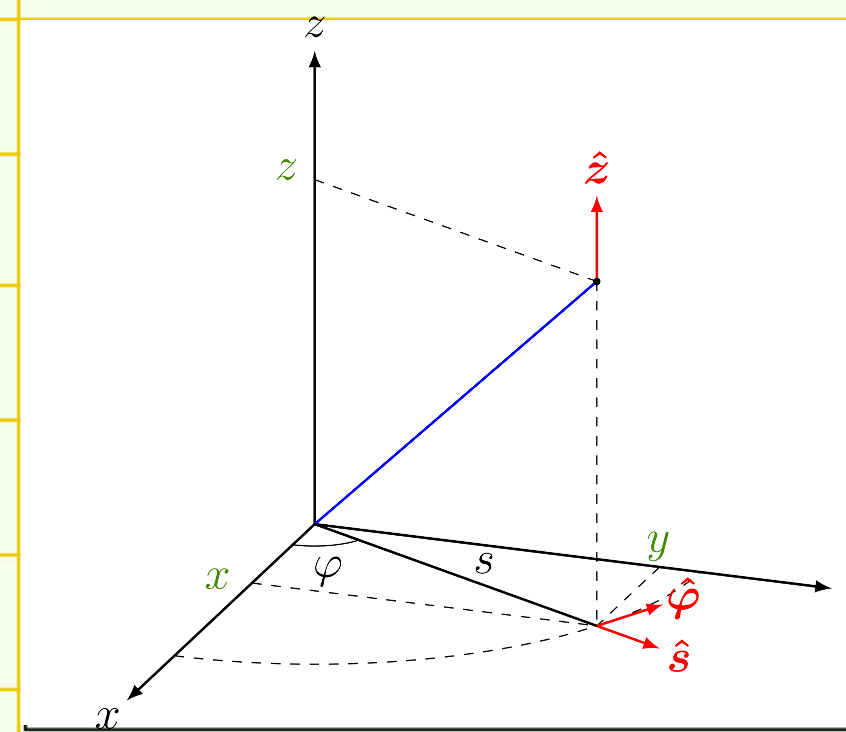
# Eletromagnetismo

07 de março

Análise vetorial

# Coordenadas cilíndricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$




$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

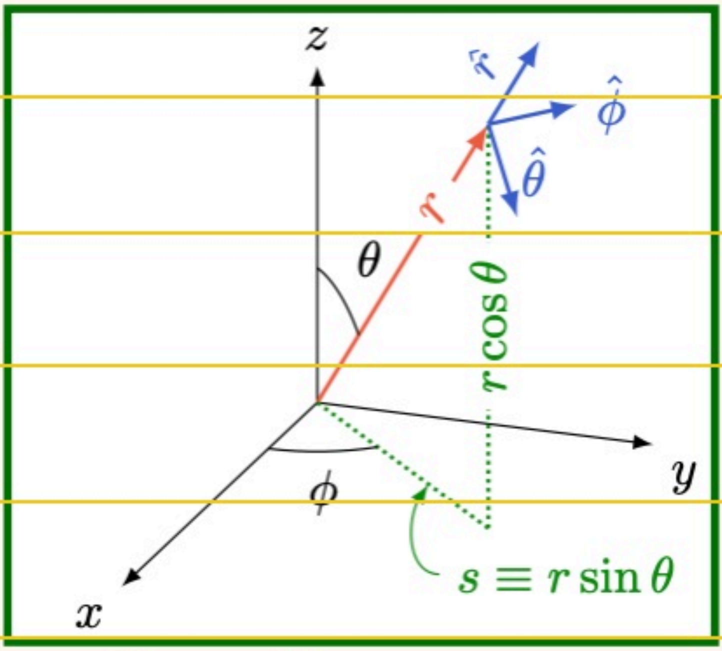
$$\vec{\nabla}\cdot\vec{v} = \frac{1}{s}\frac{\partial(s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla}\times\vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial(s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

# Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$




$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

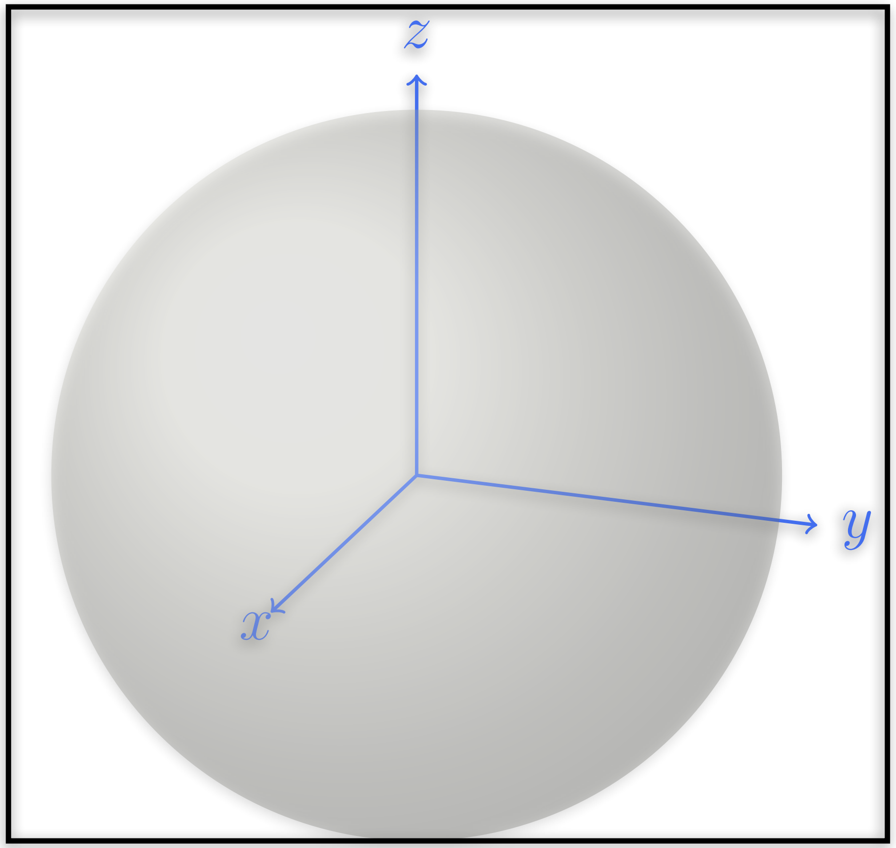
$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

