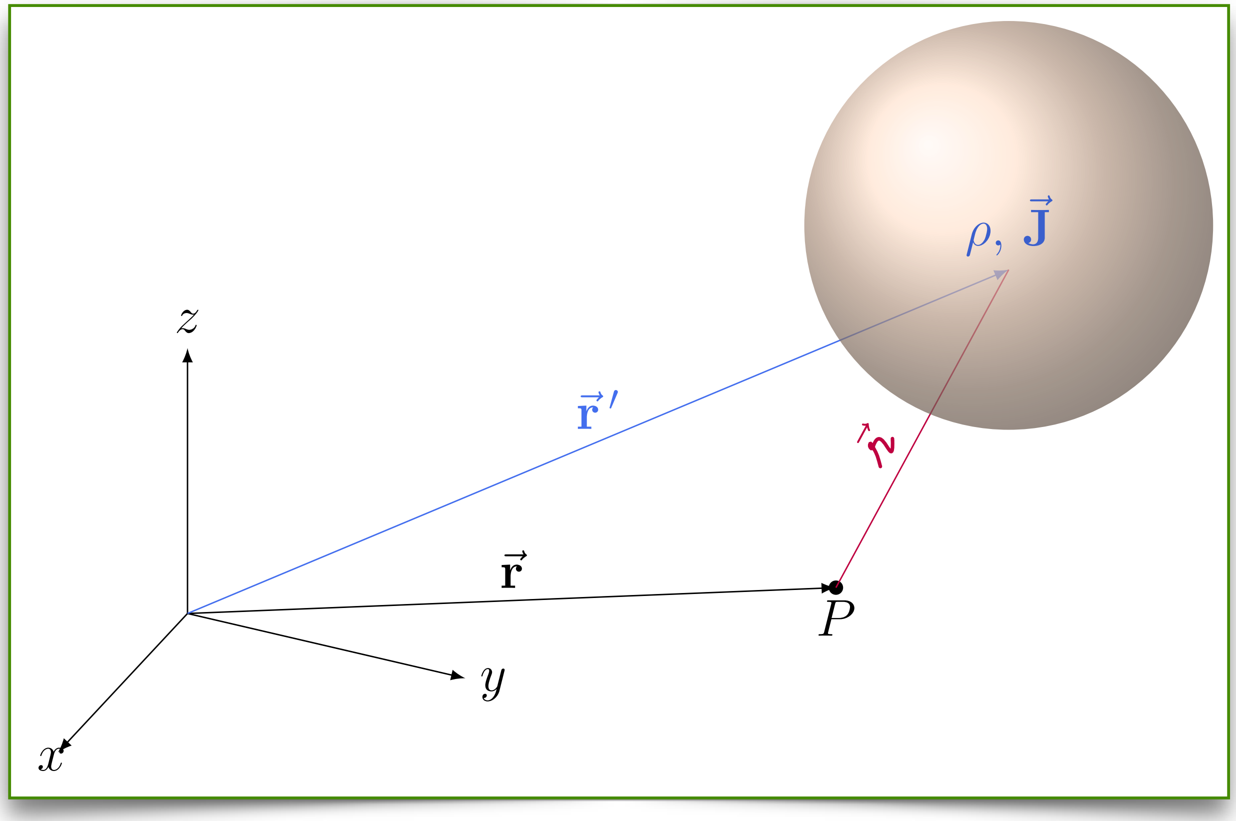
Eletromagnetismo Avançado

8 dezembro
Radiação

Radiação de distribuição de cargas

$r \gg c\tau \gg R$

$\tau \equiv$ tempo característico



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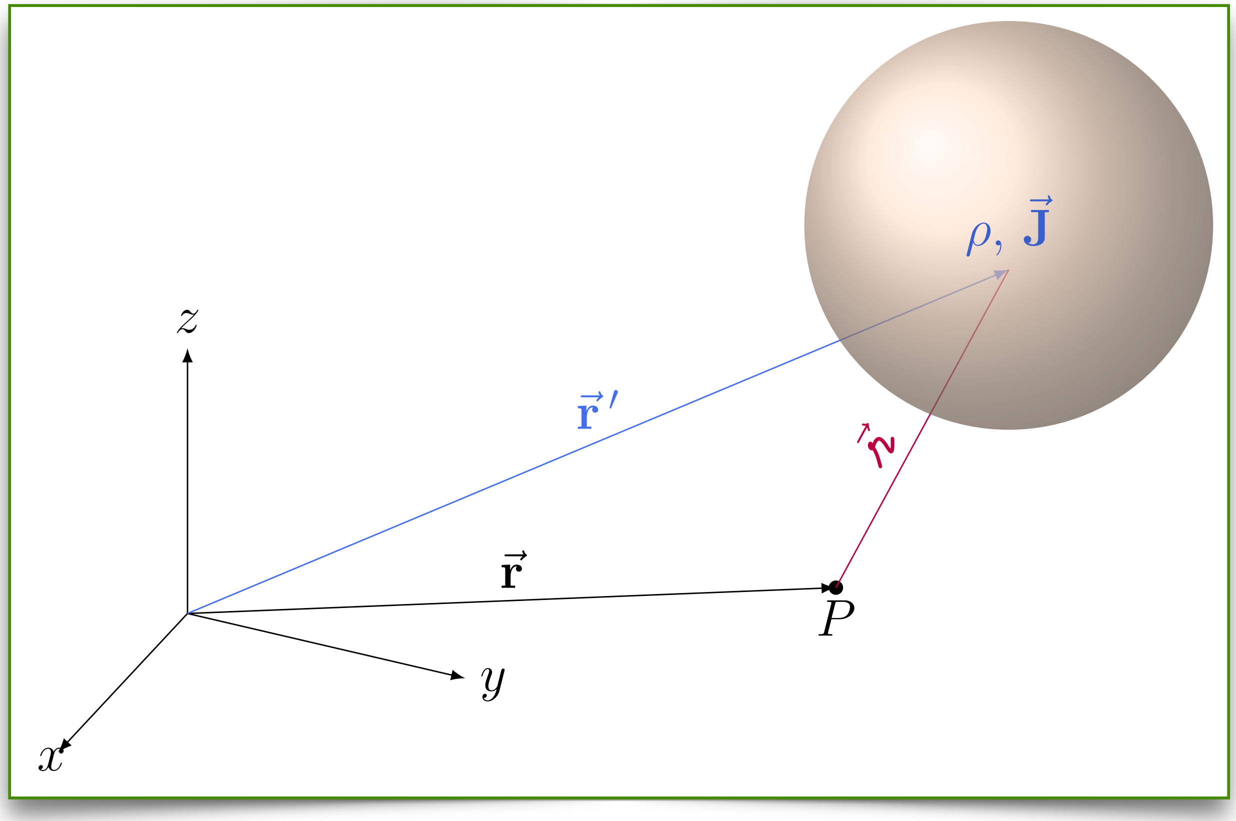

$V(\vec{\mathbf{r}}, t) = ?$

$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = ?$

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$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}\,', t_r)}{\mathscr{r}} \, d\tau'$$

