Eletricidade e Eletromagnetismo II

Aula III:

Circuitos RC
Resolução de Equações diferenciais de primeira ordem separáveis
Balanço de energia
Aplicações em filtro de sinais e temporização

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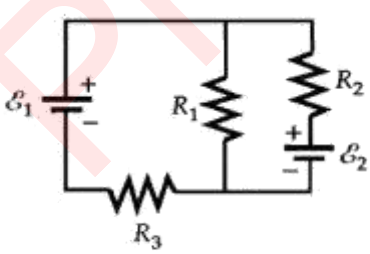
Fontes:

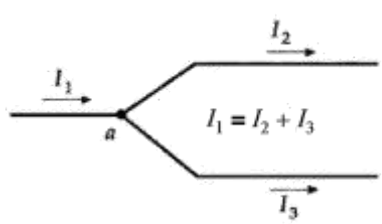
Serway, Capitulo 26
Tipler, Capitulo 35
Halliday, Capitulo 27

Circuitos Elétricos

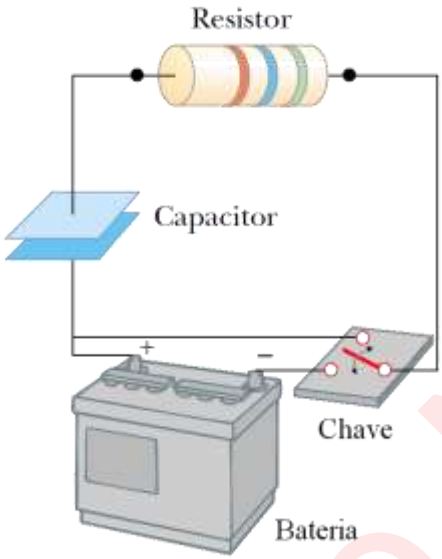
Leis de Kirchhoff

- Quando um circuito fechado é percorrido, a soma algébrica das mudanças no potencial deve ser zero
- Em qualquer ponto de junção no circuito onde a corrente pode se dividir, a soma das correntes que entram an junção deve ser igual a soma das correntes que saem da junção



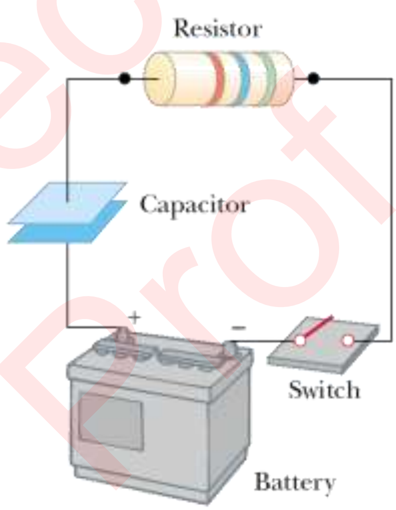


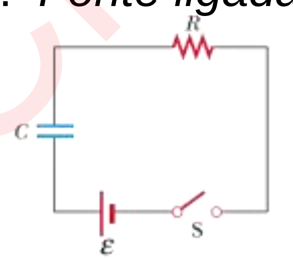
Circuito RC



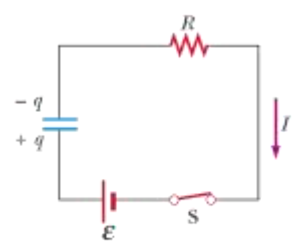
Alternando-se a posição da Chave comutadora, pode-se ligar e desligar a fonte no sistema

Carga do Capacitor : *Fonte ligada*

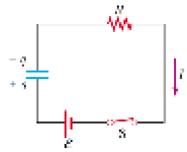




$t < 0$



$t > 0$



Assumindo que o capacitor C está inicialmente descarregado podemos aplicar a lei das malhas e obter:

$$\varepsilon - V_C - V_R = 0$$

$$V_C = \frac{q}{C} \qquad V_R = IR$$

$$\varepsilon - \frac{q}{C} - IR = 0$$

Duas variáveis, *I* e *q*

São Independentes?

NÃO!!!

