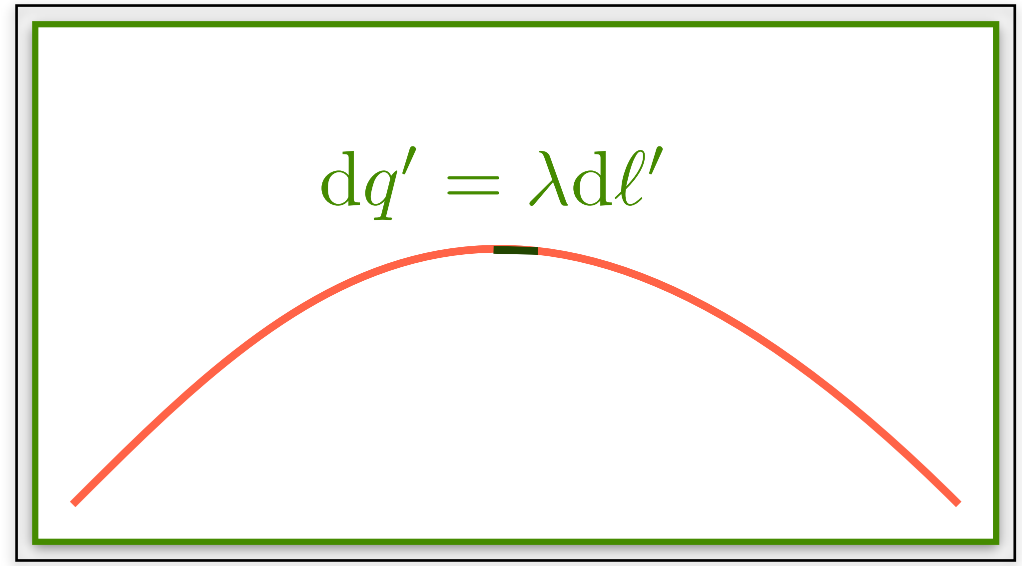
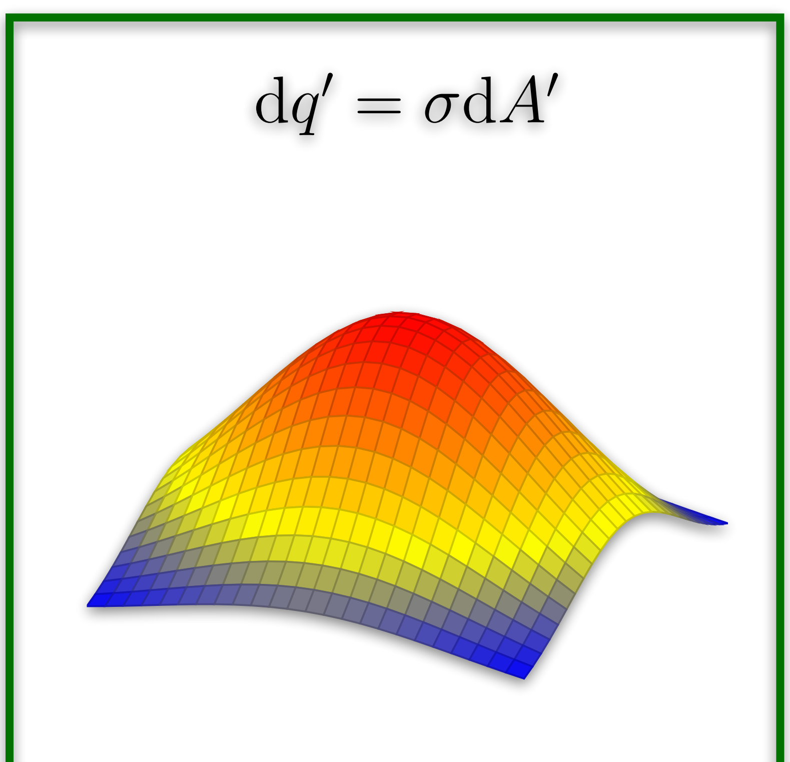
Eletromagnetismo

