Eletromagnetismo

05 de março

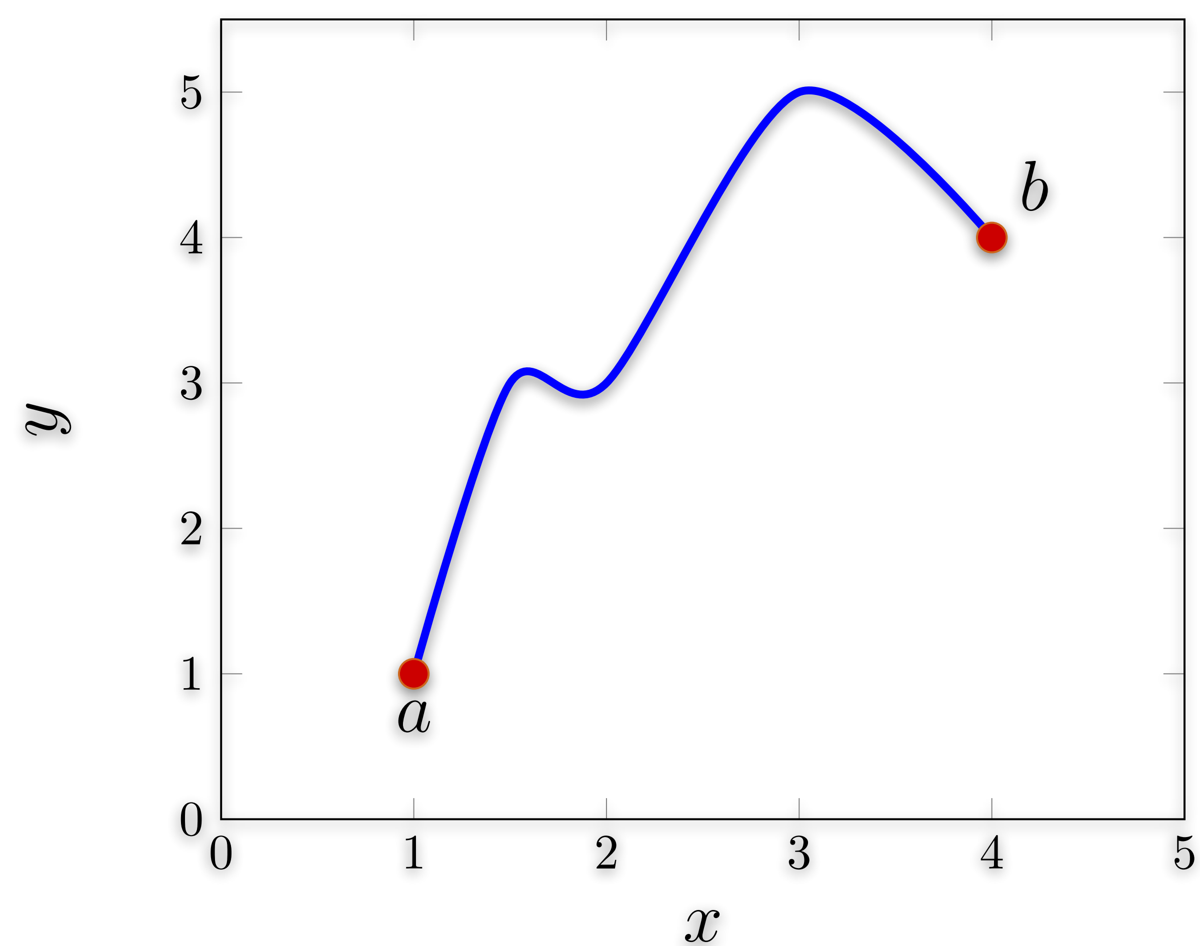
Análise vetorial

Análise vetorial

$$\int_{\mathcal{C}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

Teorema do gradiente

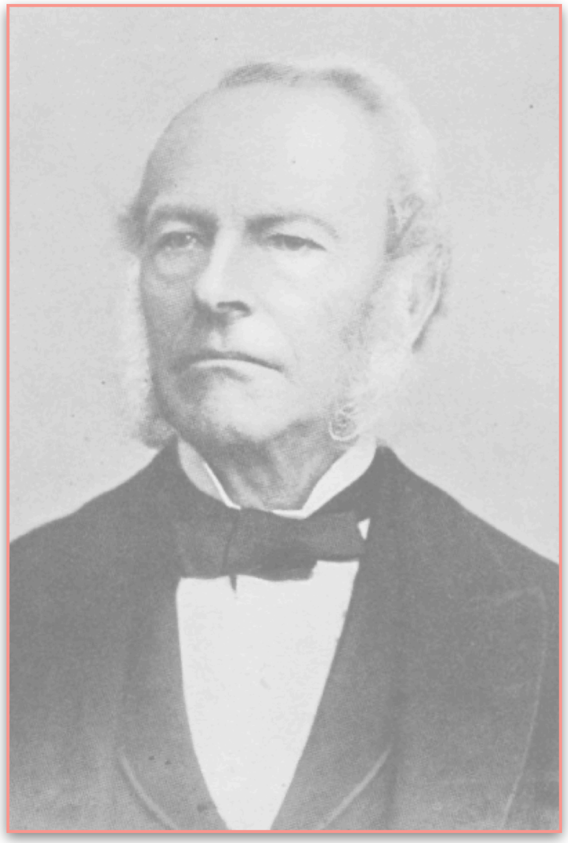
$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}$$



