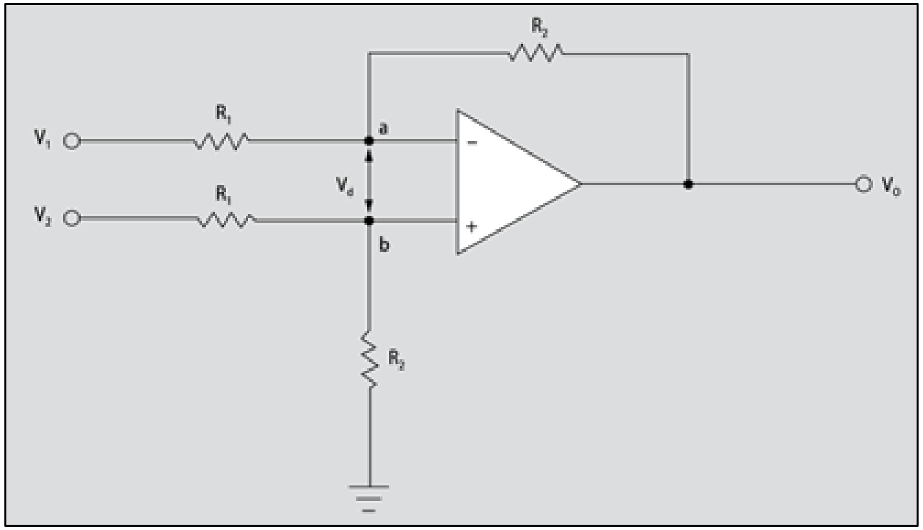
Single Op Amp Difference Amplifier

A Single Op-Amp Difference Amplifier



$\boxed{1}$ **Nó a:**

$$\frac{V_1 - V_a}{R_1} + \frac{V_o - V_a}{R_2} = 0$$

$\boxed{2}$ **Nó b:**

$$\frac{V_2 - V_b}{R_1} - \frac{V_b}{R_2} = 0$$

$\boxed{3}$ $V_a = V_b$ (virtual ground)

$\boxed{1}$ e $\boxed{2}$ e $\boxed{3}$ $\Rightarrow$

$$\boxed{v_0 = \underbrace{\frac{R_2}{R_1}}_{A_d}\underbrace{(v_2 - v_1)}_{v_{Id}}}$$

4

Although ideally the difference amplifier will amplify only the differential input signal v_{Id} and reject completely the common-mode input signal v_{Icm}, practical circuits will have an output voltage v_O given by

$$\boxed{v_0 = \underbrace{\frac{R_2}{R_1}}_{A_d}\underbrace{(v_2 - v_1)}_{v_{Id}}}$$

$$\boxed{v_o = A_d v_{Id} + A_{cm} v_{Icm}} \left\{ \begin{array}{l} v_{Id} = v_2 - v_1 \\ v_{Icm} = \dfrac{v_1 + v_2}{2} \end{array} \right.$$

Where A_d denotes the amplifier differential gain and A_{cm} denotes its common-mode gain (ideally zero). The efficacy of a differential amplifier is measured by the degree of its rejection of common-mode signals in preference to differential signals. This is usually quantified by a measure known as the **common-mode rejection ratio** (**CMRR**), defined as:

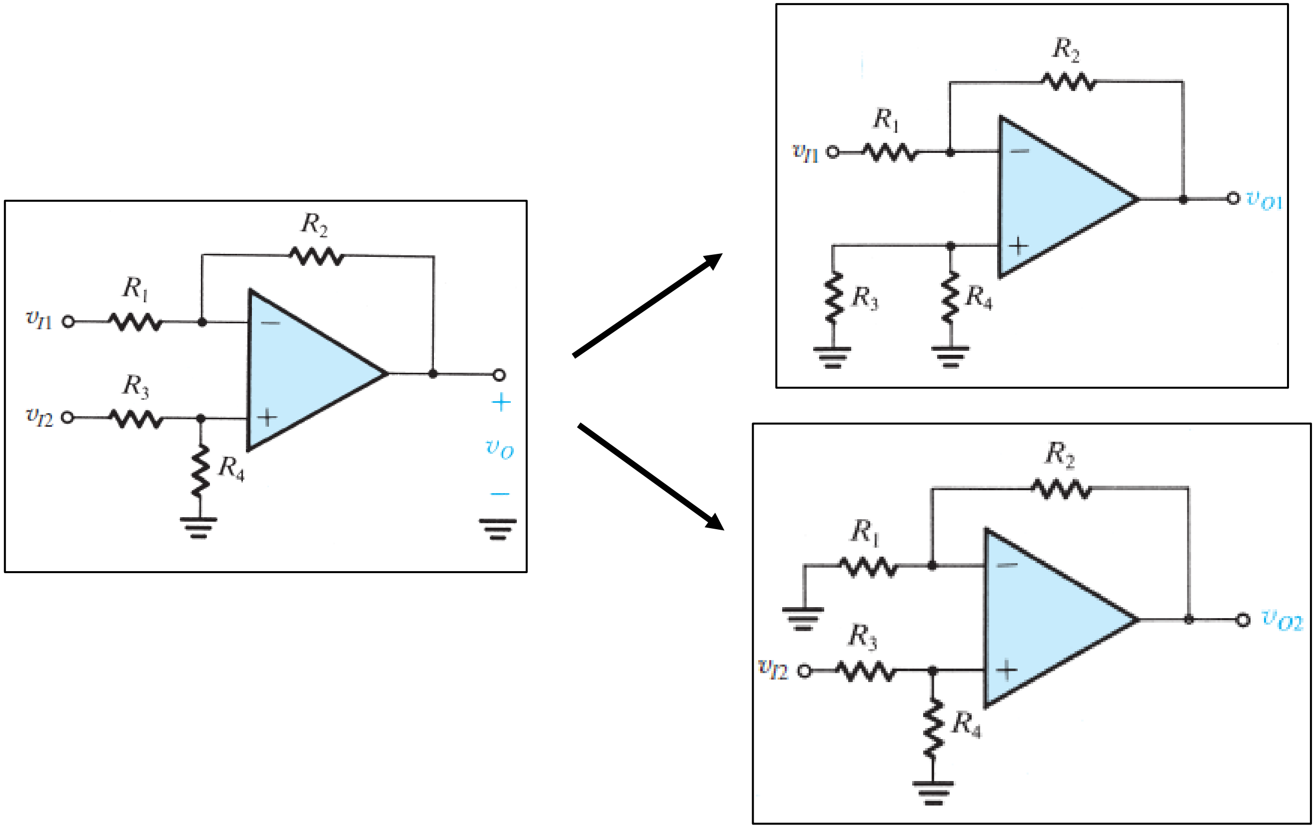
$$\boxed{\mathrm{CMRR} = \frac{|A_d|}{|A_{cm}|}}$$

$$\boxed{\mathrm{CMRR} = 20 \log \frac{|A_d|}{|A_{cm}|}} \quad \text{(dB)}$$

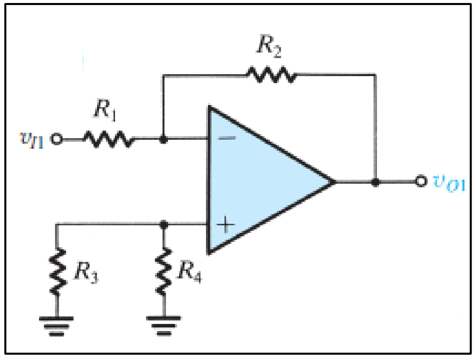
Calculation of A_d and A_{cm}

A_d Calculation

1 Specifically, we wish to determine the output voltage v_O in terms of v_{I1} and v_{I2}. Toward that end, we observe that **the circuit is linear, and thus we can use superposition**.

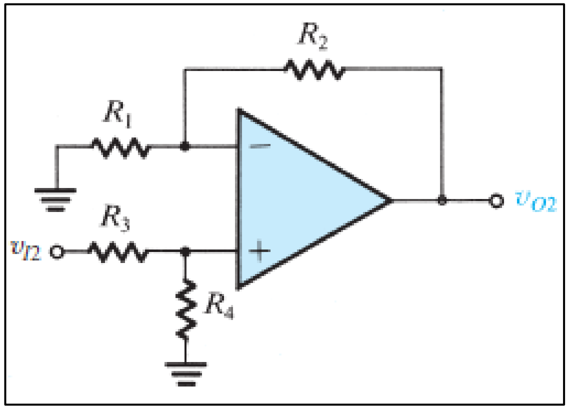


2 To apply superposition, we first reduce v_{I2} to zero, that is ground the terminal to which v_{I2} is applied, and then find the corresponding output voltage, which will be due entirely to v_{I1}. We denote this output voltage v_{O1}.