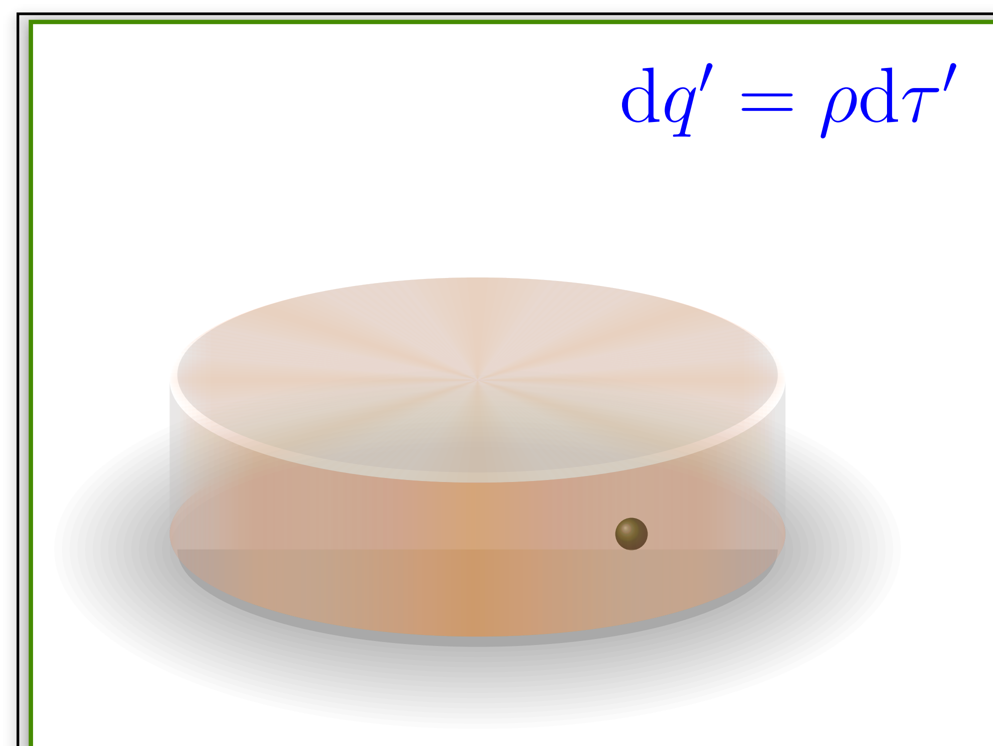
1 de março
Eletrostática

Campo elétrico de distribuição de cargas

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int \frac{\mathrm{d}q'}{\mathscr{r}^3}\vec{\mathscr{r}}$$

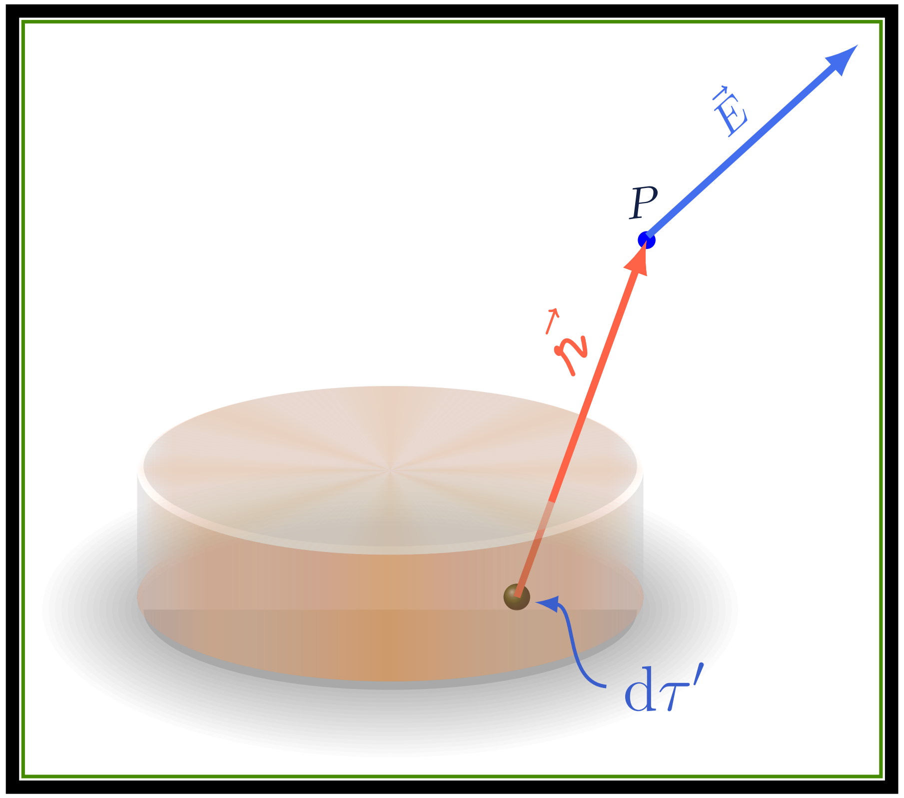






Divergente do campo elétrico

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho(\vec{\mathbf{r}}')}{\mathscr{r}^3}\vec{\mathscr{r}}\,\mathrm{d}\tau'$$



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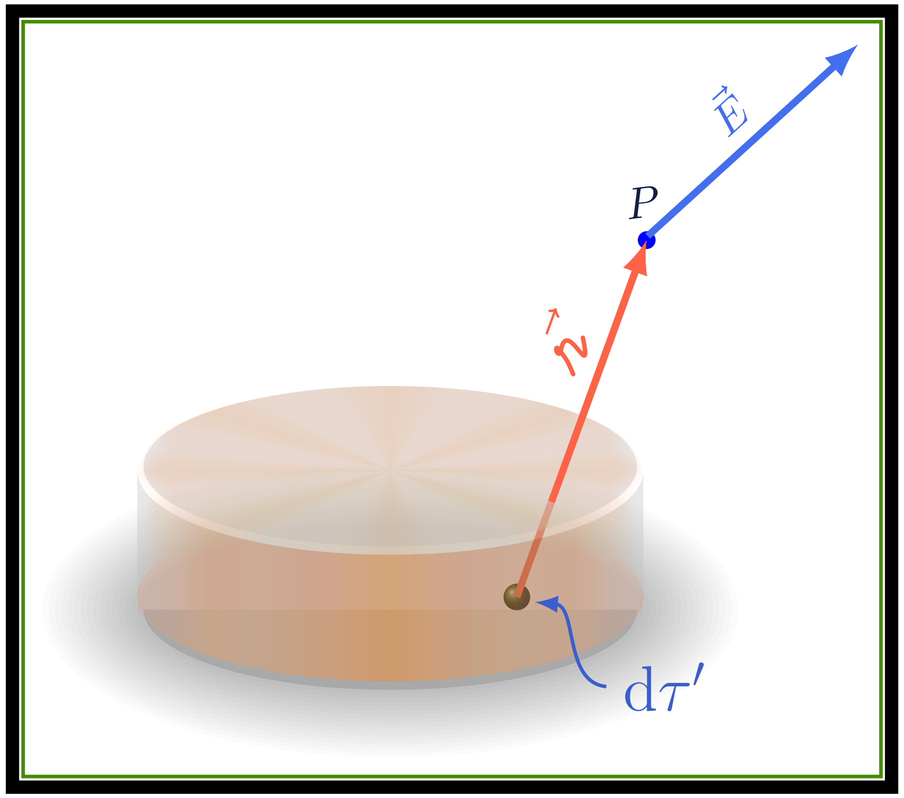

$$\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \rho(\vec{\mathbf{r}}')\vec{\boldsymbol{\nabla}}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\mathrm{d}\tau'$$

