Optimization theory & Coordination

2.183/2.184

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Coordination of movement

- The problem of choice:
  - Human body: ~200 DOF actuated by ~600 muscles
  - Most tasks can be performed in infinitely many ways
  - How does the CNS choose? What does it choose?
- How does the CNS manage movement?
  - How is the “software” organized or structured?
    - Quotes because “computation” and “software” may be no more than metaphors for neural processes

Hierarchical organization

- Neural processes are organized hierarchically
  - Evidence: “release” phenomena
    - Higher levels exploit lower-level functionality
  - Multi-stage (multi-level) process, progressively adding detail from abstract to particular
  - For motor control, planning then execution.
- What is planned?
- How is it planned?
- How is the plan executed?

Mechanics matters

- What constitutes evidence of a plan?
- We (mostly) observe its execution.
  - Perfect execution is unlikely; imperfect execution may occlude a plan
- Disentangling plan from execution is challenging
  - Our knowledge of the system used to carry out actions is inaccurate & incomplete
- One approach: look for patterns or invariances
  - Those aspects of behavior that don't change when the system performing actions varies.

Hand vs. joint coordinates

- Row 1: joint angles
- Row 2: joint angular velocity
- Row 3: joint angular acceleration
- Row 4: tangential hand velocity

- Substantially less variability in **hand Cartesian coordinates**

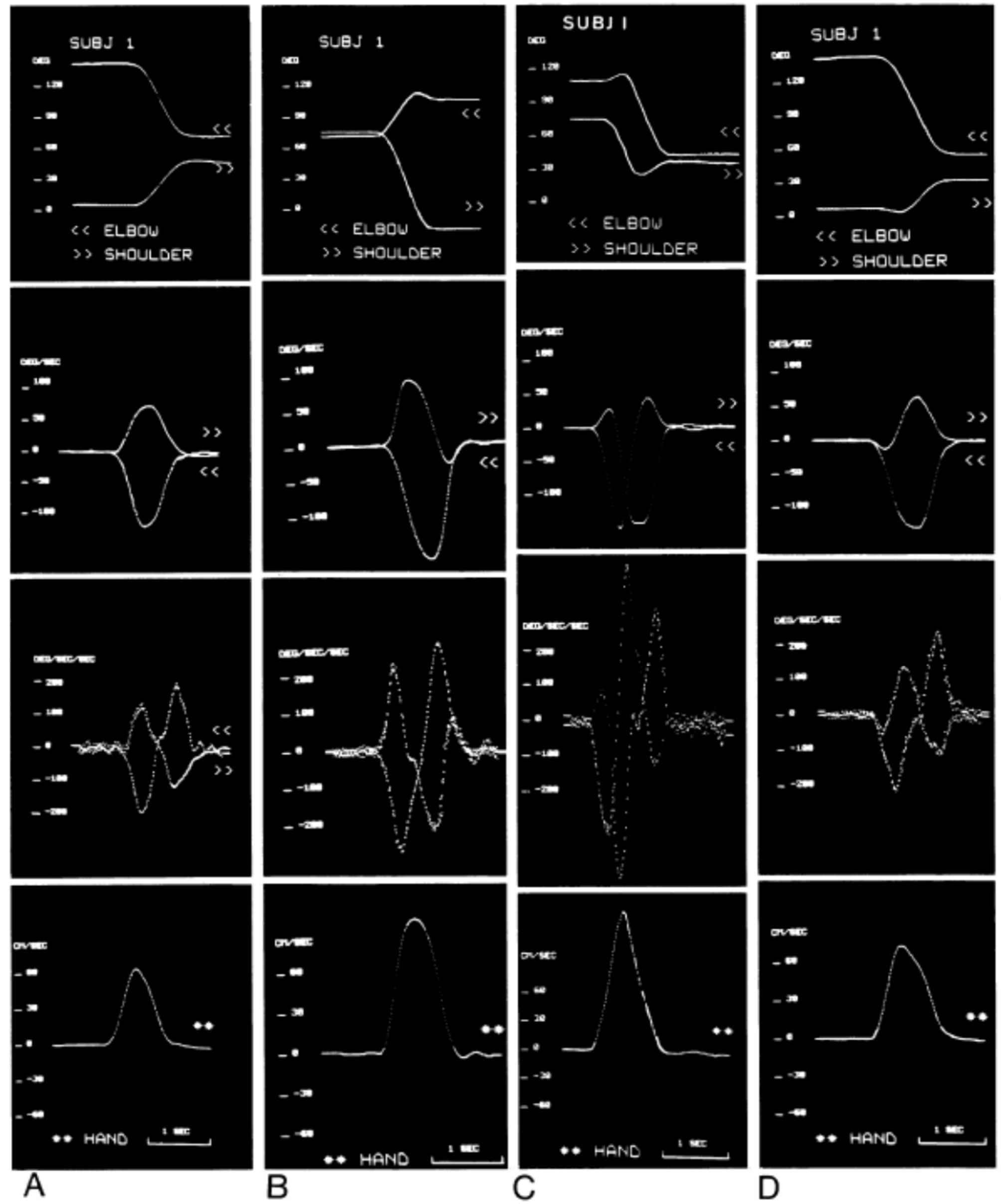


Fig. 3A–D. Joint rotation and hand trajectory. For subject 1, this figure displays the temporal patterns of four typical movements (a column for each movement) which exhibit distinctive joint angular patterns, though preserving the shape of the tangential velocity curve. For each movement, the following curves are displayed: Row 1: Joint angle (vertical scale: 30 deg); row 2: Joint angular velocity (vertical scale: 50 deg/s); row 3: Joint angular acceleration (vertical scale: 100 deg/s/s); row 4: Tangential hand velocity (vertical scale: 30 cm/s) (time scale: 1 s). **A** Target 1 to target 4. **B** Target 3 to target 5. **C** Target 2 to target 5. **D** Target 1 to target 5 (see Fig. 1 for target numbering)

Morasso, P. 1981 *Spatial Control of Arm Movements* Experimental Brain Research 43:223-227