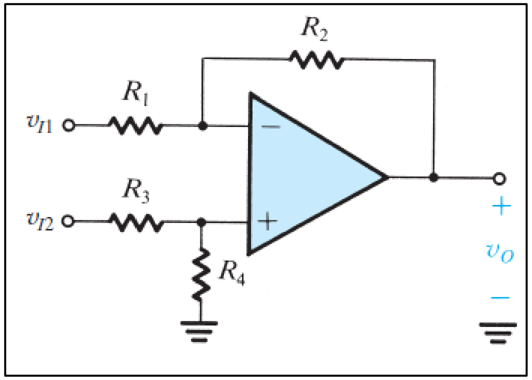
$$\boxed{v_{o1} = -\frac{R_2}{R_1} v_{I1}}$$

3 Next, we reduce v_{i1} to zero and evaluate the corresponding output voltage v_{O2}. The circuit will now take the form shown which we recognize as the noninverting configuration with an additional voltage divider, made up of R_3 and R_4, connected to the input v_{I2}.



$$v_{o2} = v_{I2} \overbrace{\frac{R_4}{R_3 + R_4}}^{v_+} \left(1 + \frac{R_2}{R_1}\right)$$

If $\frac{R_4}{R_3} = \frac{R_2}{R_1}$ $\longrightarrow$ $\boxed{v_{o2} = v_{I2} \frac{R_2}{R_1}}$



4 The superposition principle tells us that the output voltage v_O is equal to the sum of v_{O1} and v_{O2}. Thus we have:

$$\boxed{v_{o1} = -\frac{R_2}{R_1} v_{L1}}$$

$$+$$

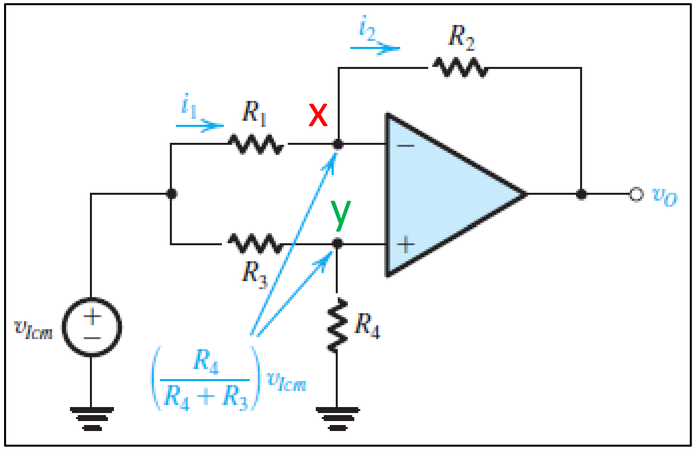
$$\boxed{v_{o2} = v_{I2} \frac{R_2}{R_1}}$$

$$\rightarrow \boxed{v_o = \frac{R_2}{R_1}(v_{I2} - v_{I1})}$$

$$v_o = A_d v_{Id} + A_{cm} v_{Icm} \rightarrow \boxed{A_d = \frac{R_2}{R_1}}$$

A_{cm} Calculation

5 Let’s next consider the circuit with only a common-mode signal applied at the input, as shown in the circuito below:



5.1 $V_x = V_y$

5.2
$$i_1 = \frac{1}{R_1}\left[v_{Icm} - \overbrace{\frac{R_4}{R_4+R_3}v_{Icm}}^{V_y}\right] = v_{Icm}\frac{R_3}{R_4+R_3}\frac{1}{R_1}$$

5.3
$$v_o = \overbrace{\frac{R_4}{R_4+R_3}v_{Icm}}^{V_y} - i_2R_2$$

5.4
$$i_2 = i_1$$

Any mismatch in the resistance ratios can make *A_{cm}* nonzero and hence CMRR finite !

5.1 to **5.4** →

$$\boxed{v_0 = \frac{R_4}{R_4+R_3}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)v_{Icm}}$$

$$v_o = A_d v_{Id} + A_{cm}v_{Icm} \longrightarrow \boxed{A_{cm} = \frac{R_4}{R_4+R_3}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)}$$

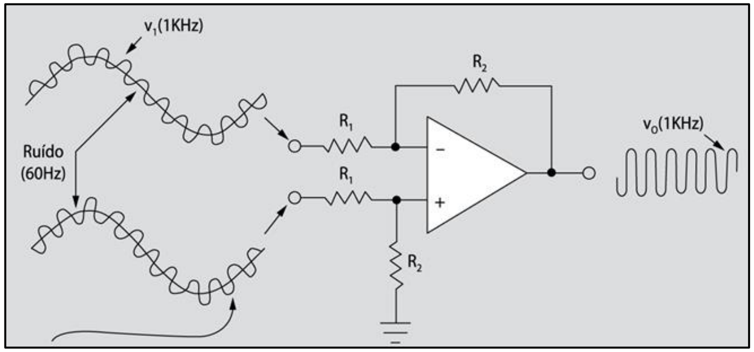
$$\frac{R_4}{R_3} = \frac{R_2}{R_1} \longrightarrow \boxed{A_{cm} = 0}$$