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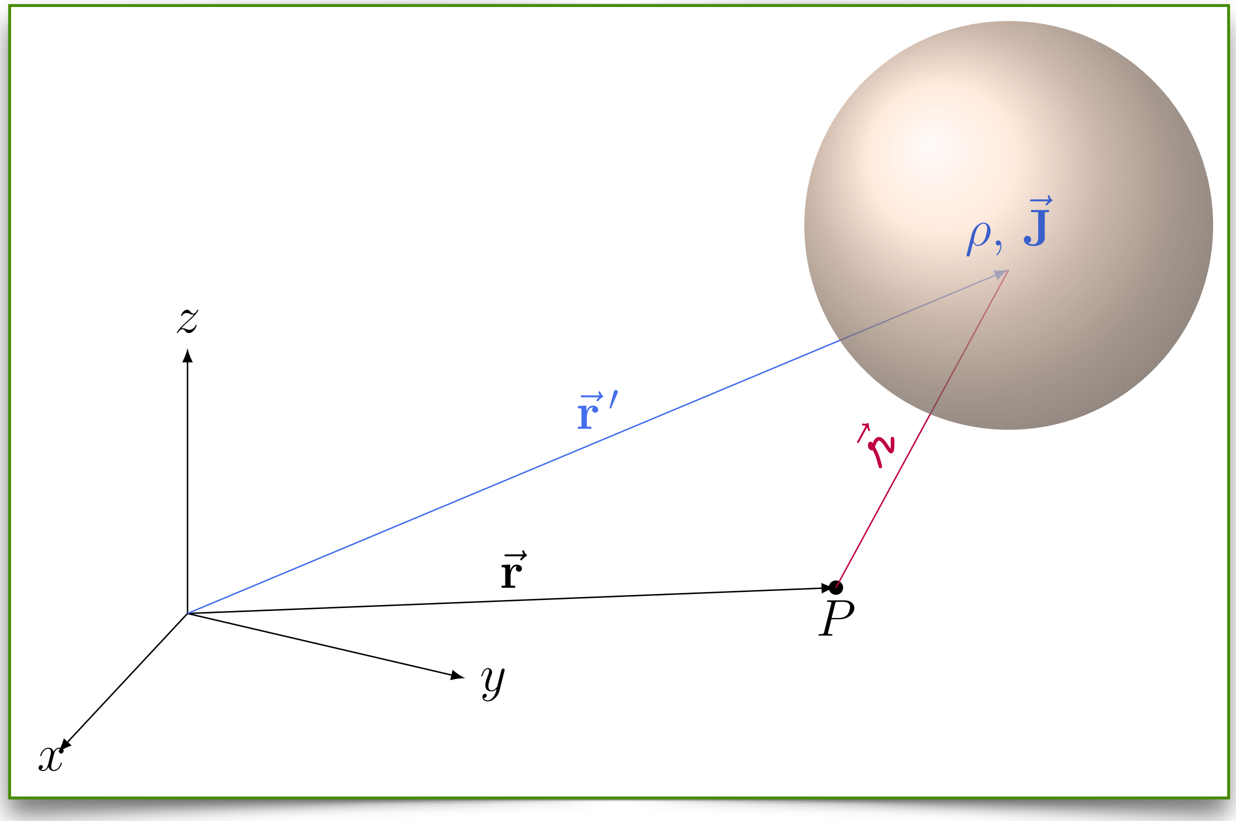

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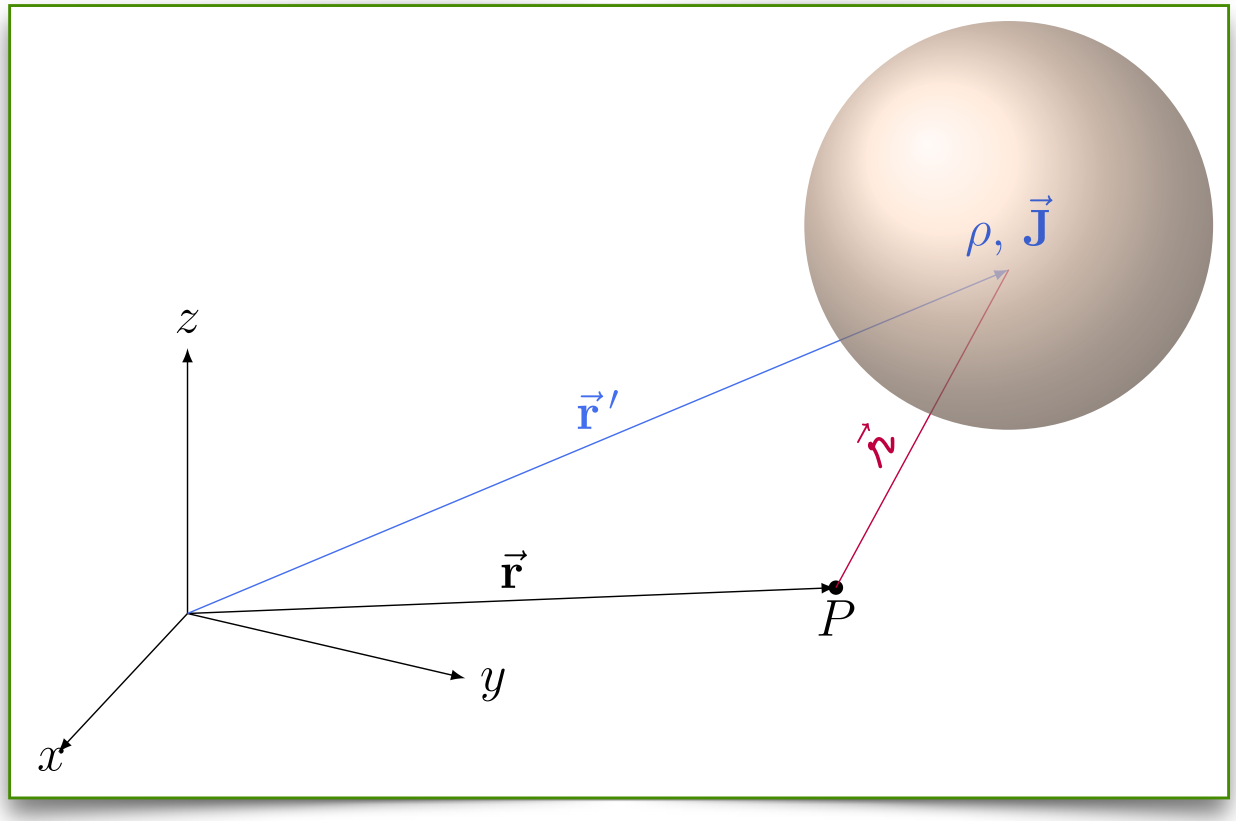