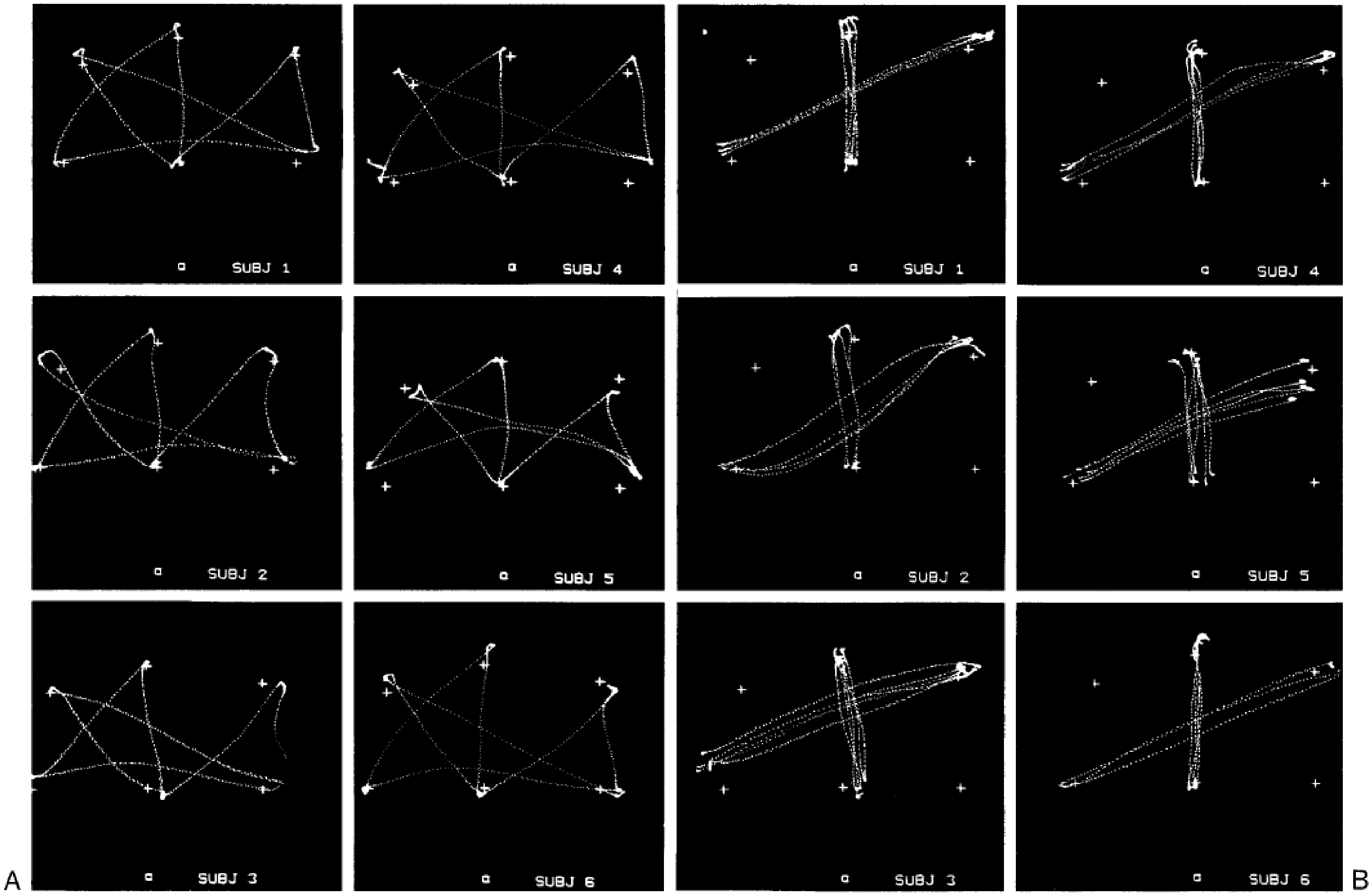


Fig. 2A, B. Spatial trajectories of the hand. The crosses indicate the target positions and the square indicates the shoulder position. The trajectory was sampled at 100 samples/s. The distance between the first and the second target (see Fig. 1 for target numbering) is 30 cm. **A** The sequence of movements is generated by the following sequence of targets: 1, 4, 2, 6, 5, 1, 3, 6. **B** Several repetitions of two movements are superimposed

Optimization theory serves as a model of neural “computation”

- Optimization theory provides a mathematical tool to model planning
  - top-down approach
    - goals first, details later
  - tends to be abstract
    - a “coarse-grained” description of the results of fine-grained neural processes
  - integrative and predictive
    - describes outcomes rather than procedures
  - highly specific and testable

Elements of optimization theory

- task goal or “cost function”
  - quantifies what is considered in planning;
  - may be used as a model of “software” organization
- model of controlled system
  - may embody dynamics
- specification of inputs available to modulate
  - what variables encode the plan
- algorithm to compute a solution
  - CAUTION! the algorithm we use may or may not have any relation to what the brain does

Kinematic cost functions

- Postulate a separation of *planning* from *execution*
  - Plan based on geometry and kinematics alone
  - Execute taking mechanics and dynamics into account

- Biological motions are characteristically smooth—one simple measure: mean-squared jerk
    - Hogan, 1982, IEEE ACC, 2:522-527 Hogan, 1984, J. Neurosci. 4(11):2738-2744.

$$C = \frac{1}{t_f - t_o} \int_{t_o}^{t_f} \left(\frac{d^3 q}{dt^3} \right)^2 dt$$

  - other measures have been explored; this seems to be the simplest that works well

Trajectory plan

- Find a trajectory $q(t)$ to minimize C
  - solve via, e.g., Euler-Poisson equation ⇐ More on this shortly
- Yields specific, testable predictions
  - $q(t)$ is a quintic polynomial in time

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

    - boundary conditions determine constants
- Start and stop at rest:
  - symmetric speed profile
    - generally true, some asymmetry reported
  - peak speed/mean speed = 1.875
    - consistent with observation

Is this just “curve-fitting”?