Importance in Instrumentation

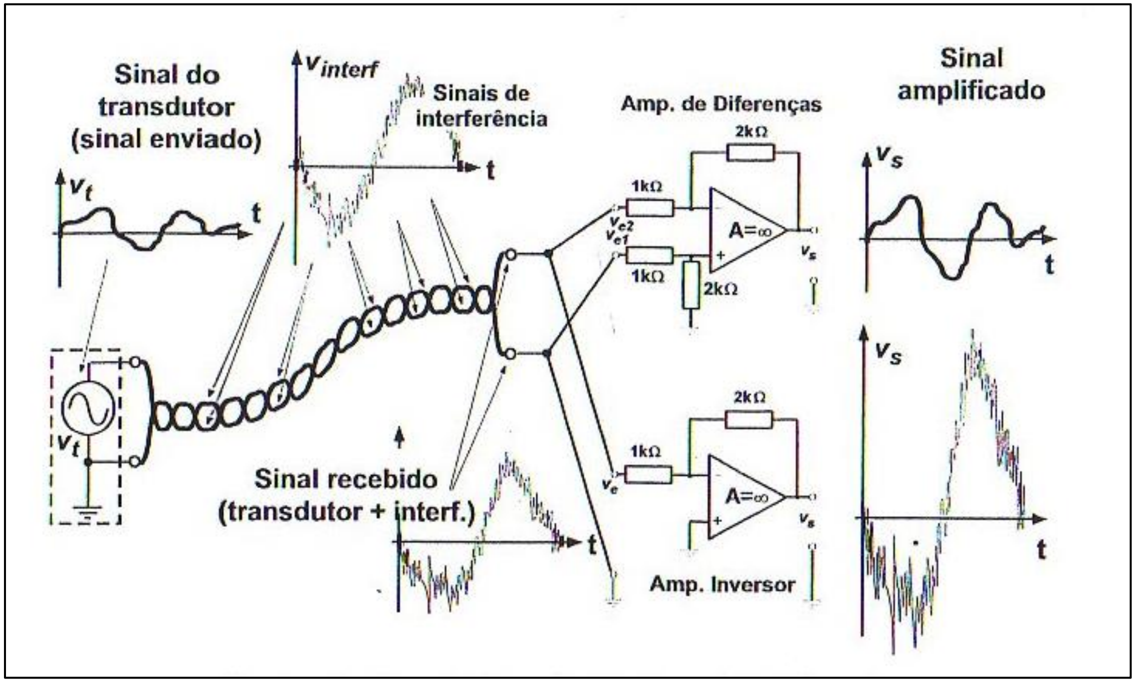
A Single-Op Difference Amplifier

A difference amplifier is one that responds to the difference between the two signals applied at its input and ideally rejects signals that are common to the two inputs.

Difference Amplifier



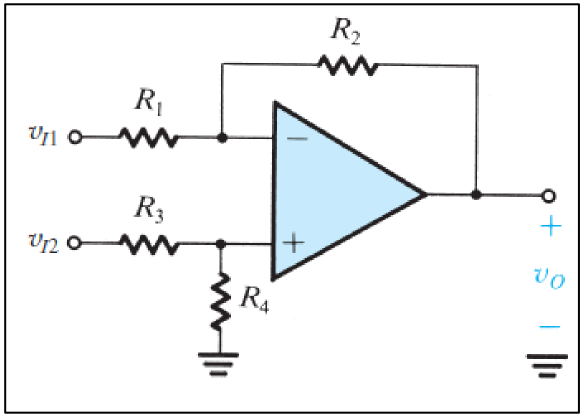
Difference Amplifier
X
Inversor Amplifier



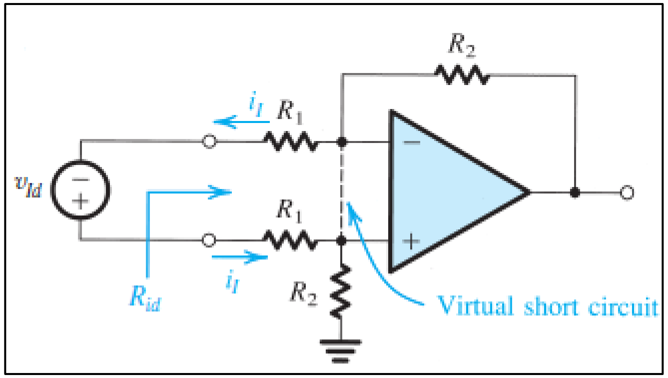
Input Resistance

Input Resistance

1 **In addition to rejecting common-mode signals, a difference amplifier is usually required to have a high input resistance**. To find the input resistance between the two input terminals (i.e., the resistance seen by v_{Id}), called the **differential input resistance** R_{id}, consider the circuit below. Here we have assumed that the resistors are selected so that:



$$\left.\begin{aligned} R_3 &= R_1 \\ R_4 &= R_2 \end{aligned}\right\} \longrightarrow$$