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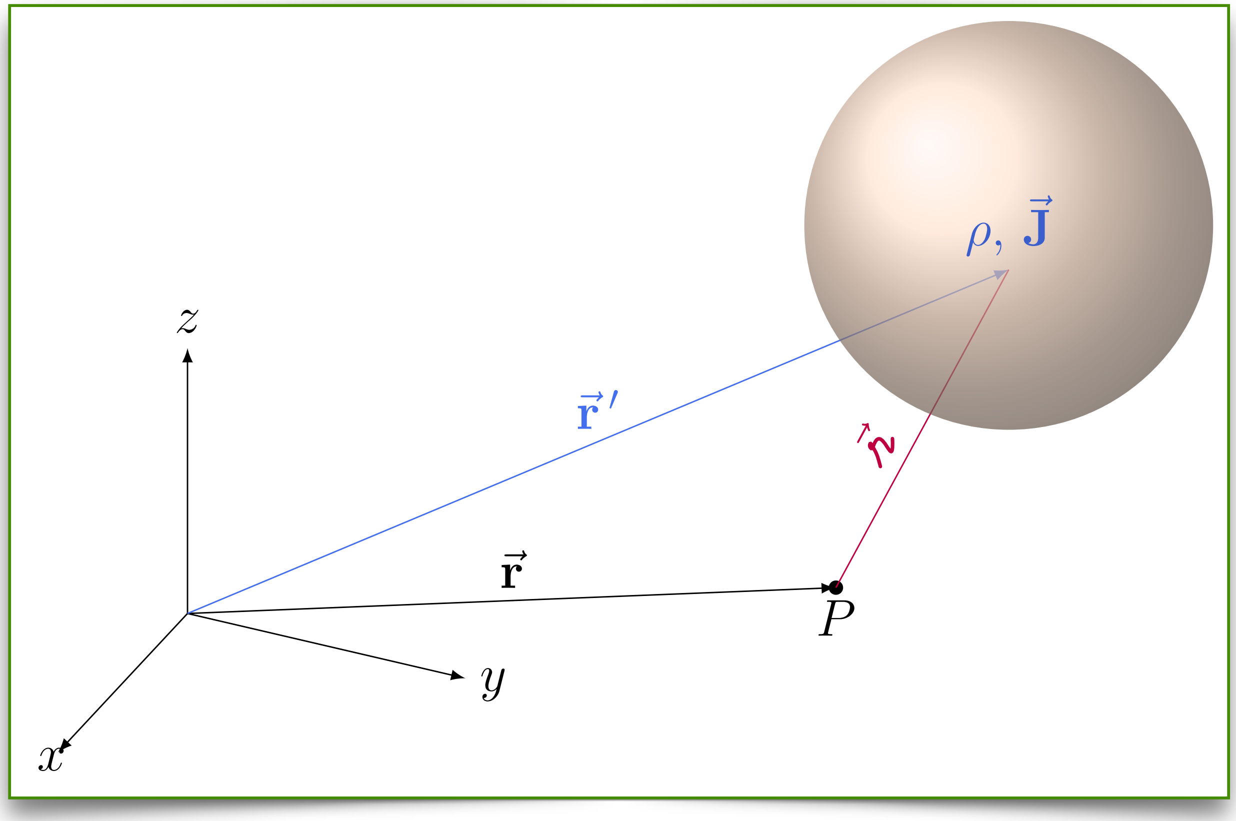

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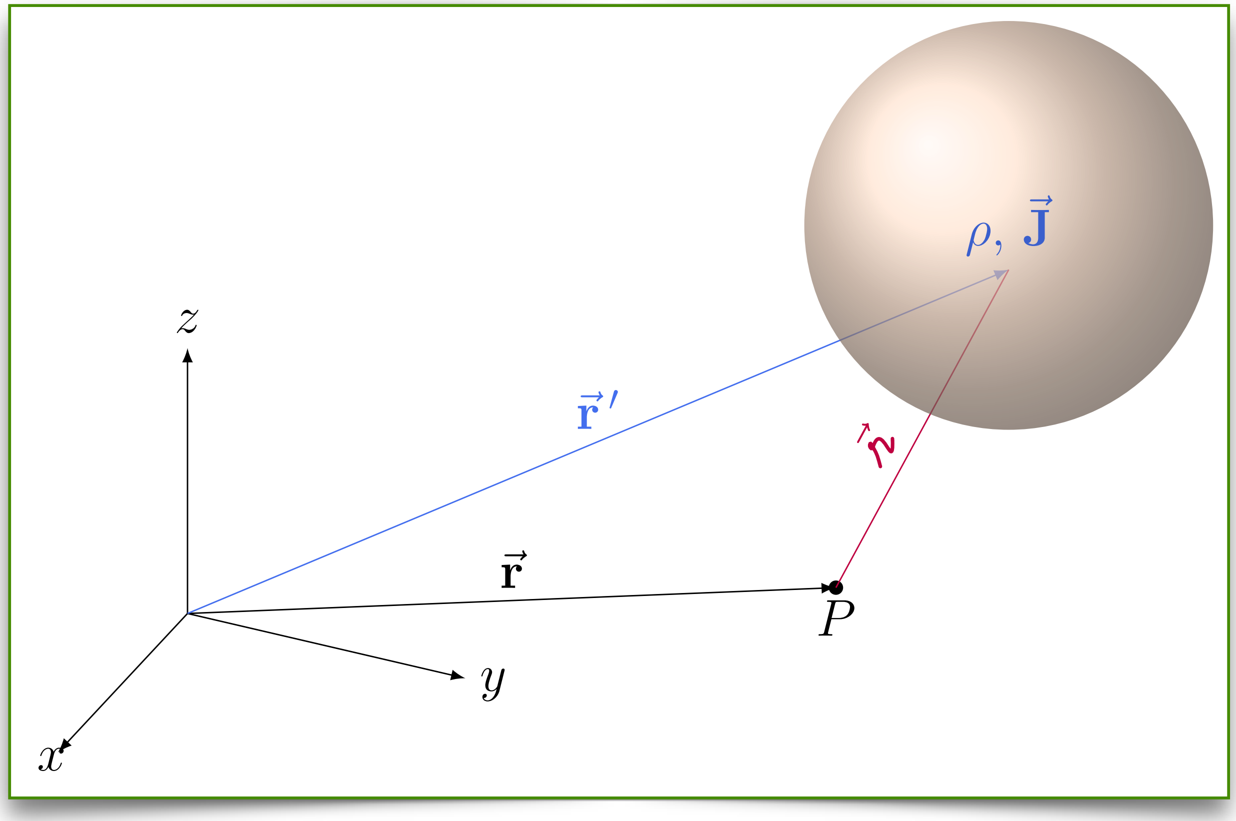

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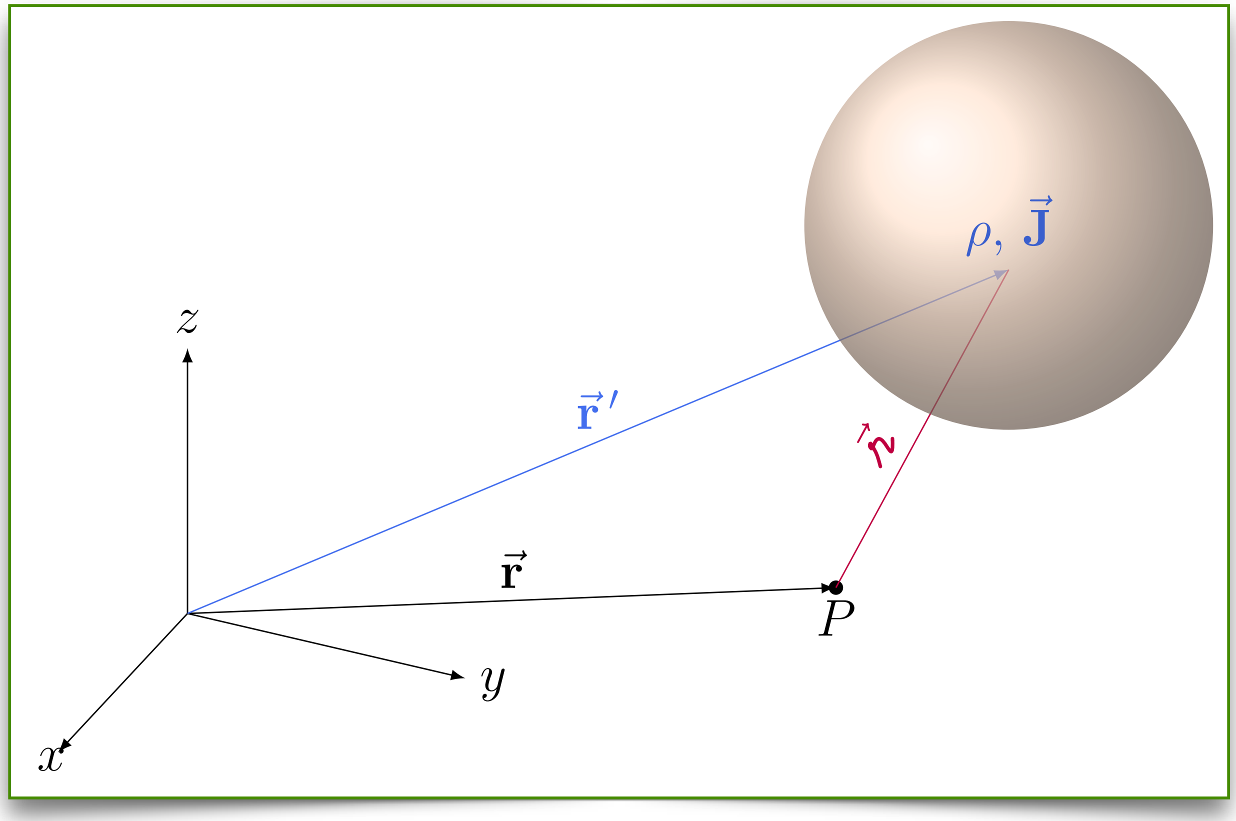