- No:
  - the theory makes testable predictions
    - not just interpolations of a data set
  - the solutions are constrained by the assumptions
    - e.g. about the controlled system
  - the theory may afford new insight

Multi-joint movements

- Multi-joint mechanics is (a lot) richer than single-joint mechanics;
- the main reason is the (complicated) geometry of spatial kinematic chains.

$$I(\ddot{q}) + H(\dot{q}, q) + D(\dot{q}, q) + G(q) = \tau$$

More on this shortly

q: joint angles

τ: joint torques

I: inertia tensor

H: coriolis & centrifugal accelerations

D: dissipative forces

G: gravity forces

Kinematic cost functions

- Maximum smoothness (minimum-jerk) theory
  - e.g., Flash & Hogan, 1985, J. Neurosci. 5(7):1688-1703

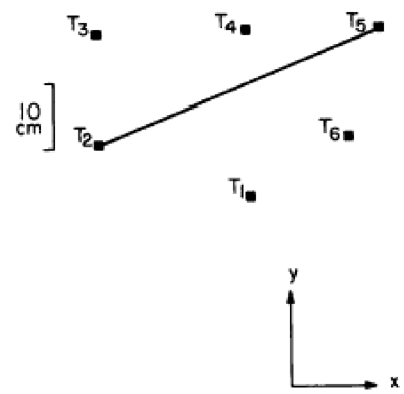
$$C = \frac{1}{t_f - t_o} \int_{t_0}^{t_f} \left(\left| \frac{d^3 \mathbf{r}}{dt^3} \right|^2 \right) dt$$

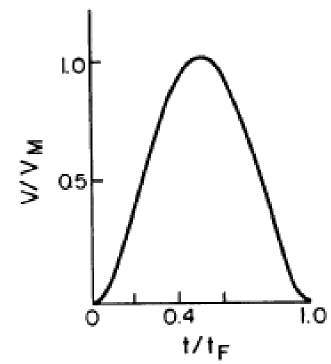
r: vector of limb coordinates

- The coordinate frame matters
  - e.g, joint angles vs. hand coordinates
  - predicted behavior is **sensitive to the frame** in which smoothness is measured.

Smooth moves

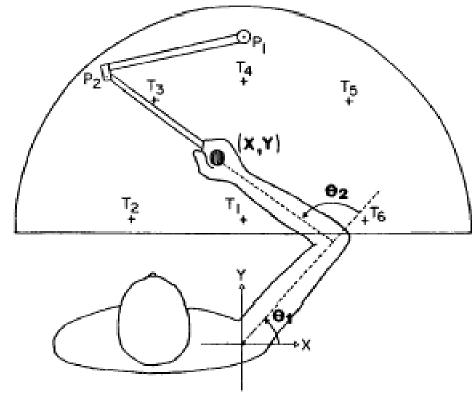
- Idea:
  - use optimization theory as a summary model of micro-structured neural computation
- Hypothesis:
  - Ordinary arm motions are as smooth as possible
    - Ordinary: well within the maximum-performance envelope
- Smooth:
  - Minimize mean-squared jerk
  - In world (visual?) coordinates
- Predictions:
  - Point-to-point reaches are straight
  - Symmetric bell-shaped speed profile
    - Hogan '82, '84, Flash & Hogan '85

