1º Simpósio NOVACAP

A tecnologia nas coberturas das arenas de 2014

Tensoestruturas:

ideias básicas e algumas aplicações aos projetos de estádios

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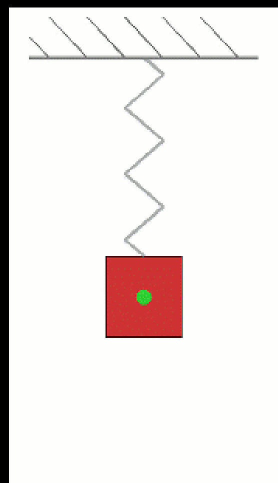
NOVACAP

Brasília, 04/12/2012


Taut Structures:

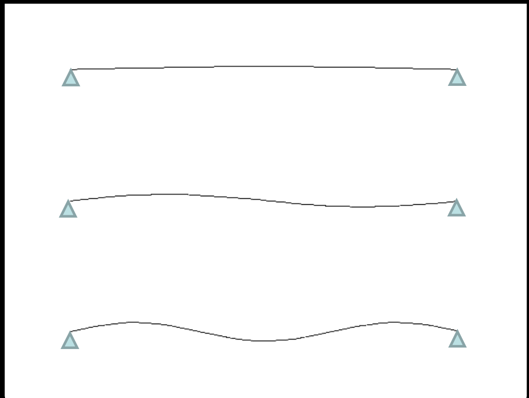
"those that require their elements to be ***taut***, instead of ***slack*** or ***wrinkled***, to work properly.

A spring-mass system:



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

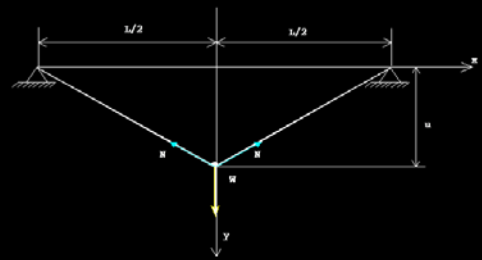
A vibrating string:



$$f = n\pi \sqrt{\frac{\left(\frac{T}{L}\right)}{m}}$$

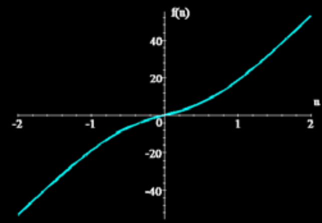
$$k_g \sim \frac{T}{L}$$

A transversaly loaded string:



$$k = \frac{EA}{\ell_r} \qquad \ell_r < L$$

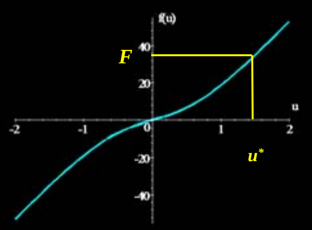
$$P(u) = 4k\left(1 - \frac{\ell_r}{\sqrt{L^2 + 4u^2}}\right)u$$



Non-linear equilibrium:

Given F, find u^* such that

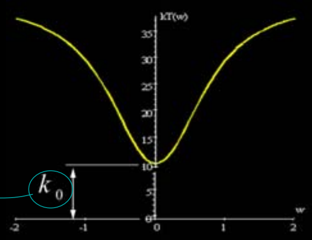
$$g\left(u^*\right) = P\left(u^*\right) - F = 0$$



Newton´s Method:

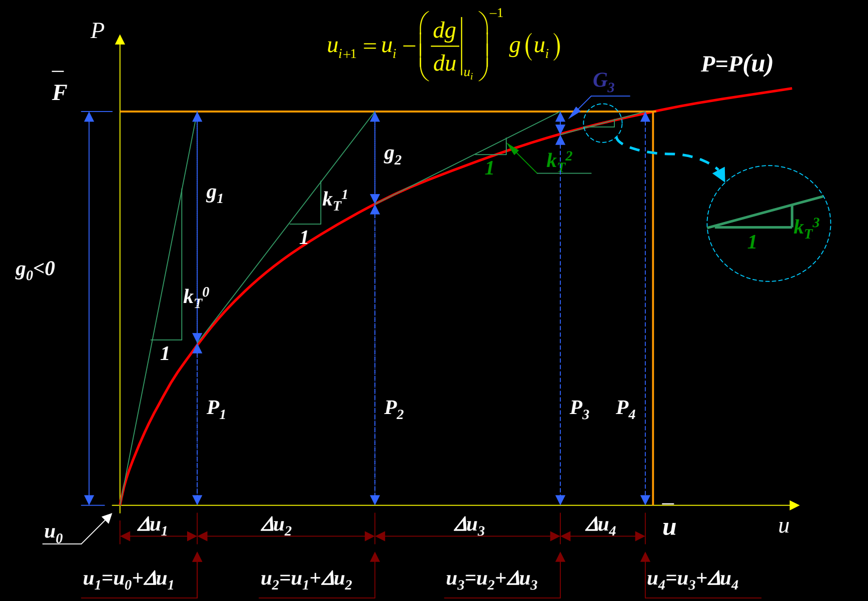
$$u_{i+1} = u_i - \left(\left.\frac{dg}{du}\right|_{u_i}\right)^{-1} g\left(u_i\right)$$

Tangent stiffness $k_{t_i} = \left.\frac{dg}{du}\right|_{u_i}$



$$k_0 = \frac{4T_0}{L}$$

Newton´s Method for a scalar problem $g=P(u)-F=0$



Geometrically Non-Linear Equilibrium, for many DOF:

Find $\mathbf{u}^*$ such that $\mathbf{g}\left(\mathbf{u}^*\right) = \mathbf{P}\left(\mathbf{u}^*\right) - \mathbf{F}\left(\mathbf{u}^*\right) = \mathbf{0}$

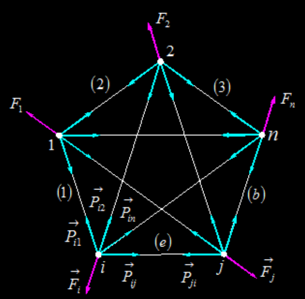
$\mathbf{P} = \mathbf{P}(\mathbf{u})$	Internal Load Vector
$\mathbf{F} = \mathbf{F}(\mathbf{u})$	External Load Vector
$\mathbf{g} = \mathbf{g}(\mathbf{u})$	Unbalanced ('Error') Load Vector

Newton's method for many DOFs:

$$\mathbf{u}_{i+1} = \mathbf{u}_i - \left(\left.\frac{\partial \mathbf{g}}{\partial \mathbf{u}}\right|_{\mathbf{u}_i}\right)^{-1} \mathbf{g}(\mathbf{u}_i) = \mathbf{u}_i - \left(\mathbf{K}_t^{\,i}\right)^{-1} \mathbf{g}(\mathbf{u}_i)$$

$$\mathbf{K}_t^{\,i} = \left.\frac{\partial \mathbf{g}}{\partial \mathbf{u}}\right|_{\mathbf{u}_i}$$ **Tangent stiffness matrix**

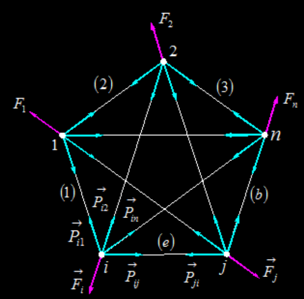
A System of Central Forces



$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_n \end{bmatrix} \quad ; \quad \mathbf{P} = \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_n \end{bmatrix}$$

$$\mathbf{p}_i = \sum_{j=1}^{n} \mathbf{p}_{ij} = \sum_{j=1}^{n} N_{ij}\,\mathbf{v}_{ij} = \sum_{j=1}^{n} N_{ij} \frac{\mathbf{x}_j - \mathbf{x}_i}{\left\|\mathbf{x}_j - \mathbf{x}_i\right\|} \quad , \quad i = 1, \ldots, n$$

A System of Central Forces



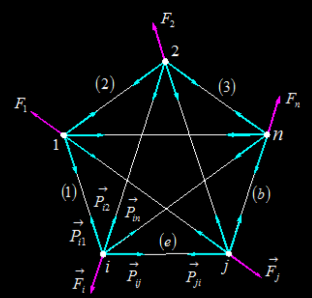
$$\mathbf{P} = \mathbf{C}\,\mathbf{N}$$

"Geometric operator" ($\mathbf{C}$)

Vector of internal <u>element</u> loads ($\mathbf{N}$)

$$\mathbf{C} = \sum_{(e)=1}^{b} \mathbf{C}^{(e)} = \sum_{(e)=1}^{b} \begin{array}{c} \\ 1 \\ i \\ \vdots \\ j \\ \vdots \\ n \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & \cdots & (e) & \cdots & b \end{array} \\ \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & -\mathbf{v}^{(e)} & \cdots & 0 \\ \vdots & \vdots & 0 & \vdots & 0 \\ 0 & 0 & \mathbf{v}^{(e)} & 0 & 0 \\ \vdots & \vdots & 0 & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}_{3n\times b} \end{array} \qquad \mathbf{N} = \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_b \end{bmatrix}_{b\times 1}$$

A System of Central Forces



$$\mathbf{P} = \mathbf{C}\,\mathbf{N}$$

"Geometric operator"