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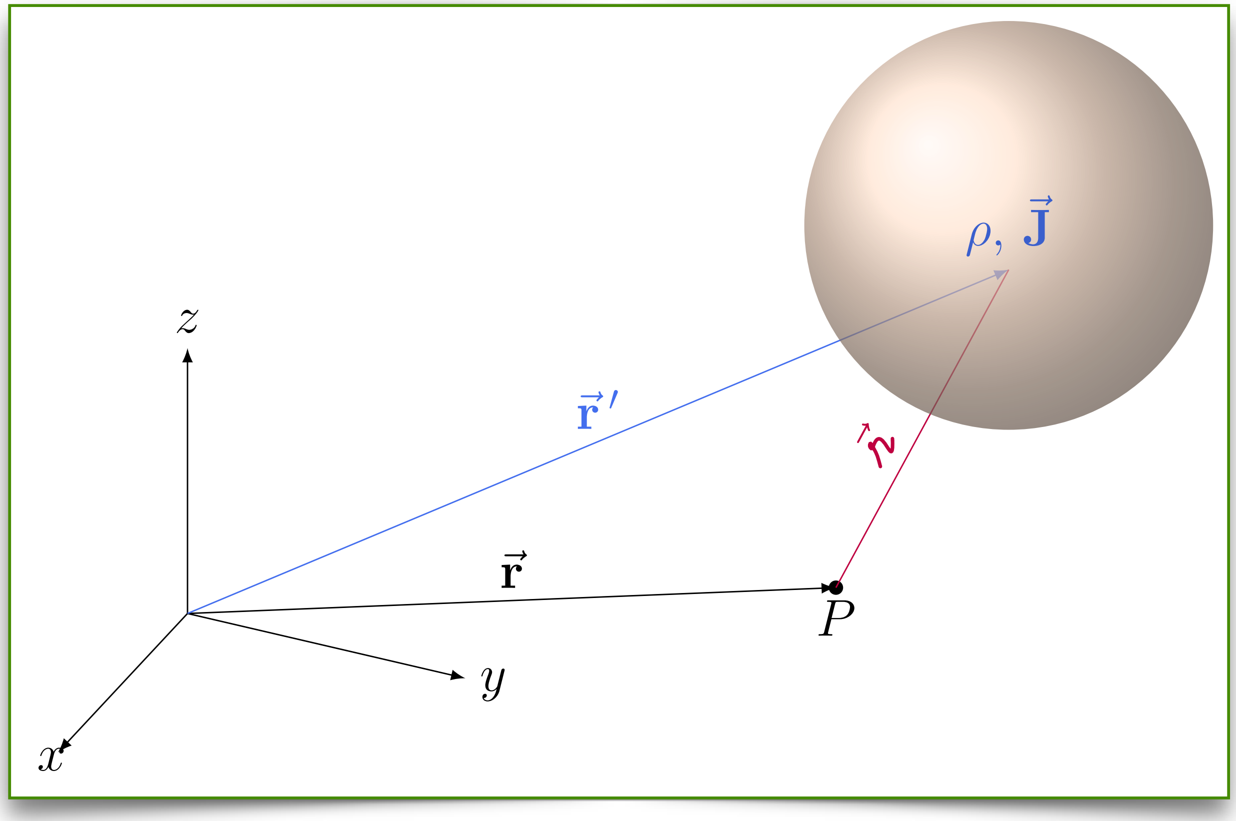

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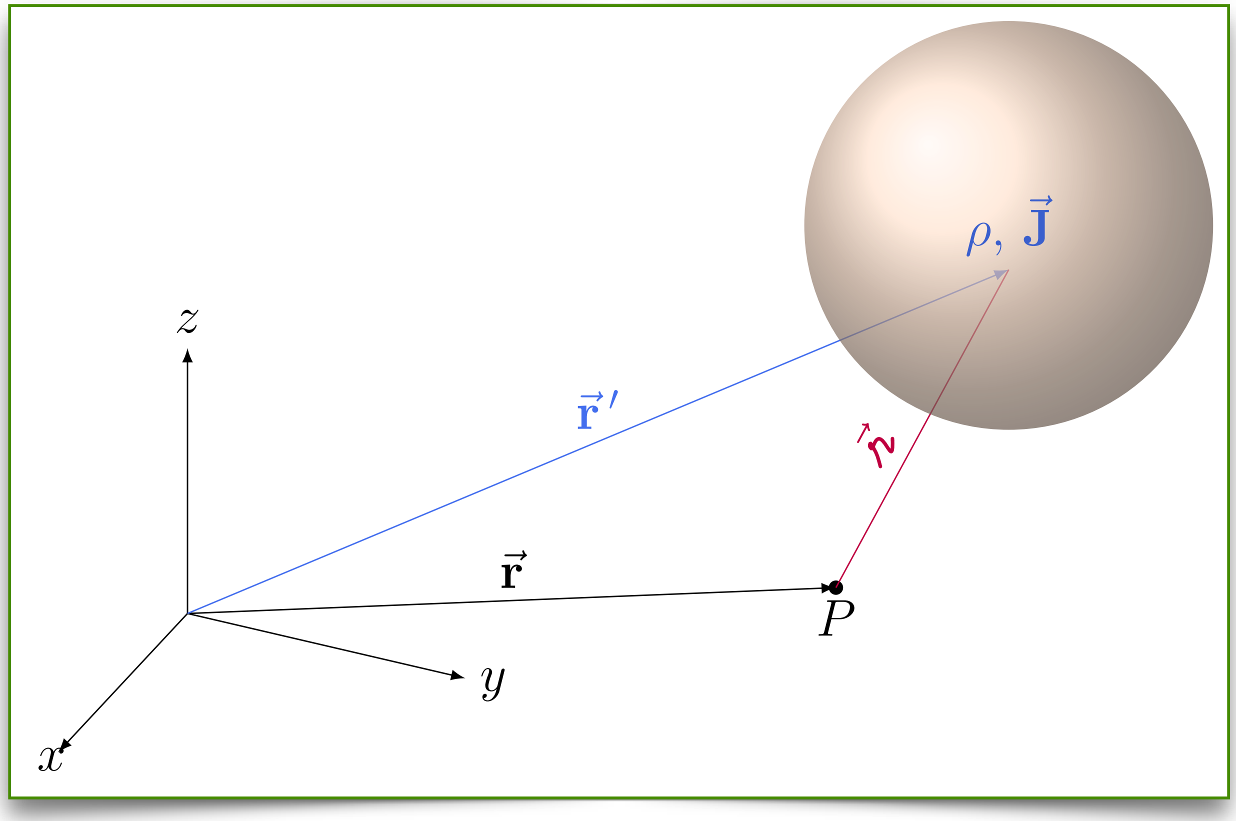