Análise vetorial

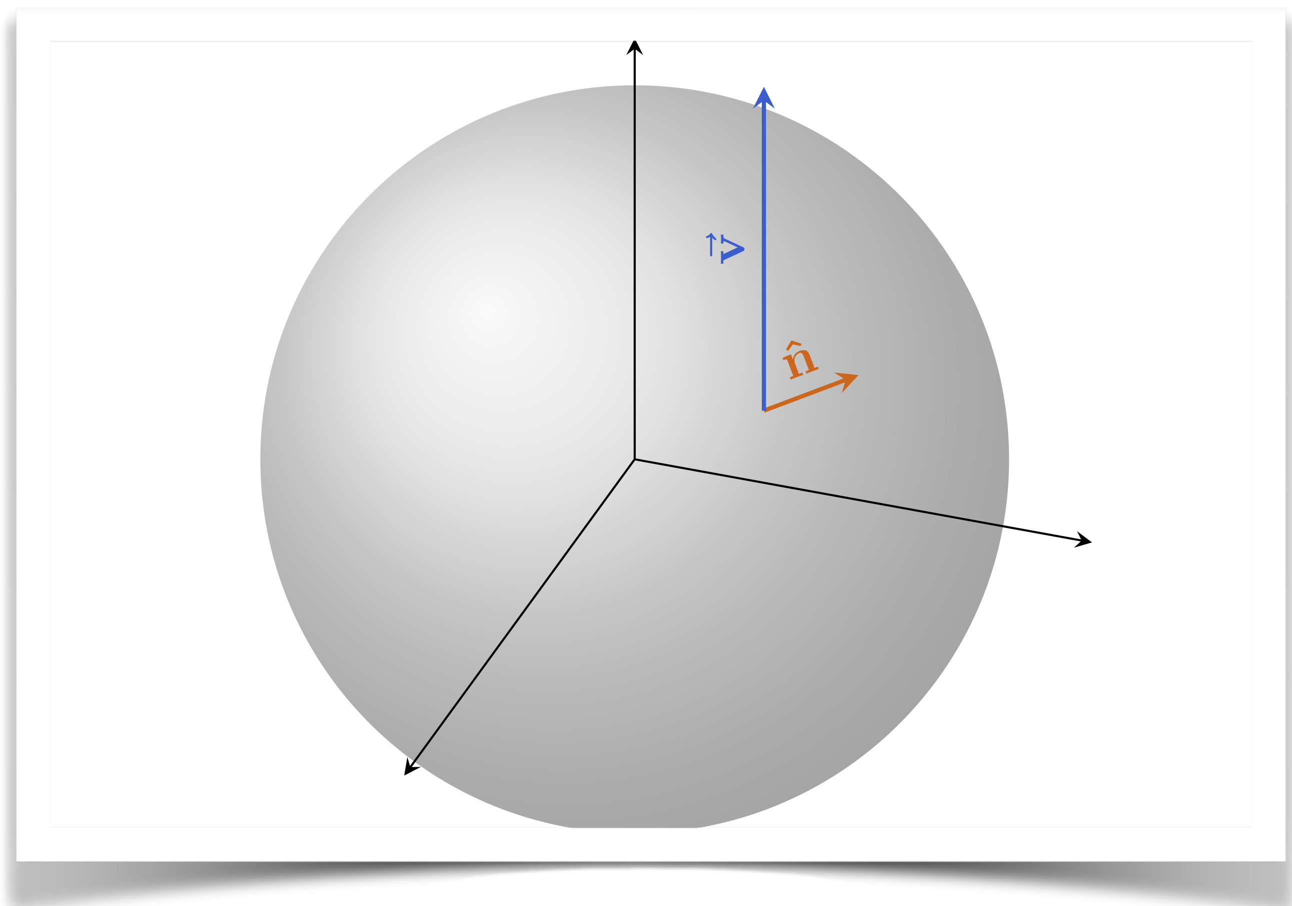
$$\int_S \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{v}} \cdot \hat{n}\, \mathrm{d}A = \oint \vec{\mathbf{v}} \cdot \mathrm{d}\vec{\ell}$$

Teorema de Stokes



Análise vetorial

$$\int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} \; \mathrm{d}\tau = \int \vec{\mathbf{v}} \cdot \hat{n} \; \mathrm{d}A$$