Divergente do campo elétrico

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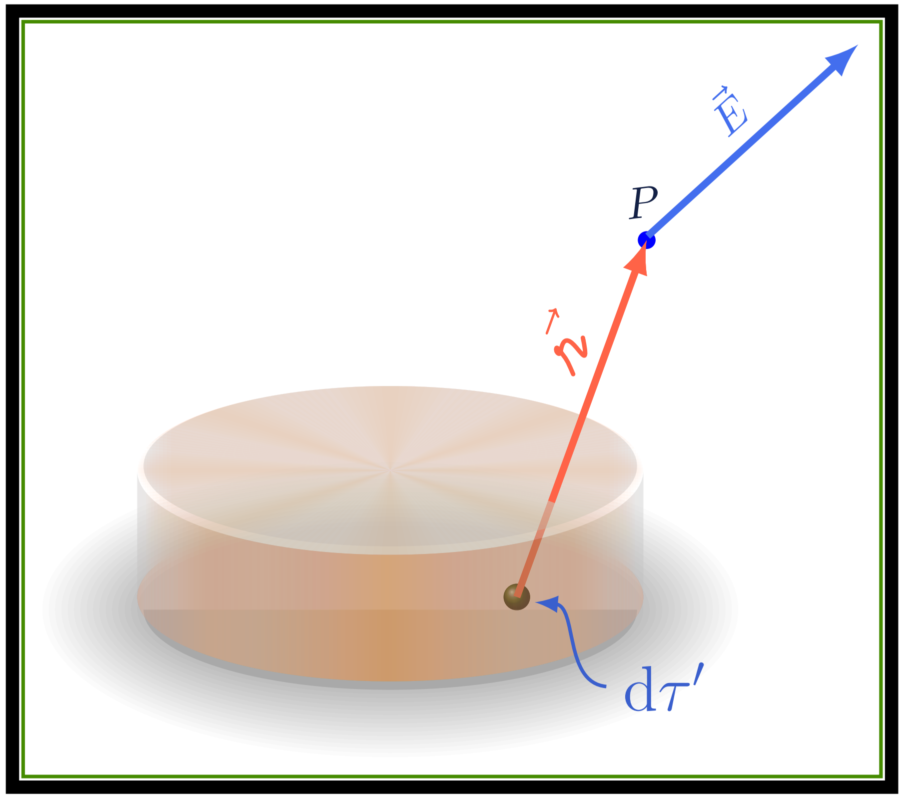

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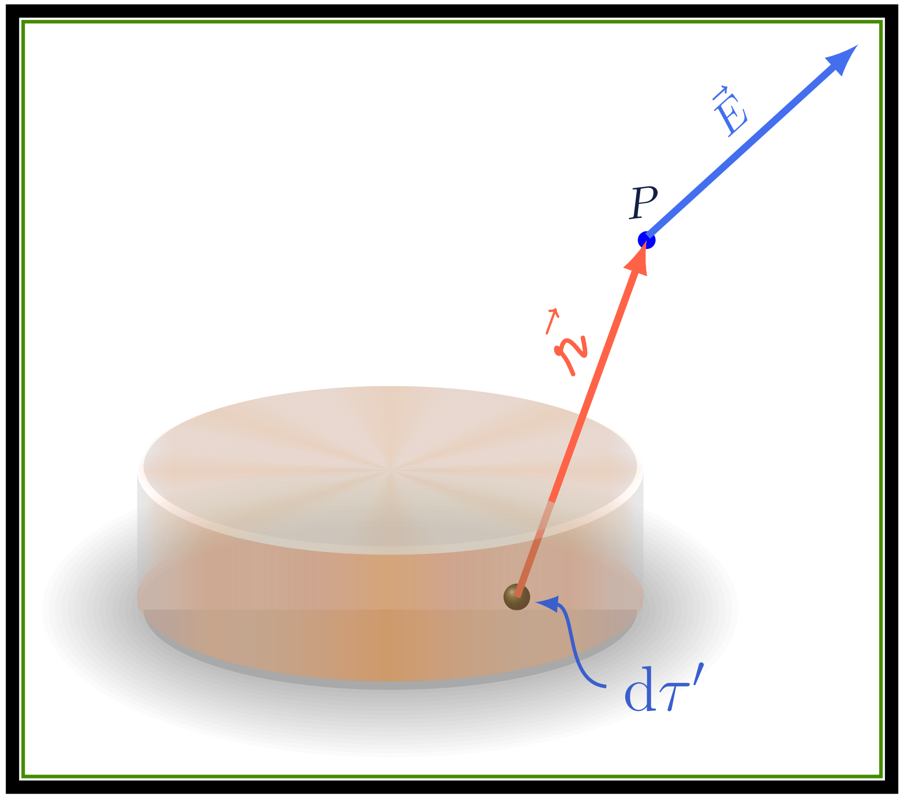
$$\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \rho(\vec{\mathbf{r}}')4\pi\delta(\vec{\mathscr{r}})\,\mathrm{d}\tau'$$

