1. $$\left(\frac{\partial}{\partial T}(F/T)\right)_V = \frac{1}{T}\left(\frac{\partial F}{\partial T}\right)_V + F\cdot\left(\frac{\partial}{\partial T}\left(\frac{1}{T}\right)\right)_V \overset{*}{=}$$

$$\overset{*}{=} \frac{1}{T}\cancel{(-S)} + (U-\cancel{TS})\left(-\frac{1}{T^2}\right) = -\frac{U}{T^2}$$

$$\therefore \boxed{U = -T^2\left(\frac{\partial}{\partial T}\left(\frac{F}{T}\right)\right)_V}$$

$*$: POIS $F=U-TS$

E

$dF=-SdT-PdV+\mu dN$

$\Downarrow$

$S=-\left(\frac{\partial F}{\partial T}\right)_{V,N}$

2. COMO $H=U+PV$, $G=U-TS+PV$,

$dG=-SdT+VdP+\mu dN$ E $S=-\left(\frac{\partial G}{\partial T}\right)_{P,N}$,

$$H=G+TS \Rightarrow H=G-T\left(\frac{\partial G}{\partial T}\right)_{P,N}.$$

$$\left(\frac{\partial}{\partial T}\left(\frac{G}{T}\right)\right)_P = \frac{1}{T}\left(\frac{\partial G}{\partial T}\right)_P + G\cdot\left(\frac{\partial}{\partial T}\left(\frac{1}{T}\right)\right)_P =$$

$$= \frac{1}{T}\cancel{(-S)} + (H-\cancel{TS})\left(-\frac{1}{T^2}\right) = -\frac{H}{T^2}$$

$$\therefore \boxed{H=-T^2\left(\frac{\partial}{\partial T}\left(\frac{G}{T}\right)\right)_P}$$

3. $C_P = \left(\frac{\partial H}{\partial T}\right)_P$.

MAS $dG = -S\,dT + V\,dP + \mu\,dN$, DE MODO QUE

$S = -\left(\frac{\partial G}{\partial T}\right)_{P,N}$ E $H = G + T.S = G - T\left(\frac{\partial G}{\partial T}\right)_{P,N}$.

ENTÃO $C_P = \left(\frac{\partial H}{\partial T}\right)_P = \left[\frac{\partial}{\partial T}\left(G - T\left(\frac{\partial G}{\partial T}\right)_P\right)\right]_P =$

$= \cancel{\left(\frac{\partial G}{\partial T}\right)_P} - 1.\cancel{\left(\frac{\partial G}{\partial T}\right)_P} - T\left(\frac{\partial^2 G}{\partial T^2}\right)_P$.

ASSIM, $\left(\frac{\partial C_P}{\partial P}\right)_T = \frac{\partial}{\partial P}\left[-T\frac{\partial^2 G}{\partial T^2}\right] =$

$= -T\left(\frac{\partial^2}{\partial T^2}\left[\left(\frac{\partial G}{\partial P}\right)_T\right]\right)_P = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P$.

$\therefore \boxed{\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P}$.

4. COMO $dF = -SdT - PdV + \mu dN$ E

$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$, $F = U - TS \Rightarrow U = F - T\left(\frac{\partial F}{\partial T}\right)_{V,N}$.

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N} = \left[\frac{\partial}{\partial T}\left(F - T\left(\frac{\partial F}{\partial T}\right)_{V,N}\right)\right]_{V,N} =$$

$$= \cancel{\left(\frac{\partial F}{\partial T}\right)_{V,N}} - 1 \cdot \cancel{\left(\frac{\partial F}{\partial T}\right)_{V,N}} - T\left(\frac{\partial^2 F}{\partial T^2}\right)_{V,N}.$$

ASSIM,

$$\left(\frac{\partial C_V}{\partial V}\right)_T = \frac{\partial}{\partial V}\left[-T\frac{\partial^2 F}{\partial T^2}\right] = -T\frac{\partial^2}{\partial T^2}\left(\frac{\partial F}{\partial V}\right) = -T\frac{\partial^2 (-P)}{\partial T^2}$$

$$\therefore \boxed{\left(\frac{\partial C_V}{\partial V}\right)_T = +T\left(\frac{\partial^2 P}{\partial T^2}\right)_V}$$