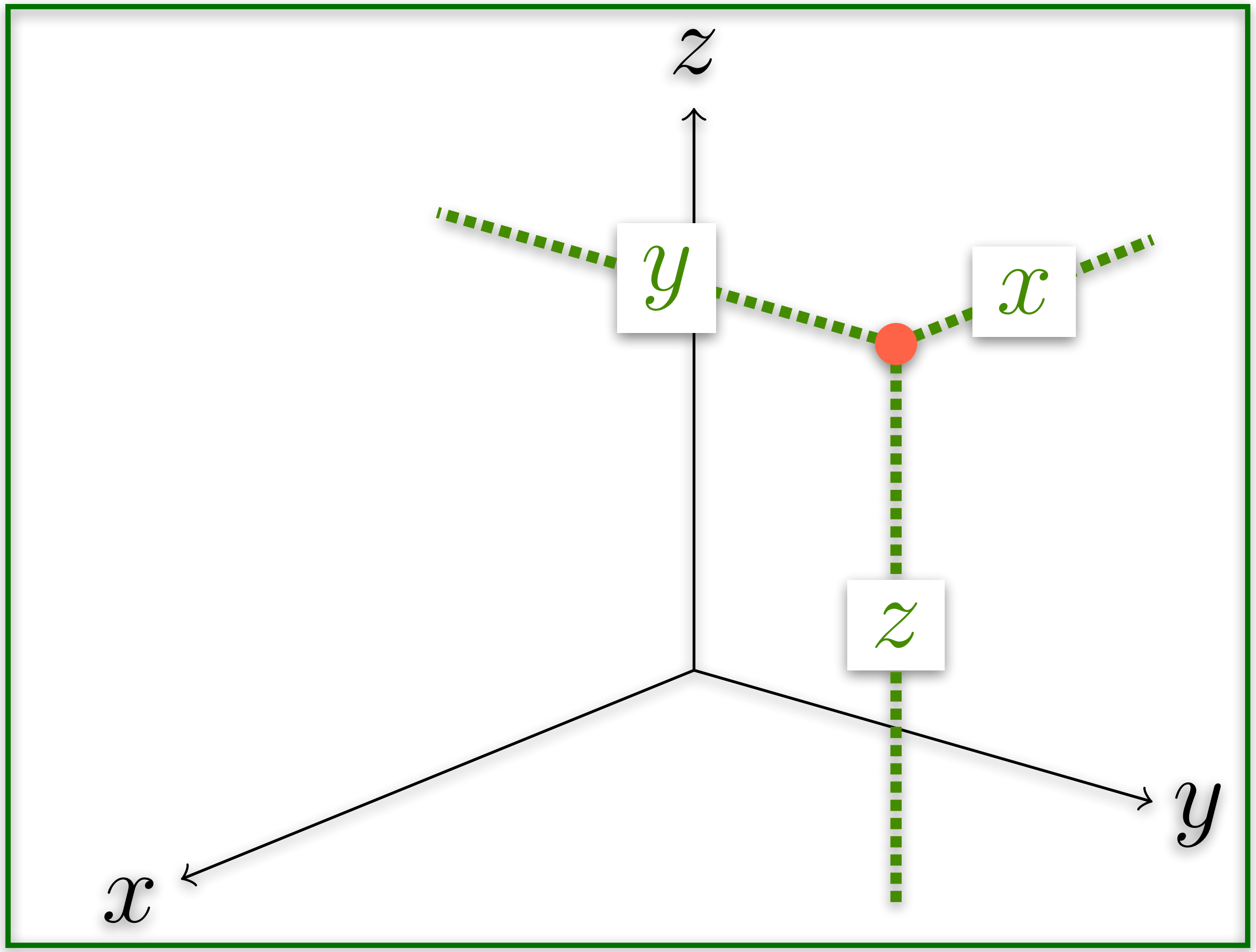


Teorema de Gauss

Análise vetorial

Coordenadas cartesianas

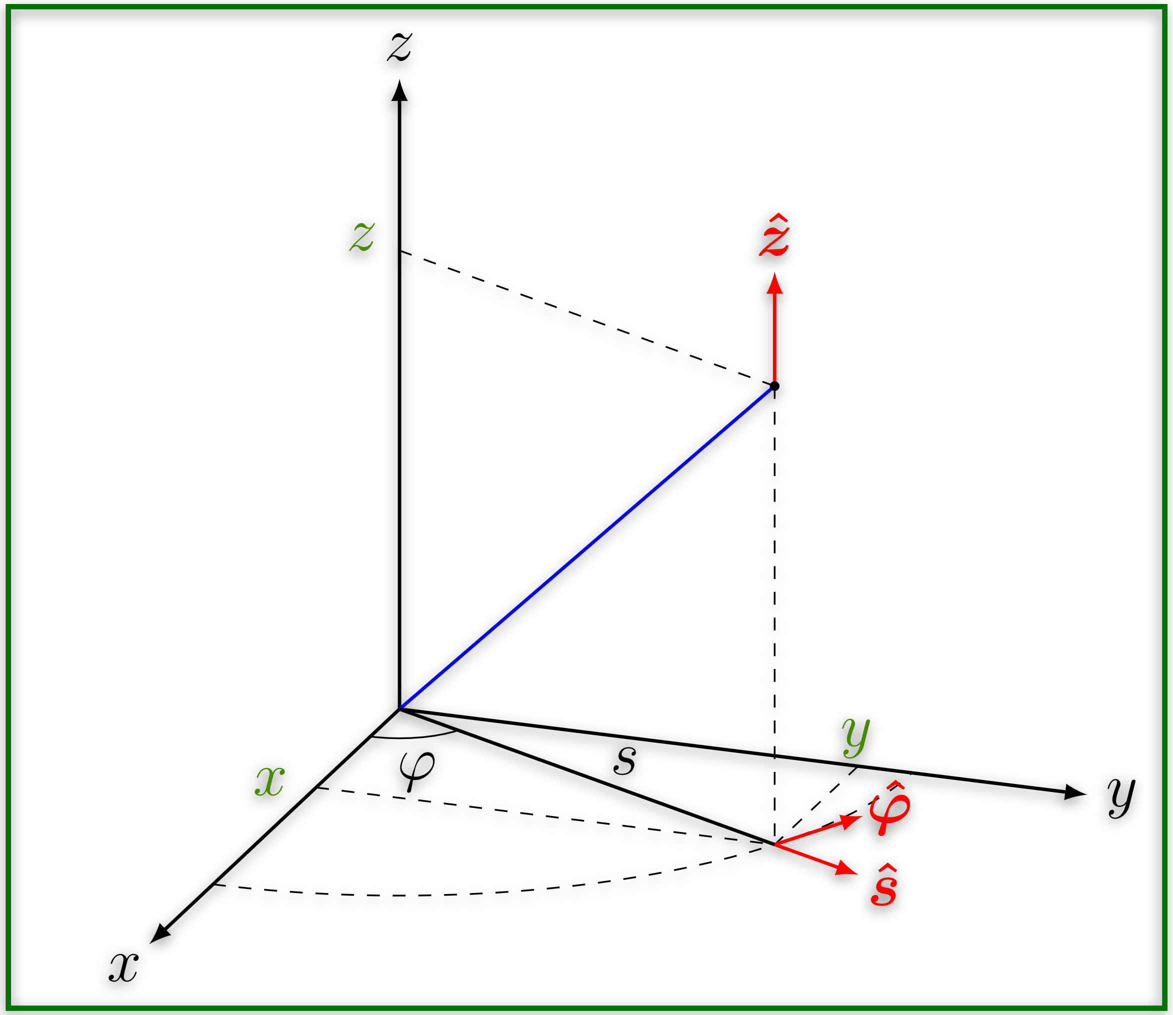
$$d\vec{\ell} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$



Análise vetorial

Coordenadas cilíndricas

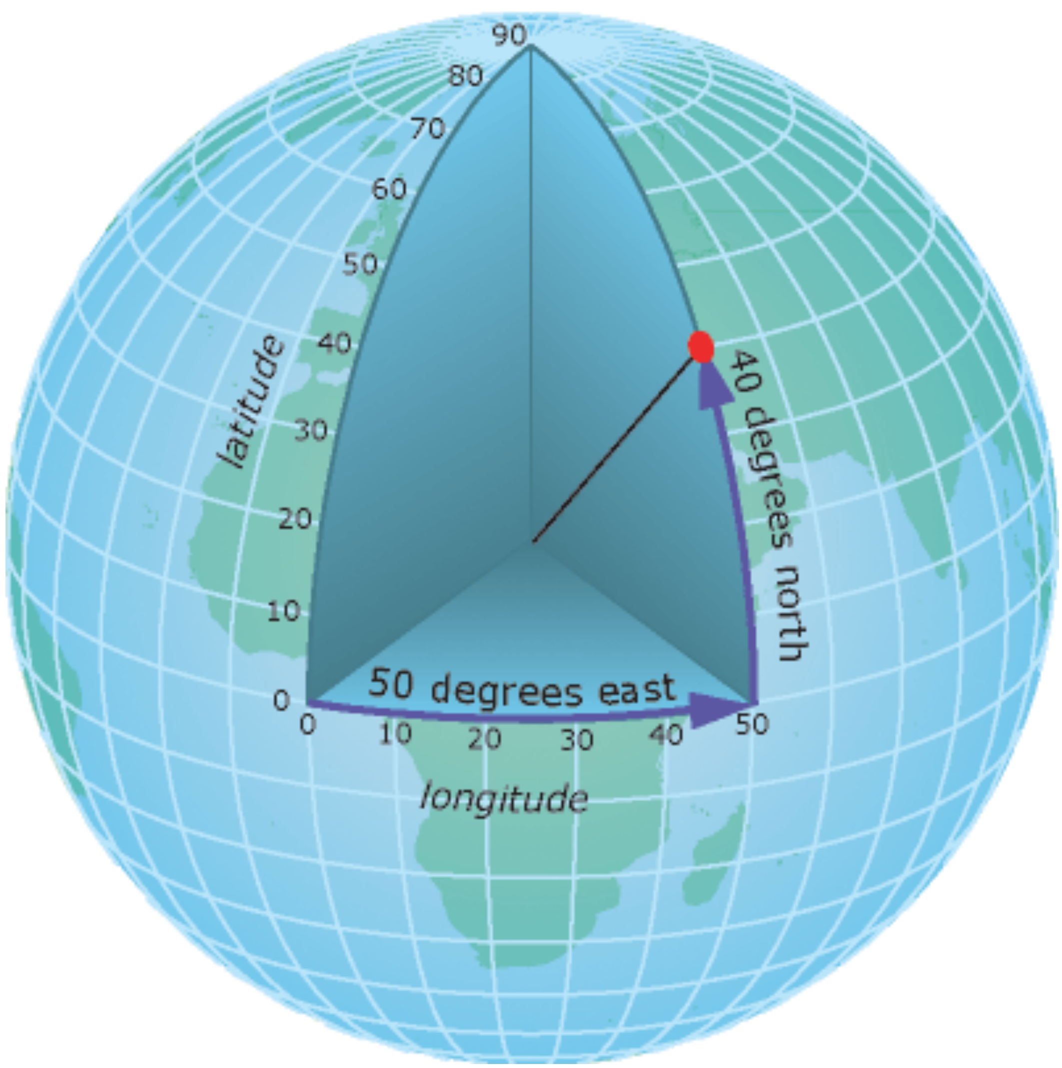
$$\mathrm{d}\vec{\ell} = ds\,\hat{\mathbf{s}} + s\,\mathrm{d}\varphi\,\hat{\varphi} + \mathrm{d}z\,\hat{\mathbf{z}}$$