2 Since the two input terminals of the op amp track each other in potential, we may write a loop equation and obtain:

$$\left.\begin{aligned} v_{id} &= R_1 i_I + R_1 i_I \\ R_{id} &= \frac{v_{id}}{i_I} \end{aligned}\right\} \longrightarrow \boxed{R_{id} = 2R_1}$$

Disadvantages

Disadvantages

1

$$v_o = \frac{R_2}{R_1}(v_{I2} - v_{I1})$$

$$R_{id} = 2R_1$$

If the amplifier is required to have a large differential gain, then R_1 will be relatively small and the input resistance will be correspondingly low.

if R_1 is chosen with a high value, let's say 1 MΩ, it would require a higher R_2 value for getting a desirable gain which makes the circuit impractically !

2 Another drawback of the circuit is that it is not easy to vary the differential gain of the amplifier because there are two R_1 resistances.

Both of these drawbacks are overcome with another kind of difference amplifier named **instrumentation amplifier !**

CMRR Measurement

$$v_o = A_d v_d + A_c v_c$$

$$v_d = v_1 - v_2$$

$$v_c = \frac{v_1 + v_2}{2}$$

A_d - diferential gain

A_c - common mode gain

1 If v_c =0

$$\left.\begin{array}{l} A_d = \frac{v_o}{v_d} \\ \rightarrow v_1 = - v_2 \\ v_d = v_1 - v_2 = 2\, v_1 \end{array}\right\} \rightarrow A_d = \left|\frac{v_o}{2v_1}\right|$$

If v_1 = 0,5V $\rightarrow$ $A_d = |v_{od}|$

2 if v_d =0

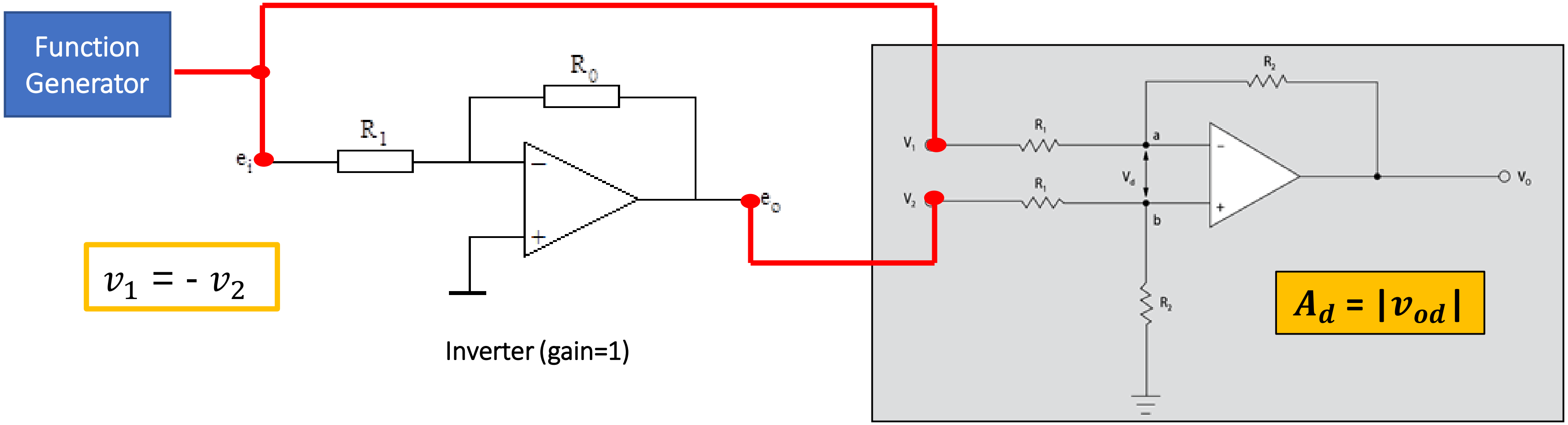
$$\left.\begin{array}{l} A_c = \frac{v_o}{v_c} \\ \rightarrow v_1 = v_2 \\ v_c = v_2 \end{array}\right\} \rightarrow A_c = \left|\frac{v_o}{v_1}\right|$$