5. $dU = TdS - PdV + \mu dN \Rightarrow$

$$\Rightarrow dS = \frac{1}{T}dU + \frac{P}{T}dV - \frac{\mu}{T}dN .$$

COMO $G = U - TS + PV$ E $U = TS - PV + \mu N$, $G = \mu N$ E O POTENCIAL QUÍMICO É A ENERGIA LIVRE DE GIBBS POR PARTÍCULA,

$g \equiv \frac{G}{N} = \mu$. BASTA CALCULARMOS μ E EXPRESSÁ-LO COMO FUNÇÃO DE T E P.

$$\mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V} =$$

$$= -T . \frac{\partial}{\partial N}\left\{ N.c + N . \frac{a^{1/4} . V^{1/4} . U^{1/2}}{N^{3/4}} \right\} =$$

$$= -T . \left\{ c + \frac{1}{4} N^{-3/4} . a^{1/4} . V^{1/4} . U^{1/2} \right\} .$$

MAS $\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N} = \frac{\partial}{\partial U}\left\{ N.c + a^{1/4} . U^{1/2} V^{1/4} . N^{1/4} \right\} =$

$$= \frac{1}{2} a^{1/4} . U^{-1/2} . V^{1/4} . N^{1/4}$$

E

5. (CONT.)

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{U,N} = \frac{\partial}{\partial V}\left\{N.c + a^{1/4}.U^{1/2}.V^{1/4}.N^{1/4}\right\} =$$

$$= \frac{1}{4}a^{1/4}.U^{1/2}.V^{-3/4}.N^{1/4} \quad .$$

U E V PODEM SER EXPRESSOS EM TERMOS DE T E P.

$$\frac{1}{T}\cdot\frac{P}{T} = \frac{1}{8}a^{1/2}.V^{-1/2}.N^{1/2} \quad E$$

$$\left(\frac{1}{T}\right)^3\cdot\frac{P}{T} = \frac{1}{32}a.U^{-1}.N \quad , \quad \text{DE MODO QUE}$$

$$V^{1/2} = \frac{1}{8}\cdot\frac{T^2}{P}a^{1/2}.N^{1/2} \quad E$$

$$U = \frac{1}{32}\cdot\frac{T^4}{P}aN \; .$$

FINALIZANDO, $\mu = -T\left\{c + \frac{1}{4}a^{1/4}\left(\frac{1}{32}\frac{T^4}{P}aN\right)^{1/2}\cdot\right.$

$$\left.\cdot\left(\frac{1}{8}\cdot\frac{T^2}{P}a^{1/2}.N^{1/2}\right)^{1/2}.N^{-3/4}\right\} = -T\left\{c + \frac{1}{64}a\frac{T^3}{P}\right\} \; .$$

6. $\begin{cases} P = -\dfrac{u}{v - 2Auv} \\ T = 2C \dfrac{u^{1/2} v^{1/2}}{1 - 2Au} e^{Au} \end{cases} \sim$

$\sim \begin{cases} \dfrac{1}{T} = \dfrac{1}{2C} \cdot \dfrac{1 - 2Au}{u^{1/2} v^{1/2}} e^{-Au} \\ \dfrac{P}{T} = -\dfrac{1}{2C} \cdot \dfrac{u^{1/2}}{v^{3/2}} e^{-Au} \end{cases}$

$$ds = \frac{1}{T} du + \frac{P}{T} dv$$

SE $s(u,v) = D . u^n . v^m . e^{-Au}$, É PRECISO QUE

$m.D u^n v^{m-1} . e^{-Au} = \dfrac{P}{T} = -\dfrac{1}{2C} u^{1/2} . v^{-3/2} . e^{-Au} \Rightarrow$

$\Rightarrow \begin{cases} mD = -1/2C \\ n = 1/2 \\ m - 1 = -3/2 \end{cases} \sim \begin{cases} m = -1/2 \\ n = +1/2 \\ D = 1/C \end{cases}$

ASSIM, $s(u,v) = \dfrac{1}{C} u^{1/2} . v^{-1/2} . e^{-Au}$ E

$S(U,V,N) = N . s = N . \dfrac{1}{C} \left(\dfrac{U}{N}\right)^{1/2} . \left(\dfrac{V}{N}\right)^{-1/2} . e^{-AU/N}$

6. (CONT.)

$$\therefore \boxed{S(U,V,N) = \frac{1}{c} U^{1/2} \cdot V^{-1/2} \cdot N \cdot e^{-AU/N}}$$