3a prova de Dinâmica Estocástica -3st Exam- 15/12/2023

Instruções para a realização da avaliação: A prova deverá ser enviada até 19:00h de 15/01/2023 para os emails fiorecarlos.cf@gmail.com e iagomamede@usp.br . Ela pode ser realizada com a consulta de livros e/ou materiais, porém as respostas e interpretações são **individuais**.

Todas as respostas deverão ser devidamente justificadas com base em argumentos físicos e matemáticos adequados. A não justificativa das respostas implicará na não integralidade da nota. Utilizem o idioma que for melhor para você.

Para ajudar na correção e garantir que tudo corra bem, procurem seguir as seguintes recomendações:

- A prova P3 é OPCIONAL. Caso o aluno faça a prova P3, será considerada somente duas das melhores notas;
- Procurem mandar a prova em um único arquivo PDF, pode ser foto ou scan. Sugestão de app: Adobe Scan;
- Verifiquem a ordem das páginas, a orientação delas e a legibilidade do texto;
- mandem para o email do professor e do monitor e confiram o envio.

Boa prova!

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1. Monte Carlo simulations and phase transitions (7,0)

In the original MV, each site i of the lattice can have spins $+1$ or -1 and with probability $1-f$ it tends to align itself with its local neighborhood majority and, with complementary probability f, the majority rule is not followed. The transition rate from σ_i to $-\sigma_i$ is given as follows:

$$w_i(\sigma) = \frac{1}{2}\left\{1 - (1-2f)\sigma_i S(\sum_{j=1} \sigma_j)\right\}, \tag{1}$$

where $\sum_{j=1}^{z} \sigma_j$ are the nearest neighbor sites of the site i and $S(X) = \pm 1$ and 0, according to $X > 0$, < 0 and $X = 0$, respectively. In the case of a square lattice, $z = 4$.

By increasing the misalignment parameter f, a continuous order-disorder phase transition takes place at f_c, irrespective the lattice topology. The order parameter for such phase transition is the magnetization per spin $m = \langle \sigma_i \rangle$.

a) For a square lattice and system sizes $L = 10, 20, 30$ and 40, show curves of the magnetization m versus f, for f ranging from 0.01 to 0.25 in increments of $\Delta f = 0.01$.

b) Plot the behavior of the order parameter variance $\chi = L^2[\langle m^2 \rangle - \langle m \rangle^2]$ for the above system sizes and verify that its maximum becomes sharper as L increases.

c) In order to locate the critical point f_c, evaluate the reduced fourth-order cumulant $U_4 = 1 - \frac{\langle m^4 \rangle}{3\langle m^2 \rangle^2}$ and plot its behavior for the same system size as before. Verify that all curves cross at the vicinity of the critical point.

d) Once the critical point f_c is found, evaluate the critical exponents β/ν and γ/ν and verify that they are the same to the Ising model in a bidimensional (square) lattice.

2. Three state model under a constant driving (3,0)

Consider a three state model placed in contact with a thermal bath of temperature T ($\beta = 1/k_B T$) under the presence of a driving of strength f. In such case, transition rates obey the following rule $W_{ij} = \Gamma e^{-(\epsilon_i - \epsilon_j - \beta f)/2}$ when $\{ij\} = \{21\}, \{32\}$ and $\{13\}$ and $W_{ij} = e^{-(\epsilon_i - \epsilon_j + \beta f)/2}$, otherwise (for $i \neq j$).

a) From the master equation formalism, write down the equations for the time evolution of probability $p_i(t)$, where $\{i\} \in \{1, 2, 3\}$.

b) For $\epsilon_1 = \epsilon_2 = \epsilon_3$, find the steady probability distribution and show that the steady entropy production is given by $\sigma = 2k_B \Gamma \beta f \sinh(\beta f)/2$.

c) Near the equilibrium regime ($\beta f << 1$), show that the entropy can be rewritten as $\sigma = L(\beta f)^2$, where $L \geq 0$ denotes its Onsager coefficient.