If v_1 = 1V $\rightarrow$ $A_c = |v_{oc}|$

$$\rightarrow \mathbf{CMRR} = \left|\frac{\mathbf{A_d}}{\mathbf{A_c}}\right|$$

A_d Measurement

3 A_d measurement using $|v_1^{max}| = |v_2^{max}| = 0,5V$.



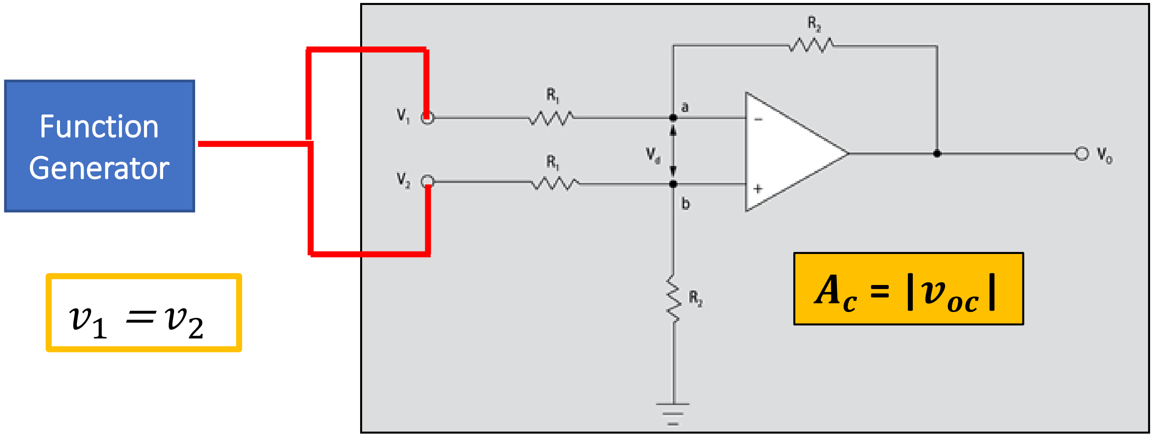
$v_1 = - v_2$

Inverter (gain=1)

$A_d = |v_{od}|$

A_c Measurement

4 A_c measurement using $v_1^{max} = v_2^{max} = 1V$.



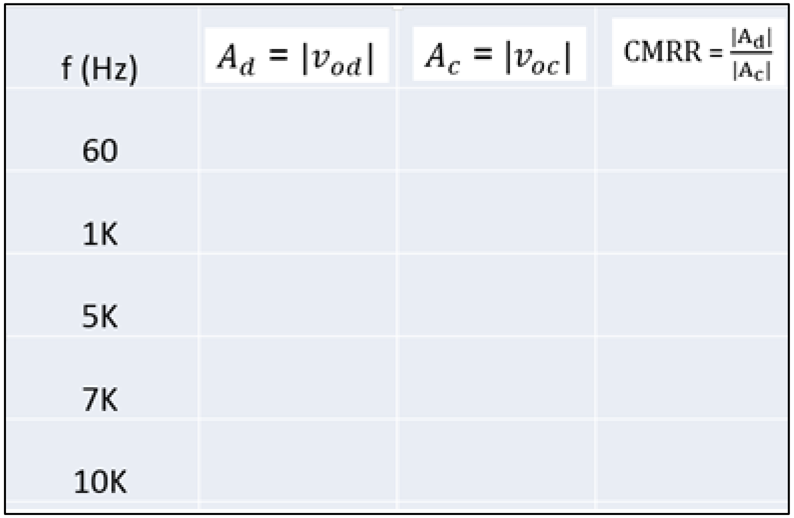
$v_1 = v_2$

$A_c = |v_{oc}|$

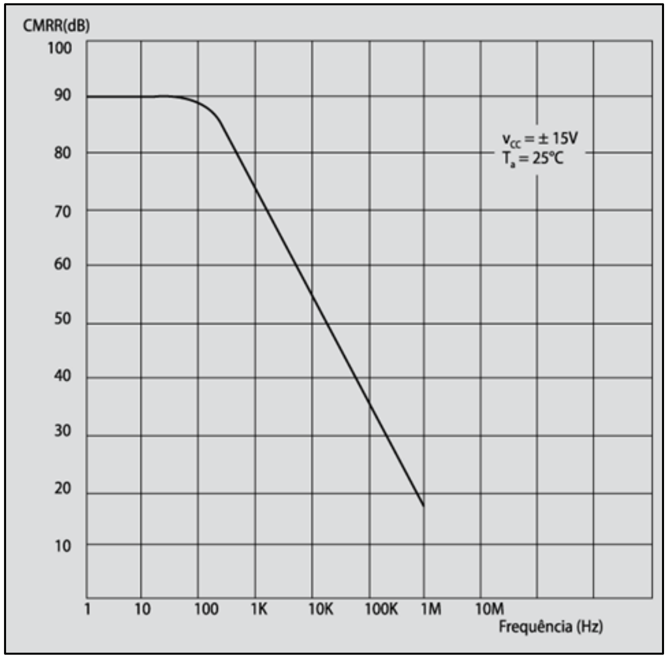
$$\text{CMRR} = \left|\frac{A_d}{A_c}\right|$$