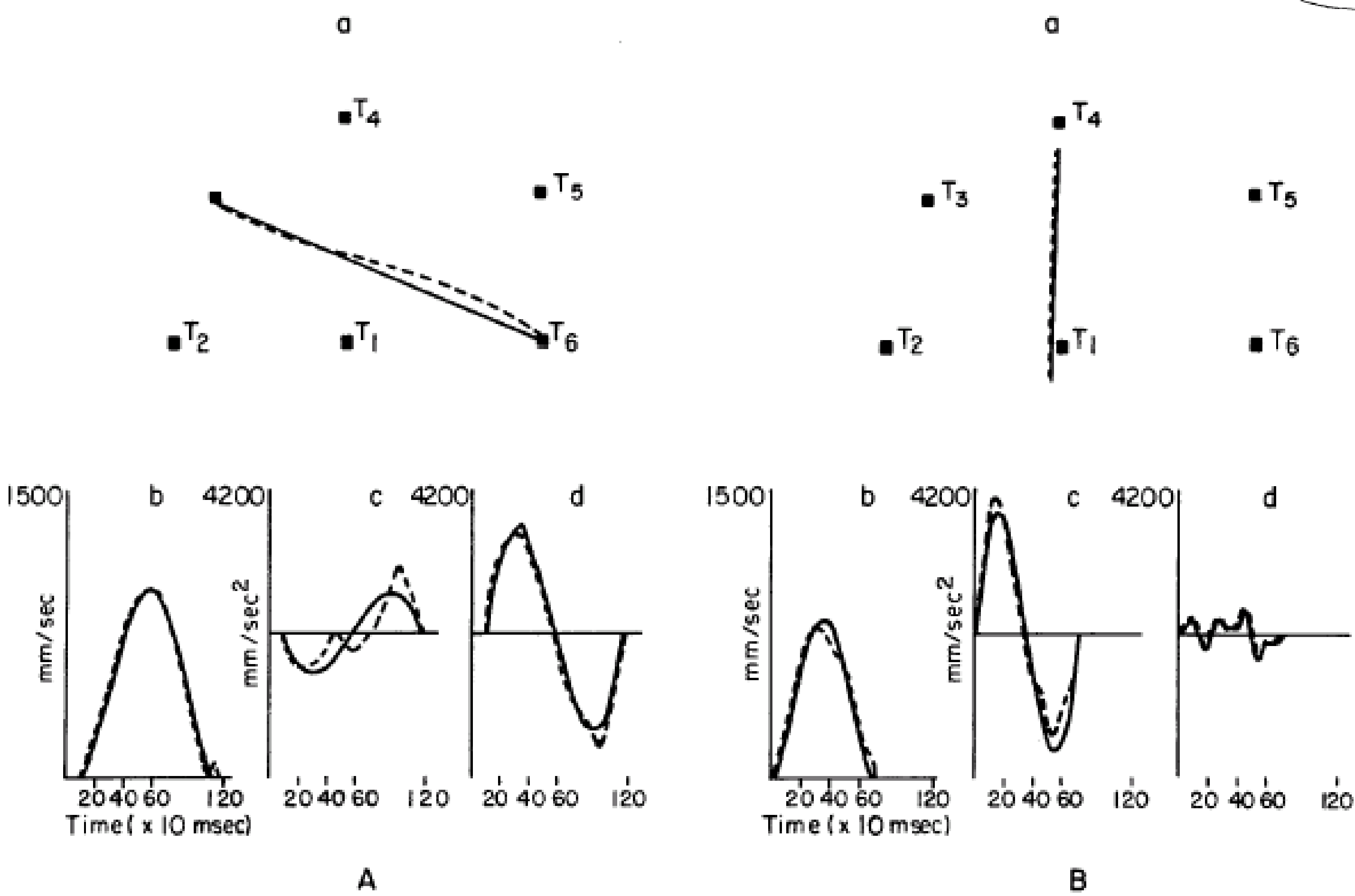




$$\mathbf{r}(t) = \arg\min \frac{1}{t_{final} - t_{initial}} \int_{t_{initial}}^{t_{final}} \left\| \frac{d^3\mathbf{r}}{dt^3} \right\|^2 dt$$