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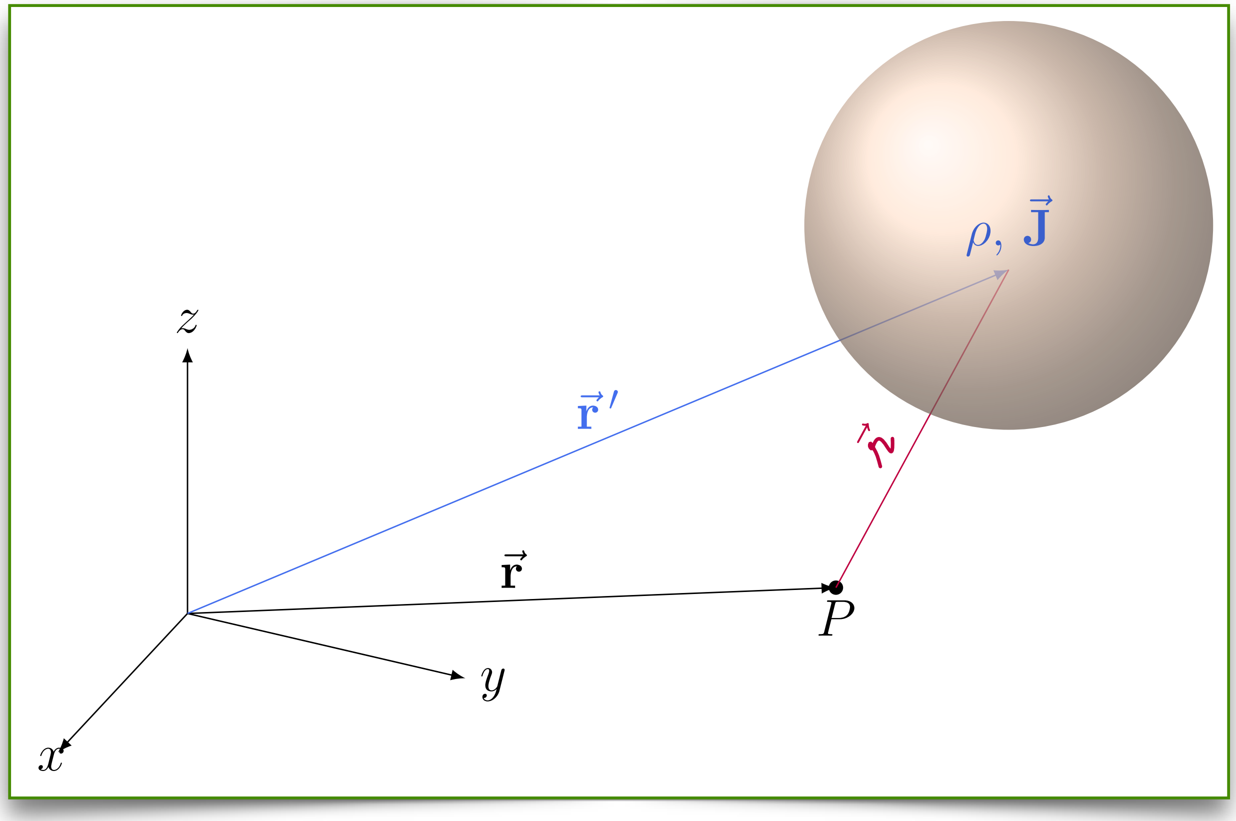

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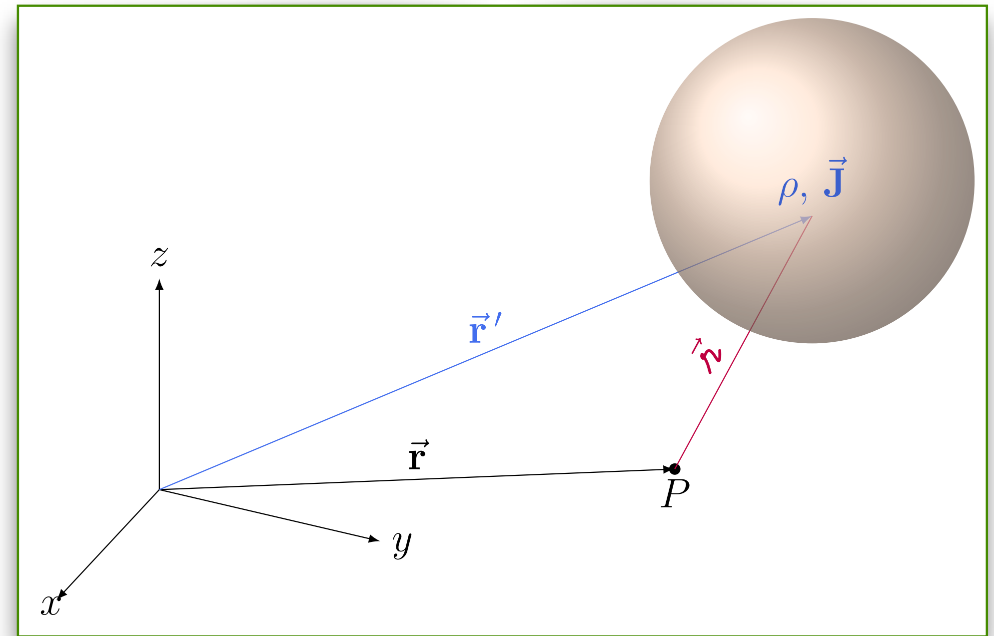

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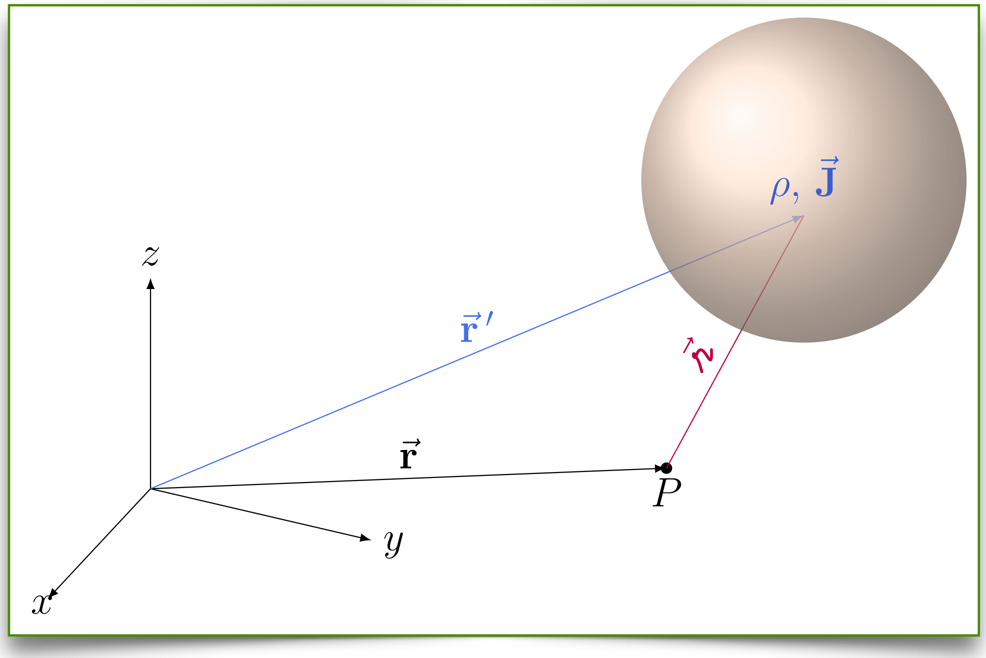