CMRR measurement is made in different frequencies !

$$\text{CMRR} = \left|\frac{A_d}{A_c}\right|$$



f (Hz)	$A_d = \lvert v_{od} \rvert$	$A_c = \lvert v_{oc} \rvert$	$\text{CMRR} = \frac{\lvert A_d \rvert}{\lvert A_c \rvert}$
60			
1K			
5K			
7K			
10K			

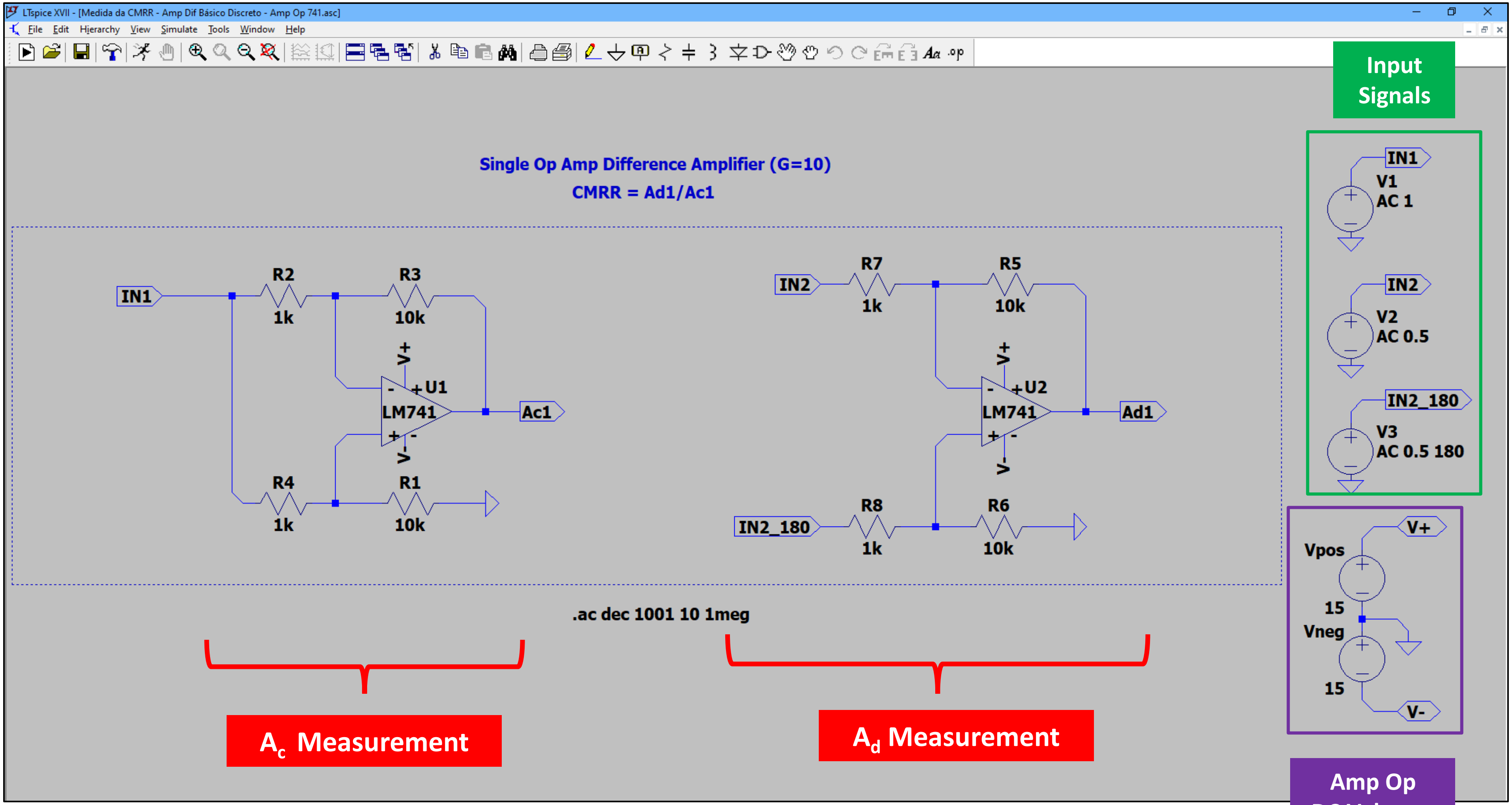


Typical CMRR Curve

CMRR Measurement in the LTSPice

CMRR measurement results in simulation are very close to the ones using integrated circuits!