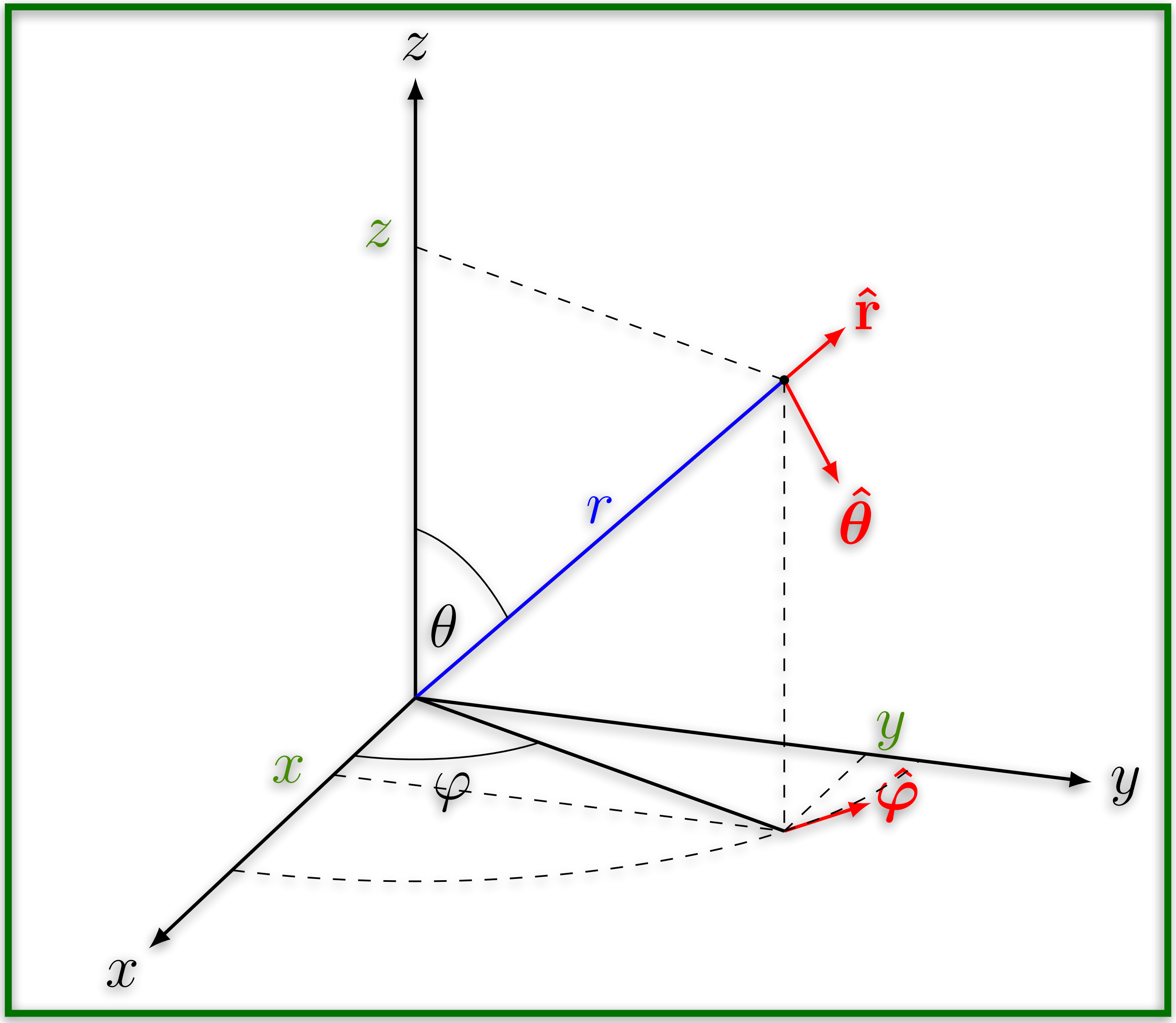


Análise vetorial

Coordenadas esféricas

$$d\vec{\ell} = dr\hat{\mathbf{r}} + r\,d\theta\hat{\theta} + r\sin(\theta)d\varphi\hat{\varphi}$$





Análise vetorial

Expressão para o gradiente

$$\int_{\mathcal{C}} \vec{\boldsymbol{\nabla}} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

Análise vetorial

Expressão para o gradiente

$$\int_{\mathcal{C}} \vec{\boldsymbol{\nabla}} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

$$\vec{\boldsymbol{\nabla}} T \cdot \mathrm{d}\vec{\ell} = \mathrm{d}T$$

Análise vetorial

Expressão para o gradiente

$$\int_{\mathcal{C}} \vec{\boldsymbol{\nabla}} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

$$\vec{\boldsymbol{\nabla}} T \cdot \mathrm{d}\vec{\ell} = \mathrm{d}T$$

$$\vec{\boldsymbol{\nabla}} T)_x \mathrm{d}x + \vec{\boldsymbol{\nabla}} T)_y \mathrm{d}y + \vec{\boldsymbol{\nabla}} T)_z \mathrm{d}z = \frac{\partial T}{\partial x}\mathrm{d}x + \frac{\partial T}{\partial y}\mathrm{d}y + \frac{\partial T}{\partial z}\mathrm{d}z$$

Análise vetorial

Expressão para o gradiente

$$\int_{\mathcal{C}} \vec{\boldsymbol{\nabla}} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

$$\vec{\boldsymbol{\nabla}} T \cdot \mathrm{d}\vec{\ell} = \mathrm{d}T$$

$$\vec{\boldsymbol{\nabla}} T)_x \mathrm{d}x + \vec{\boldsymbol{\nabla}} T)_y \mathrm{d}y + \vec{\boldsymbol{\nabla}} T)_z \mathrm{d}z = \frac{\partial T}{\partial x}\mathrm{d}x + \frac{\partial T}{\partial y}\mathrm{d}y + \frac{\partial T}{\partial z}\mathrm{d}z$$

$$\Rightarrow \quad \vec{\boldsymbol{\nabla}} T)_x = \frac{\partial T}{\partial x}$$