# Pratique o que aprendeu

$$\vec{\mathbf{v}}(\vec{\mathbf{r}}) = \frac{\hat{\mathbf{r}}}{r^2}$$

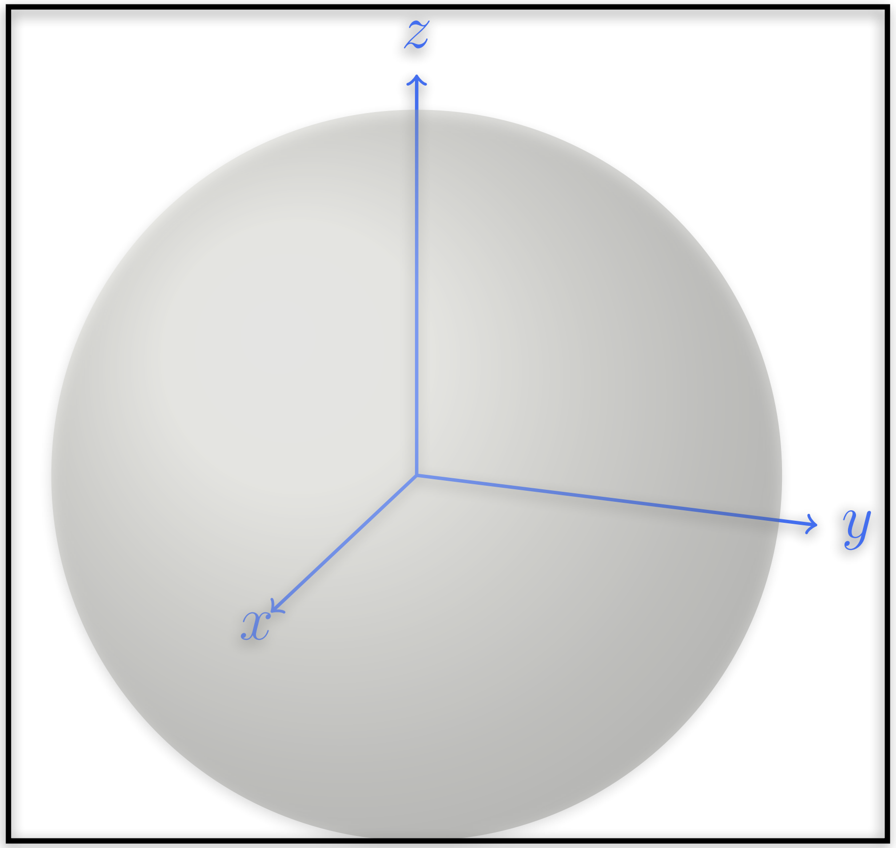
$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = ?$$



# Pratique o que aprendeu

$$\vec{\mathbf{v}}(\vec{\mathbf{r}}) = \frac{\hat{\mathbf{r}}}{r^2}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = \delta^3(\vec{\mathbf{r}})?$$



