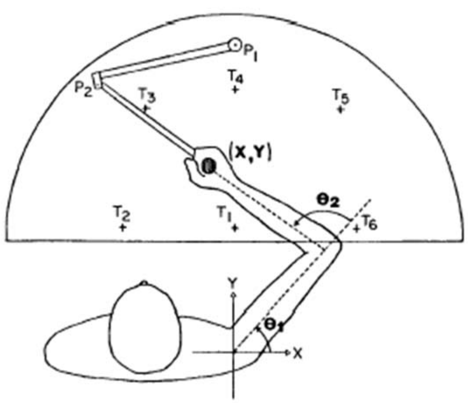
Multi-joint Equilibrium-Point/ Virtual Trajectory Control

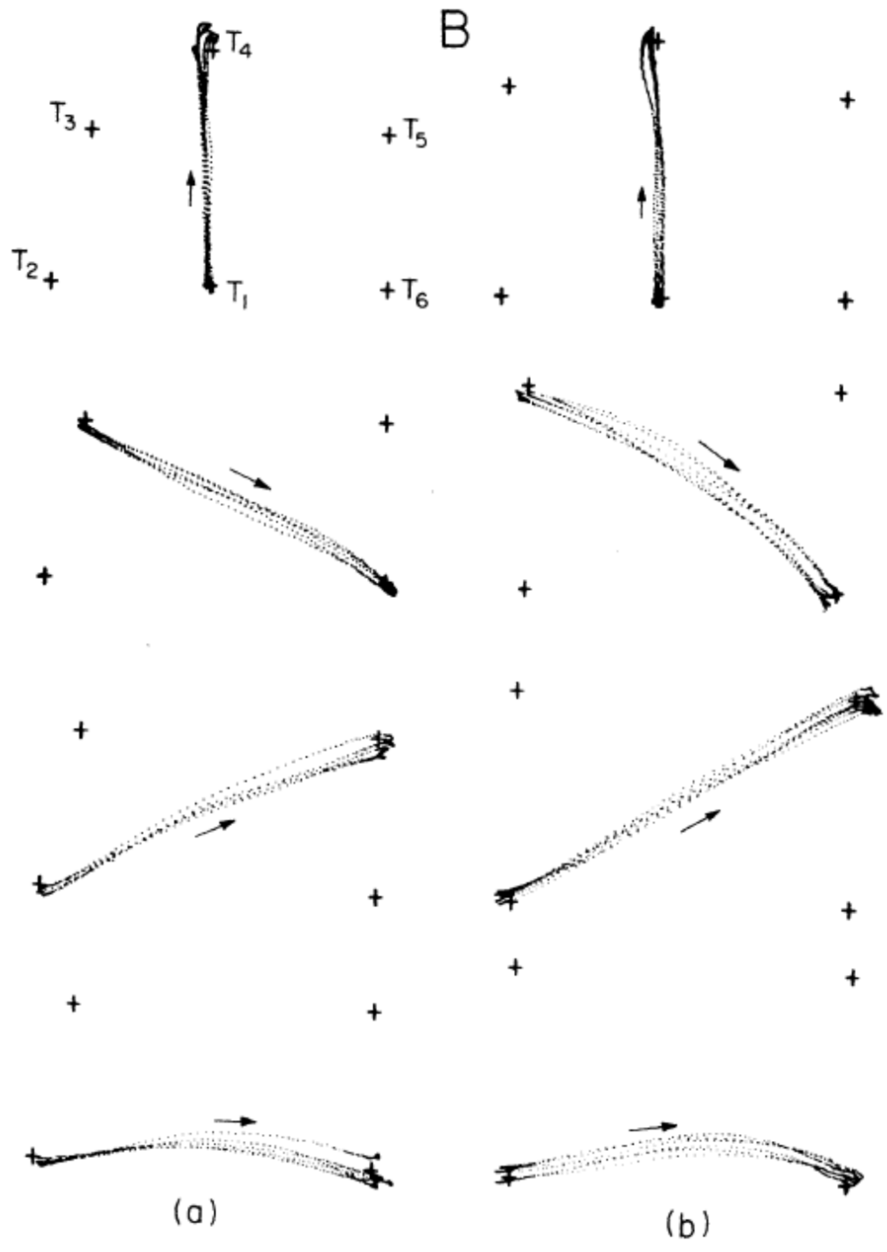
2.183/2.184

Discrete Reaching

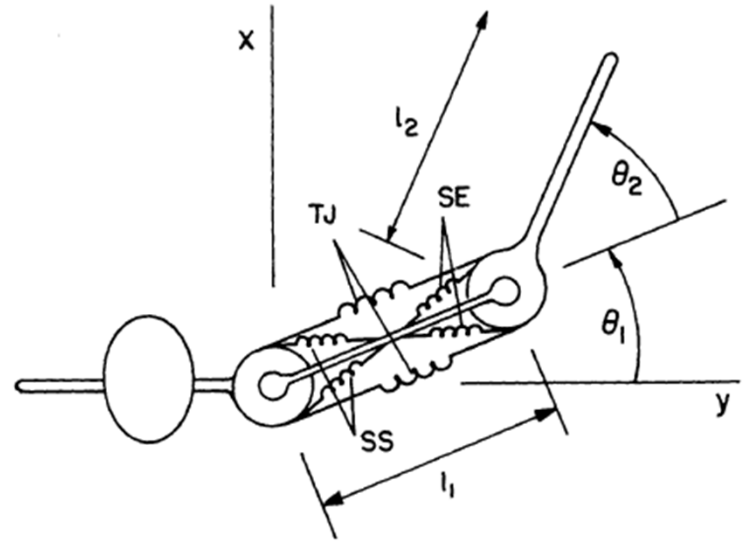
- Two unimpaired human subjects reaching to visual targets in a horizontal plane
- Repeatable kinematic behavior
- Largely straight paths



T. Flash (1987) The Control of Hand Equilibrium Trajectories in Multi-Joint Arm Movements. Biological Cybernetics 57:257-274



Two-joint Dynamic Model



- Model the skeleton as a configuration-dependent inertia driven by torque
  - No gravity
  - No friction

$$\mathbf{I}(\boldsymbol{\theta})\dot{\boldsymbol{\omega}} + \mathbf{C}(\boldsymbol{\theta},\boldsymbol{\omega})\boldsymbol{\omega} = \boldsymbol{\tau}_{actuator}$$

$$\boldsymbol{\tau}_{actuator} = \mathbf{K}(\boldsymbol{\theta}_o - \boldsymbol{\theta}) - \mathbf{B}\boldsymbol{\omega}$$

θ: joint angles, a.k.a. configuration variables, generalized coordinates

ω: generalized velocities, joint angular velocities

τ: joint torques, a.k.a. generalized forces

I: configuration-dependent inertia

C: inertial coupling (Coriolis & centrifugal accelerations)