Análise vetorial

Expressão para o gradiente

$$\int_{\mathcal{C}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

$$\vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = \mathrm{d}T$$

$$\vec{\nabla} T)_x \mathrm{d}x + \vec{\nabla} T)_y \mathrm{d}y + \vec{\nabla} T)_z \mathrm{d}z = \frac{\partial T}{\partial x}\mathrm{d}x + \frac{\partial T}{\partial y}\mathrm{d}y + \frac{\partial T}{\partial z}\mathrm{d}z$$

$$\boxed{\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}}$$

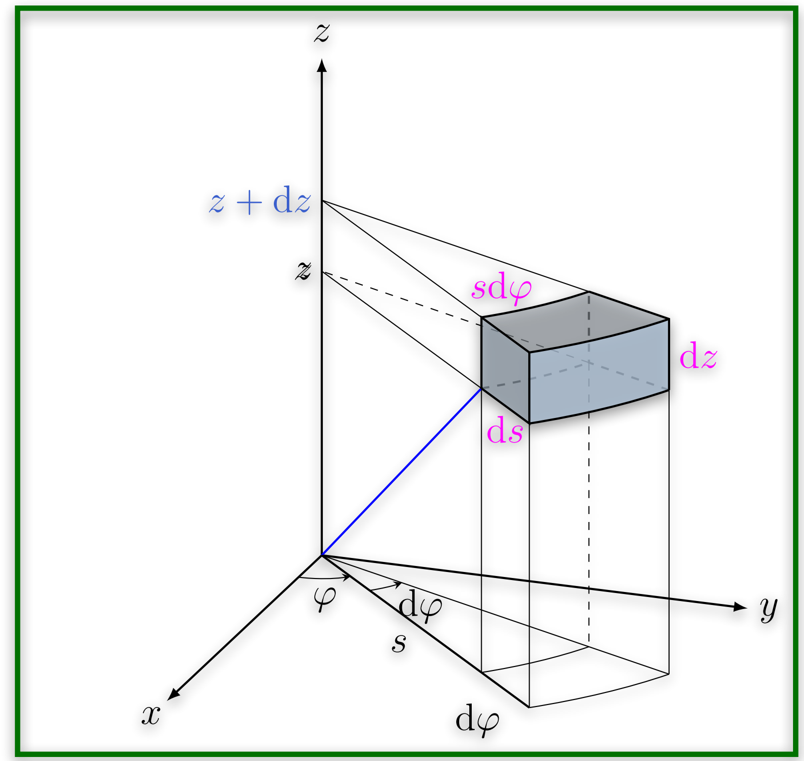
Cartesianas

Análise vetorial

Expressão para o gradiente

$$\int_{\mathcal{C}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

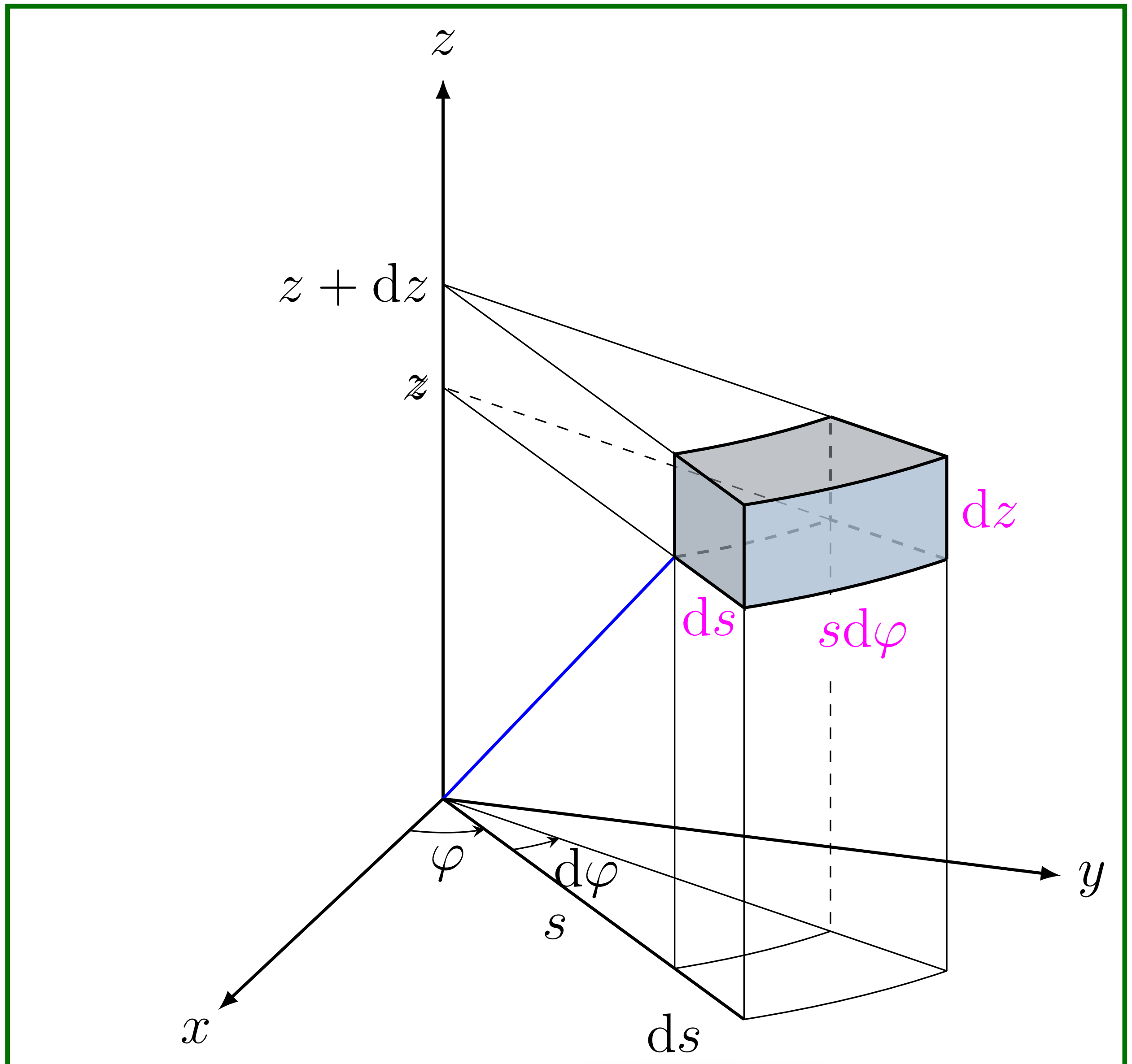
$$\vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = \mathrm{d}T$$