Divergente do campo elétrico

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho(\vec{\mathbf{r}}')}{\boldsymbol{\mathscr{r}}^3}\vec{\boldsymbol{\mathscr{r}}}\,\mathrm{d}\tau'$$



$$\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \rho(\vec{\mathbf{r}}')\vec{\boldsymbol{\nabla}}\cdot\left(\frac{\hat{\boldsymbol{\mathscr{r}}}}{\boldsymbol{\mathscr{r}}^2}\right)\mathrm{d}\tau'$$

$$\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \rho(\vec{\mathbf{r}}')4\pi\delta(\vec{\boldsymbol{\mathscr{r}}})\,\mathrm{d}\tau'$$

$$\vec{\boldsymbol{\mathscr{r}}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}' \Rightarrow \quad \delta(\vec{\boldsymbol{\mathscr{r}}}) = \delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}')$$

$$\rho(\vec{\mathbf{r}'})\delta(\vec{\boldsymbol{\mathscr{r}}}) \equiv \rho(\vec{\mathbf{r}})\delta(\vec{\boldsymbol{\mathscr{r}}})$$

Divergente do campo elétrico

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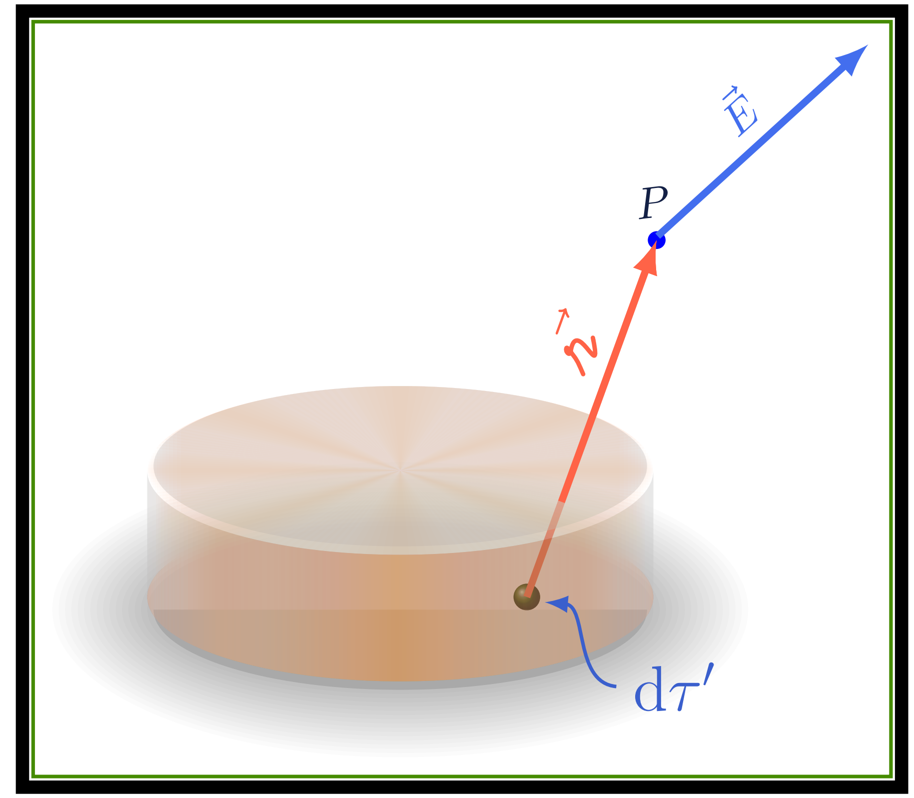

$$\vec{\nabla}\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \rho(\vec{\mathbf{r}}')\vec{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\mathrm{d}\tau'$$

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$$\vec{\nabla}\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}4\pi\rho(\vec{\mathbf{r}})\int_{\mathcal{V}} \delta(\vec{\mathscr{r}})\,\mathrm{d}\tau'$$