# Função delta

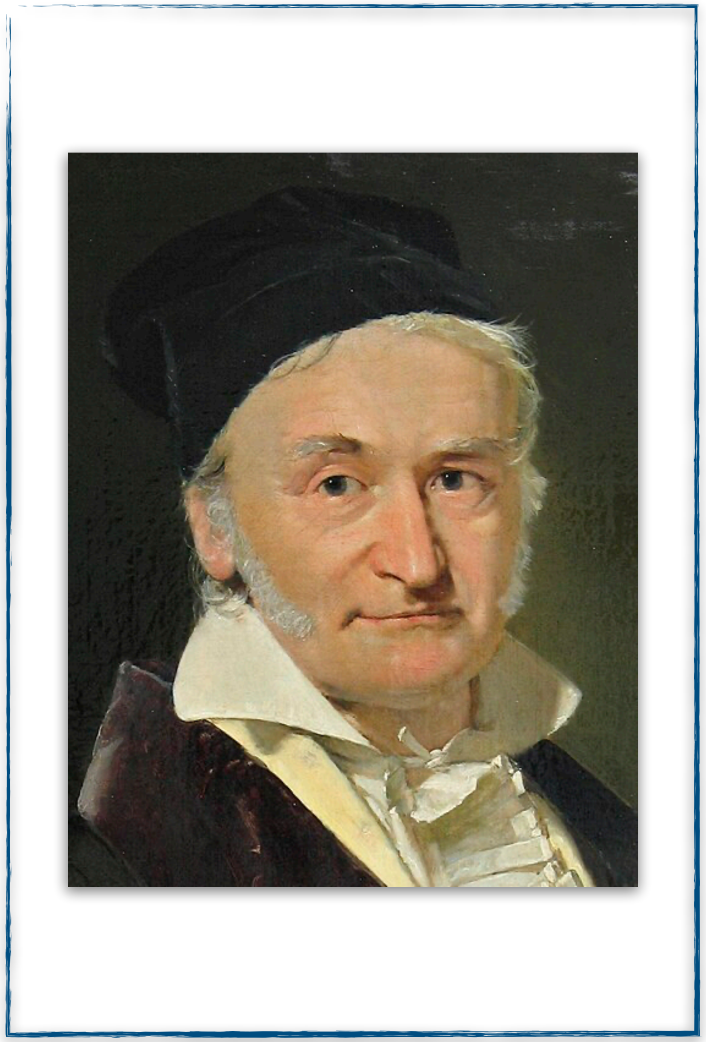
## Três dimensões

$$\delta^3(\vec{\mathbf{r}}) = \begin{cases} 0 & \vec{\mathbf{r}} \neq 0 \\ \text{indefinida} & \vec{\mathbf{r}} = 0 \end{cases}$$

$$\int_{\mathcal{V}} f(\vec{\mathbf{r}})\delta^3(\vec{\mathbf{r}})\mathrm{d}\tau = f(0)$$

# Análise vetorial

$$\int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} \; \mathrm{d}\tau = \int \vec{\mathbf{v}} \cdot \hat{n} \; \mathrm{d}A$$



Teorema de Gauss

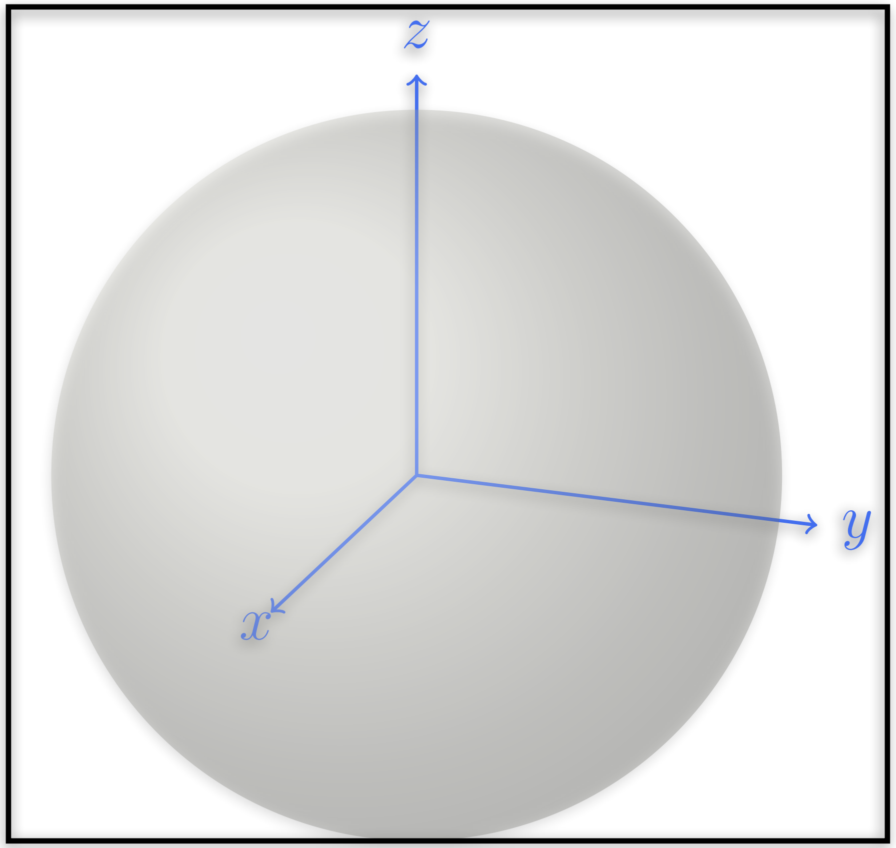
# Pratique o que aprendeu

$$\vec{\mathbf{v}}(\vec{\mathbf{r}}) = \frac{\hat{\mathbf{r}}}{r^2}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = \delta^3(\vec{\mathbf{r}})?$$

$$\int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}}\, \mathrm{d}\tau = \int_{\mathcal{S}} \vec{\mathbf{v}} \cdot \hat{\mathbf{n}}\, dA$$

$$\int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}}\, \mathrm{d}\tau = \int_{\mathcal{S}} \frac{1}{r^2}\, \mathrm{d}A$$



