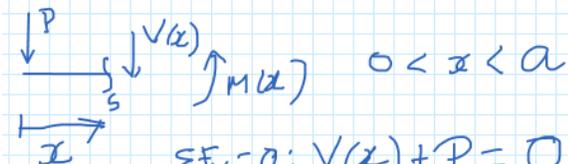
Note que entre AB e BC há uma força vertical concentrada.

Dividir corte em 2 trechos (S_1 e S_2):



a) reações

$$\sum M_C = 0: M_R = 2Pa + Pa = 3Pa$$

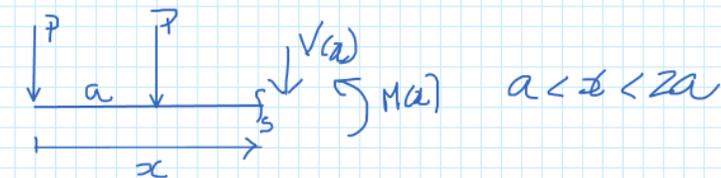


$$\sum F_y = 0: V(x) + P = 0$$

$$V(x) = -P$$

$$\sum M_S = 0: M(x) + P \cdot x = 0$$

$$M(x) = -P \cdot x \quad \left. \begin{array}{l} M(0) = 0 \\ M(a) = -Pa \end{array} \right\}$$



$$\sum F_y = 0: P + P + V(x) = 0$$

$$V(x) = -2P$$

$$\sum M_S = 0: M(x) + P \cdot (x - a) + P \cdot x = 0$$

$$M(x) = -2Px + P \cdot a \quad \left. \begin{array}{l} M(a) = -2Pa \\ M(2a) = -3Pa \end{array} \right\}$$

Exemplo 13: Duas forças verticais

- trecho AB $0 \leq x < a$

$$N(x) = 0$$

$$V(x) = -P$$

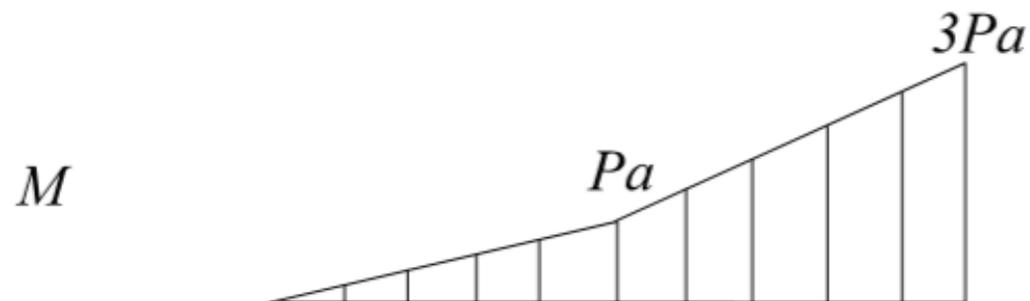
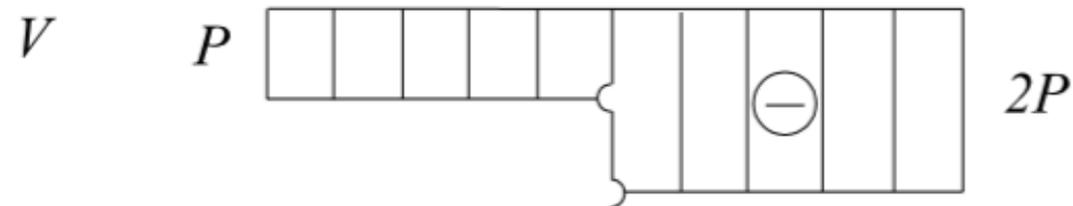
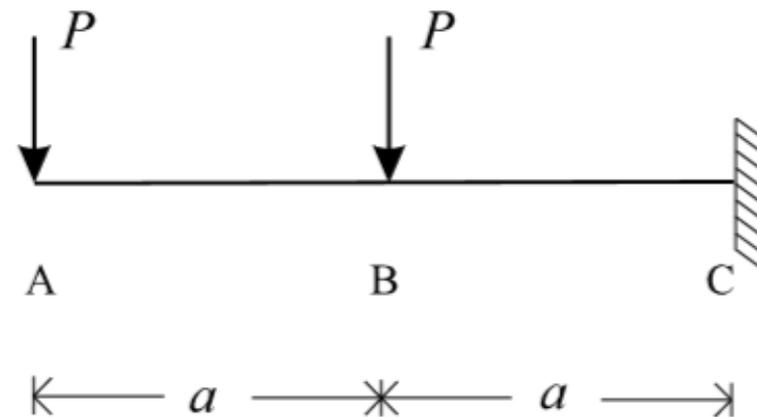
$$M(x) = -Px$$