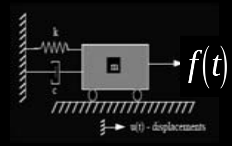
Vector of internal element loads

$$\mathbf{K}_t = \frac{\partial}{\partial \mathbf{u}}(\mathbf{C}\mathbf{N}-\mathbf{F}) = \mathbf{C}\frac{\partial \mathbf{N}}{\partial \mathbf{u}} + \mathbf{N}^T\frac{\partial \mathbf{C}}{\partial \mathbf{u}} - \frac{\partial \mathbf{F}}{\partial \mathbf{u}}$$

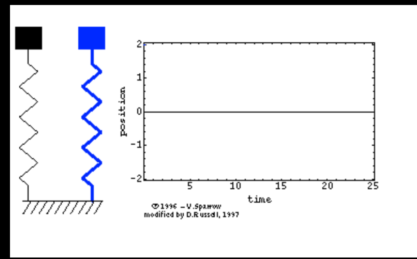
$$\mathbf{K}_t = \mathbf{K}_c + \mathbf{K}_g + \mathbf{K}_{ext}$$

The Dynamic Relaxation Method

A single DOF oscillator:

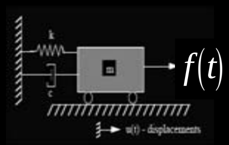


$$m\ddot{u} + c\dot{u} + p(u) = f(t)$$



The Dynamic Relaxation Method

A single DOF oscillator:

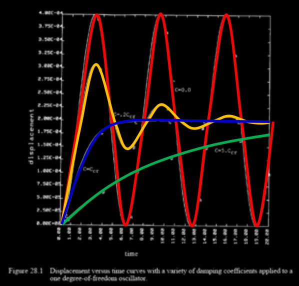


$$m\ddot{u} + c\dot{u} + p(u) = f(t)$$

A linear SDOF oscillator under a step force:

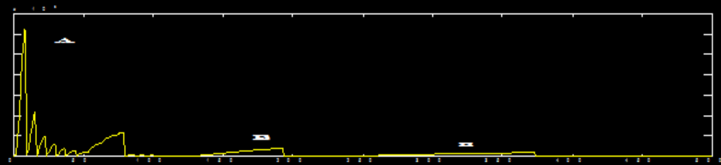
$$p(u) = ku$$

$$f(t) = \begin{cases} 0, & t < 0 \\ F, & t \geq 0 \end{cases}$$

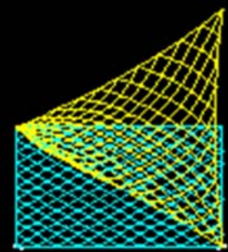


The Dynamic Relaxation Method

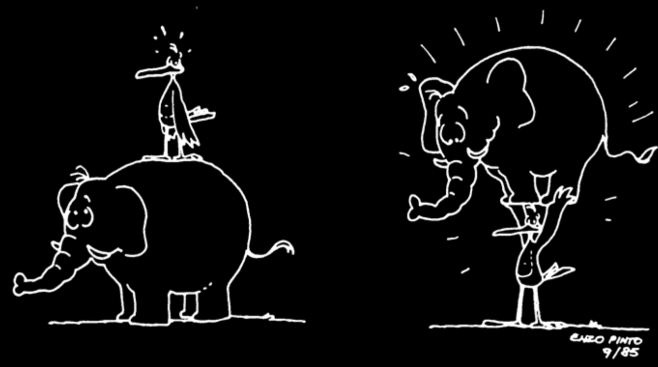
Kinetic Damping:



Transient of kinetic energy during the shape finding of a cable network via DR, with kinetic damping

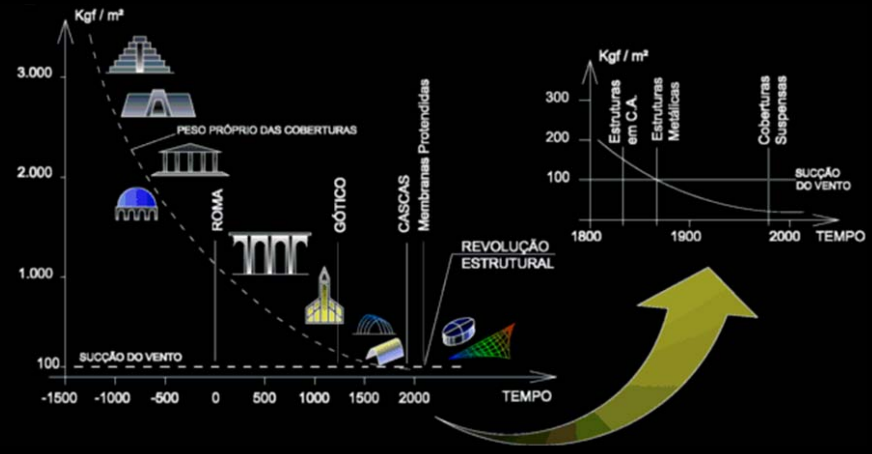


“Light structures”



“Peso portante << Peso portado”
(Majowiecki, 1994)

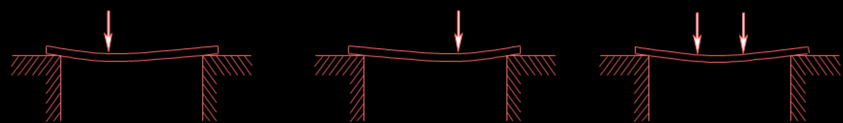
“Light structures”



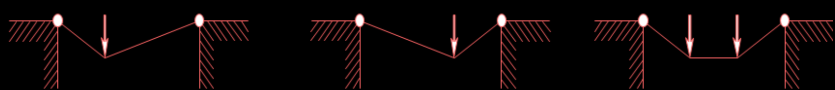
“Light structures, structures of light” – H. Berger

Suvarnabhumi Airport, Bangkok, Thailand

Taut Structures are ‘flexible’:



(a) a stiff ‘structure’ , such as a bridge, does not change drastically its shape, when loading varies



(b) A ‘flexible’ structure, such as a cable, can change drastically its shape, when loading varies

Earhquake in Eichuan, China (May, 2008)

Flexible structures must conform to

Funicular shapes:

Those that equilibrate a set of loads, without bending moments

Flexible structures must conform to

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Funicular shapes:

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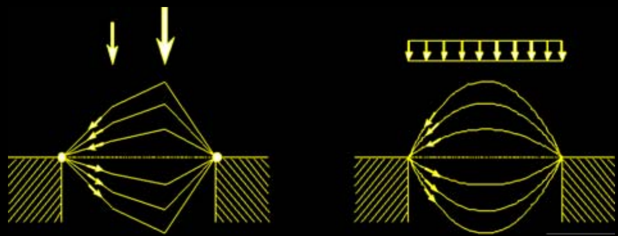
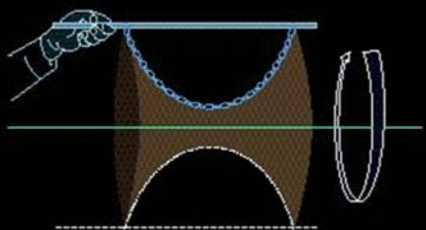
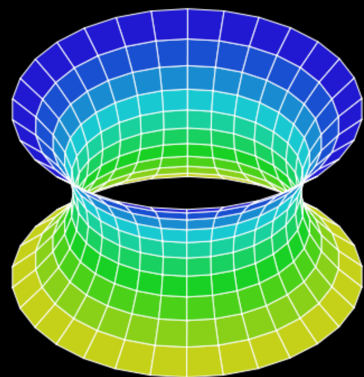
Funicular Shapes:



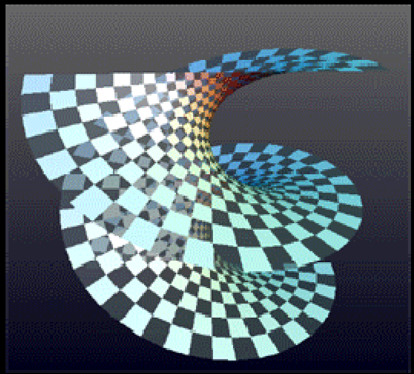
The Catenaria



The Catenoid:



The Helicatenoid



A gyroid (Alan Schoen, 1970)

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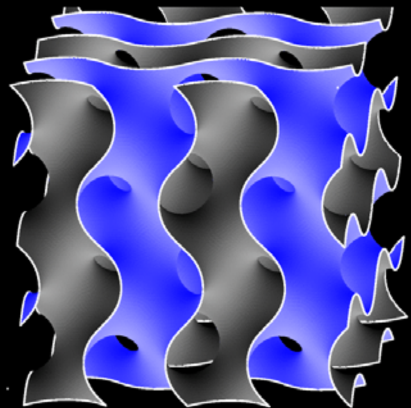
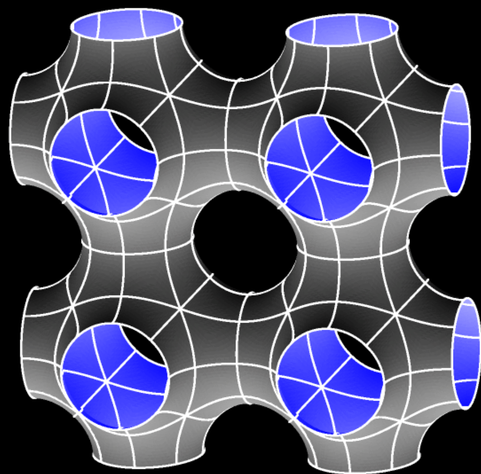
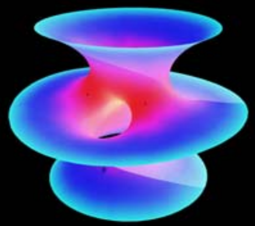