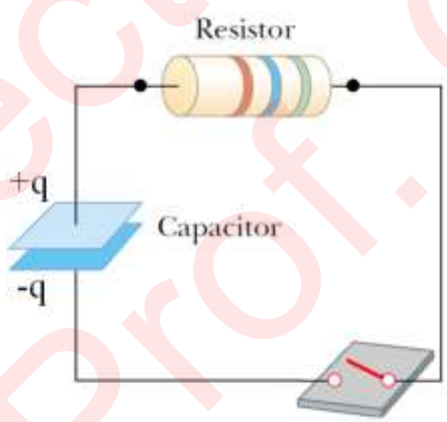
Corrente elétrica: $I = \frac{dq}{dt}$

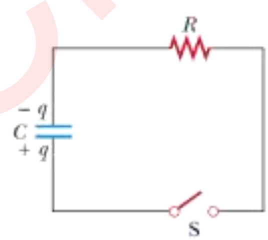
$$R\frac{dq}{dt} + \frac{q}{C} = \varepsilon$$

Equação Diferencial de primeira ordem com sinal externo

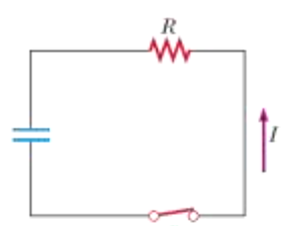
Como Resolver?

Descarga do Capacitor

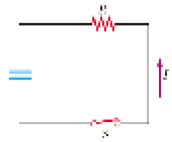




$t < 0$



$t > 0$



Agora o capacitor está completamente carregado. Como não existe mais a fonte a lei das malhas fornece:

$$V_C + V_R = 0$$

$$V_C = \frac{q}{C} \qquad V_R = IR$$

$$\boxed{\frac{q}{C} + IR = 0}$$

Duas variáveis, *I* e *q*

São Independentes?

NÃO!!!

Corrente elétrica: $I = \frac{dq}{dt}$

$$\boxed{R\frac{dq}{dt} + \frac{q}{C} = 0}$$

Equação Diferencial de primeira ordem homogênea

Como Resolver?

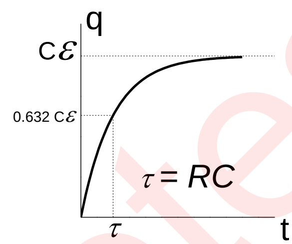
Precisamos Resolver:

Carga do Capacitor $$R\frac{dq}{dt} + \frac{q}{C} = \varepsilon$$

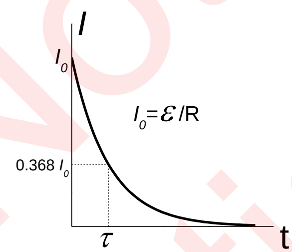
Descarga do Capacitor $$R\frac{dq}{dt} + \frac{q}{C} = 0$$

Solução obtida: Carga do Capacitor

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

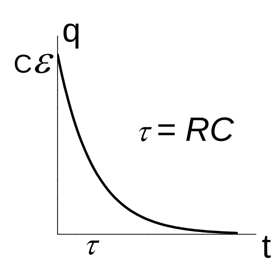


$$I = \frac{dq}{dt} \Rightarrow I = \left(\frac{\varepsilon}{R}\right)e^{-\frac{t}{RC}}$$

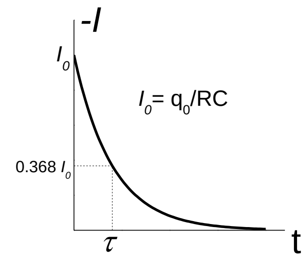


Solução obtida: Descarga do Capacitor

$$q = q_0\, e^{-\frac{t}{RC}}$$



$$I = \frac{dq}{dt} \Rightarrow I = \left(-\frac{q_0}{RC}\right)e^{-\frac{t}{RC}}$$