K: stiffness

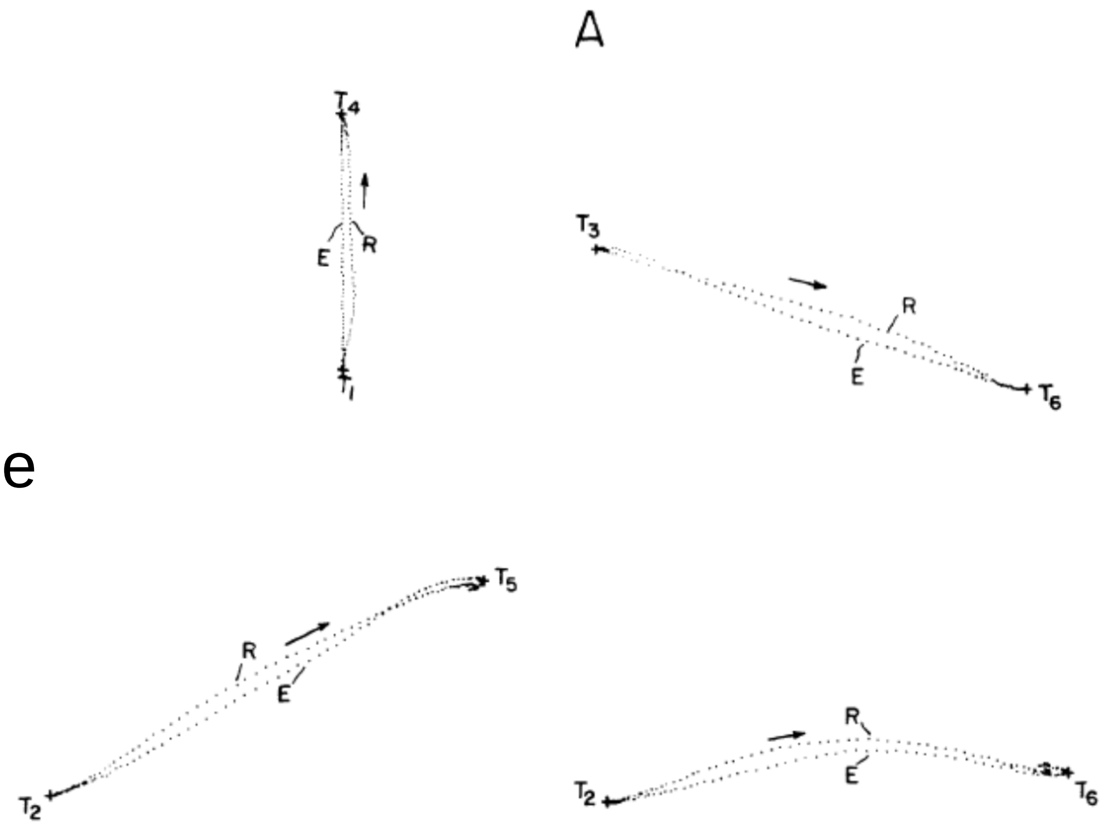
B: damping

- Model neuromuscular torque generation as a two-dimensional spring and damper
  - Spring between actual and virtual configuration
  - Damper to ground
  - More on this later

Calculation Details

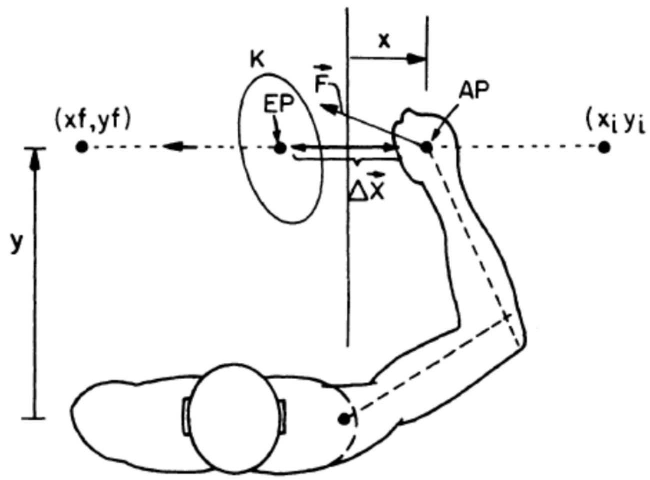
- R: real data
- E: equilibrium-point (virtual trajectory)

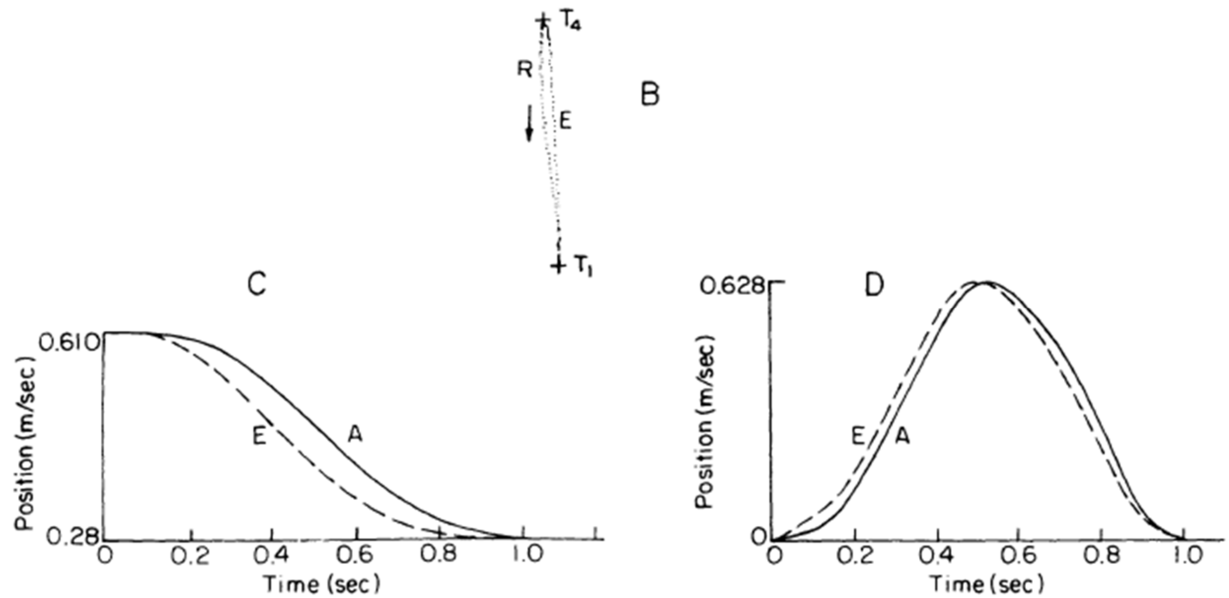
- E derived by "back-calculating" using the model