A Schwarz-P surface



Costa's Surface (1982):

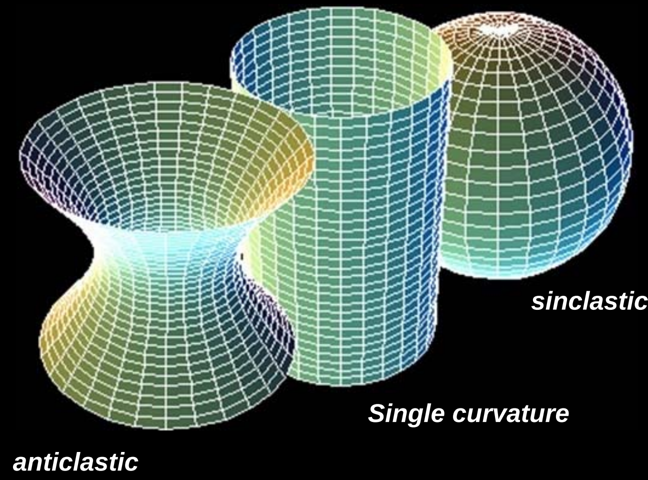


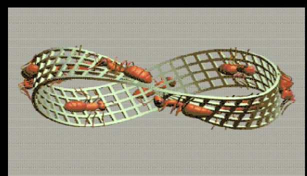
Helaman Ferguson, 1999

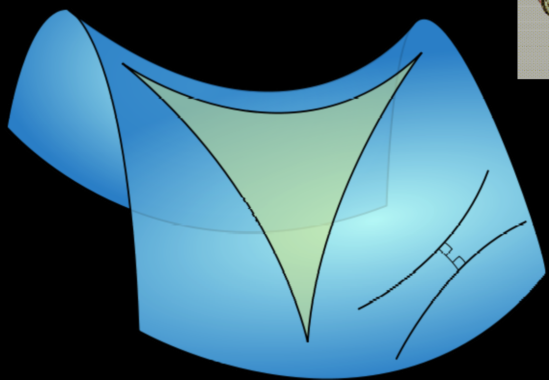
Helaman Ferguson, 2008

AUSTRALIAN WILDLIFE HEALTH CENTRE

Double curvature superfaces

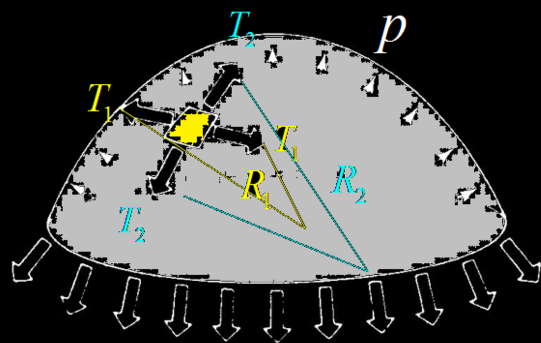




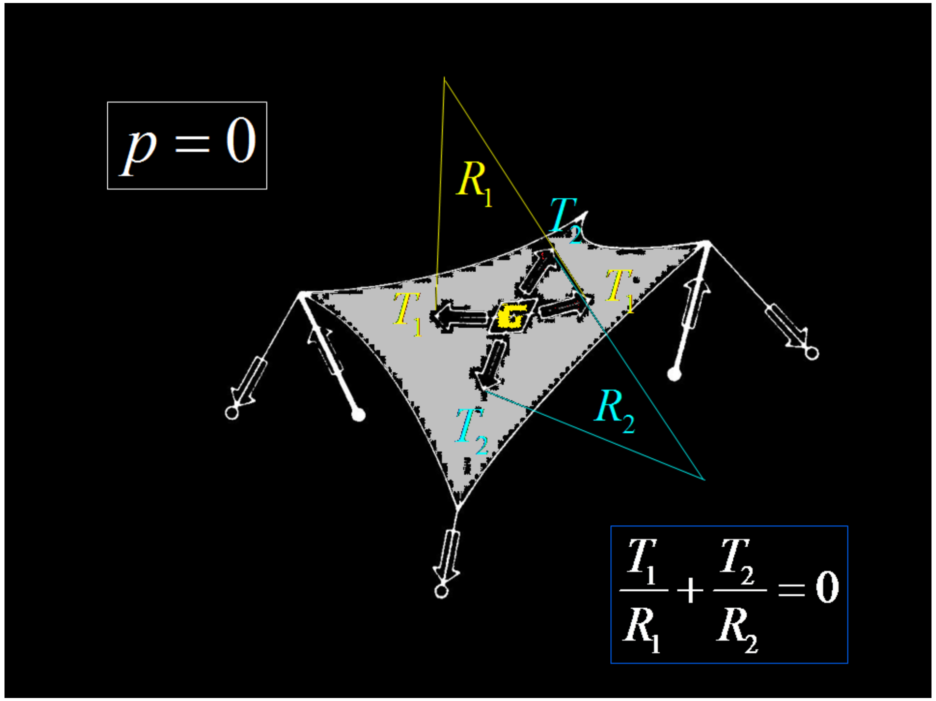


A triangle immersed in a saddle-shape plane (a hyperbolic paraboloid), as well as two diverging ultraparallel lines.

Equação de Laplace-Young (equação das bolhas de sabão, ou das membranas):



$$\frac{T_1}{R_1} + \frac{T_2}{R_2} = p$$

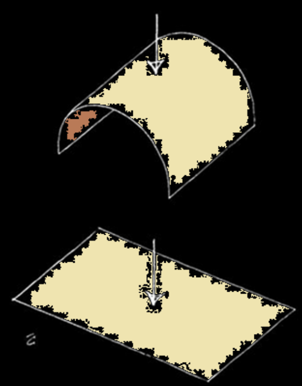


A paradox:

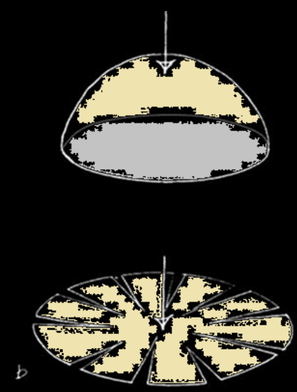
'Flexible' structures → formal stiffness

'Stiff' structures → formal flexibility

Planification



Single curvature surfaces can be flattened without distortion;

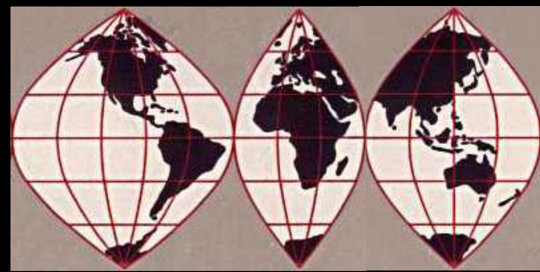


Double curvature surfaces undergoe distortion due to flattening

Mercator projection

Winkel projection

Mercator projection



Interrupted sinusoidal map

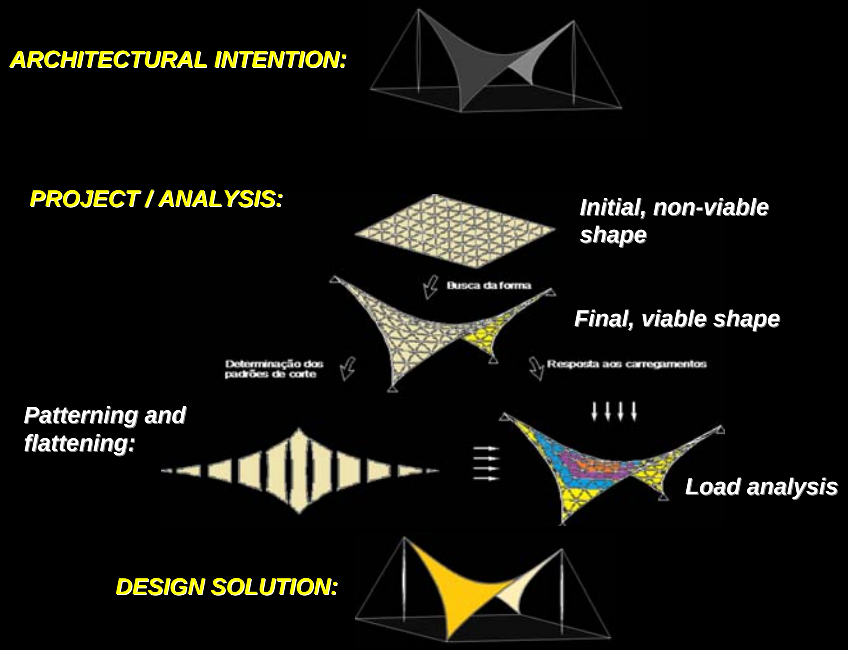
Mercator projection

Fueller´s Dymaxion Projection

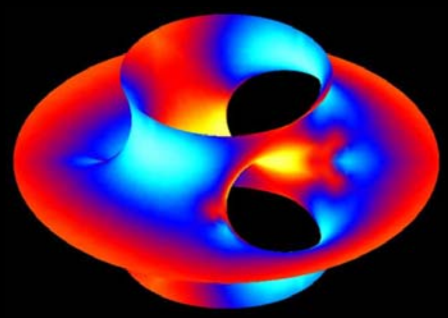
The Design Process of Taut Structures

"No other class of architectural structural systems is as dependent upon the use of digital computers as are tensile membrane structures".