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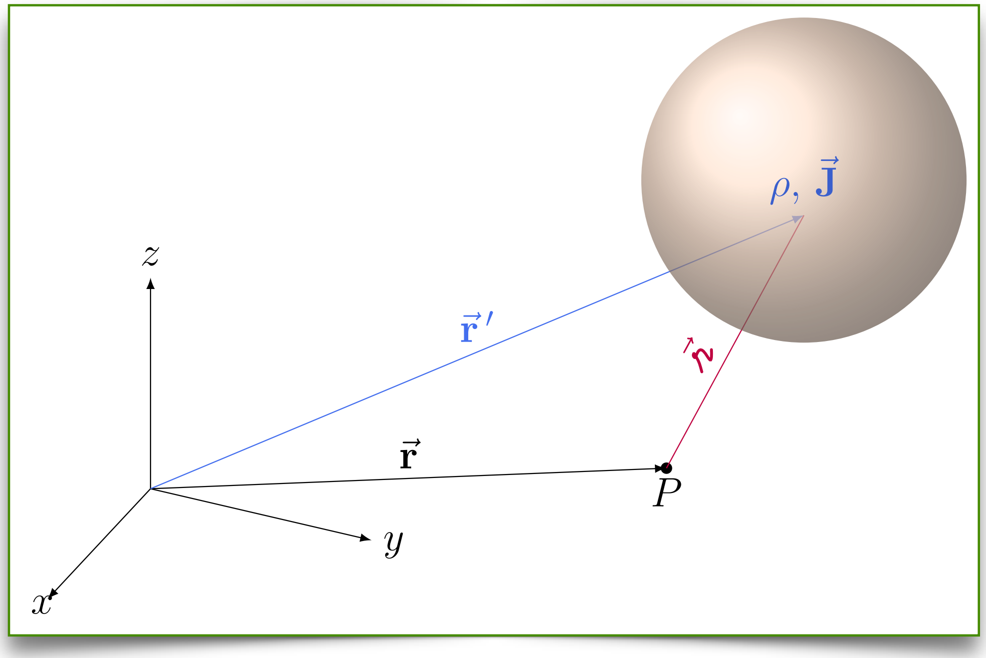

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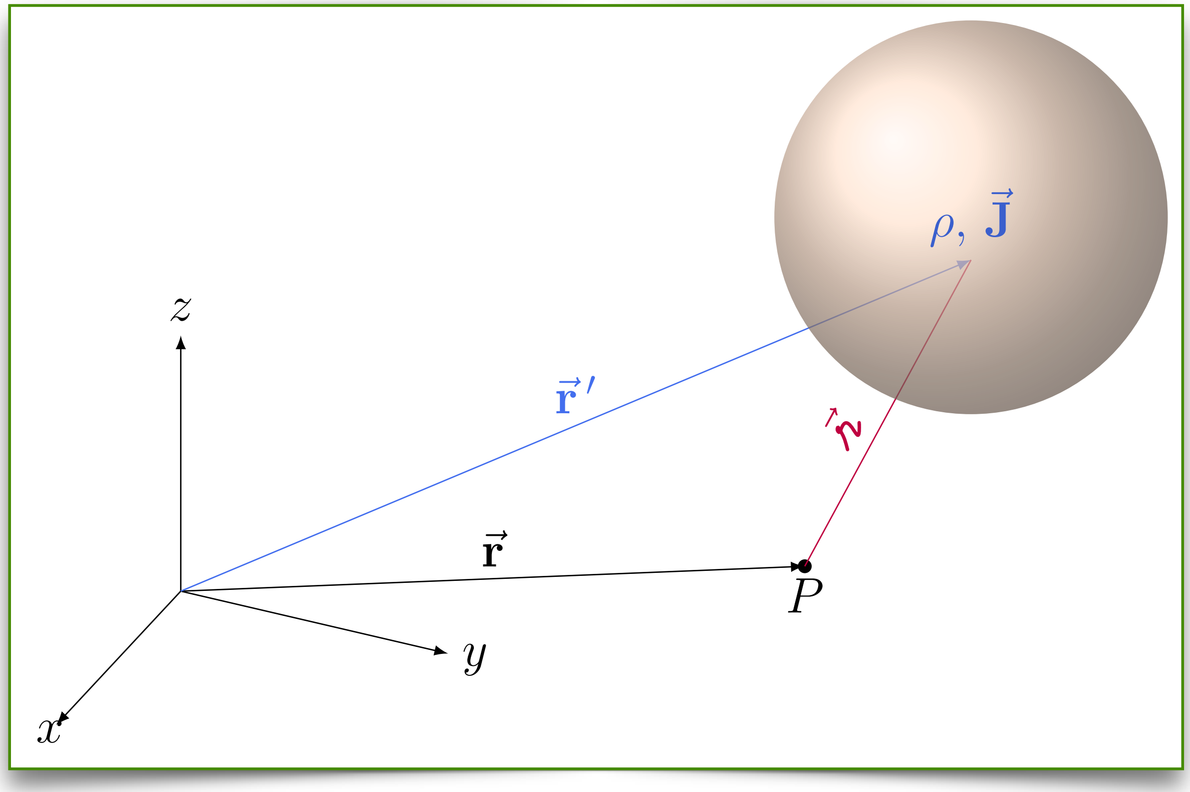

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}\,', t_r)}{\mathscr{r}} \, \mathrm{d}\tau'$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi r} \int \vec{\mathbf{J}}(\vec{\mathbf{r}}\,', t_r) \, \mathrm{d}\tau'$$

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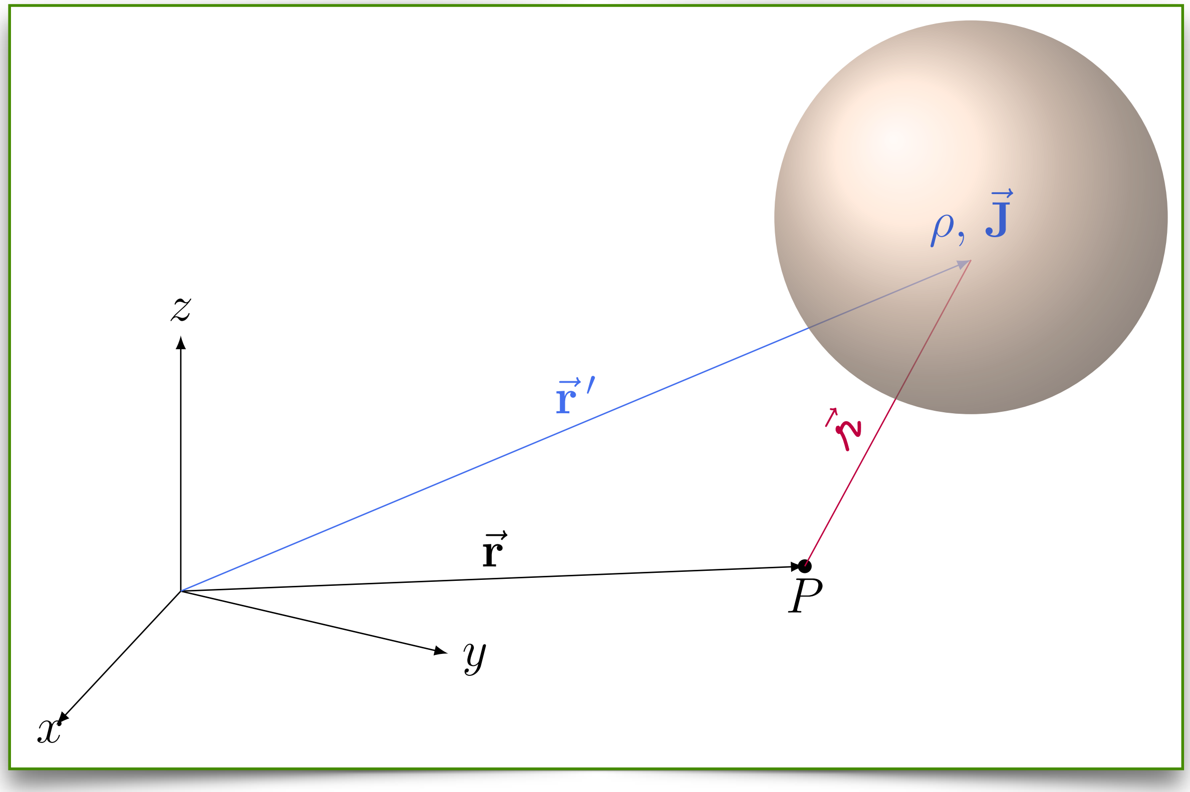