- trecho BC $a < x \leq 2a$

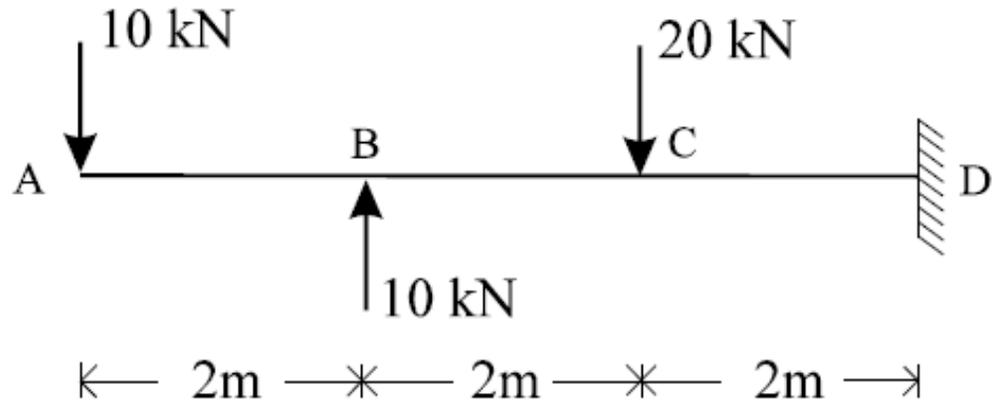
$$N(x) = 0$$

$$V(x) = -2P$$

$$M(x) = -Px - P(x - a)$$



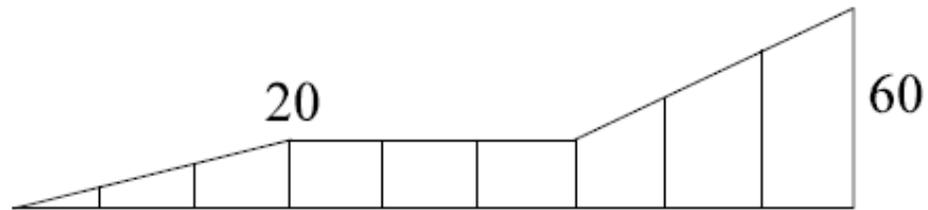
Exemplo 14: Três forças verticais



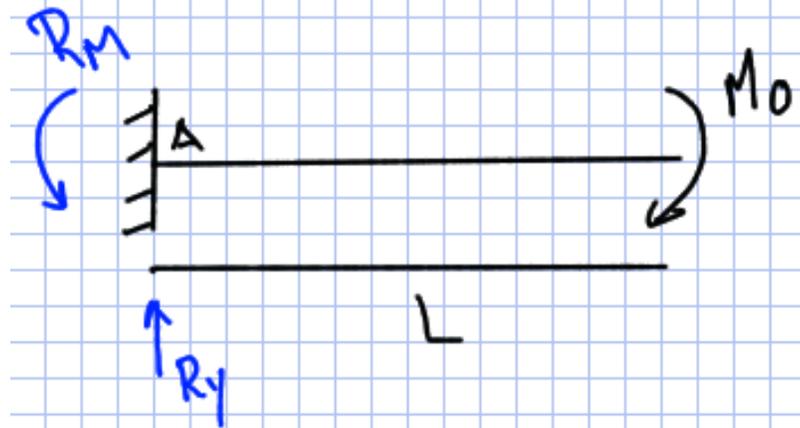
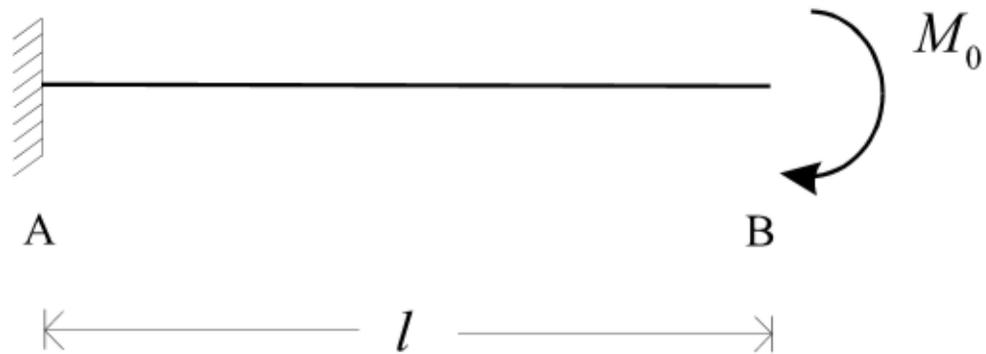
V (kN)



M (kNm)