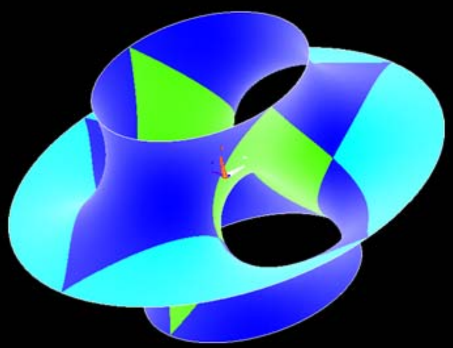
David Campbel [*ASCE Second Civil Engineering Automation Conference,* 1991].



Costa's Surface:

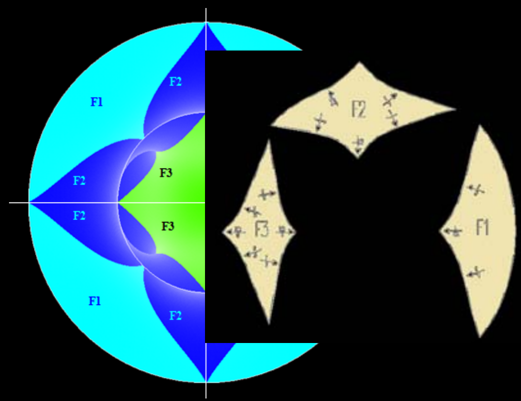


Costa's Surface:



Symmetries & Patterns

Costa's Surface:

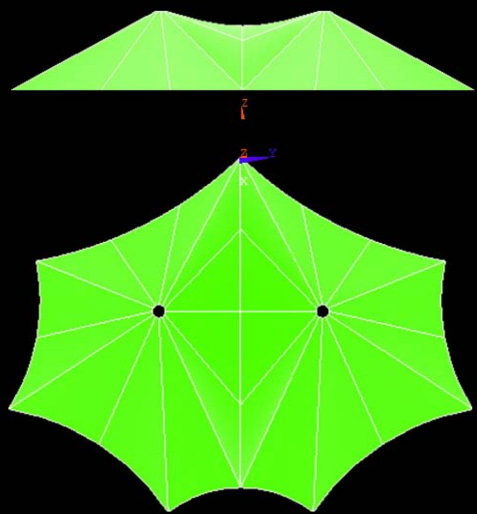


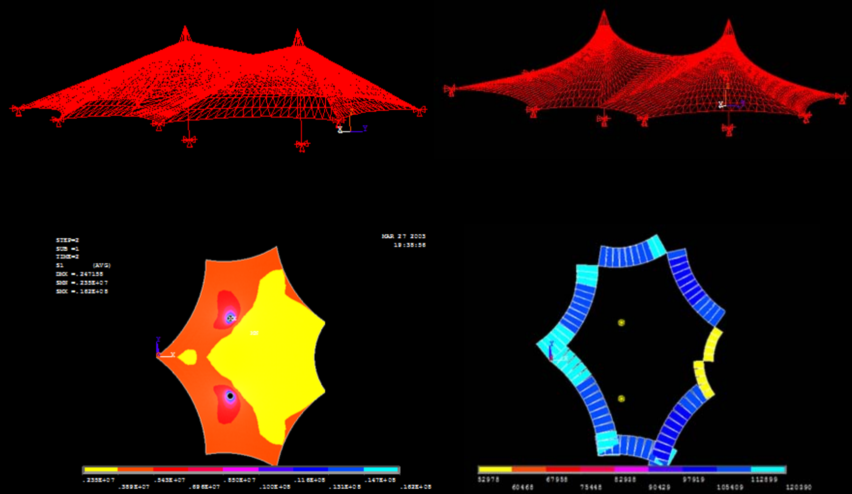
Symmetries & Patterns

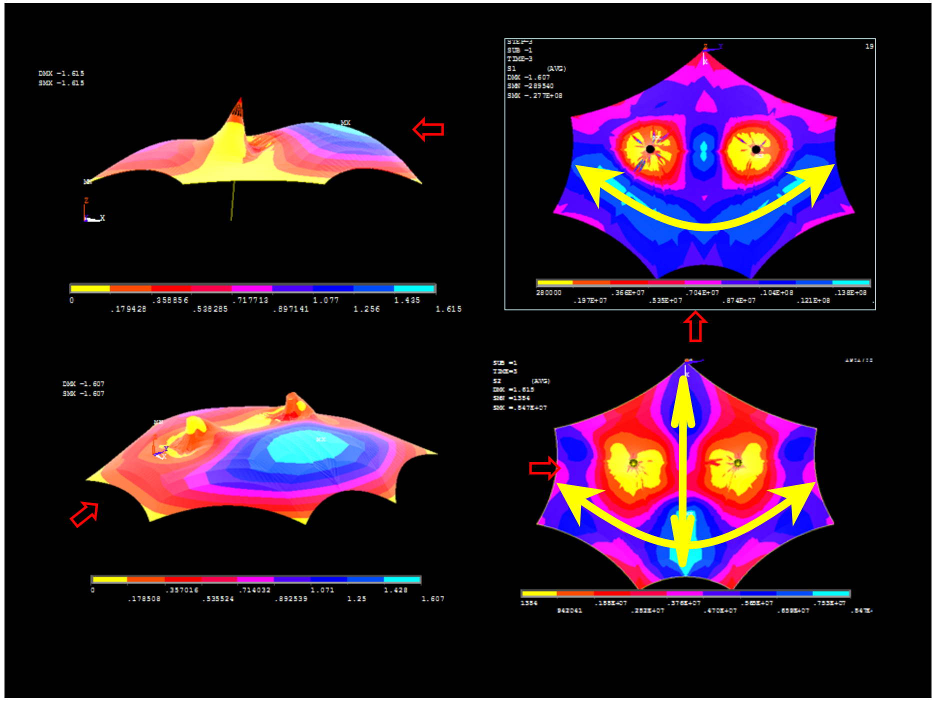
Costa's Surface:

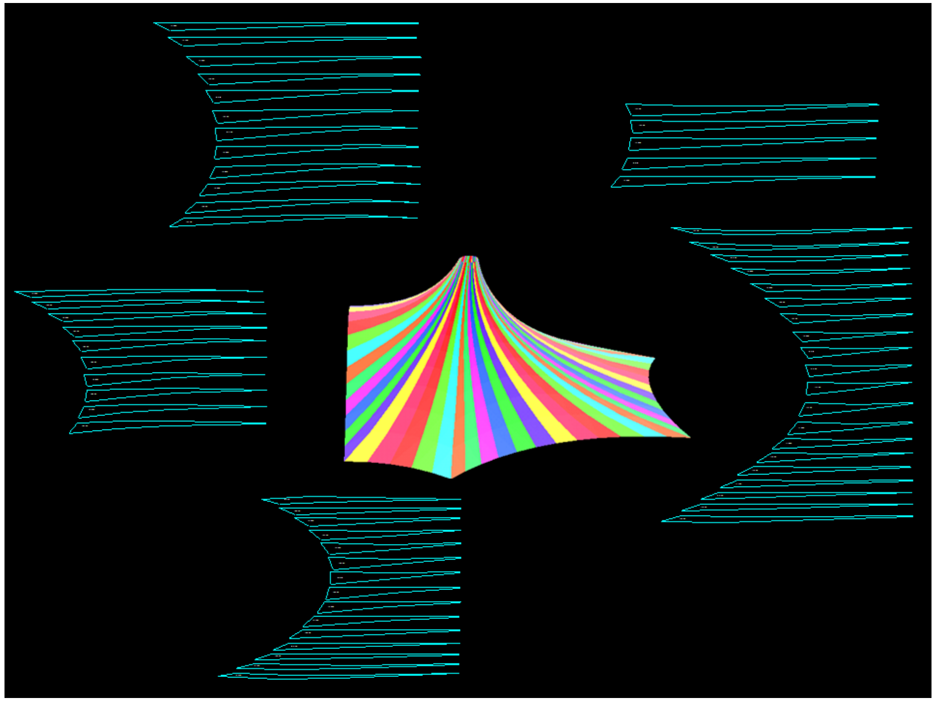
Realization: Lycra sculpture at EPUSP (2008)