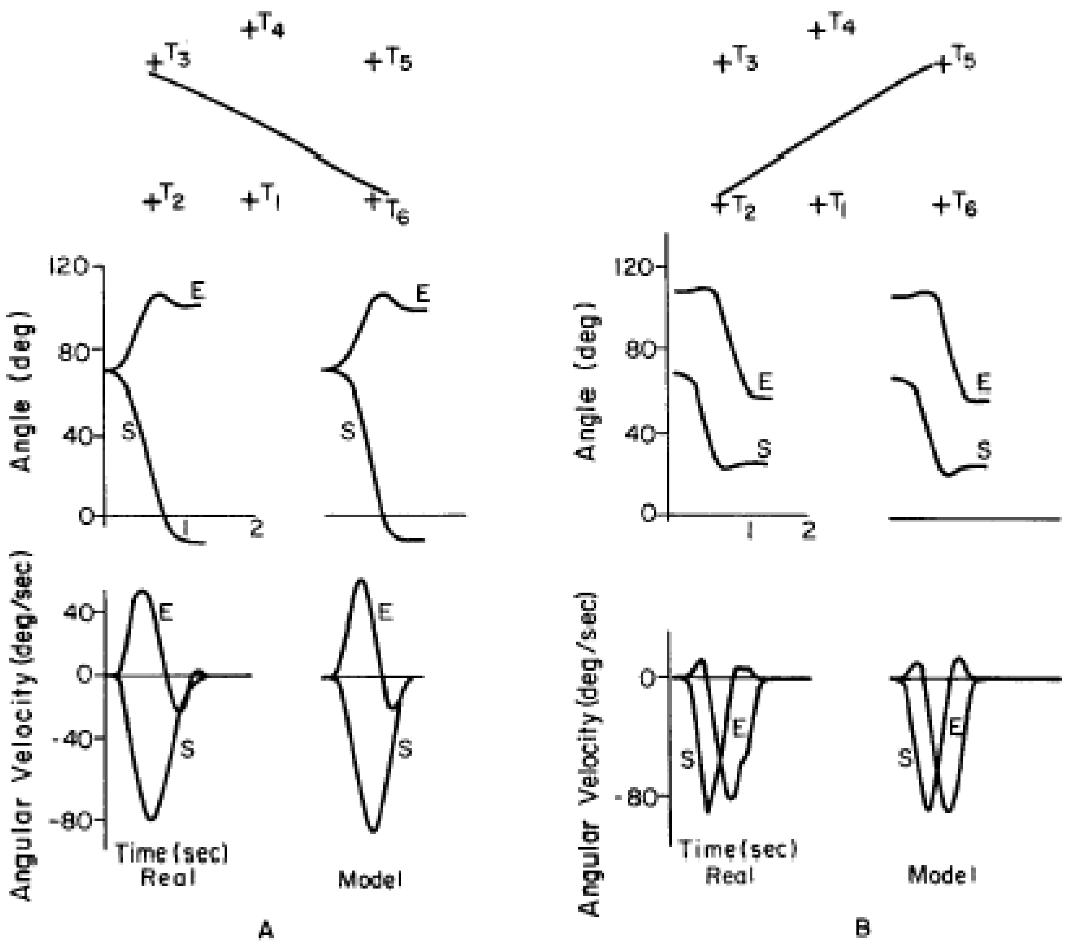
Theory vs. data





Hand vs. joint coordinates

- In joint coordinates, kinematic patterns vary
  - Speed profile may have multiple peaks
- In hand coordinates, kinematic patterns do not vary
  - Speed profiles have one peak (unimodal)