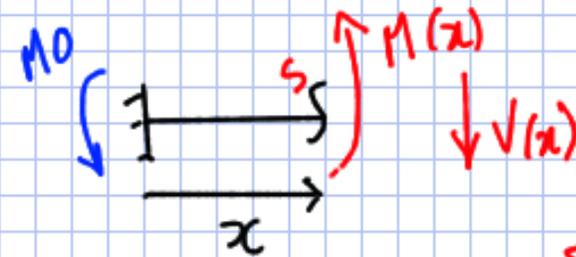


Exemplo 15: Momento concentrado



$$\sum F_y = 0: R_y = 0$$

$$\sum M_A = 0: R_M = M_0$$



$$\sum F_y = 0$$

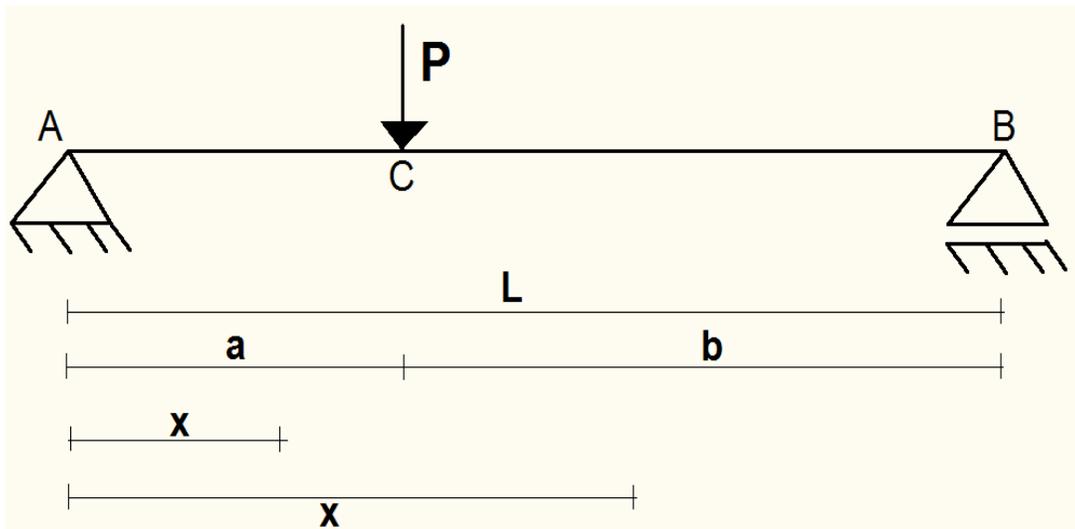
$$V(x) = 0$$

$$\sum M_S = 0$$

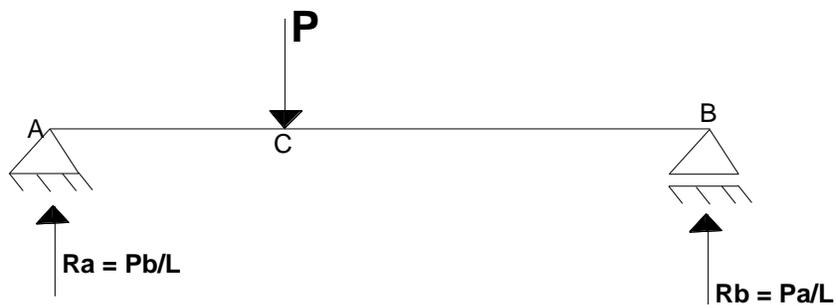
$$M(x) + M_0 = 0$$

$$M(x) = -M_0$$

Exemplo 16: Viga bi-apoiada



1. Obter reações:



2. Esforços em cada trecho:

Determinação das equações nos cortes de cada trecho:

Trecho 1: $0 < x < a$

$$\sum F_y = 0$$

$$R_a - V(x) = 0 \rightarrow V(x) = R_a$$

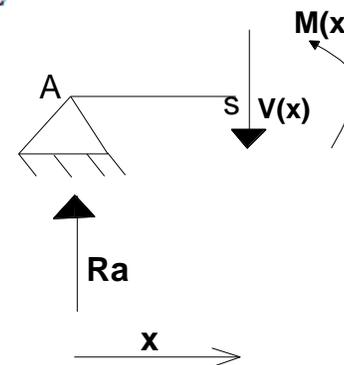
$$V(x) = P \cdot b / L \text{ (constante)}$$

$$\sum M_s = 0$$

$$M(x) - R_a \cdot x = 0 \rightarrow M(x) = R_a \cdot x$$

$$M(x) = P \cdot b \cdot x / L \text{ (reta)}$$

$$\text{Para } x = a : M(a) = P \cdot b \cdot a / L$$



Exemplo 16: Viga bi-apoiada

Trecho 2: $a < x < L$

$$\sum F_y = 0$$

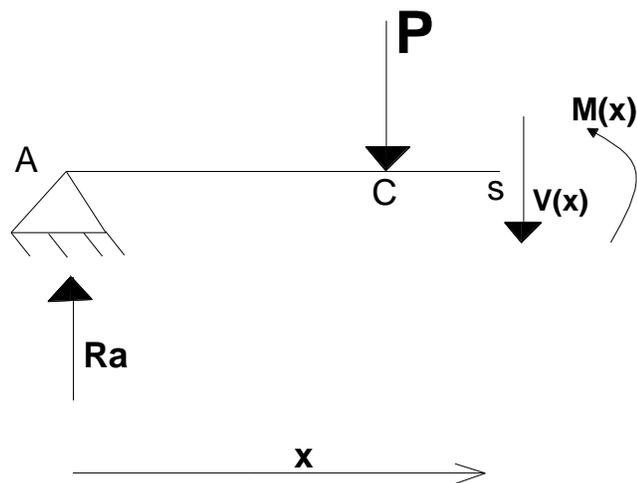
$$R_a - P - V(x) = 0 \rightarrow V(x) = R_a - P = P \cdot b/L - P = P(b/L - 1) = -P \cdot a/L$$