$$\vec{\nabla}\cdot\vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$

Lei de Gauss

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$

Lei de Gauss

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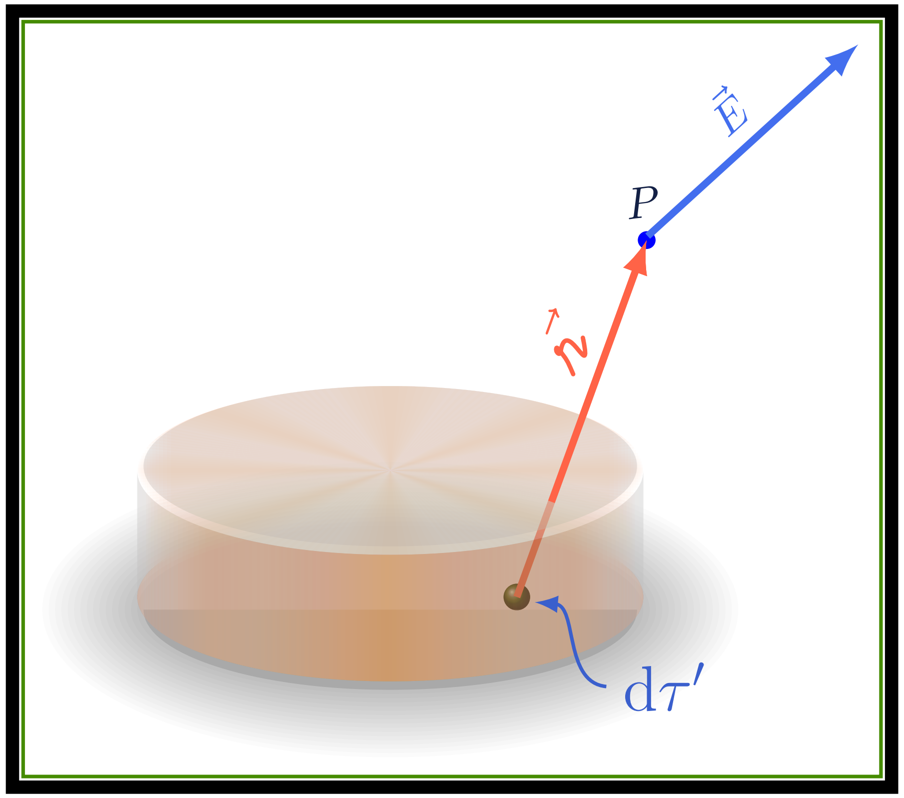
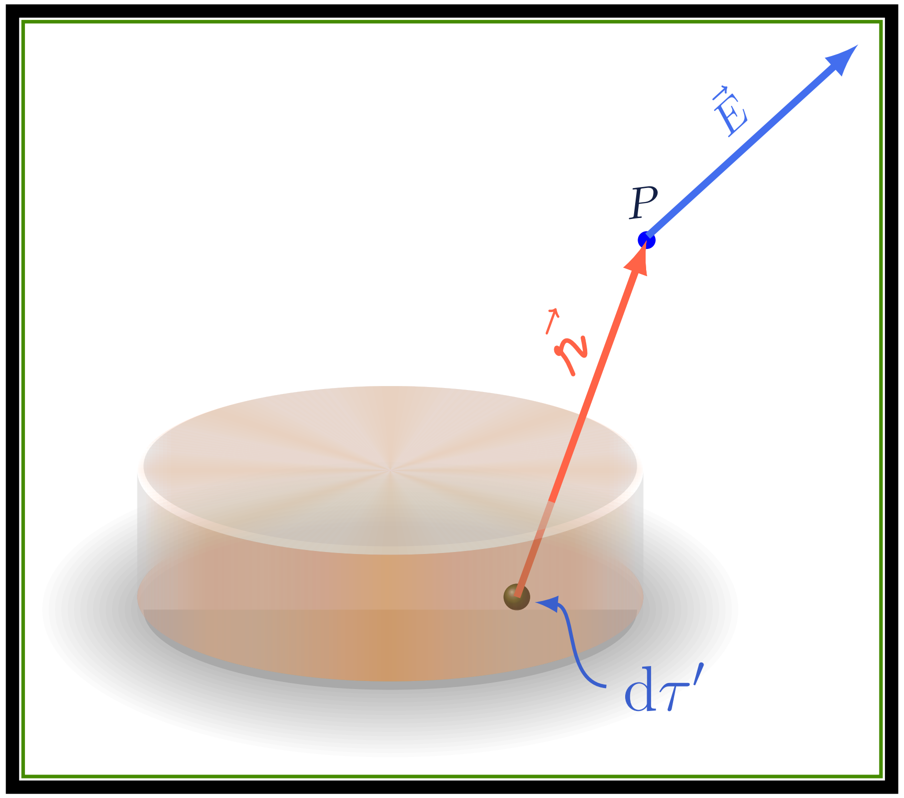
$$\int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}}\, \mathrm{d}\tau = \int_{\mathcal{V}} \frac{\rho}{\epsilon_0}\, \mathrm{d}\tau$$



Lei de Gauss

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$

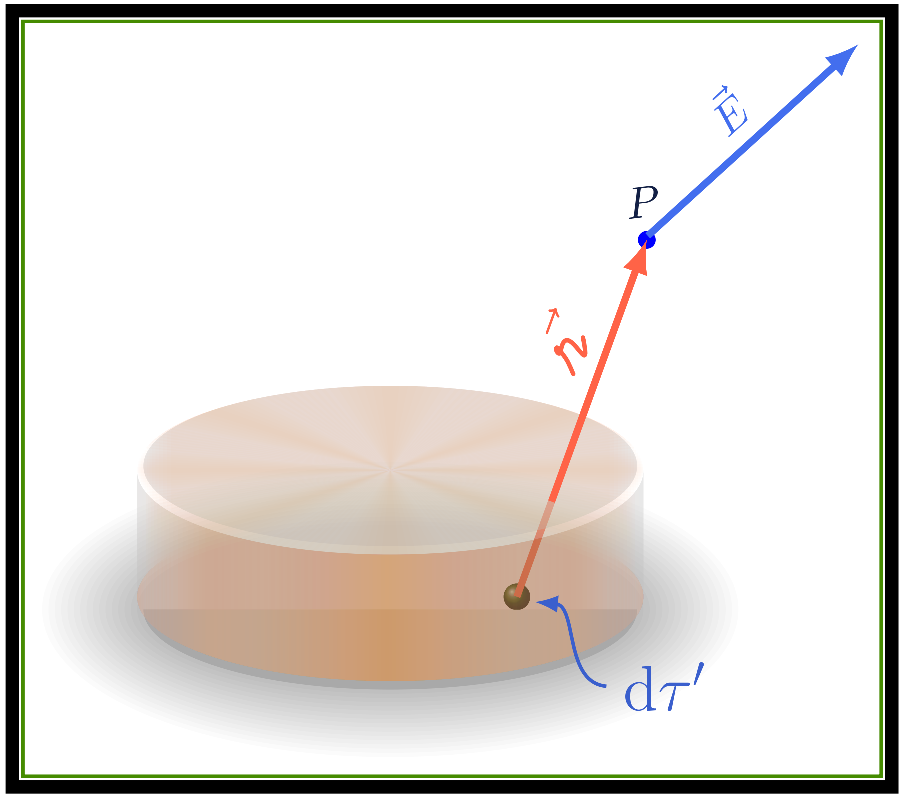


$$\int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}} \, \mathrm{d}\tau = \int_{\mathcal{V}} \frac{\rho}{\epsilon_0} \, \mathrm{d}\tau$$

$$\int_{\mathcal{S}} \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} \, \mathrm{d}A = \frac{Q}{\epsilon_0}$$