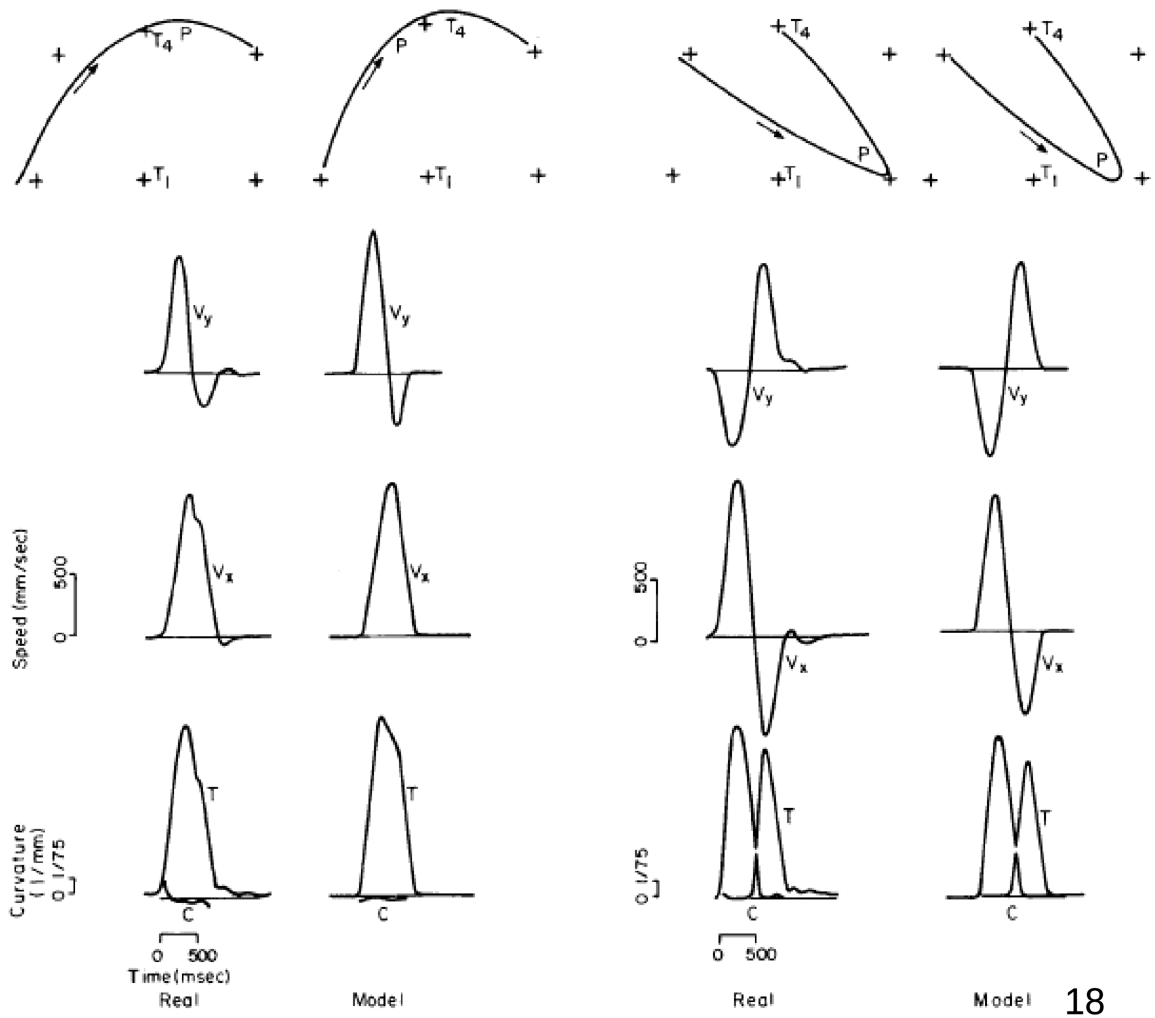


Constraints vs. choices

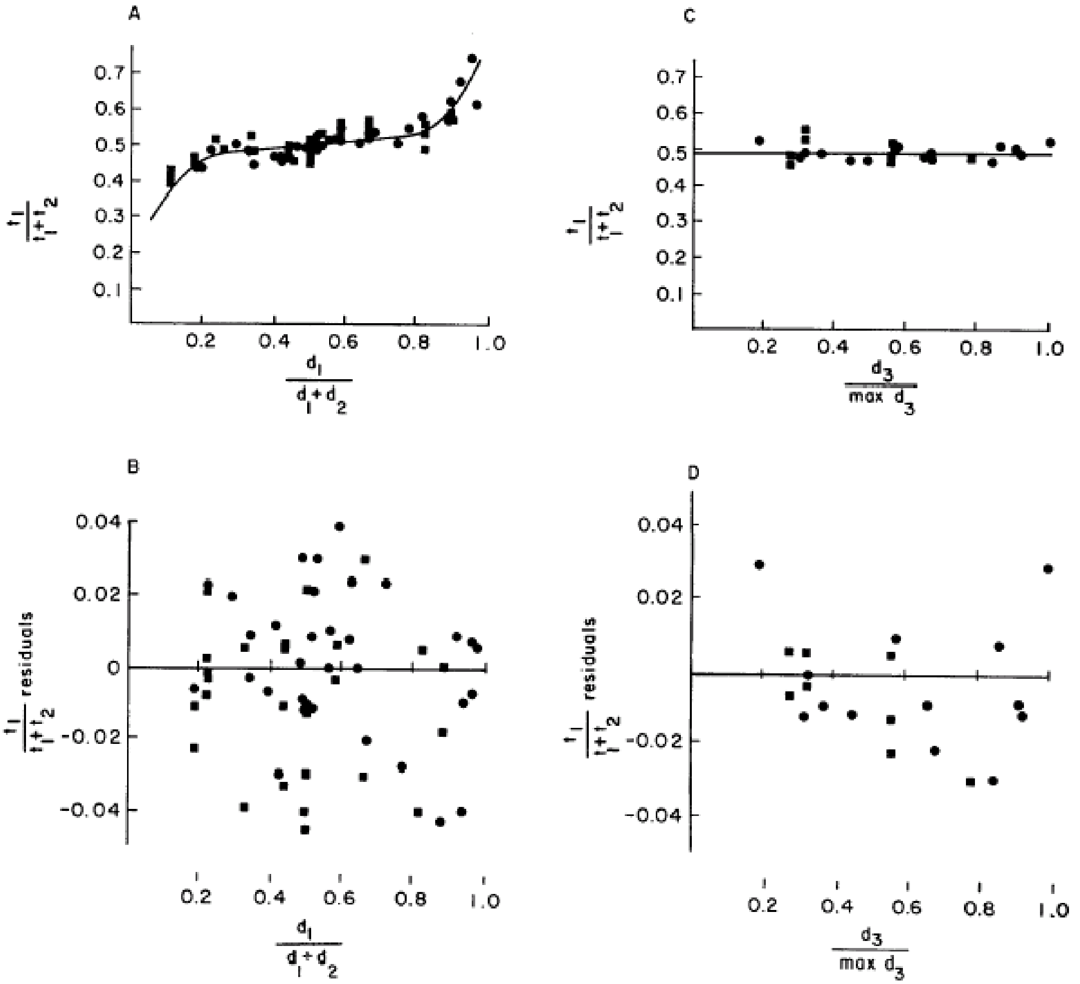
- Predictions:
  - start and stop at rest:
    - straight line,
    - symmetric speed profile
  - generalize to curved motions by adding “via” points
    - curvature peaks at speed minima
  - invariant under rotation and translation
- these are choices, not constrained by mechanics
- good agreement with observation
  - success suggests planning is of hand path in visually-relevant coordinates