$$V(x) = -P \cdot a/L \text{ (constante)}$$

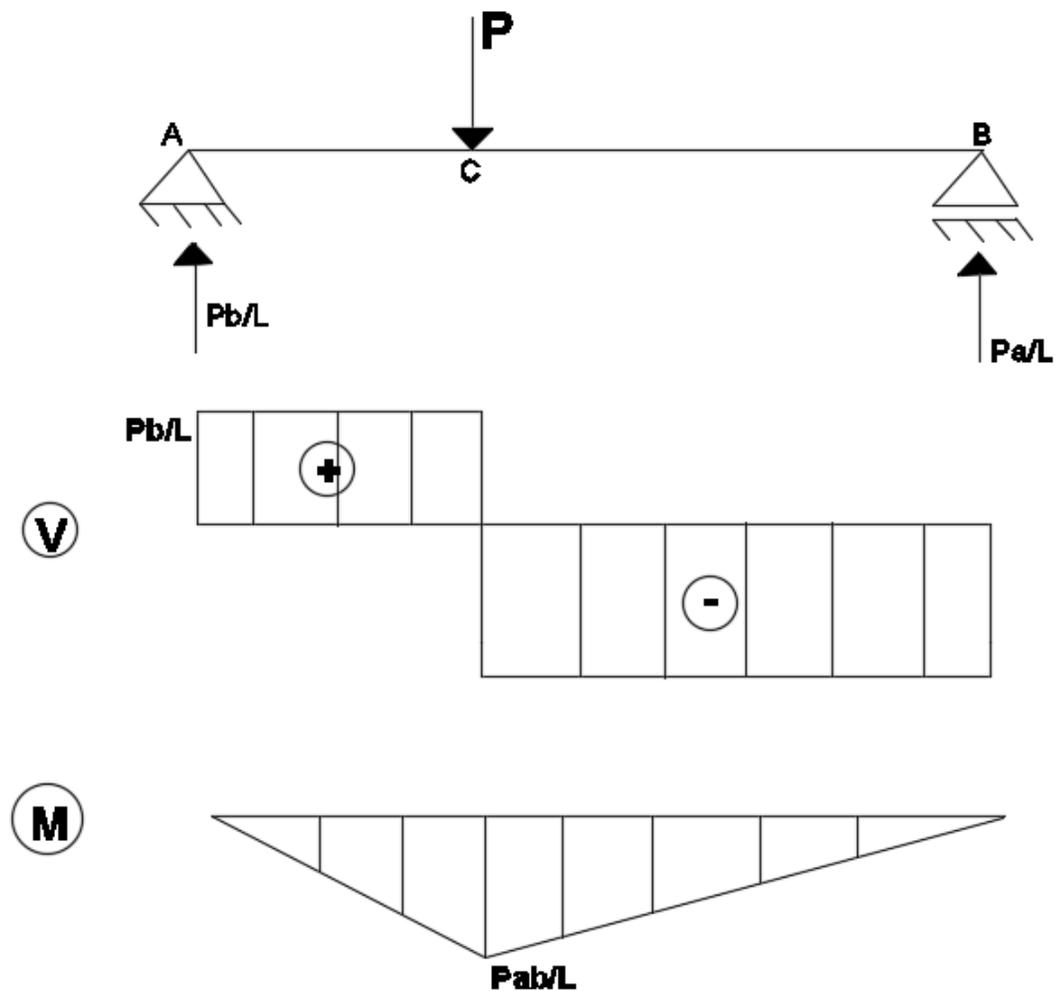
$$\sum M_s = 0$$

$$M(x) + P \cdot (x - a) - R_a \cdot x = 0 \rightarrow M(x) = P \cdot b \cdot x/L - P(x - a)$$

$$M(x) = P \cdot a - (P \cdot a/L) \cdot x \text{ (reta)}$$



3. Diagramas:



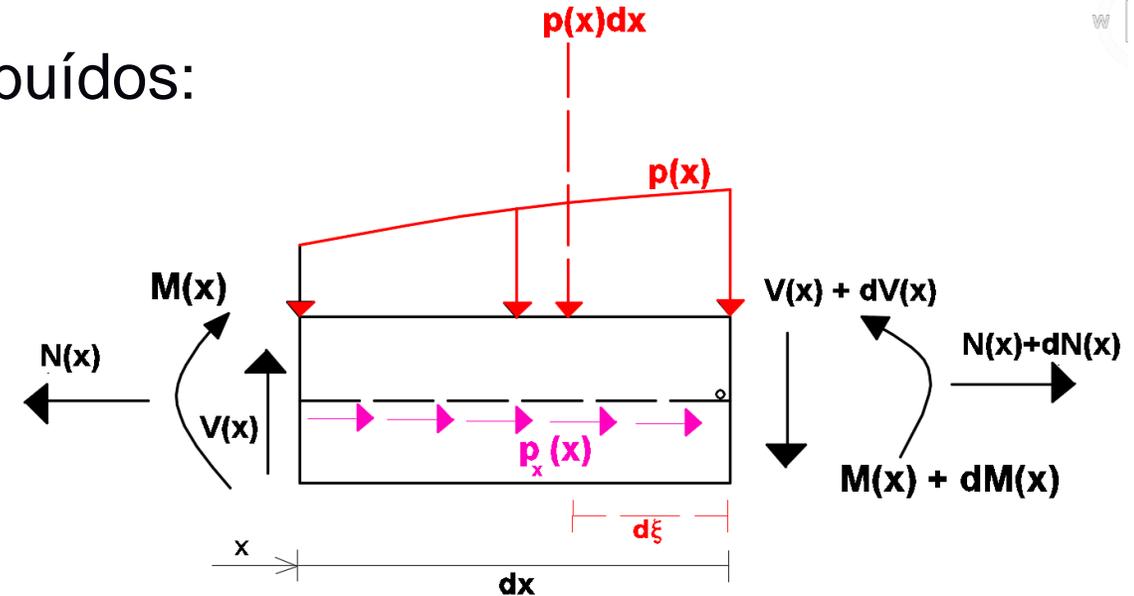
Equação Diferencial de Equilíbrio

Seja um trecho da barra sujeito a carreg. distribuídos:

a) Paralelo ao eixo, p_x

$$\sum F_x = 0 \quad dN(x) + p_x(x)dx = 0$$

$$\frac{dN(x)}{dx} = -p_x(x)$$



b) Perpendicular ao eixo, p

$$\sum F_y = 0$$

$$\sum M_o = 0$$

$$(M + dM) + [p(x) \cdot dx] \cdot d\xi - M - V \cdot dx = 0$$

$$V - p(x) \cdot dx - (V + dV) = 0$$

$$\frac{dM(x)}{dx} = V(x)$$

$$\frac{dV(x)}{dx} = -p(x)$$

$$\frac{d^2 M(x)}{dx^2} = -p(x)$$

Equação Diferencial de Equilíbrio

$$\frac{dV(x)}{dx} = -p(x) \quad \frac{d^2M(x)}{dx^2} = -p(x) \quad \frac{dM(x)}{dx} = V(x)$$

a) Caso $p(x) = 0$

Sem carga distribuída no trecho $x_1 < x < x_2$

$V(x) = C_1 = cte \rightarrow$ Função (diagrama) de esforço cortante constante

$M(x) = C_1 \cdot x + C_2 \rightarrow$ Função (diagrama) de momento fletor linear