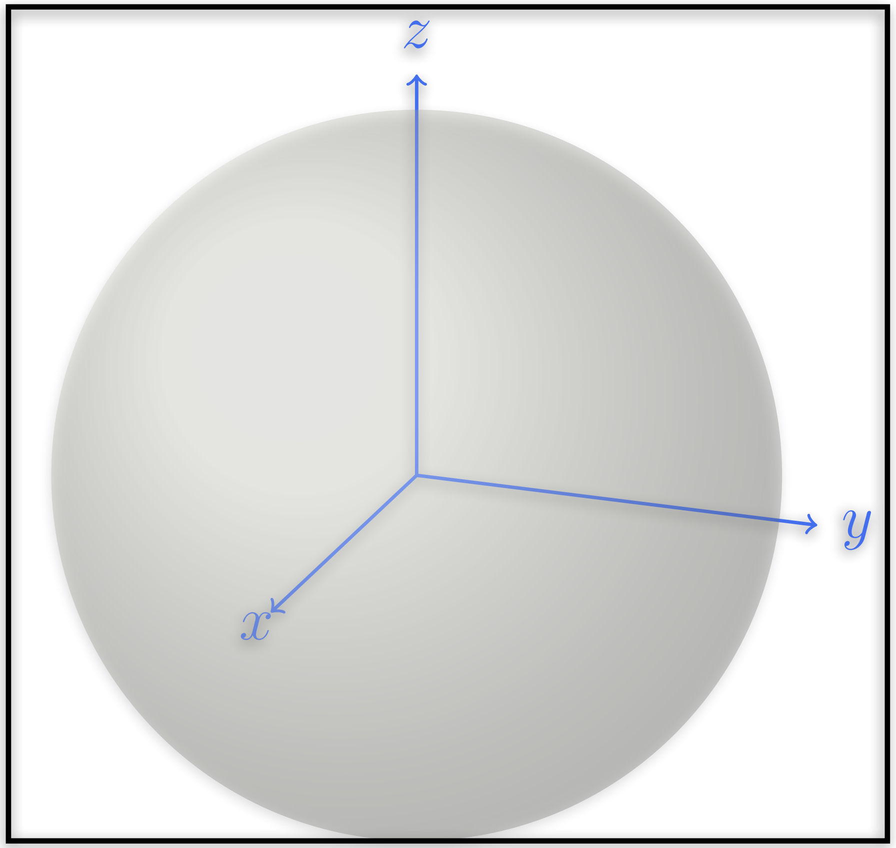
# Pratique o que aprendeu

$$\vec{\mathbf{v}}(\vec{\mathbf{r}}) = \frac{\hat{\mathbf{r}}}{r^2}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = \delta^3(\vec{\mathbf{r}})?$$

$$\int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}}\, \mathrm{d}\tau = \int_{\mathcal{S}} \vec{\mathbf{v}} \cdot \hat{\mathbf{n}}\, dA$$

$$\int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}}\, \mathrm{d}\tau = \int_{\mathcal{S}} \frac{1}{r^2}\, \mathrm{d}A = 4\pi$$



# Pratique o que aprendeu

$$\vec{\mathbf{v}}(\vec{\mathbf{r}}) = \frac{\hat{\mathbf{r}}}{r^2}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = \delta^3(\vec{\mathbf{r}})?$$

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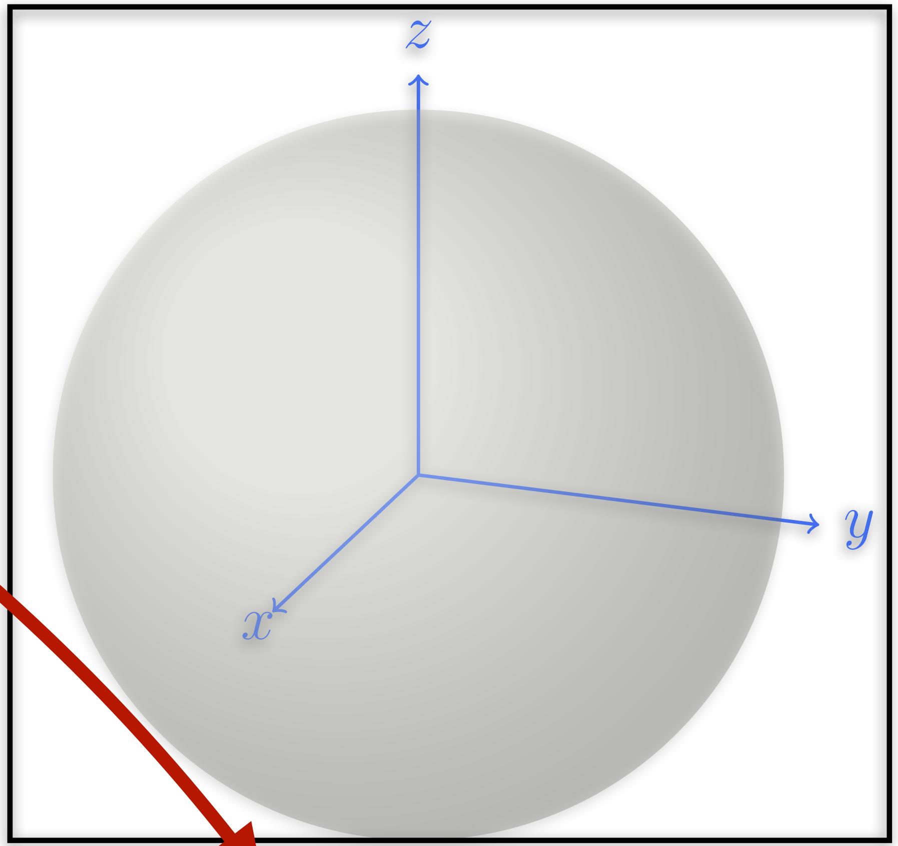


$$\Rightarrow \quad \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = 4\pi\delta^3(\vec{\mathbf{r}})$$