Balanço de energia

Energia fornecida pela Fonte:

$$dE = VI(t)dt$$

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) \qquad dq = \frac{\varepsilon}{R}e^{-\frac{t}{RC}}$$

$$E = \int_0^\infty VI(t)dt$$

$$= \varepsilon\int_0^\infty \frac{\varepsilon}{R}e^{-\frac{t}{RC}}dt$$

$$= \frac{\varepsilon^2}{R}\int_0^\infty e^{-\frac{t}{RC}}dt$$

$$= \frac{\varepsilon^2}{R}(-RC)e^{-\frac{t}{RC}}\Big|_0^\infty$$

$$= \frac{\varepsilon^2}{R}(-RC)(0-1)$$

$$= C\varepsilon^2$$

Energia Armazenada /Fornecida pelo capacitor

Carga

$$dU = Vdq = \frac{q}{C}dq$$

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) \qquad dq = \frac{\varepsilon}{R}e^{-\frac{t}{RC}}$$

$$dU = \frac{\varepsilon^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)dt$$

$$U = \int dU$$

$$= \int_0^\infty \frac{\varepsilon^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)dt$$

$$= \frac{\varepsilon^2}{R}\left[-RC(0-1) + \frac{RC}{2}(0-1)\right]$$

$$= \frac{\varepsilon^2}{R}\left[RC - \frac{RC}{2}\right] = \frac{1}{2}C\varepsilon^2$$

Descarga

$$dU = Vdq = \frac{q}{C}dq$$

$$q = C\varepsilon\, e^{-\frac{t}{RC}} \qquad dq = -\frac{\varepsilon}{R}e^{-\frac{t}{RC}}$$

$$dU = -\frac{\varepsilon^2}{R}\left(e^{-2t/RC}\right)dt$$

$$U = \int dU$$

$$= \int_0^\infty -\frac{\varepsilon^2}{R}\left(e^{-2t/RC}\right)dt$$

$$= -\frac{\varepsilon^2}{R}\left[\frac{RC}{2}(0-1)\right]$$

$$= \frac{1}{2}C\varepsilon^2$$

Onde está a outra metade da energia?

Energia Dissipada no Resistor

Carga

$$dU = RI(t)^2\,dt$$

$$I = \left(\frac{\varepsilon}{R}\right)e^{-\frac{t}{RC}}$$

$$U = \int RI(t)^2\,dt$$

$$= \int_0^\infty R\left(\frac{\varepsilon}{R}e^{-\frac{t}{RC}}\right)^2 dt$$

$$= \frac{\varepsilon^2}{R}\int_0^\infty e^{-\frac{2t}{RC}}dt$$

$$= \frac{\varepsilon^2}{R}\left[-\frac{RC}{2}(0-1)\right] = \frac{1}{2}C\varepsilon^2$$

Descarga

$$dU = RI(t)^2\,dt$$

$$I = \left(-\frac{\varepsilon}{R}\right)e^{-\frac{t}{RC}}$$

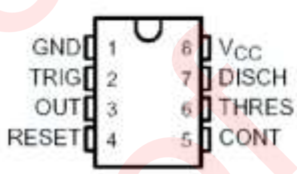
$$U = \int RI(t)^2\,dt$$

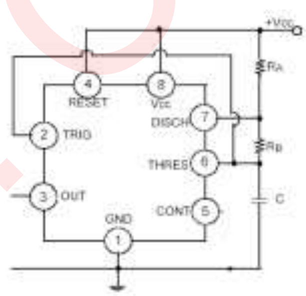
$$= \int_0^\infty R\left(-\frac{\varepsilon}{R}e^{-\frac{t}{RC}}\right)^2 dt$$

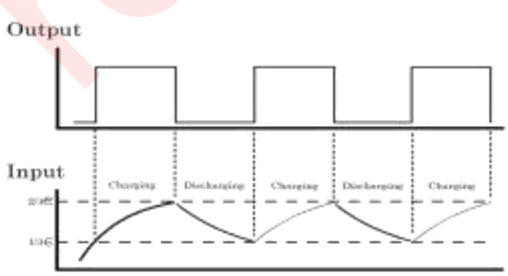
$$= \frac{\varepsilon^2}{R}\int_0^\infty e^{-\frac{2t}{RC}}dt$$

$$= \frac{\varepsilon^2}{R}\left[-\frac{RC}{2}(0-1)\right] = \frac{1}{2}C\varepsilon^2$$

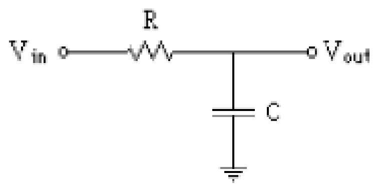
Aplicação: Temporizador RC







Filtro "Passa baixa"