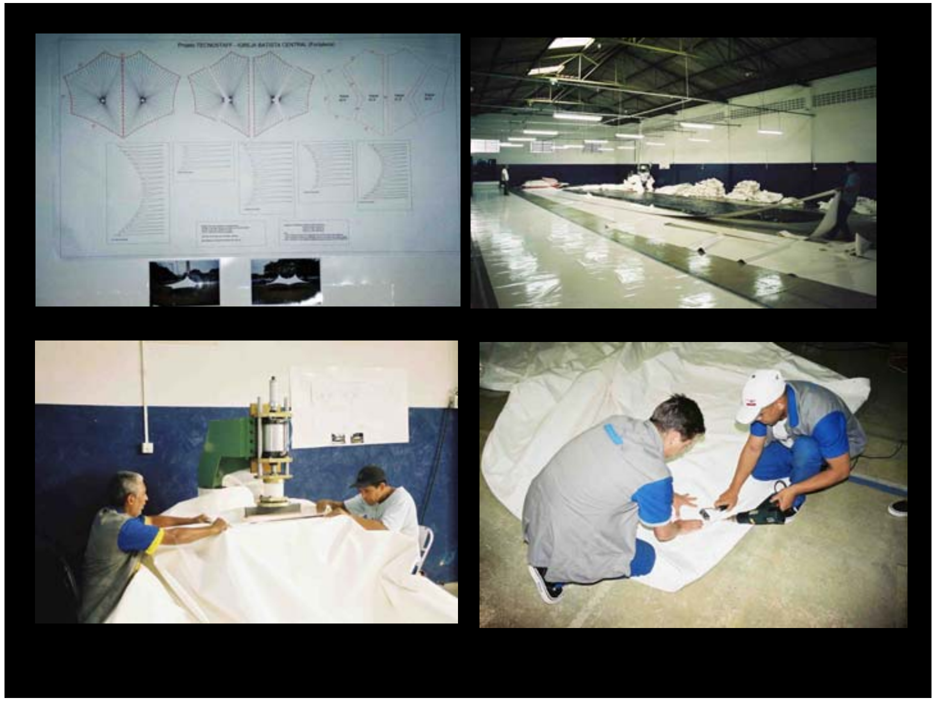
Igreja Batista Central Fortaleza (2003)



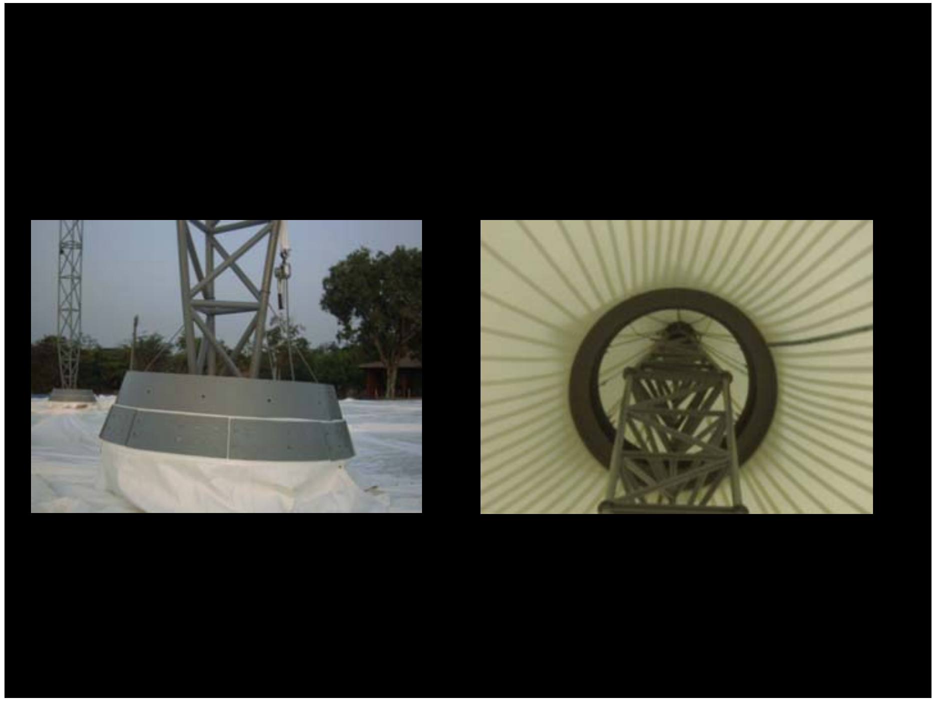






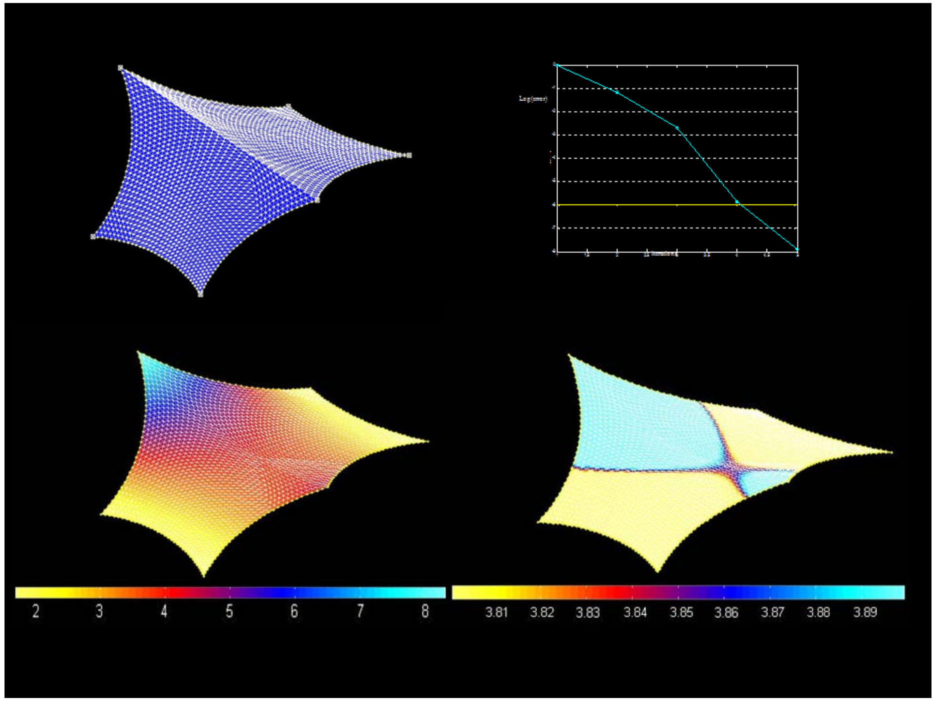



Formatto



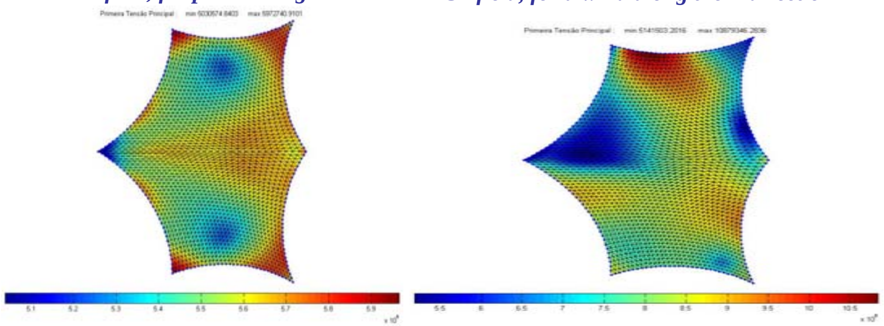

IBC, Inauguration , Nov 27, 2003

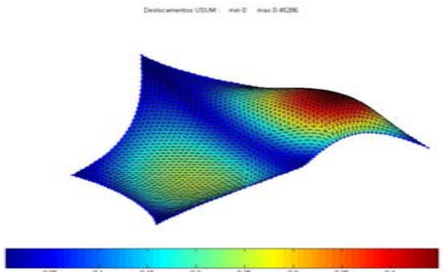
The membrane roof of the “Memorial dos Povos” of Belém do Pará



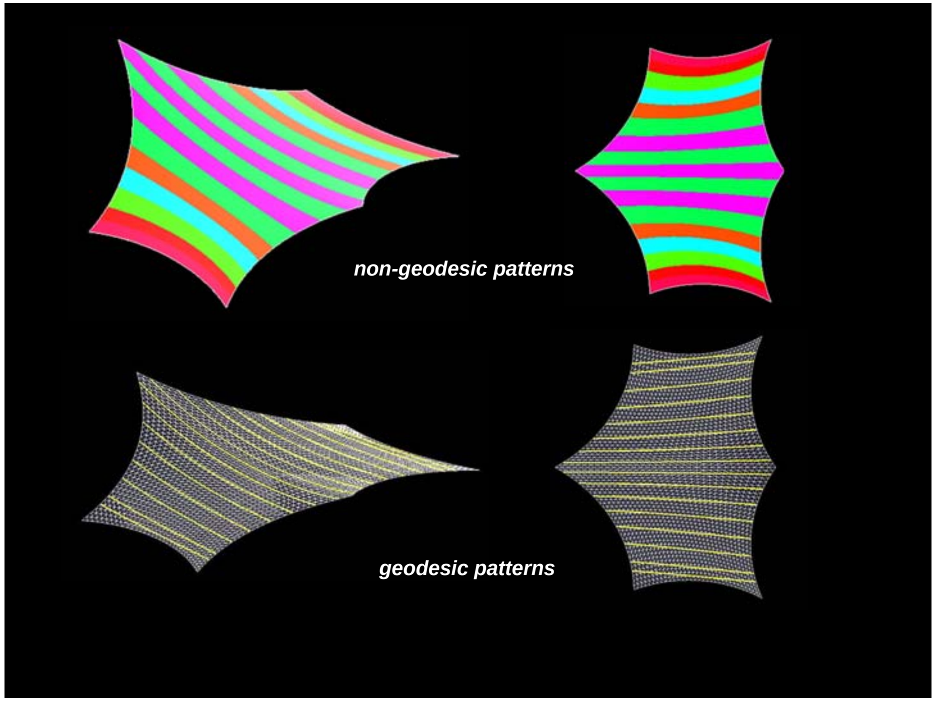
S1 field, for prestressing loads

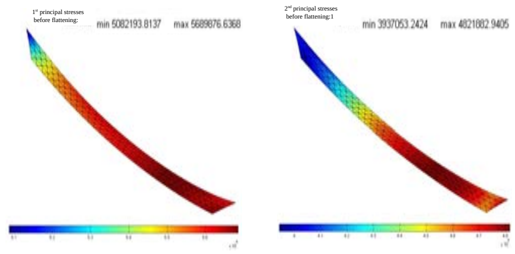
S1 field, for a wind along the Y direction