# Divergente em coordenadas esféricas

$$\int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}}\ \mathrm{d}\tau = \int \vec{\mathbf{v}} \cdot \hat{n}\ \mathrm{d}A$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} \Delta\tau = \Delta(v_n A)$$

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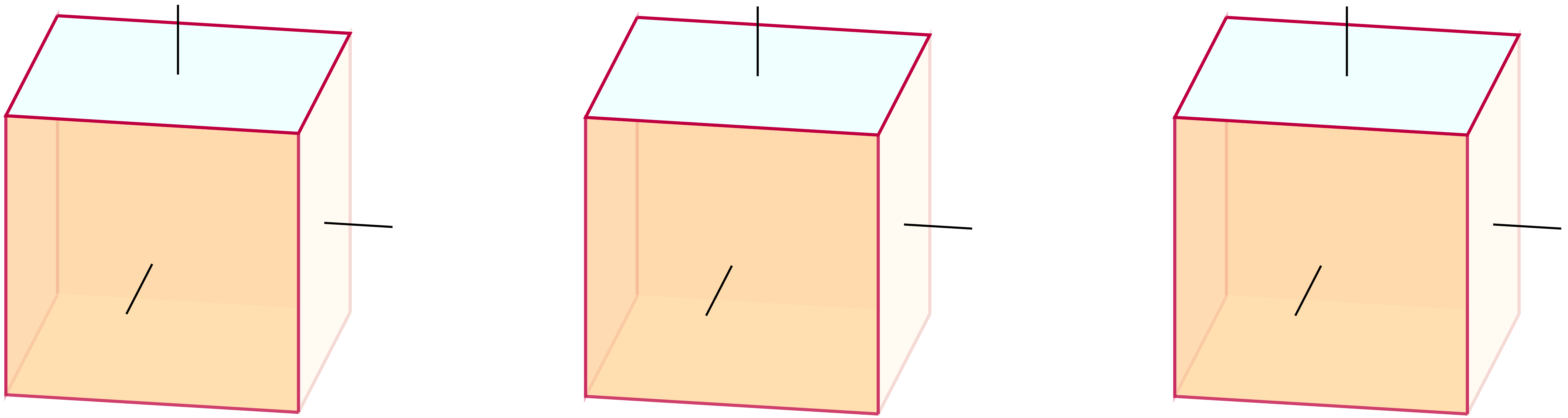

Cartesianas, para treinar

# Divergente em coordenadas esféricas

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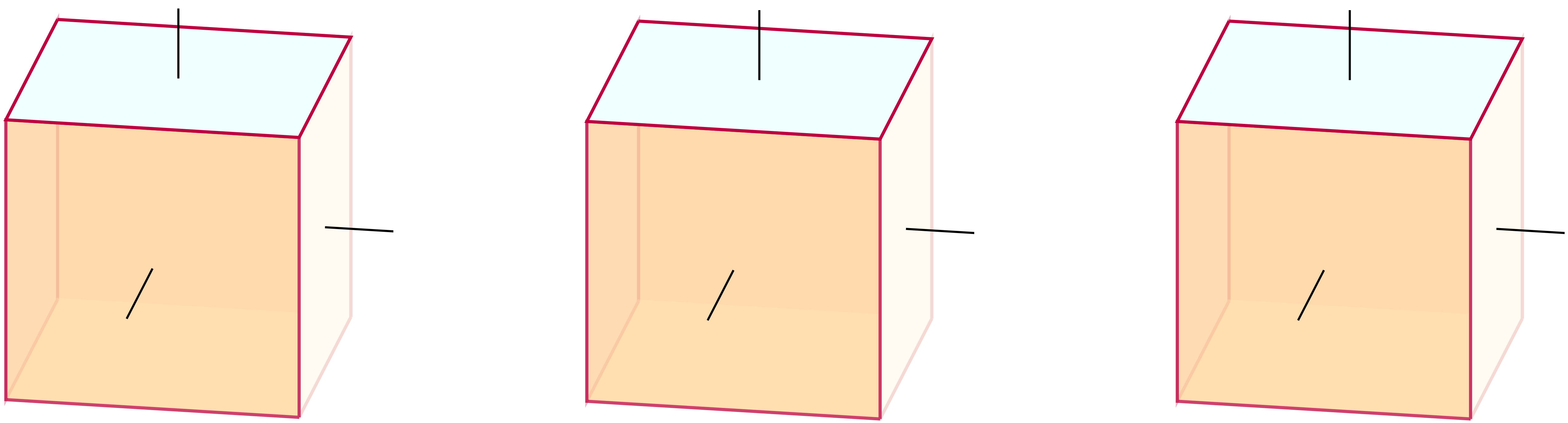

$$\left(\vec{\nabla} \cdot \vec{\mathbf{v}}\right) \Delta\tau = \Delta(v_x \mathrm{d}y \mathrm{d}z) + \Delta(v_y \mathrm{d}x \mathrm{d}z) + \Delta(v_z \mathrm{d}x \mathrm{d}y)$$

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$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