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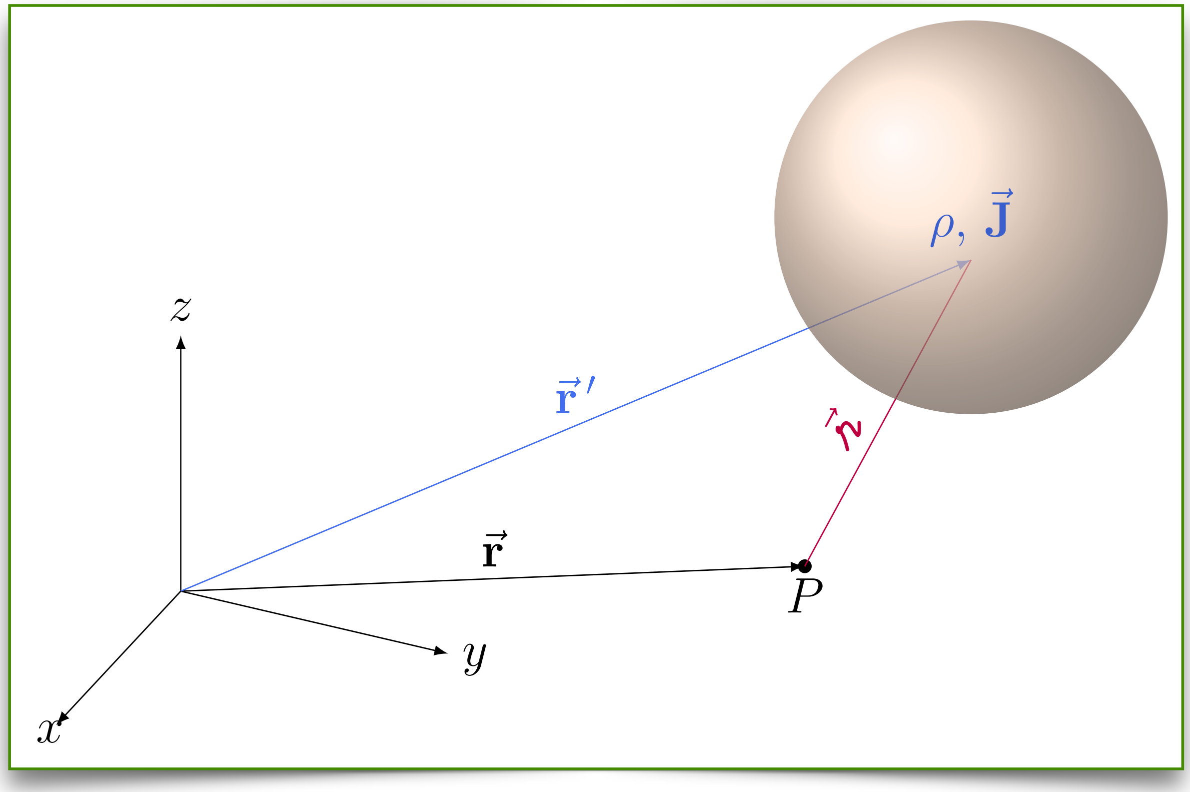

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$$\vec{\mathbf{E}} = -\vec{\nabla}V - \partial_t \vec{\mathbf{A}}$$

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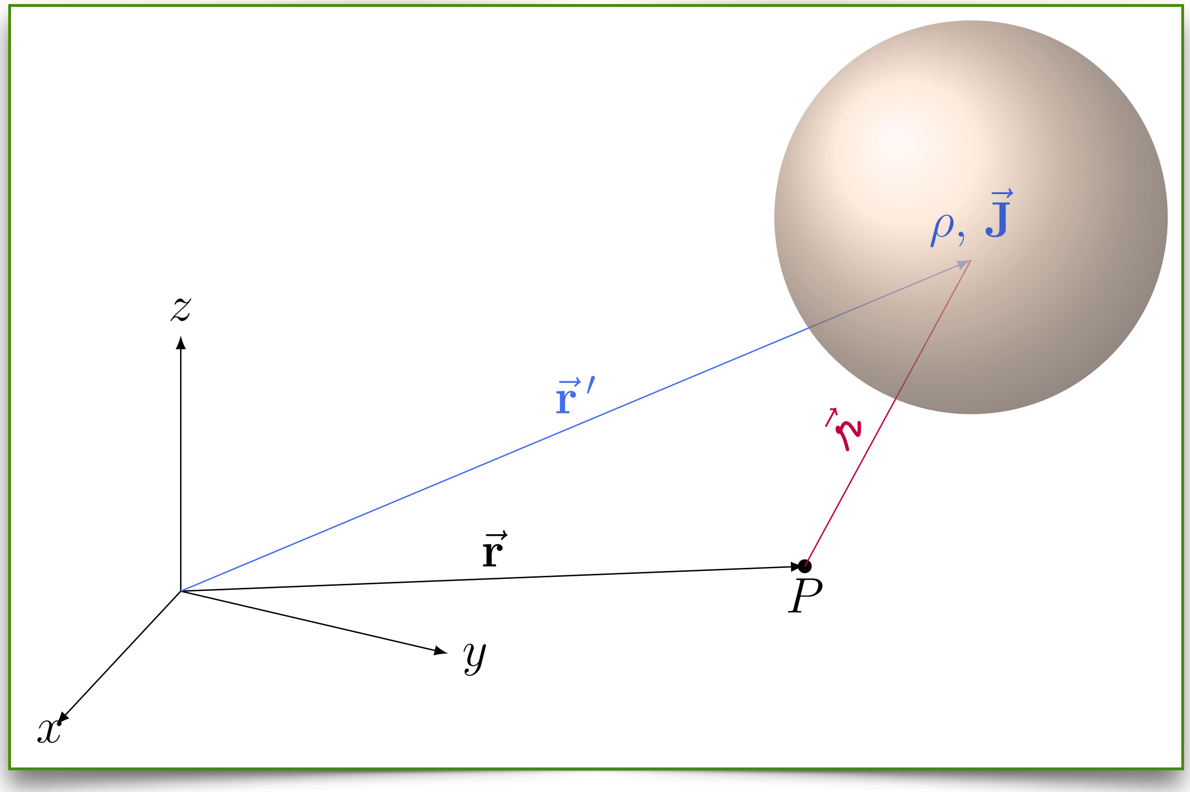

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Desconsiderar termos $\mathcal{O}(1/r^2)$ $\Rightarrow \vec{\nabla}(t - \frac{r}{c}) = -\frac{\hat{r}}{c}$