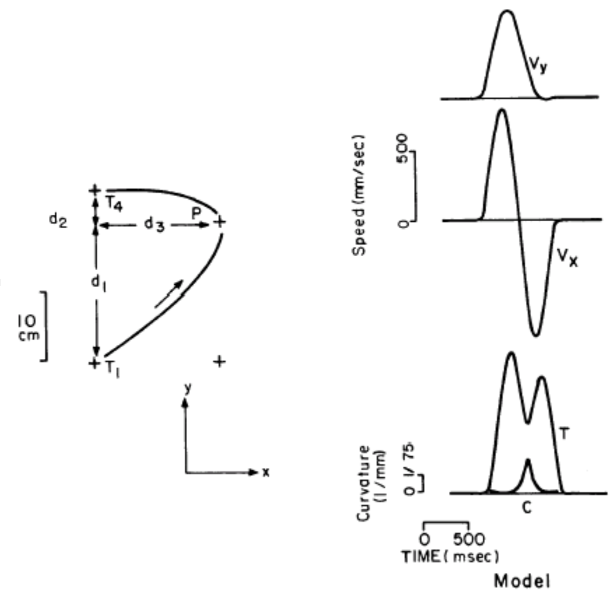
Curved motions

- Predictions:
  - Motions through a “via-point” are continuously curved
  - Speed can be multi-peaked
  - Speed “valleys” at curvature peaks
- Data closely matches theory



More predictions …





- Project via-point onto direct path from start to target
  - "longitude" d_1, "latitude" d_3
- Time to and from "via-point"
  - Varies continuously with longitudinal travel
  - Is independent of lateral displacement
- Residual error <4%