$$\text{CMRR} = \left|\frac{A_d}{A_c}\right|$$



LTspice XVII - [Medida da CMRR - Amp Dif Básico Discreto - Amp Op 741.asc]

Single Op Amp Difference Amplifier (G=10)

CMRR = Ad1/Ac1

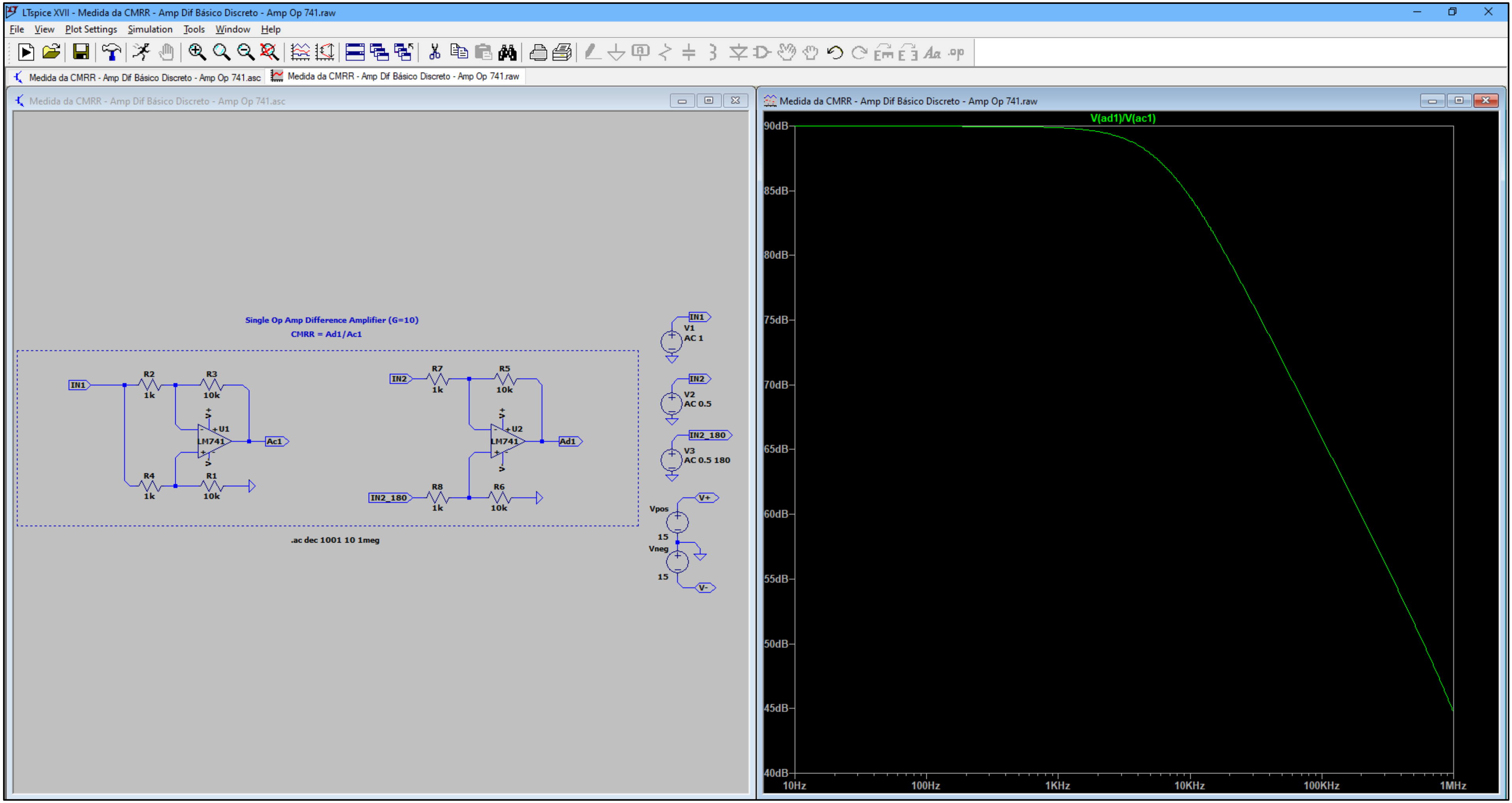
```
.ac dec 1001 10 1meg
```

Input Signals

A_c Measurement

A_d Measurement

Amp Op DC Voltages



```
.ac dec 1001 10 1meg
```