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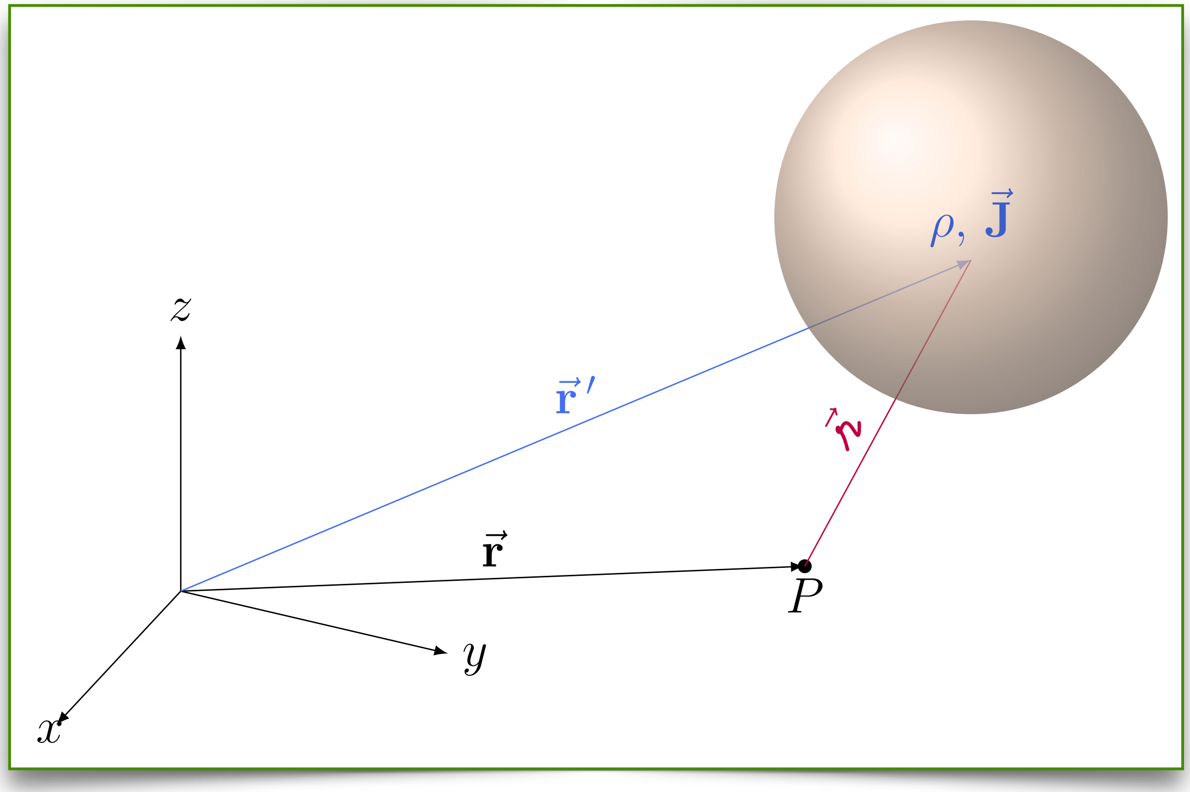
$$\vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_0 c^2}\frac{\hat{\mathbf{r}}\cdot\ddot{\vec{\mathbf{p}}}}{r}\hat{\mathbf{r}} - \frac{\mu_0}{4\pi}\frac{\ddot{\vec{\mathbf{p}}}}{r}$$



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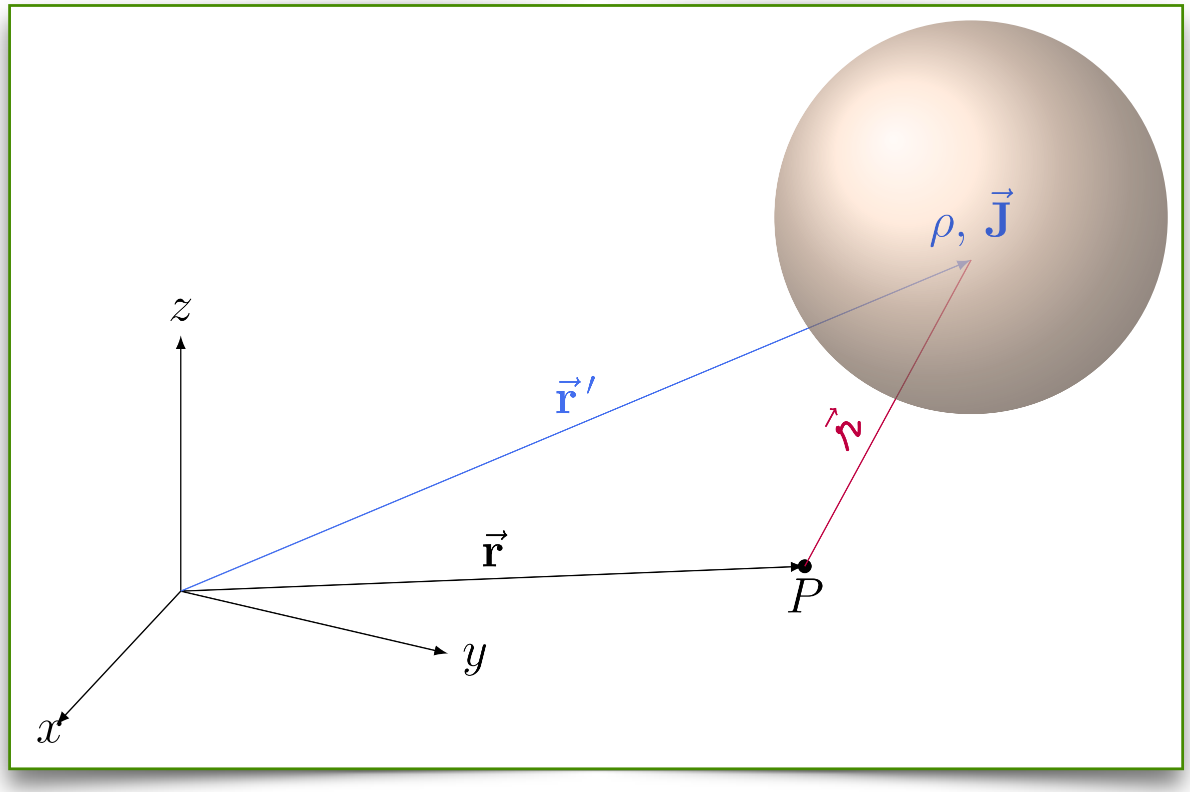
$$\vec{\mathbf{E}} = \frac{\mu_0}{4\pi r}\left((\ddot{\vec{\mathbf{p}}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \ddot{\vec{\mathbf{p}}}\right)$$

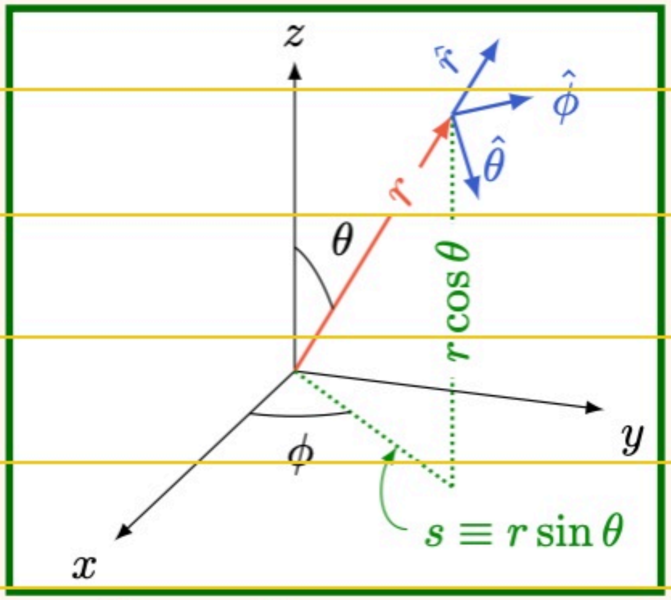
$$\vec{\mathbf{B}} = -\frac{\mu_0}{4\pi r c}(\hat{\mathbf{r}}\times\ddot{\vec{\mathbf{p}}})$$



Campo magnético

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla}t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$