Cilíndricas

Análise vetorial

Expressão para o gradiente



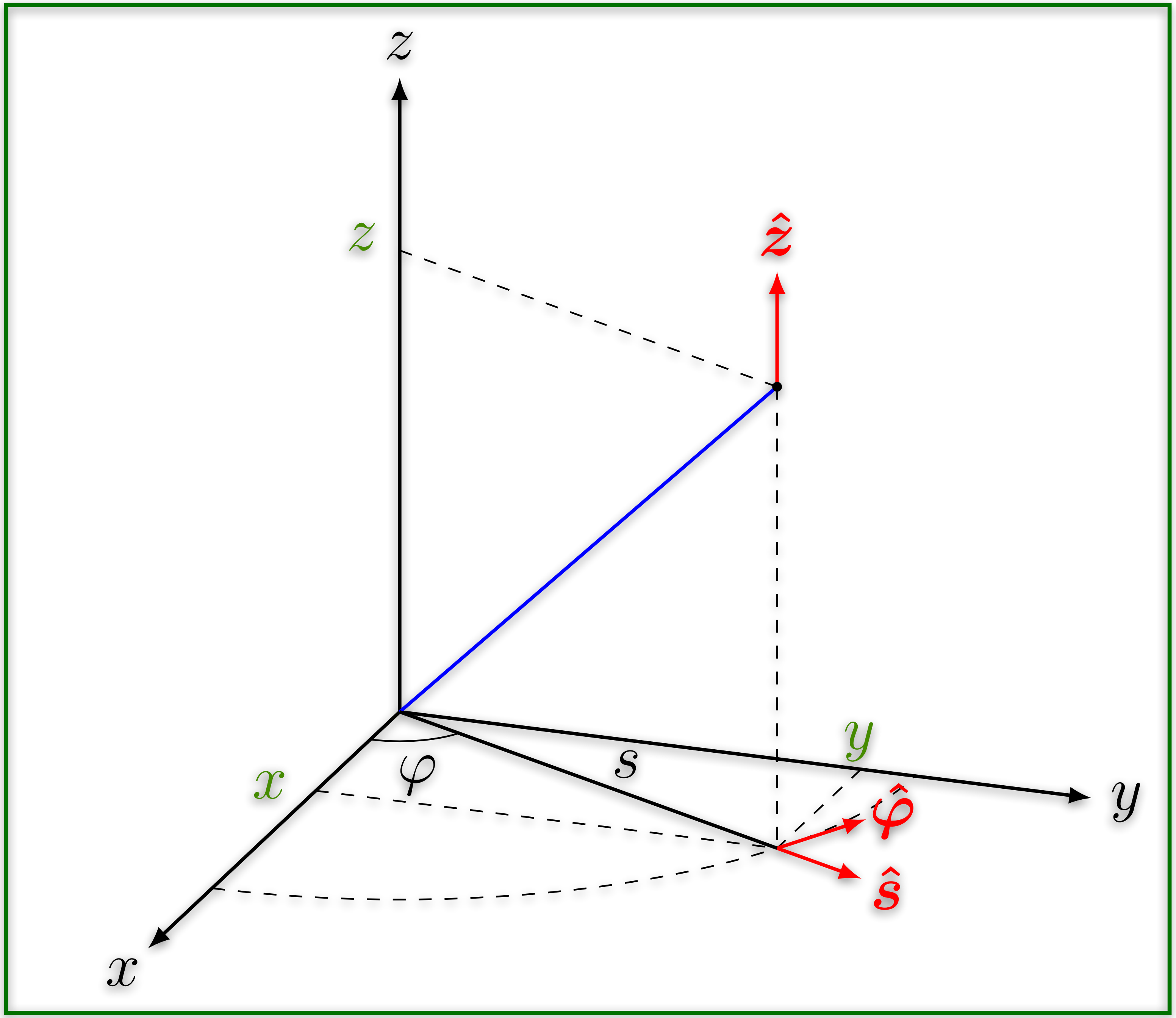
$$\vec{\nabla} T \cdot d\vec{\ell} = dT$$

$$d\vec{\ell} = ds\hat{\mathbf{s}} + s\, d\varphi\, \hat{\varphi} + dz\hat{\mathbf{z}}$$

Cilíndricas

Análise vetorial

Expressão para o gradiente



$$\vec{\boldsymbol{\nabla}} T \cdot \mathrm{d}\vec{\ell} = \mathrm{d}T$$

$$\mathrm{d}\vec{\ell} = ds\,\hat{\mathbf{s}} + s\,\mathrm{d}\varphi\,\hat{\varphi} + \mathrm{d}z\,\hat{\mathbf{z}}$$

Cilíndricas

Análise vetorial

Expressão para o gradiente

$$\int_{\mathcal{C}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

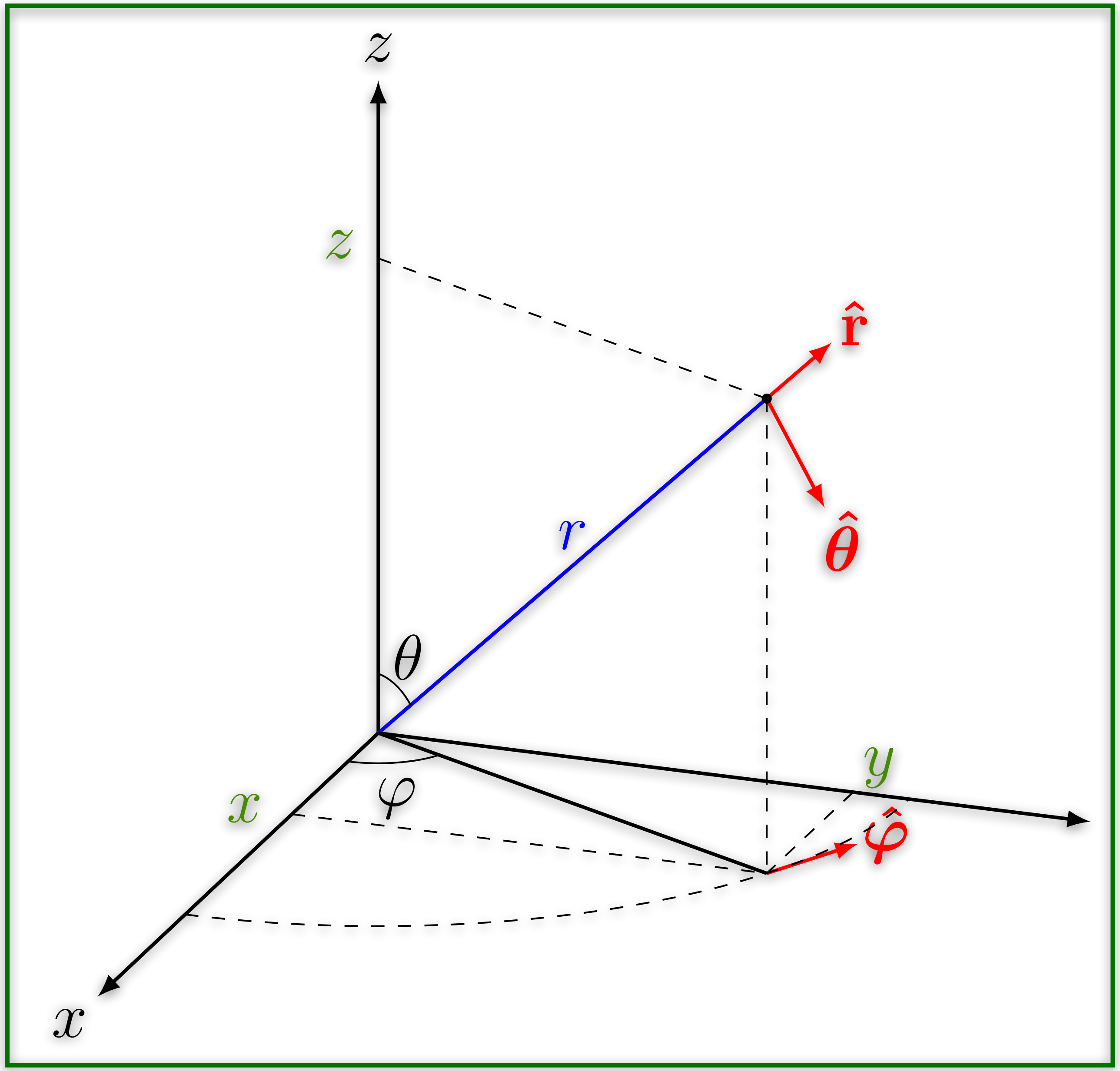
$$\mathrm{d}\vec{\ell} = ds\,\hat{\mathbf{s}} + s\,\mathrm{d}\varphi\,\hat{\varphi} + \mathrm{d}z\,\hat{\mathbf{z}}$$

