"Exemplo 2" Verificar se T é diagonalizável.

$$[T]_{can} = \begin{bmatrix} 4 & 3 & -1 \\ 2 & 5 & -1 \\ 2 & 3 & 1 \end{bmatrix} = A \qquad T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3 \qquad [T]_{can} = V$$

(1) Calcular $p_T(t)$

$$p_T(t) = -\det(T - tI_3)$$

$$\det \begin{bmatrix} 4-t & 3 & -1 \\ 2 & 5-t & -1 \\ 2 & 3 & 1-t \end{bmatrix} = (4-t)\left[(5-t)(1-t)+3\right]$$

$$-3\underset{6t-12}{\overset{-2t+4}{\left[2-2t+2\right]}} - 1\overset{+2t-4}{\left[6-10+2t\right]}$$

$$= (4-t)\left[t^2-6t+8\right] + 6t - 12 - 2t + 4.$$

$$= (4-t)(t-2)(t-4) + 4(t-2)$$

$$= (t-2)\left[(4-t)(t-4)+4\right]$$

$$= (t-2)\left[-t^2+8t-12\right]$$

$$= -(t-2)^2(t-6)$$

Logo $\boxed{p_T(t) = (t-2)^2(t-6)}$

Temos os autovalores $\lambda_1 = 2$ e $\lambda_2 = 6$

$m_a(2) = 2 \qquad m_a(6) = 1$

(2) Determinar $V(2)$ e $V(6)$

$V(2)$

$$A-2I=\begin{bmatrix}2 & 3 & -1\\ 2 & 3 & -1\\ 2 & 3 & -1\end{bmatrix}$$

Logo $\operatorname{Ker}(T-2I)=\{(x,y,z)\in\mathbb{R}^3 \mid 2x+3y-z=0\}$

$=\{(x,y,2x+3y) \mid x,y\in\mathbb{R}\}$

$=\{x(1,0,2)+y(0,1,3),\ x,y\in\mathbb{R}\}$

$=[(1,0,2),(0,1,3)]$

$V(2)=[(1,0,2),(0,1,3)]$

$\dim V(2)=2=m_g(2)=m_a(2)$

$V(6)$ $\quad 1\leq \dim V(6)\leq 1$

Já sabemos que T é diagonalizável,

Achar uma base de $V(6)$

$$A-6I=\begin{bmatrix}-2 & 3 & -1\\ 2 & -1 & -1\\ 2 & -3 & 1\end{bmatrix}$$

Escalonando essa matriz temos

$$\begin{bmatrix}-2 & 3 & -1\\ 2 & -1 & -1\\ 2 & -3 & 1\end{bmatrix}\rightarrow\begin{bmatrix}-2 & 3 & -1\\ 0 & 2 & -2\\ 0 & 0 & 0\end{bmatrix}$$

Resolvendo o sistema para acha $\operatorname{Ker}(T-6I)$

$2x=2z \Rightarrow x=z$

$-2x+3y-z=0$

$-2z+3y-z=0 \Rightarrow y=z$

$V(6)=\operatorname{Ker}(T-6I)=[(1,1,1)]$

Assim T é diagonalizável

$B = \{\underbrace{(1,0,2)}_{v_1}, \underbrace{(0,1,3)}_{v_2}, \underbrace{(1,1,1)}_{v_3}\}$ é uma base de $\mathbb{R}^3$ formada por autovetores de T.

$T(v_1) = 2v_1$
$T(v_2) = 2v_2$
$T(v_3) = 6v_3$

$$[T]_B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix} = D$$

$\boxed{\text{Determinar } P \in M_3(\mathbb{R}) \text{ tal que } P^{-1}AP = D}$

$$\underset{B}{\mathbb{R}^3} \xrightarrow{I} \underset{can}{\mathbb{R}^3} \xrightarrow{T} \underset{can}{\mathbb{R}^3} \xrightarrow{I} \underset{B}{\mathbb{R}^3}$$

$$D = [T]_B = [I]_{can,B}\,[T]_B\,[I]_{B,can}$$

$$[I]_{B,can} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} = P$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix} = P^{-1}AP.$$