# Divergente em coordenadas esféricas

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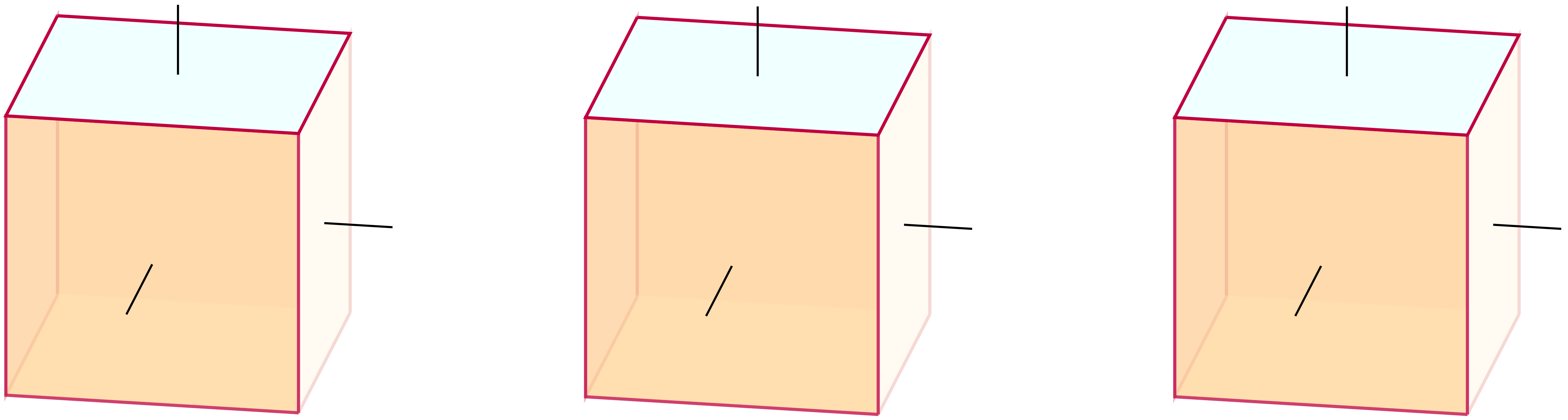

Coordenadas esféricas

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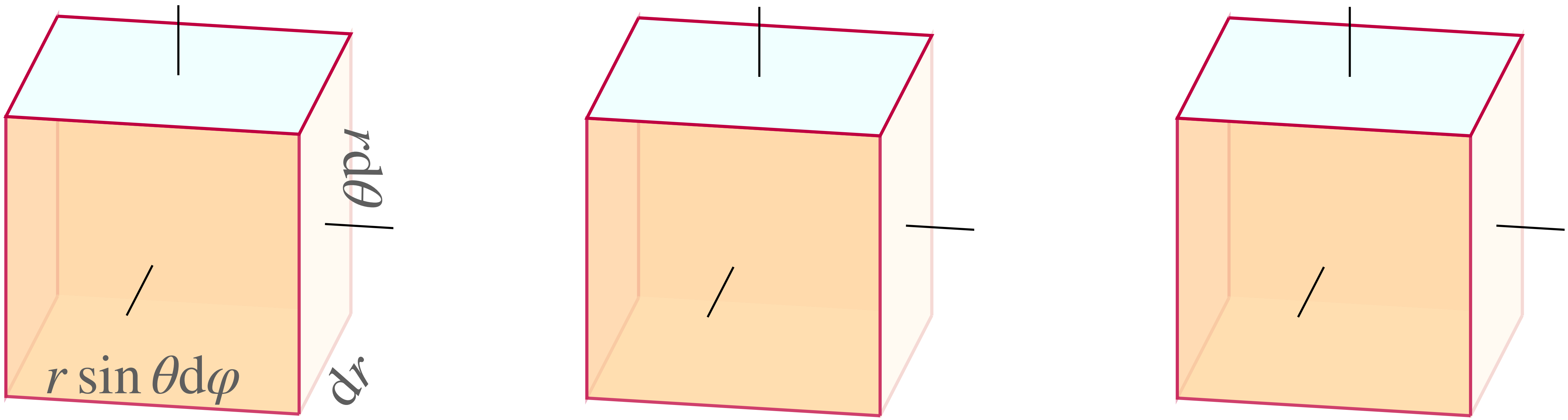


$$\left(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}}\right) \Delta\tau = \Delta(v_r r\mathrm{d}\theta \, r \sin\theta \mathrm{d}\varphi) + \Delta(v_\theta \mathrm{d}r \, r \sin\theta \mathrm{d}\varphi) + \Delta(v_\varphi \mathrm{d}r \, r\mathrm{d}\theta)$$