$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial \varphi}\hat{\varphi} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}$$

Cilíndricas

Análise vetorial

Expressão para o gradiente



$$\int_{\mathcal{C}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

$$\mathrm{d}\vec{\ell} = dr\hat{\mathbf{r}} + r\mathrm{d}\theta\hat{\theta} + r\sin(\theta)\mathrm{d}\varphi\hat{\varphi}$$

Esféricas

Análise vetorial

Expressão para o gradiente

$$\int_{\mathcal{C}} \vec{\boldsymbol{\nabla}} T \cdot \mathrm{d}\vec{\ell} = T_b - T_a$$

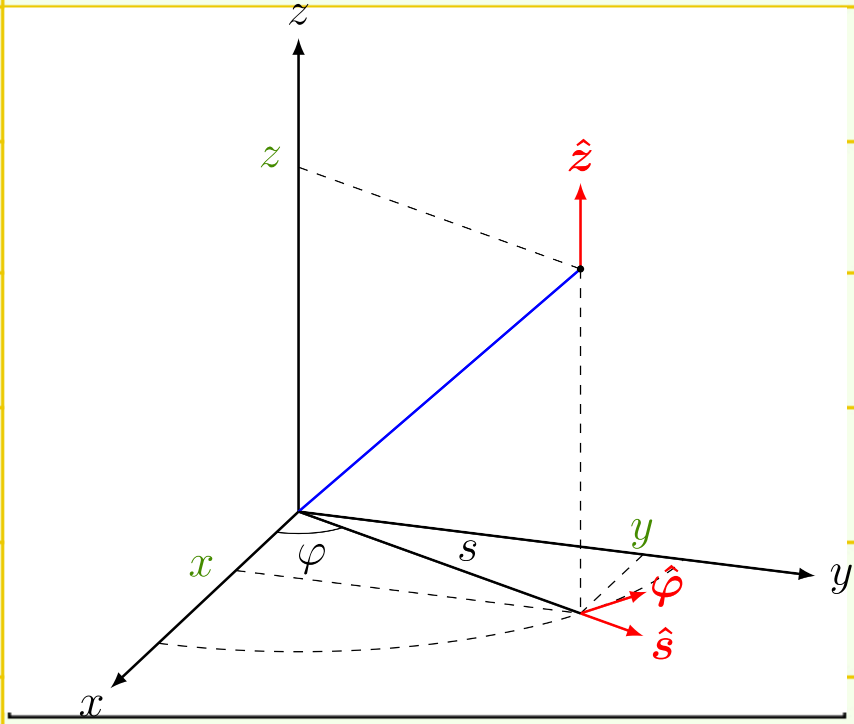
$$\mathrm{d}\vec{\ell} = ds\,\hat{\mathbf{s}} + s\,\mathrm{d}\varphi\,\hat{\varphi} + \mathrm{d}z\,\hat{\mathbf{z}}$$

$$\vec{\boldsymbol{\nabla}} T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \varphi}\hat{\varphi}$$

Esféricas

Coordenadas cilíndricas

$$d\vec{\ell} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$



$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

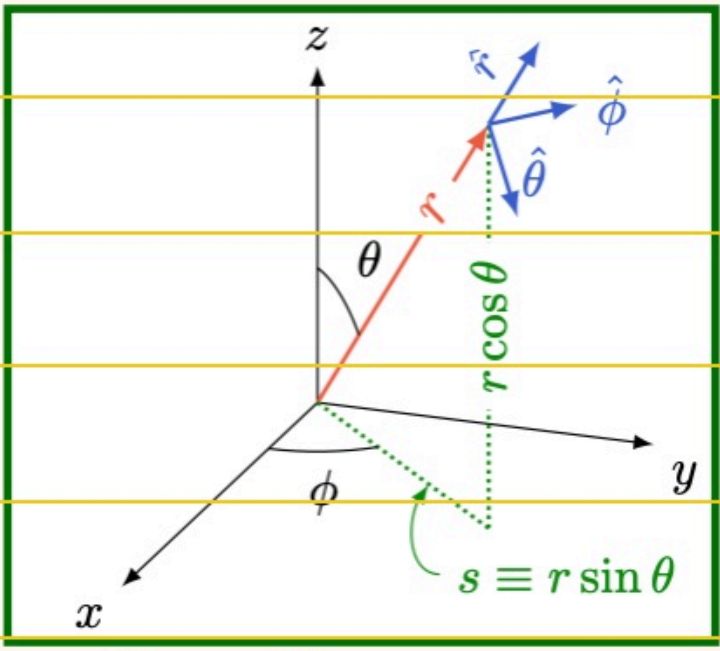
$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