Lei de Gauss

$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$



$$\int_{\mathcal{V}} \vec{\nabla} \cdot \vec{\mathbf{E}}\, \mathrm{d}\tau = \int_{\mathcal{V}} \frac{\rho}{\epsilon_0}\, \mathrm{d}\tau$$

$$\int_{\mathcal{S}} \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, \mathrm{d}A = \frac{Q}{\epsilon_0}$$

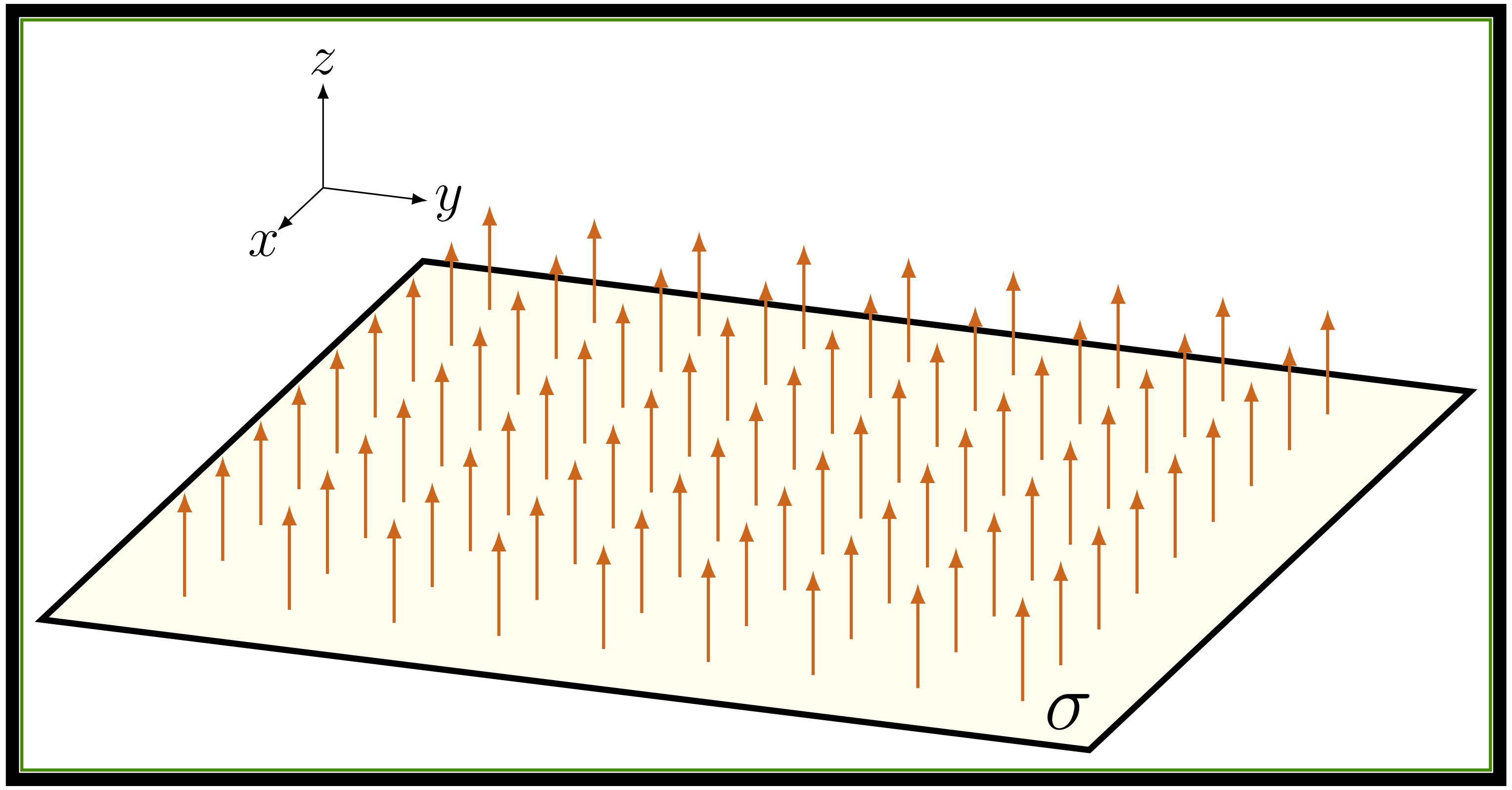
Lei de Gauss

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$

$$\int_{\mathcal{S}} \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} \, \mathrm{d}A = \frac{Q}{\epsilon_0}$$