Model Performance

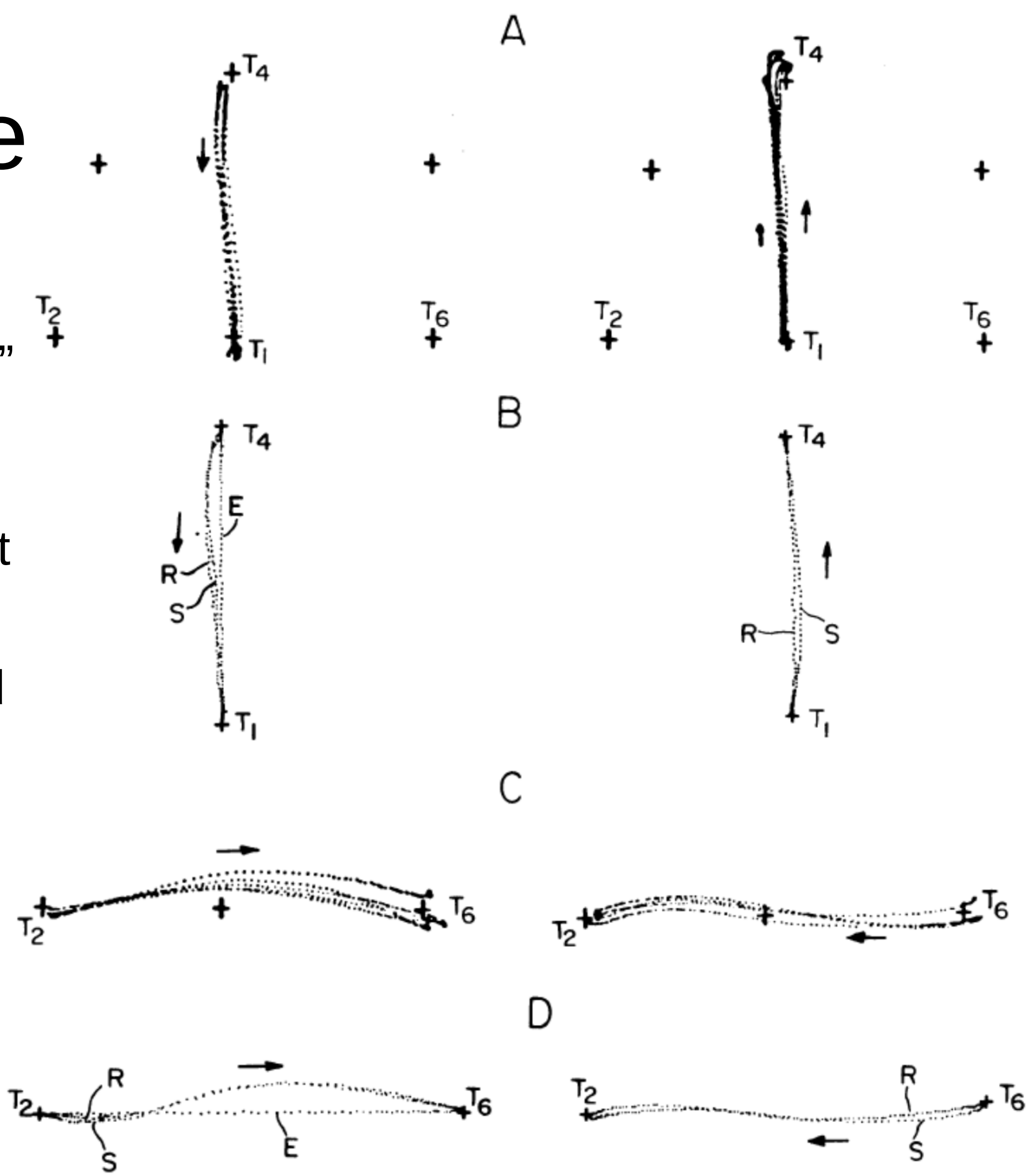
- Trajectory details
  - Tangential speed resembles single-joint observations
    - Smooth “bell-shape”, single peak
  - Actual trajectory (A) lags equilibrium point (E)
  - E derived by “back-calculating” using the model