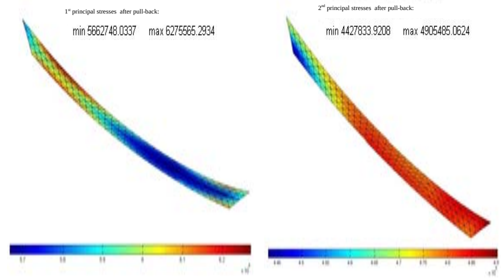


displacement norms, for a wind along the Y direction

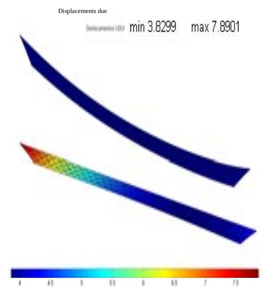




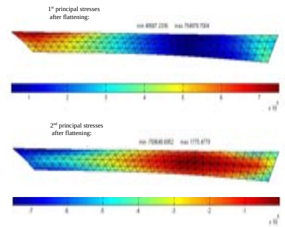
Initial prestresses



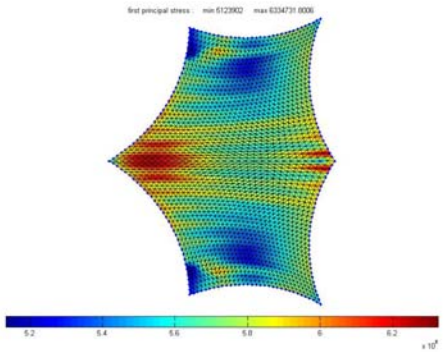
Stresses after pull-back



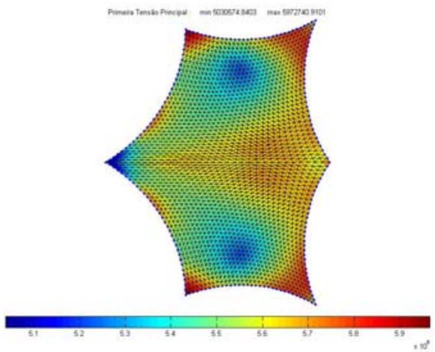
Flattening process



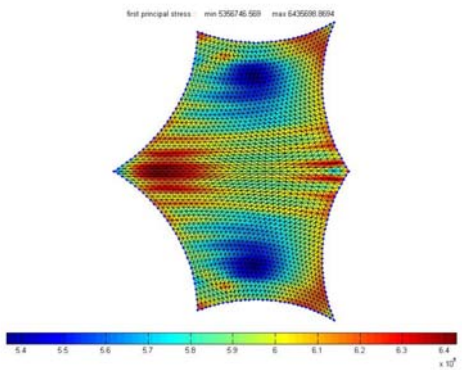
Residual stresses



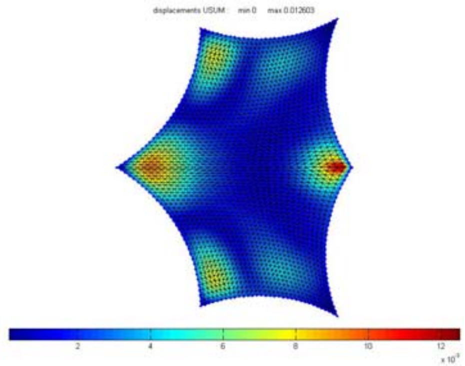
Maximum first principal stresses after planification and pull-back



Maximum first principal stresses for the prestress load case, as initially calculated

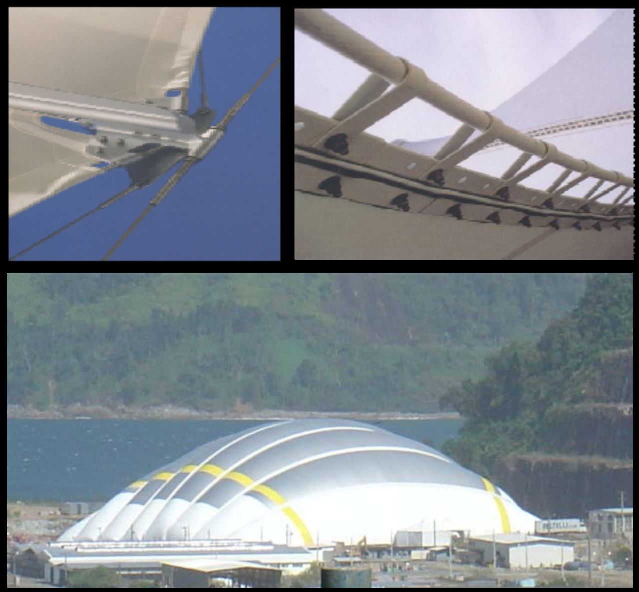


Maximum first principal stresses after relaxation of pull-back stresses



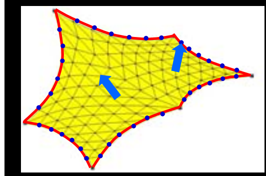
Displacements due to relaxation of pull-back stresses

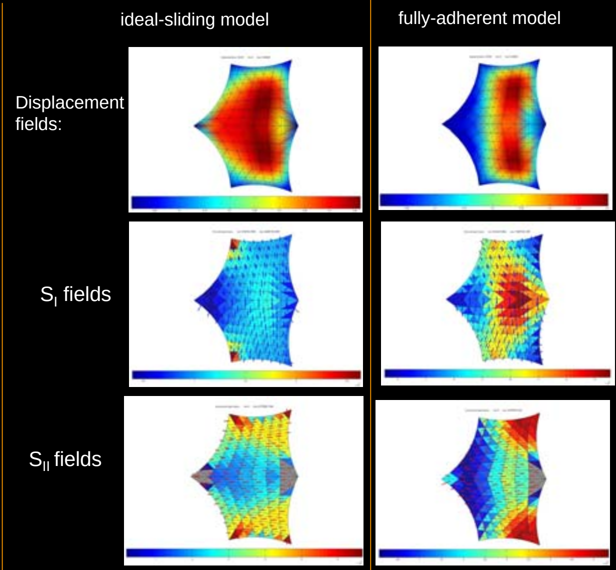
SLIDING-CABLES

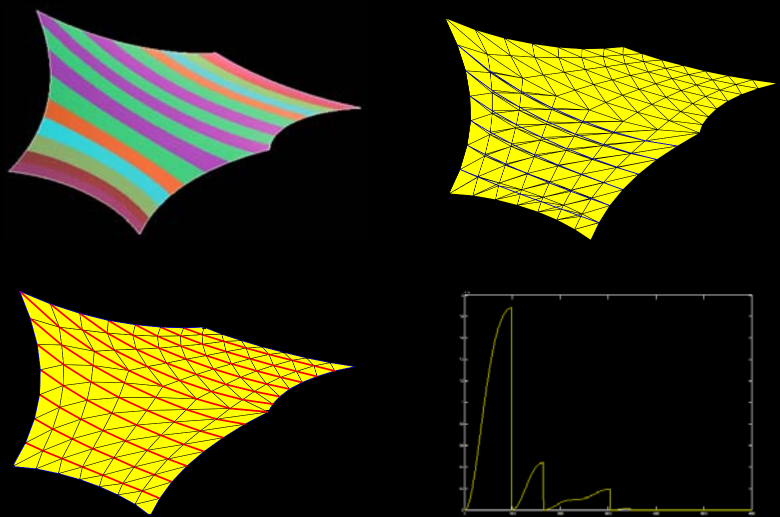


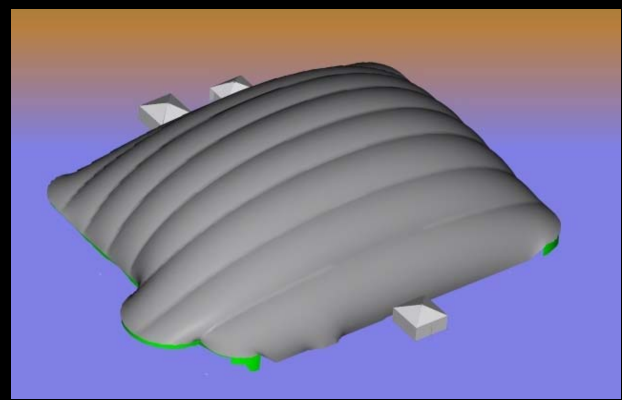
(September 2009)