b) Caso $p(x) = p = cte$

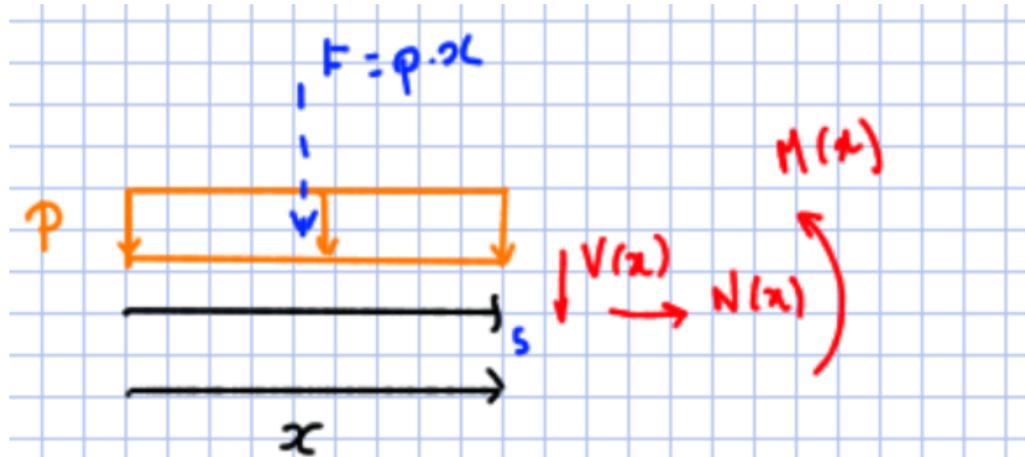
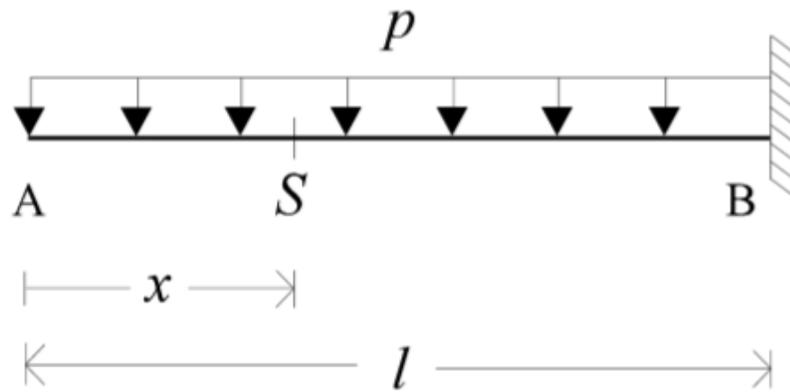
Carga distribuída uniforme no trecho $x_1 < x < x_2$

$V(x) = -px + C_1 \rightarrow$ Função (diagrama) de esforço cortante linear

$M(x) = -p \frac{x^2}{2} + C_1 \cdot x + C_2 \rightarrow$ Função (diagrama) de momento fletor é parábola

c) Generalização para $\forall p(x)$ é imediata

Exemplo 17: Carregamento distribuído constantemente



$$\sum F_x = 0: N(x) = 0$$

$$\sum F_y = 0: V(x) = -p \cdot x$$

$$\sum M_S = 0: M(x) + (p \cdot x) \frac{x}{2} = 0$$

$$0 < x < l$$

Exemplo 17: Carregamento distribuído constantemente

Substituindo valores dos extremos do trecho:

- $V(x) = -px$

$$V(0) = 0$$

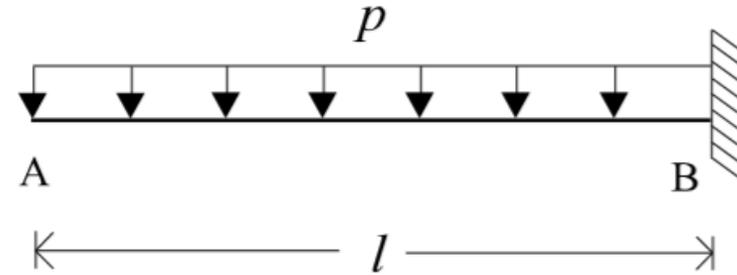
$$V(l) = -pl$$

- $M(x) = -\frac{px^2}{2}$

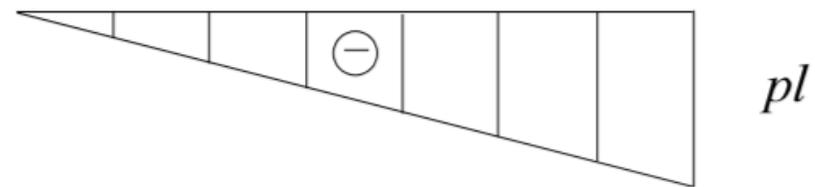
$$M(0) = 0$$

$$M(l) = -\frac{pl^2}{2}$$

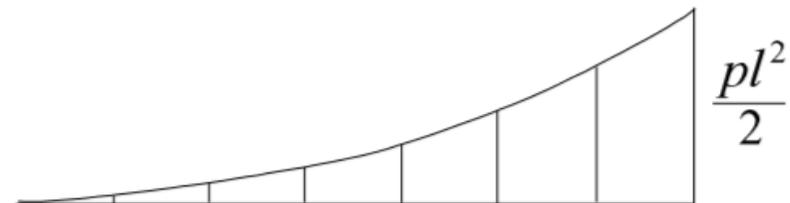
Diagramas:



V

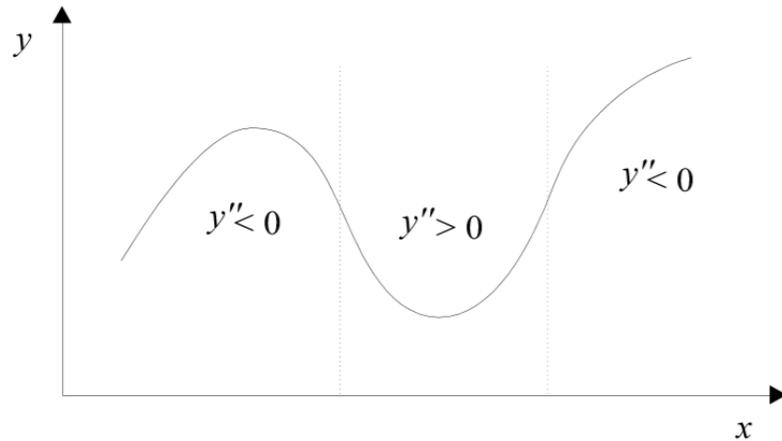


M

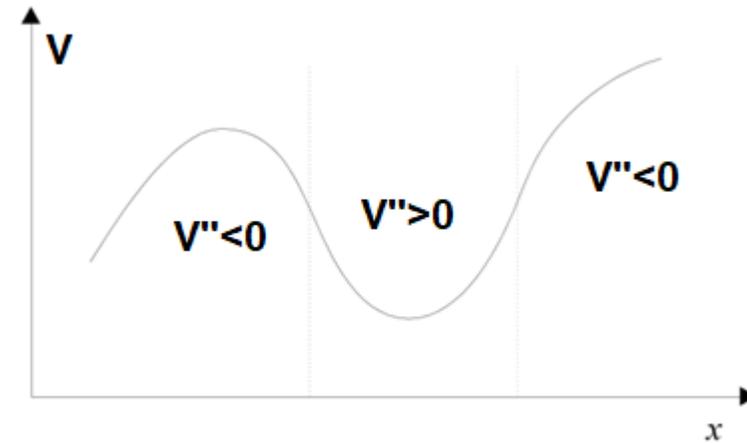


Estudo da concavidade de funções

Lembrar do cálculo:

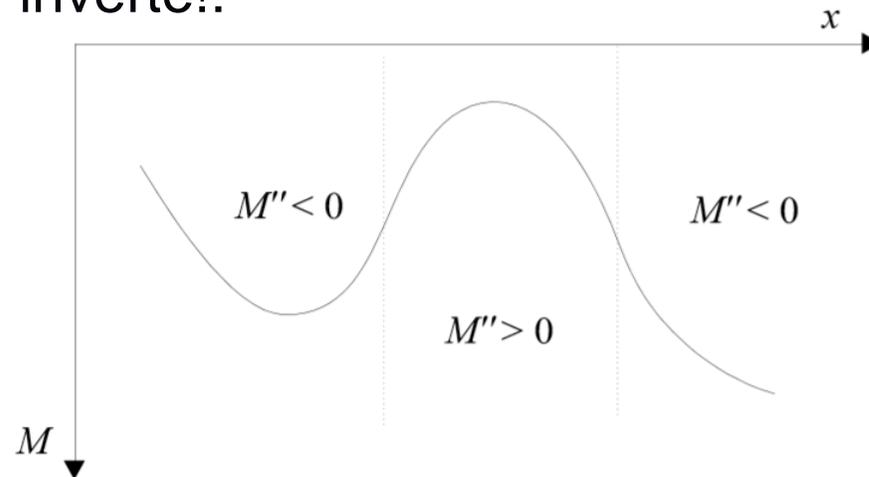


Para o cortante: imediato



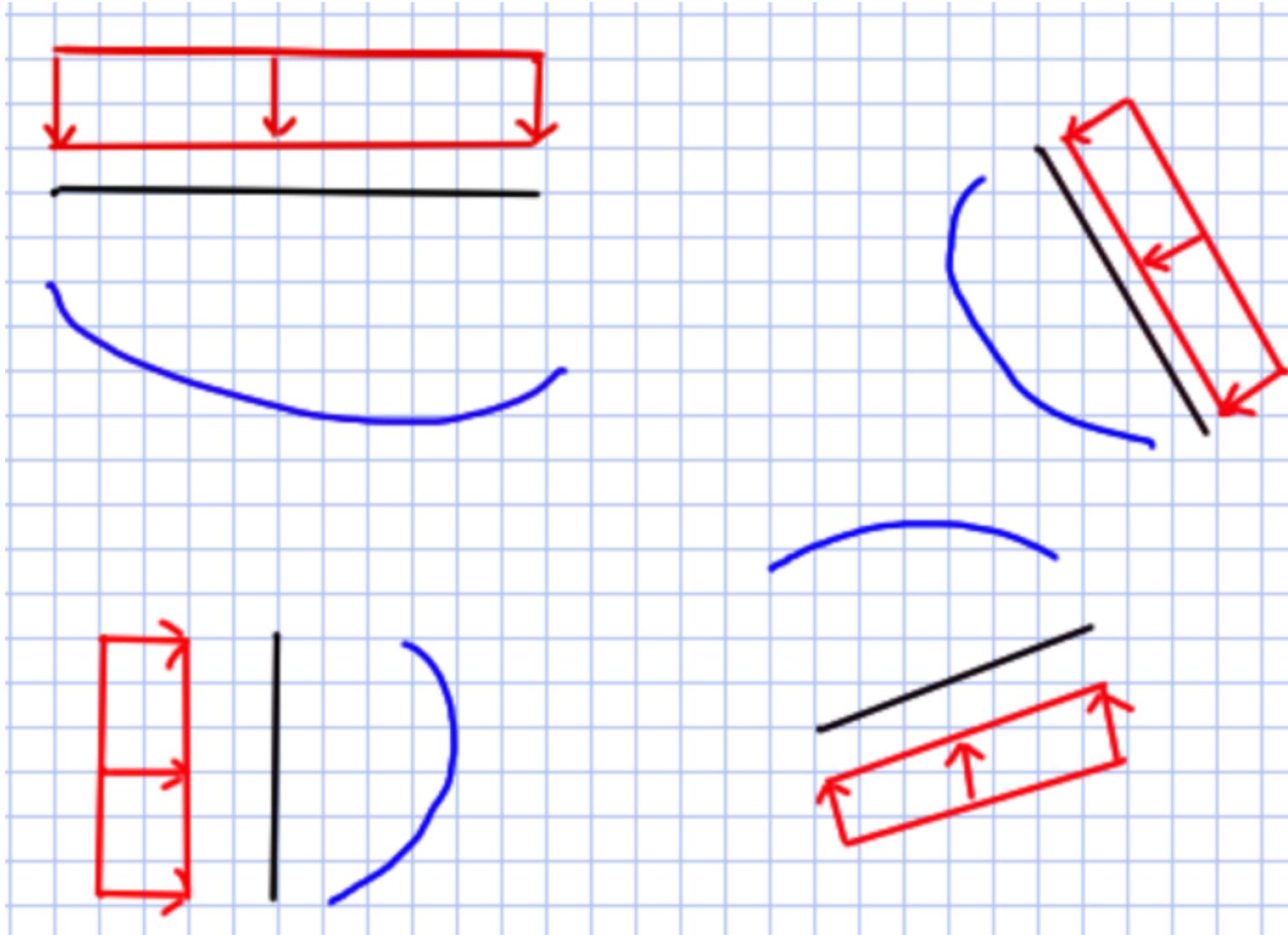
Para o momento, inverte!:

$$\frac{d^2 M(x)}{dx^2} = -p(x)$$



Estudo da concavidade de funções

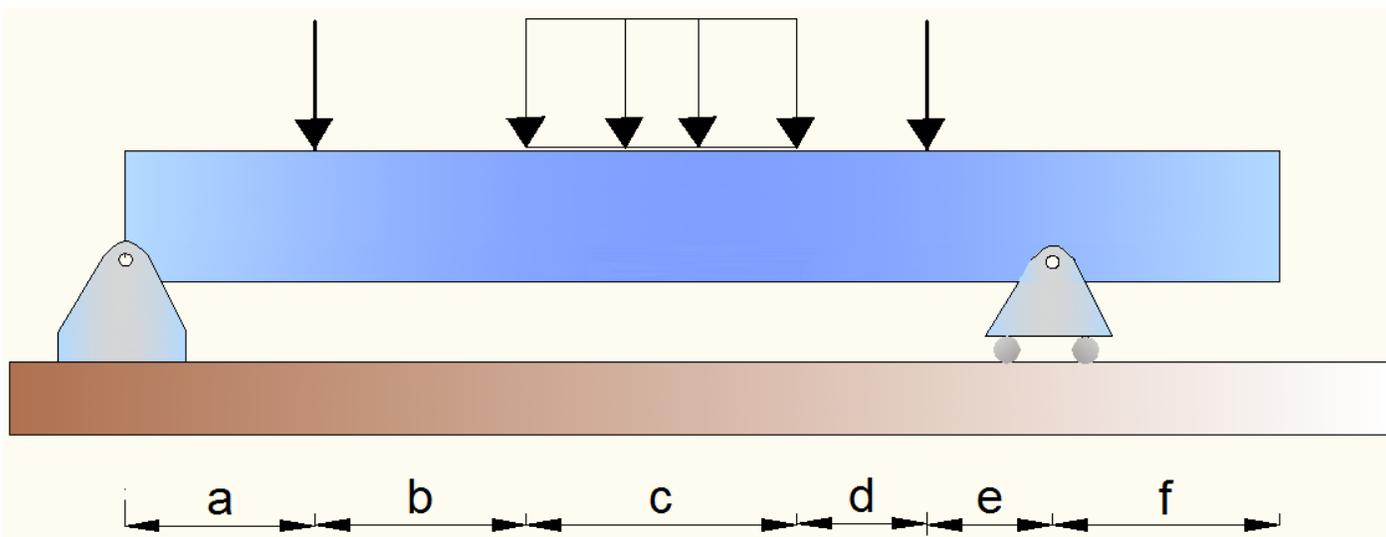
$$\frac{d^2 M(x)}{dx^2} = -p(x) < 0$$



DIAGRAMAS DE ESFORÇOS CORTANTE E MOMENTO FLETOR (DCM)

Roteiro para obter $V(x)$ e $M(x)$:

1. Dividir estrutura em trechos onde não há alteração de ações/reações;



Trecho 1: $0 < x < a$

Trecho 2: $a < x < a + b$

Trecho 3: $a + b < x < a + b + c$

Trecho 4: $a + b + c < x < a + b + c + d$

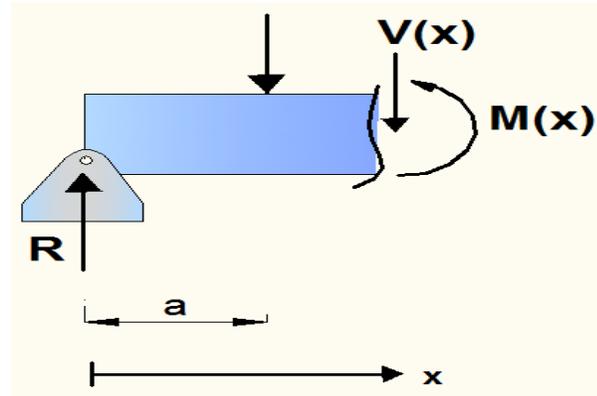
Trecho 5: $a + b + c + d < x < a + b + c + d + e$