Divergente do campo elétrico

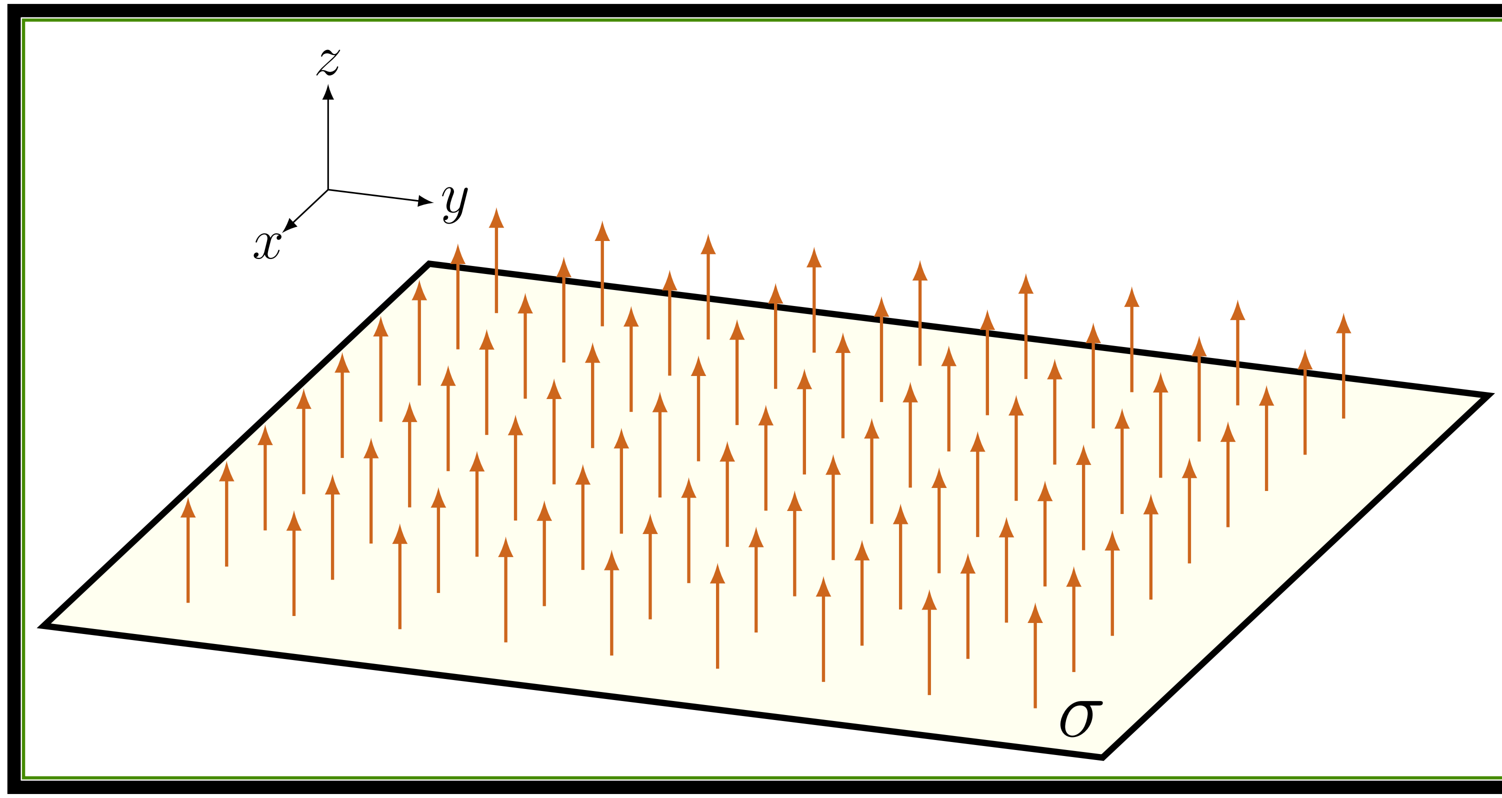
$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$



$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

Divergente do campo elétrico

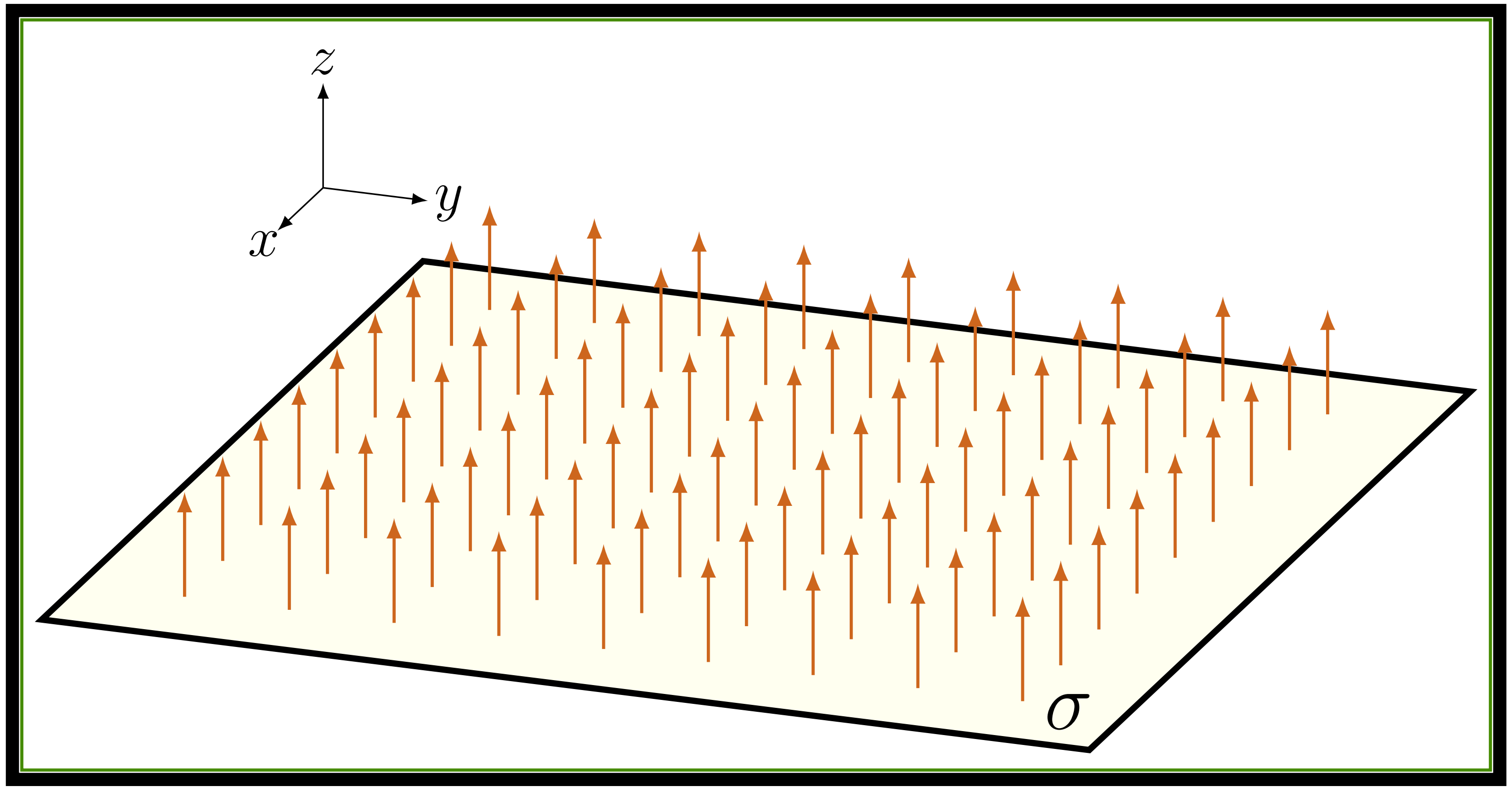
$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$