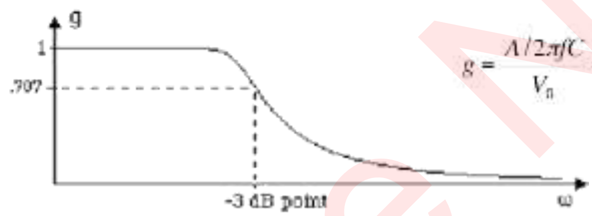


$$V_0 \sin[2\pi f t] = IR + \frac{Q}{C}$$

$$2\pi f V_0 \cos[2\pi f t] = \frac{dI}{dt} R + \frac{I}{C}$$

$$I[t] = A\cos[2\pi f t - \alpha]$$

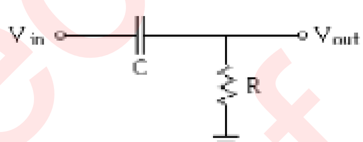
$$V[t] = Q[t]/C = \frac{\int I[t]dt}{C} = \frac{A\sin[2\pi f t - \alpha]}{2\pi f C}$$



$$g = \frac{A/2\pi f C}{V_0} = \frac{1}{\sqrt{1 + (RC2\pi f)^2}}$$

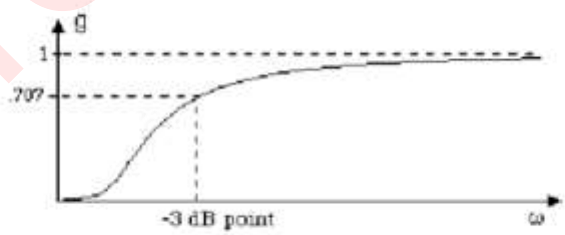
$$f_{-3dB} = \frac{1}{2\pi RC}$$

Filtro "Passa Alta"



$$V[t] = RA\cos[2\pi f t - \alpha]$$

$$g = \frac{1}{\sqrt{1/(RC2\pi f)^2 + 1}}$$



Solução Alternativa : Carga do Capacitor

$$R\frac{dq}{dt}+\frac{q}{C}=\varepsilon$$

$$\frac{dq}{dt}=\frac{\varepsilon}{R}-\frac{q}{RC}$$

$$\frac{dq}{dt}=\frac{C\varepsilon-q}{RC}$$

$$\frac{dq}{C\varepsilon-q}=\frac{1}{RC}dt$$

$$\int_0^q\frac{dq}{C\varepsilon-q}=\int_0^t\frac{1}{RC}dt$$

$$-\ln\left(C\varepsilon-q\right)\Big|_0^q=\frac{t}{RC}\Big|_0^t$$

$$\ln\left(C\varepsilon\right)-\ln\left(C\varepsilon-q\right)=\frac{t}{RC}$$

$$\ln\left(\frac{C\varepsilon}{C\varepsilon-q}\right)=\frac{t}{RC}\quad,\ln(x)=a\Rightarrow x=e^a$$

$$\frac{C\varepsilon}{C\varepsilon-q}=e^{\frac{t}{RC}}$$

$$C\varepsilon-q=C\varepsilon\,e^{-\frac{t}{RC}}$$

$$q=C\varepsilon-C\varepsilon\,e^{-\frac{t}{RC}}$$

$$\boxed{q=C\varepsilon\left(1-e^{-\frac{t}{RC}}\right)}$$

Solução Alternativa: Descarga do Capacitor

$$R\frac{dq}{dt}+\frac{q}{C}=0$$

$$\frac{dq}{dt}=-\frac{q}{RC}$$

$$\frac{dq}{dt}=-\frac{q}{RC}$$

$$\frac{dq}{q}=-\frac{1}{RC}dt$$

$$\int_Q^q\frac{dq}{q}=-\int_0^t\frac{1}{RC}dt$$

$$\ln\left(q\right)\Big|_Q^q=-\frac{t}{RC}\Big|_0^t$$

$$\ln\left(q\right)-\ln(Q)=-\frac{t}{RC}\quad,\ln(x)=a\Rightarrow x=e^a$$

$$\ln\left(\frac{q}{Q}\right)=-\frac{t}{RC}$$

$$\frac{q}{Q}=e^{-\frac{t}{RC}}$$

$$\boxed{q=Q\,e^{-\frac{t}{RC}}}$$

$Q=C\varepsilon$,
quando a carga do capacitor tiver sido completa