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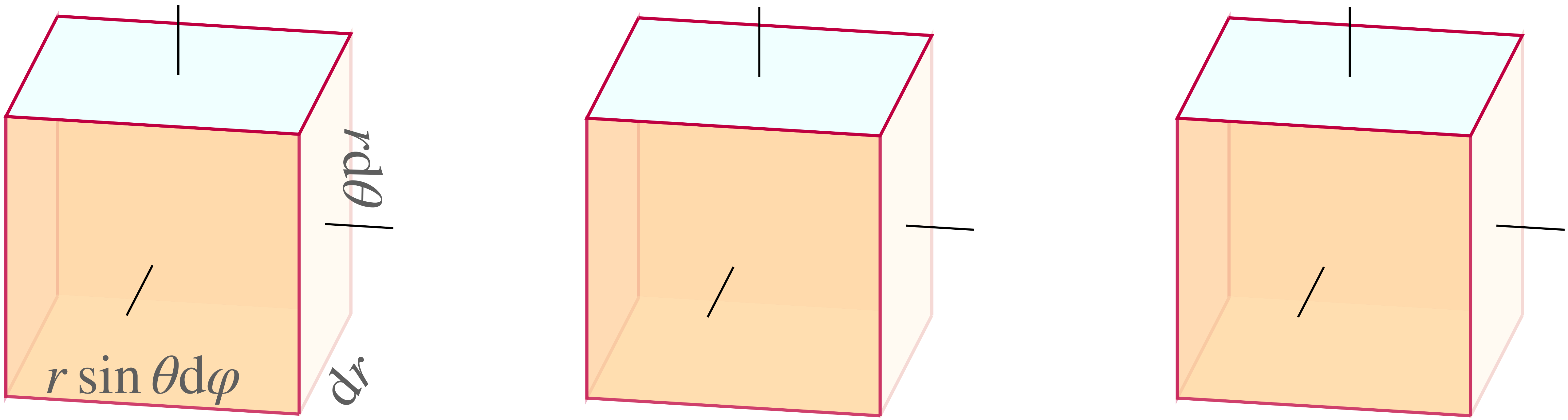


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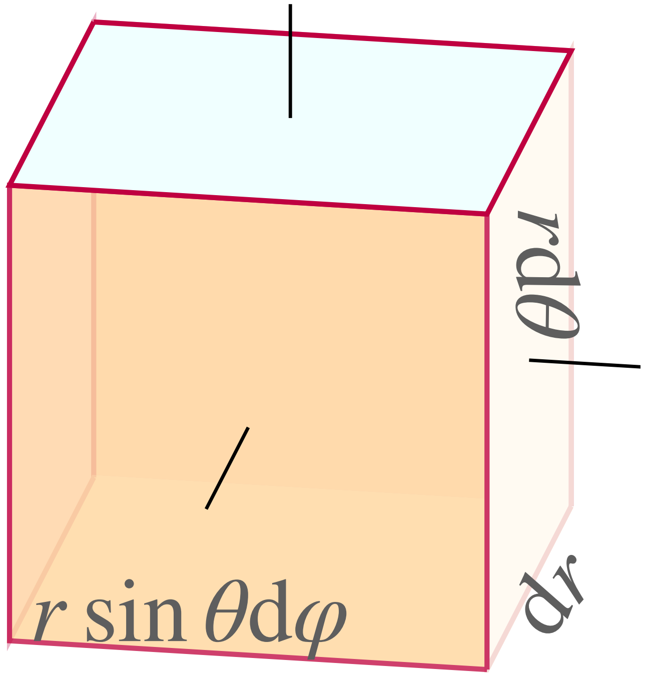
$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = \frac{\Delta(v_r r\mathrm{d}\theta\, r\sin\theta\mathrm{d}\varphi)}{\mathrm{d}r\, r\mathrm{d}\theta\, r\sin\theta\mathrm{d}\varphi} + \frac{\Delta(v_\theta \mathrm{d}r\, r\sin\theta\mathrm{d}\varphi)}{\mathrm{d}r\, r\mathrm{d}\theta\, r\sin\theta\mathrm{d}\varphi} + \frac{\Delta(v_\varphi \mathrm{d}r\, r\mathrm{d}\theta)}{\mathrm{d}r\, r\mathrm{d}\theta\, r\sin\theta\mathrm{d}\varphi}$$

# Divergente em coordenadas esféricas

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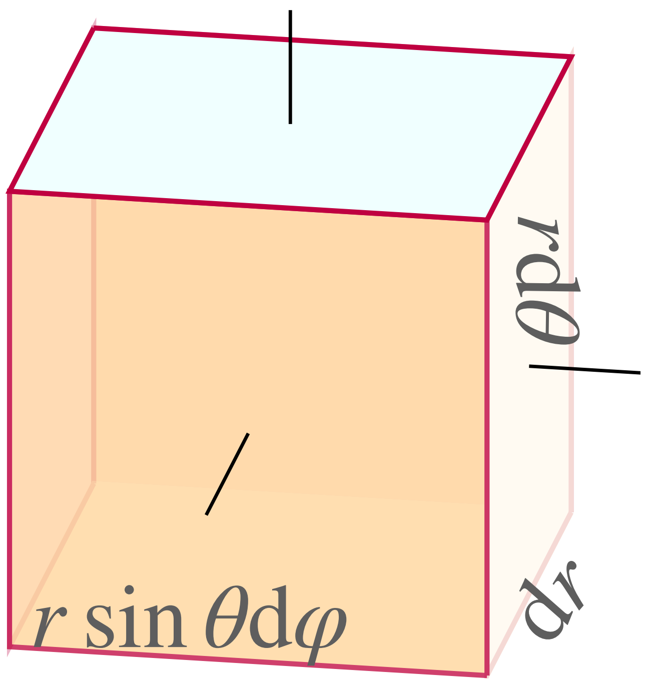
# Divergente em coordenadas esféricas