The Memorial dos Povos de Belém do Pará

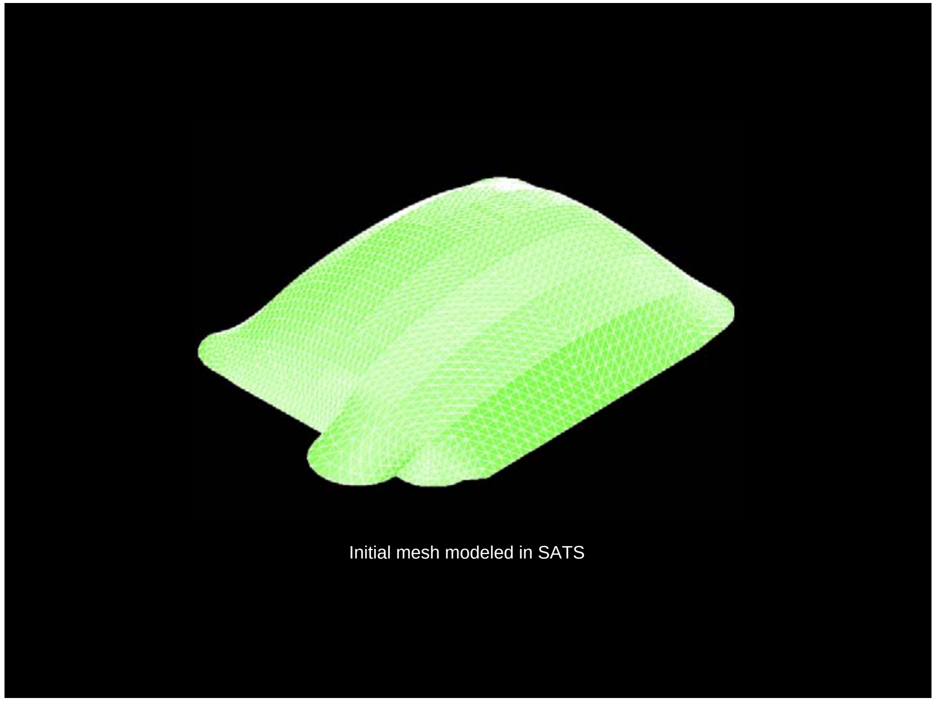


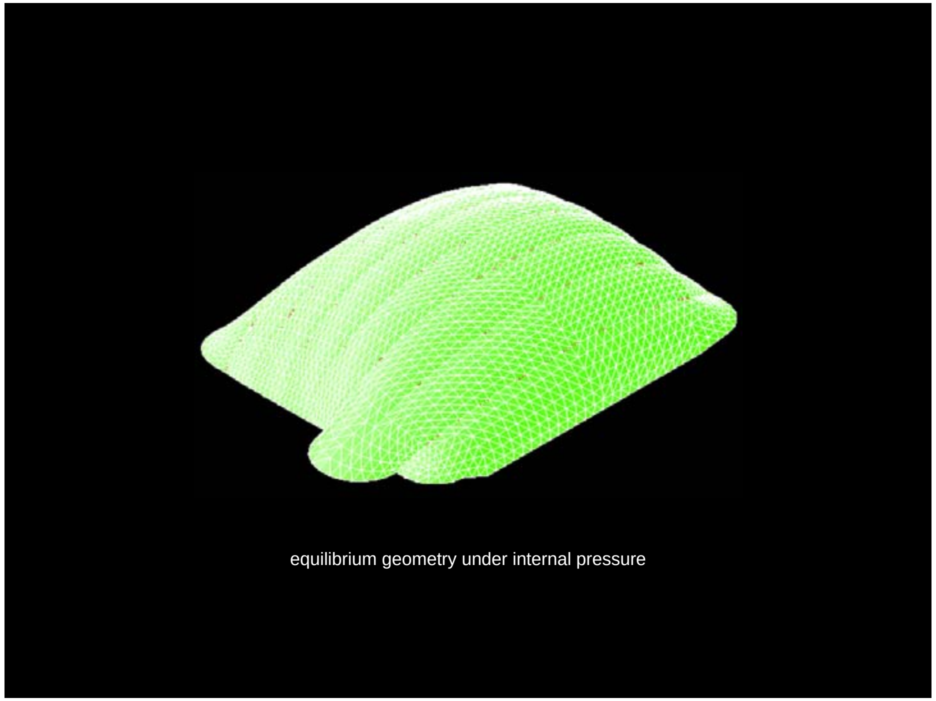


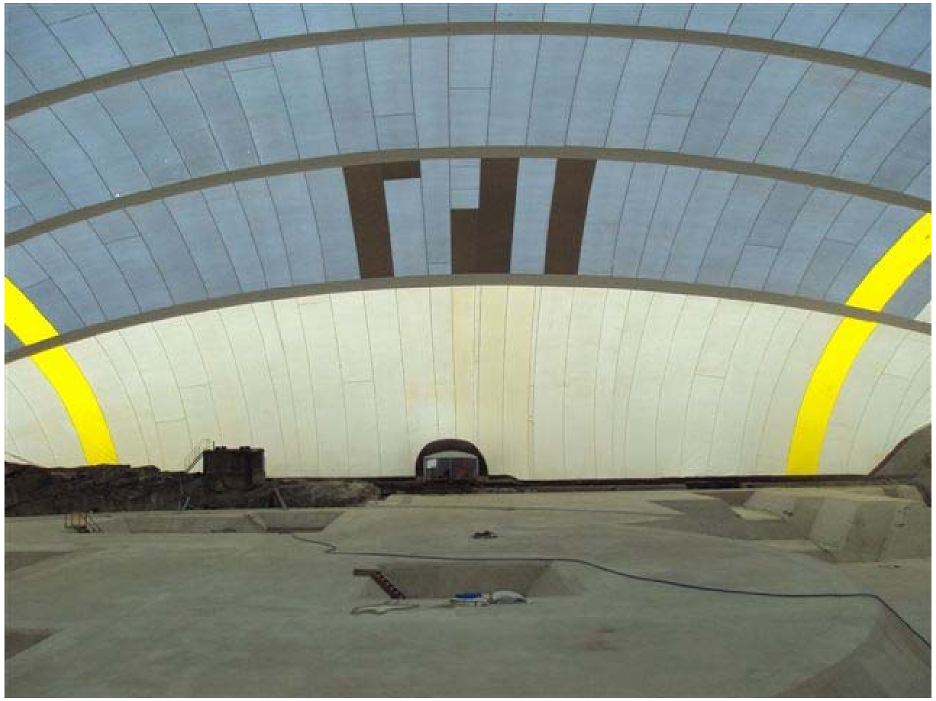




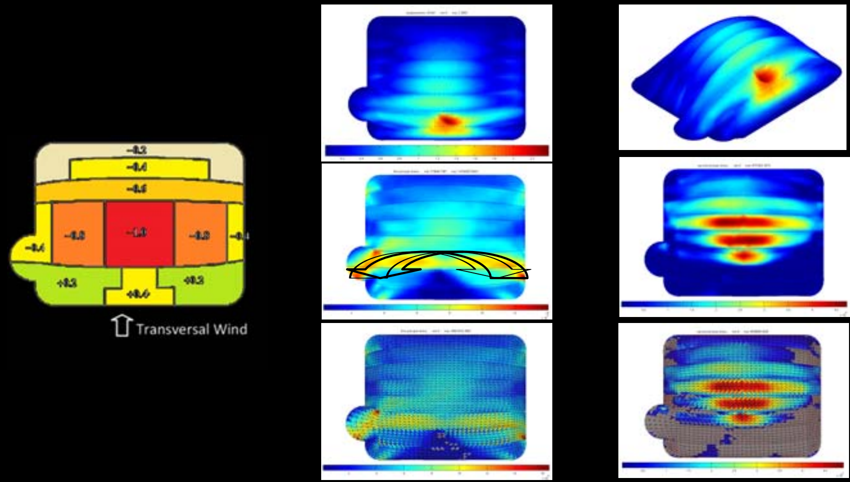
Perspective view of a large PVC -coated, polyester fabric, pneumatic envelope (110mx86mx30m) reinforced by sliding cables