Trecho 6: $a + b + c + d + e < x < a + b + c + d + e + f$

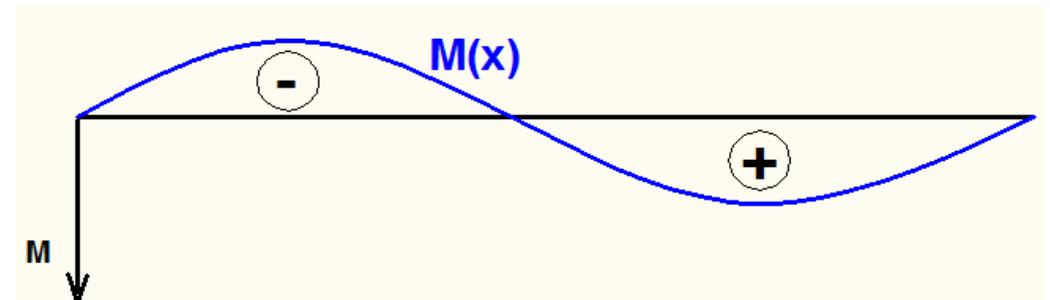
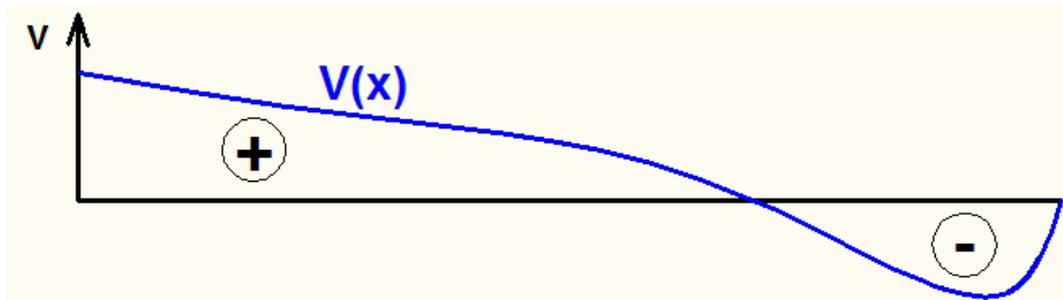
DIAGRAMAS DE ESFORÇOS CORTANTE E MOMENTO FLETOR (DCM)

Roteiro para obter $V(x)$ e $M(x)$:

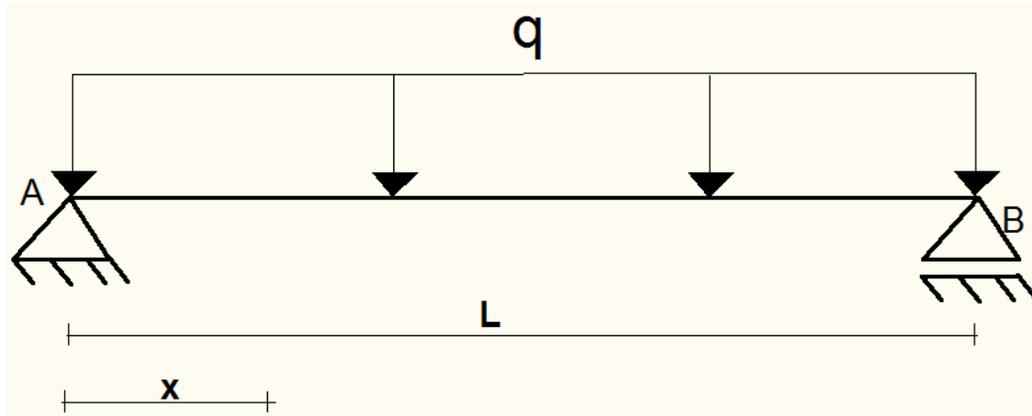
2. Fazer um corte no trecho, usar equações de equilíbrio $\Sigma F_y = 0$ e $\Sigma M_s = 0$



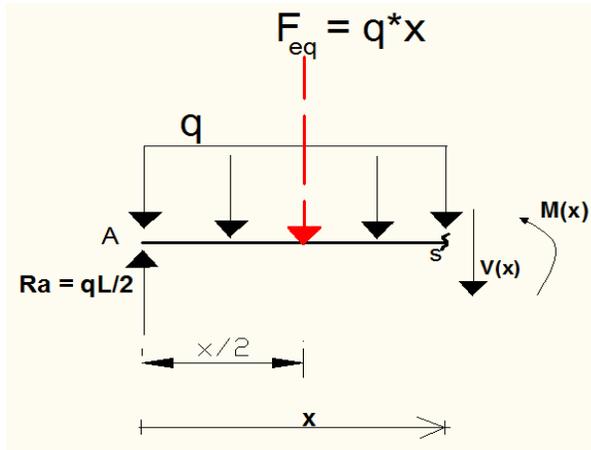
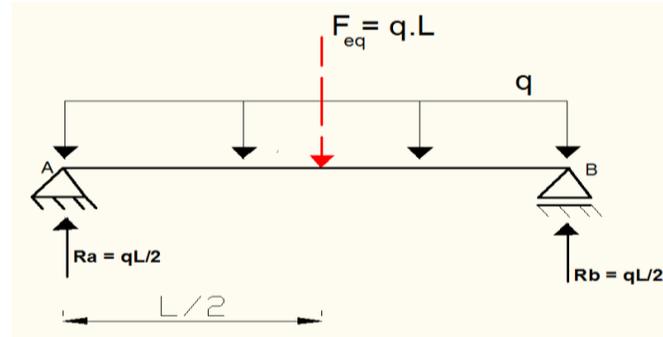
3. Desenhar $V(x)$ e $M(x)$ seguindo convenção



Exemplo 18: Viga bi-apoiada $p(x) = q$



1. Obter reações:



Determinação das equações nos cortes de cada trecho:

Trecho único: $0 < x < L$

$$\sum F_y = 0 \rightarrow R_a - q \cdot x - V(x) = 0 \rightarrow V(x) = R_a - q \cdot x$$

$$V(x) = q \cdot L/2 - q \cdot x \text{ (linear)}$$

$$\sum M_s = 0 \rightarrow M(x) + (q \cdot x) \cdot x/2 - R_a \cdot x = 0 \rightarrow M(x) = R_a \cdot x - q \cdot x^2/2$$

$$M(x) = (q \cdot L/2) \cdot x - q \cdot x^2/2 \text{ (parábola)}$$

Exemplo 18: Viga bi-apoiada $p(x) = q$

3. Diagramas:

$$\frac{dM(x)}{dx} = V(x) = 0 \rightarrow q \cdot L/2 - q \cdot x = 0 \rightarrow x = \frac{L}{2}$$

$$M(L/2) = (q \cdot L/2) \cdot L/2 - q \cdot (L/2)^2 / 2 = \frac{q \cdot L^2}{8}$$