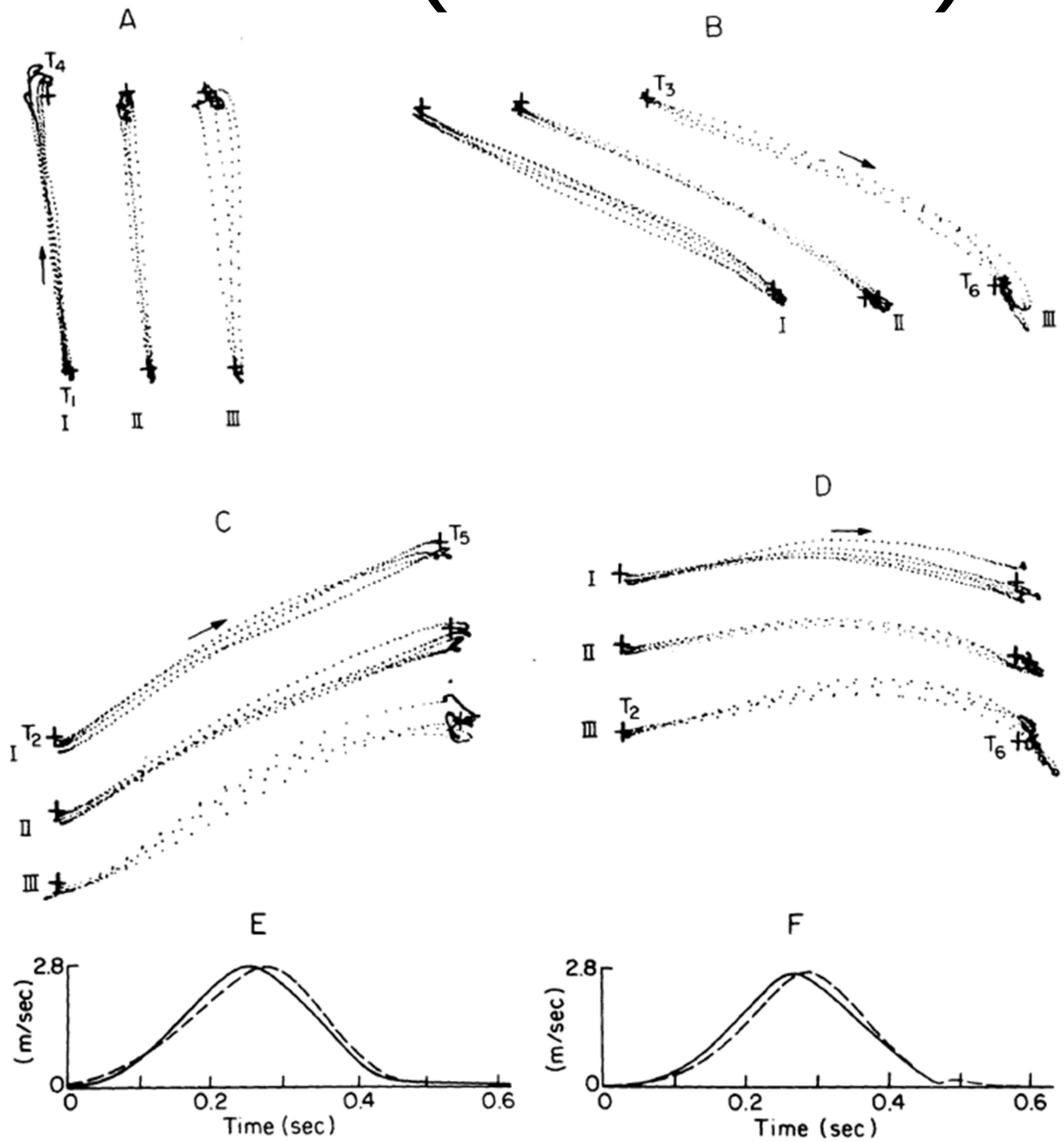
Model Competence

- Reproduces
  - Difference between “out” and “back”
  - More pronounced curvature closer to chest
  - Radial paths straighter
  - Lateral paths “s” shaped
    - R: real
    - S: simulated
    - E: equilibrium point