Initial mesh modeled in SATS


equilibrium geometry under internal pressure

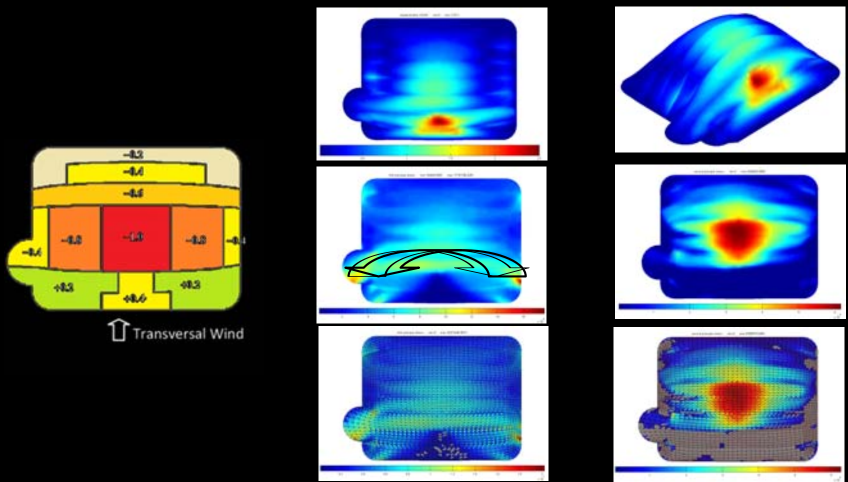


"LC1A" – Transversal wind, adherent cables



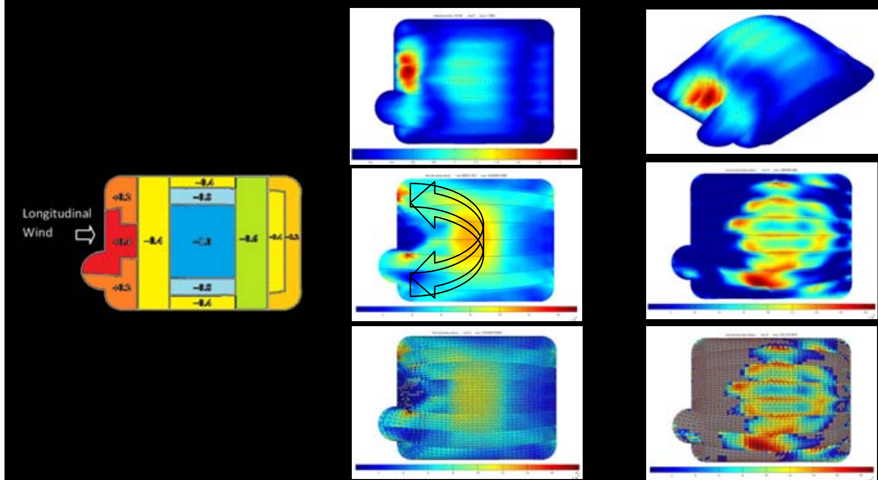
(a) field of displacement norms; *(b) idem*, isometric view; *(c)* stress field; *(d)* stress field.

"LC1S" – Transversal wind, sliding cables



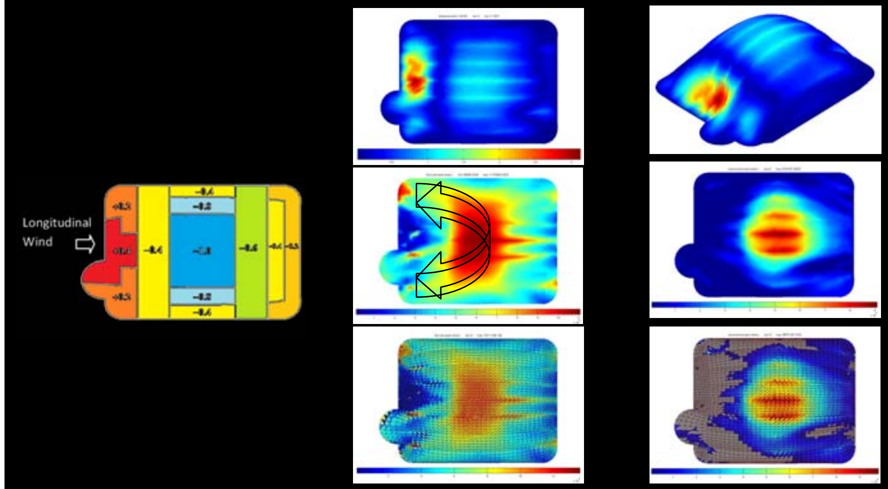
(a) field of displacement norms; *(b) idem*, isometric view; *(c)* stress field; *(d)* stress field.

"LC2A" – Longitudinal wind, adherent cables



(a) field of displacement norms; *(b) idem*, isometric view; *(c)* stress field; *(d)* stress field.

“LC2S” – Longitudinal wind, sliding cables



(a) field of displacement norms; *(b) idem*, isometric view; *(c)* stress field; *(d)* stress field.

Table 1. Initial normal loads for the longitudinal cables [kN]

Cable	1	2	3	4	5	6	7
N_0	100	60	50	40	50	80	100

Table 2. Normal loads on the adherent cables [kN]

Load Case	Cable	$N_{average}$	N_{max}	N_{min}
	1	107	117	103
	2	58	65	41
LC0A	3	47	50	43
Internal	4	40	47	20
Pressure	5	44	53	27
	6	80	120	47
	7	101	122	73
	1	198	224	187
LC1A	2	137	148	108
Transversal	3	139	157	117
Wind,	4	130	180	62
Adherent	5	154	203	117
Cables	6	138	214	89
	7	46	92	30
	1	258	290	236
LC2A	2	137	180	107
Longitudinal	3	97	145	43
Wind,	4	81	136	0
Adherent	5	100	153	12
Cables	6	205	307	131
	7	227	310	120

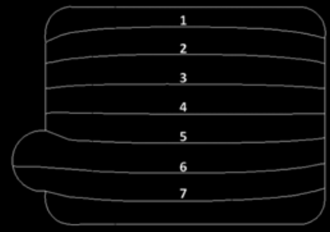
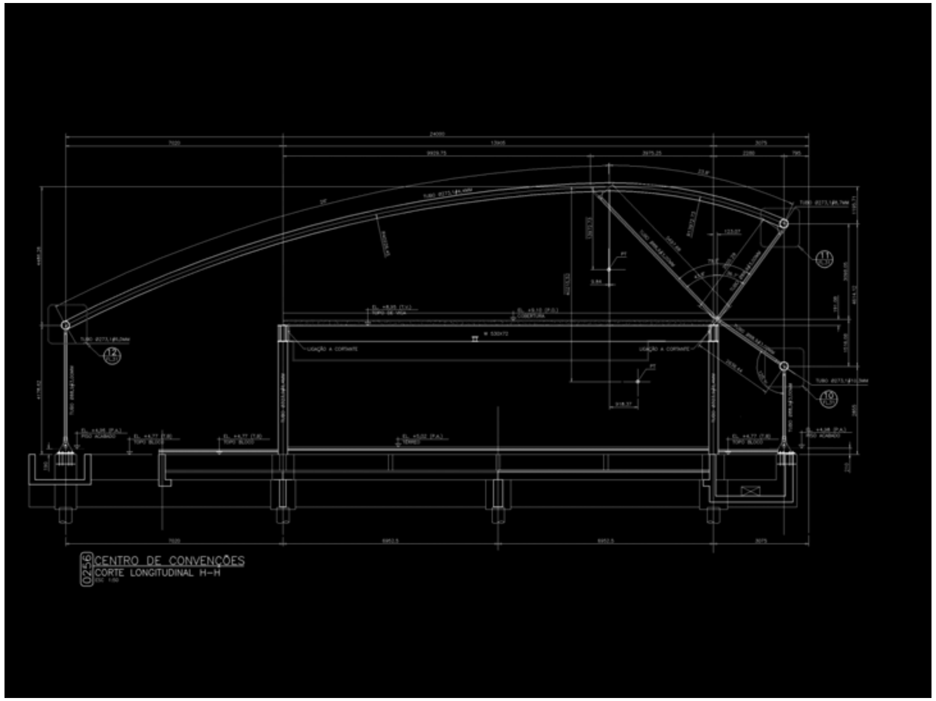
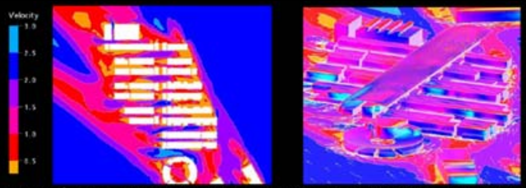


Table 3. Comparison between normal loads on adherent and sliding cables [kN]

		Internal Pressure		Wind loads			
				Trasversal wind		Longitudinal Wind	
Cable	N_0	LC0A (average)	LC0S (uniform)	LC1A (average)	LC1S (uniform)	LC2A (average)	LC2S (uniform)
1	100	107	107	198	195	258	250
2	60	58	57	137	133	137	128
3	50	47	47	139	135	97	80
4	40	40	38	130	124	81	45
5	50	44	42	154	149	100	61
6	80	80	70	138	123	205	184
7	100	101	100	46	45	227	217


0256
CENTRO DE CONVENÇÕES
CORTE LONGITUDINAL H–H



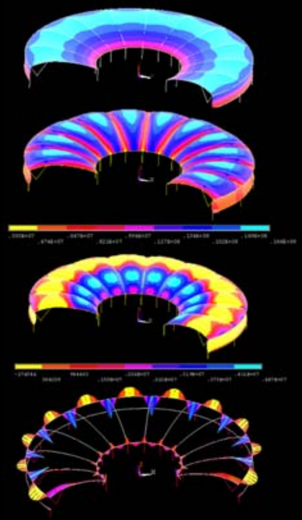


Figura 4 – À esquerda, distribuição da velocidade do vento no nível do pedestre, a 1,5m. Observar escala de velocidades de 0m/s a 3m/s. À direita, distribuição de pressões de vento sobre as envoltórias.