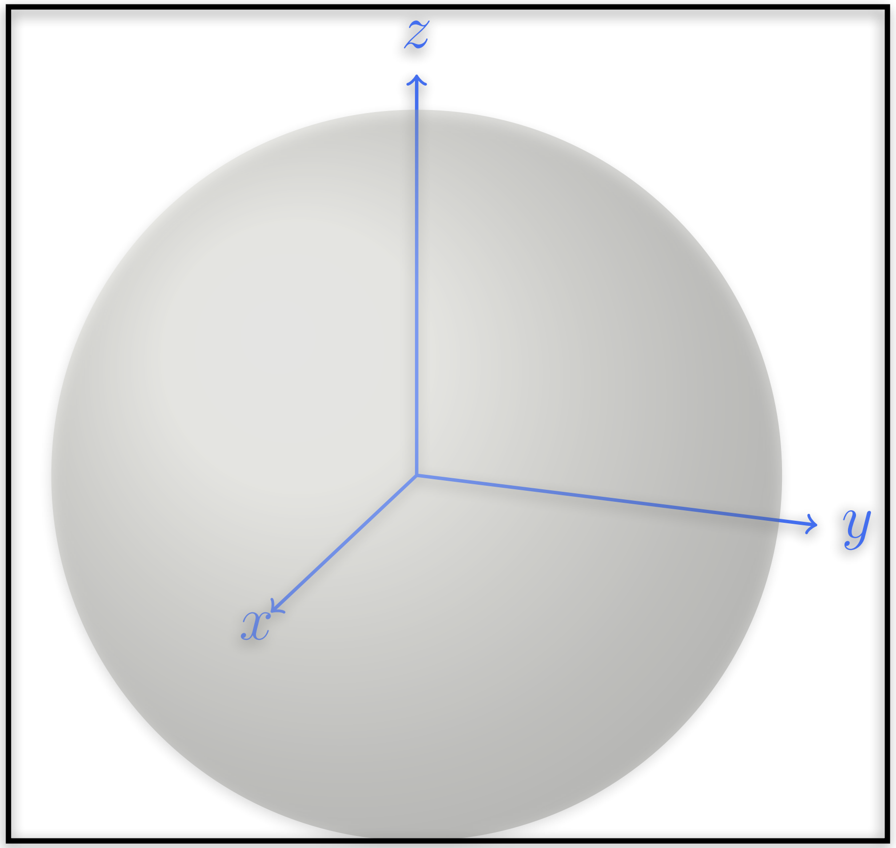
$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Pratique o que aprendeu

$$\vec{\mathbf{v}}(\vec{\mathbf{r}}) = \frac{\hat{\mathbf{r}}}{r^2} \qquad (r > 0)$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = ?$$



Função delta

(distribuição ou funcional)

$$\delta(x) = \begin{cases} 0 & (x \neq 0) \\ \text{indefinida} & (x = 0) \end{cases}$$

$$\int_{-a}^{a} f(x)\delta(x)\,\mathrm{d}x = f(0)$$

Função delta

Três dimensões

$$\delta^3(\vec{\mathbf{r}}) \equiv \delta(x)\delta(y)\delta(z)$$

$$\delta^3(\vec{\mathbf{r}}) = \begin{cases} 0 & \vec{\mathbf{r}} \neq 0 \\ \text{indefinida} & \vec{\mathbf{r}} = 0 \end{cases}$$

$$\int_{\mathcal{V}} f(\vec{\mathbf{r}})\delta^3(\vec{\mathbf{r}})\mathrm{d}\tau = f(0)$$

Pratique o que aprendeu

$$\delta(cx) = ?$$

Pratique o que aprendeu

$$\delta(cx) = ?$$

$$\delta(ax) = \begin{cases} 0 & (x \neq 0) \\ \text{indefinida} & (x = 0) \end{cases}$$ ✓

$$\int_{-a}^{a} f(x)\delta(cx)\,\mathrm{d}x = f(0)$$?

Pratique o que aprendeu

$$\delta(cx) = ? \qquad (c > 0)$$

$$\delta(cx) = \begin{cases} 0 & (x \neq 0) \\ \text{indefinida} & (x = 0) \end{cases} \qquad \checkmark$$

$$\int_{-a}^{a} f(x)\delta(cx)\,\mathrm{d}x = \frac{1}{c}\int_{-ca}^{ca} f(\frac{y}{c})\delta(y)\,\mathrm{d}y \qquad (c > 0)$$

Pratique o que aprendeu

$$\delta(cx) = ? \qquad (c > 0)$$

$$\delta(cx) = \begin{cases} 0 & (x \neq 0) \\ \text{indefinida} & (x = 0) \end{cases} \qquad \checkmark$$

$$\int_{-a}^{a} f(x)\delta(cx)\,\mathrm{d}x = \frac{1}{c}\int_{-ca}^{ca} f(\frac{y}{c})\delta(y)\,\mathrm{d}y = \frac{1}{c}f(0) \quad \Rightarrow \quad \delta(cx) = \frac{1}{c}\delta(x)$$

Pratique o que aprendeu

$$\delta(cx) = ? \qquad (c < 0)$$

$$\delta(cx) = \begin{cases} 0 & (x \neq 0) \\ \text{indefinida} & (x = 0) \end{cases} \qquad \checkmark$$

$$\int_{-a}^{a} f(x)\delta(cx)\,\mathrm{d}x = \frac{-1}{c}\int_{ca}^{-ca} f(\frac{y}{c})\delta(y)\,\mathrm{d}y \qquad (c < 0)$$

Pratique o que aprendeu

$$\delta(cx) = ?$$

$$\delta(cx) = \begin{cases} 0 & (x \neq 0) \\ \text{indefinida} & (x = 0) \end{cases} \qquad \checkmark$$

$$\int_{-a}^{a} f(x)\delta(cx)\,\mathrm{d}x = -\frac{1}{c}\int_{ca}^{-ca} f(\frac{y}{c})\delta(y)\,\mathrm{d}y = -\frac{1}{c}f(0)$$

$$\Rightarrow \quad \delta(cx) = -\frac{1}{c}\delta(x) \qquad (c < 0)$$

Pratique o que aprendeu

$$\delta(cx) = \frac{1}{c}\delta(x) \qquad (c > 0)$$

$$\delta(cx) = -\frac{1}{c}\delta(x) \qquad (c < 0)$$

$$\boxed{\delta(cx) = \frac{1}{|c|}\delta(x)}$$

Pratique o que aprendeu

$$\delta(cx) = \frac{1}{c}\delta(x) \qquad (c > 0)$$

$$\delta(cx) = -\frac{1}{c}\delta(x) \qquad (c < 0)$$

$$\boxed{\delta(cx) = \frac{1}{|c|}\delta(x)}$$

$c = -1 \Rightarrow \delta(-x) = \delta(x) \Rightarrow$ função par

Pratique o que aprendeu

$$\vec{\mathbf{v}}(\vec{\mathbf{r}}) = \frac{\vec{\mathbf{r}}}{r^3}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}} = ?$$