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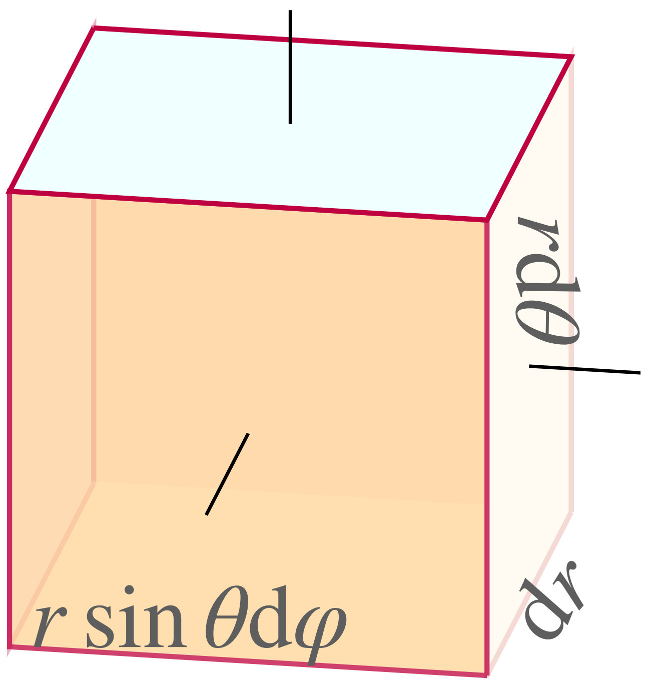
# Divergente em coordenadas esféricas

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$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$

# Minuto Pecha-Kucha

## Por que o arco-íris é redondo?

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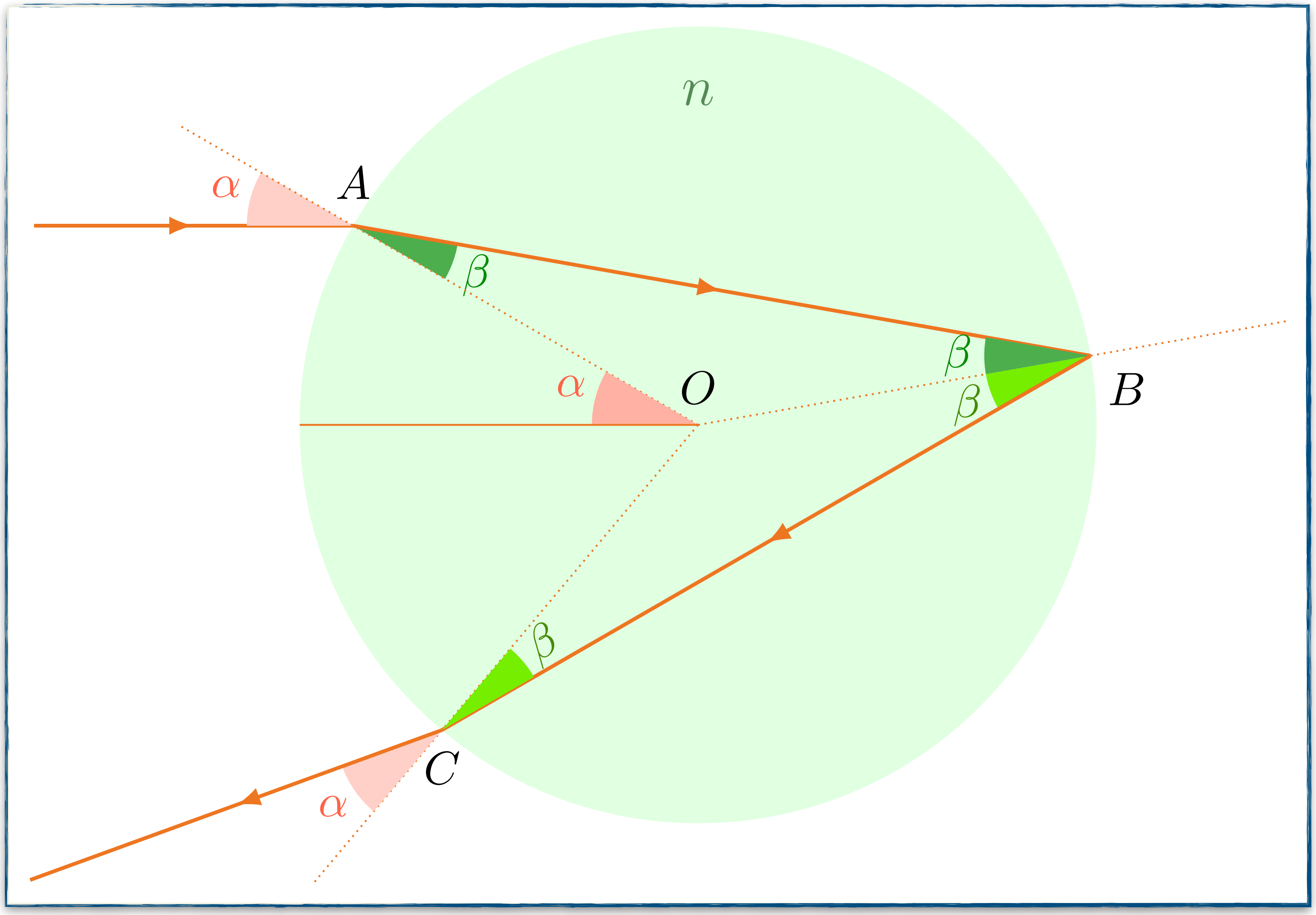

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