$$\frac{\partial \not{E}_x}{\partial x} + \frac{\partial \not{E}_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

Divergente do campo elétrico

$$\vec{\nabla} \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$$

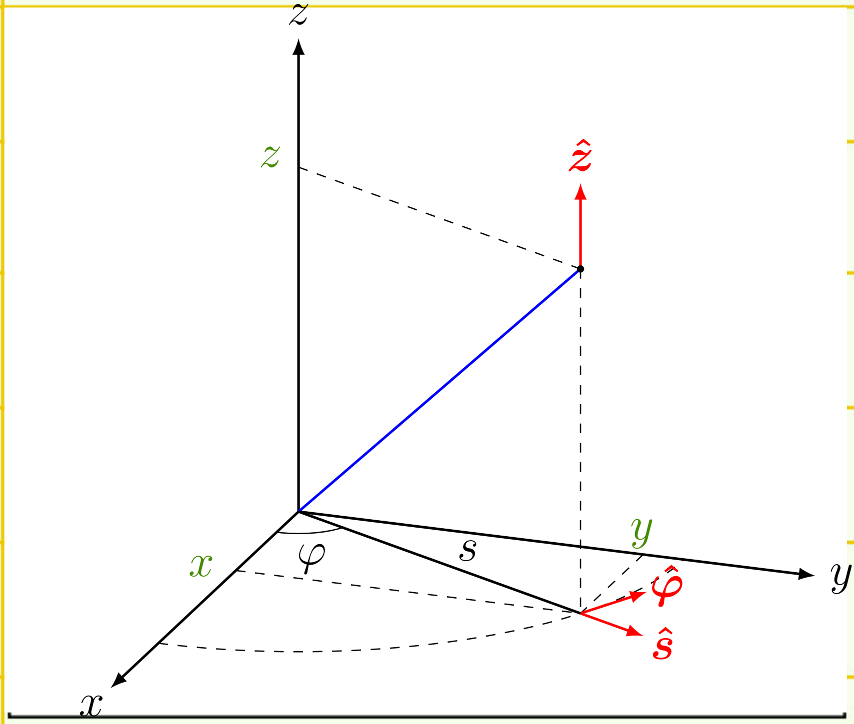


$$\frac{\partial \cancel{E}_x}{\partial x} + \frac{\partial \cancel{E}_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\Rightarrow \quad E_z = \text{constante}$$

Coordenadas cilíndricas

$$d\vec{\ell} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$



$$\vec{\nabla}T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

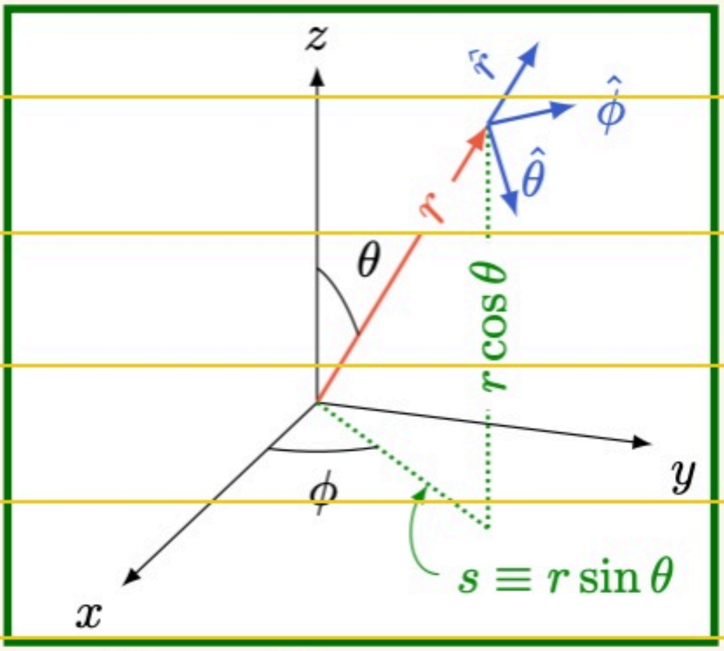
$$\vec{\nabla}\cdot\vec{v} = \frac{1}{s}\frac{\partial(s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla}\times\vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial(s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$