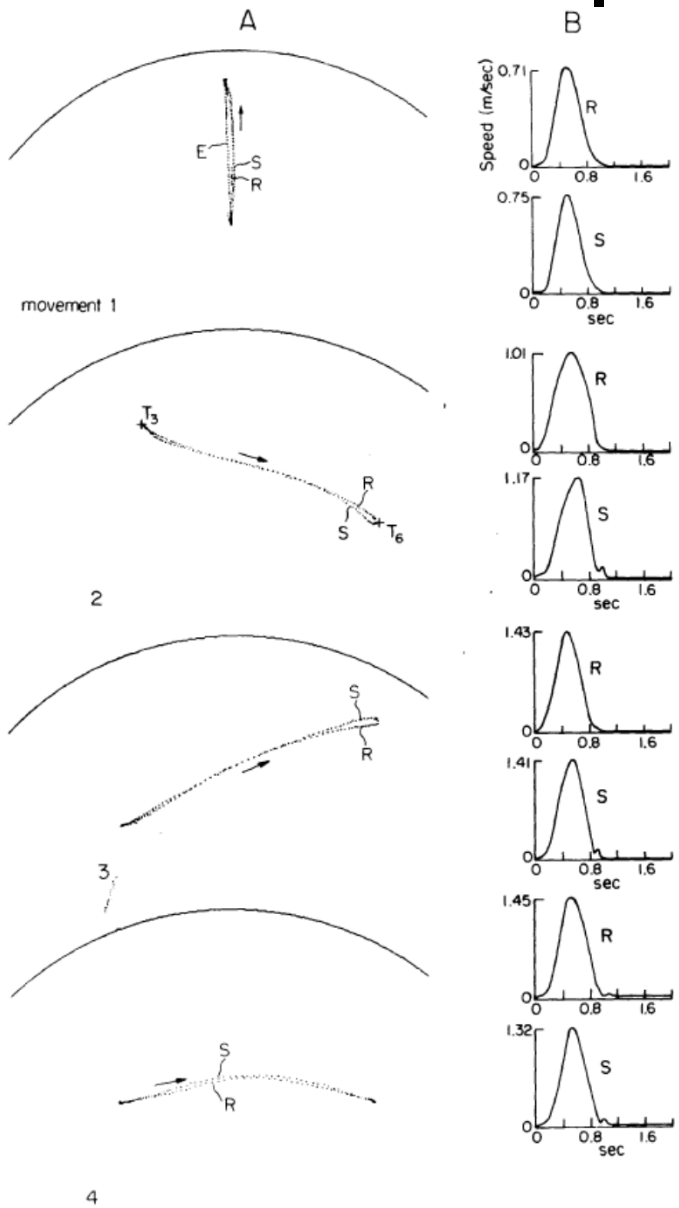


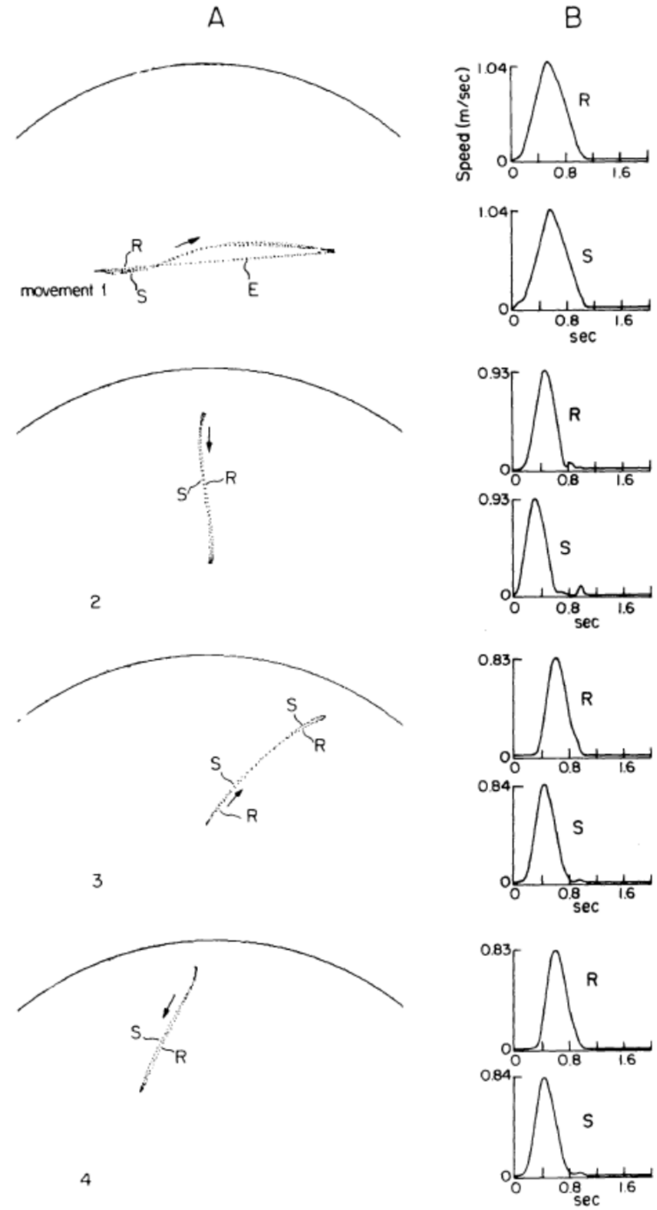
Model Competence (continued)

- Model works at different speeds
  - I: more than 0.8 s
  - II: 0.5 to 0.8 s
  - III: 0.4 to 0.5 s
  - E and F overlay speed profiles of slow (dashed) and fast (solid) moves



Model Competence (continued)





More on Model Competence

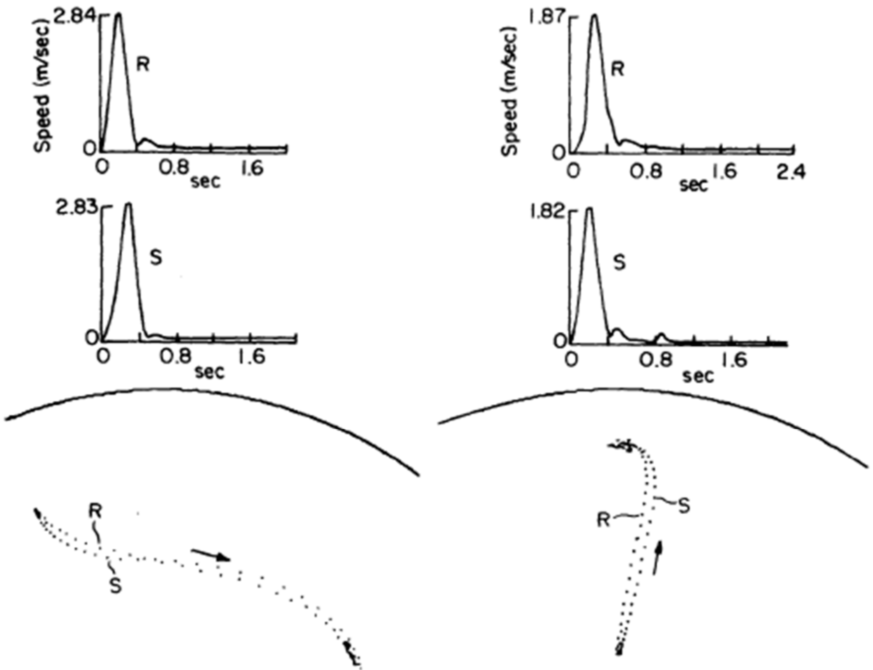


Fig. 11. Comparisons between simulated (S) and measured (R) 0.4 s trajectories. Upper part of figure: speed profiles. Lower half: hand paths. For the movement from T_3 to T_6, $G_t = 5.625$, $G_s = 3.75$, $G_e = 2.5$, $\varrho = 0.55$. For the movement from T_1 to T_4, $G_t = 3.5$, $G_s = 5.0$, $G_e = 1.0$, $\varrho = 0.5$

Two-joint Impedance

- Stiffness is directional
  - Real-valued, symmetric, positive-definite matrix
  - Represented by an ellipse
- Stiffness varies with hand location but ...
  - Stiffness is (approximately) constant in joint coordinates
- Damping is unknown, assumed proportional to stiffness
  - Consistent with a single time-constant characterizing neuromuscular dynamics