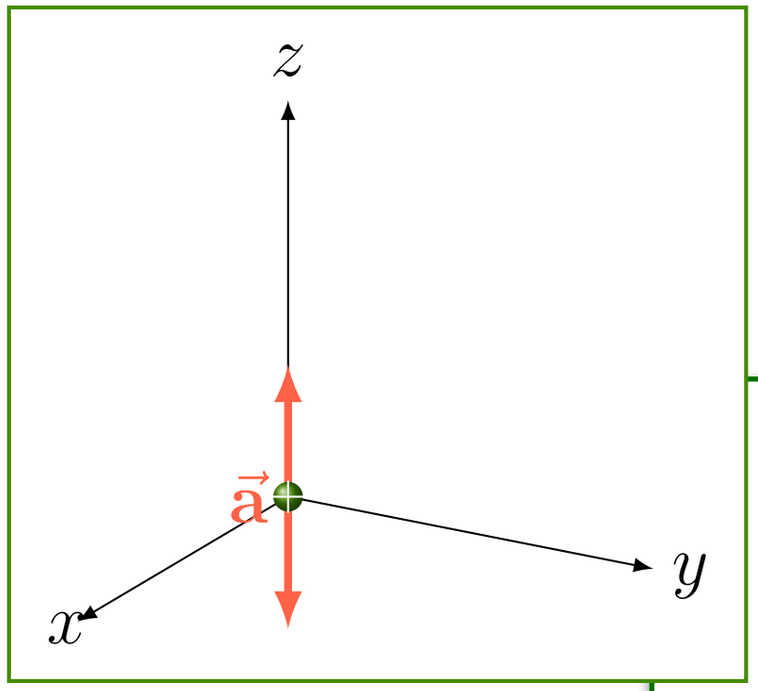
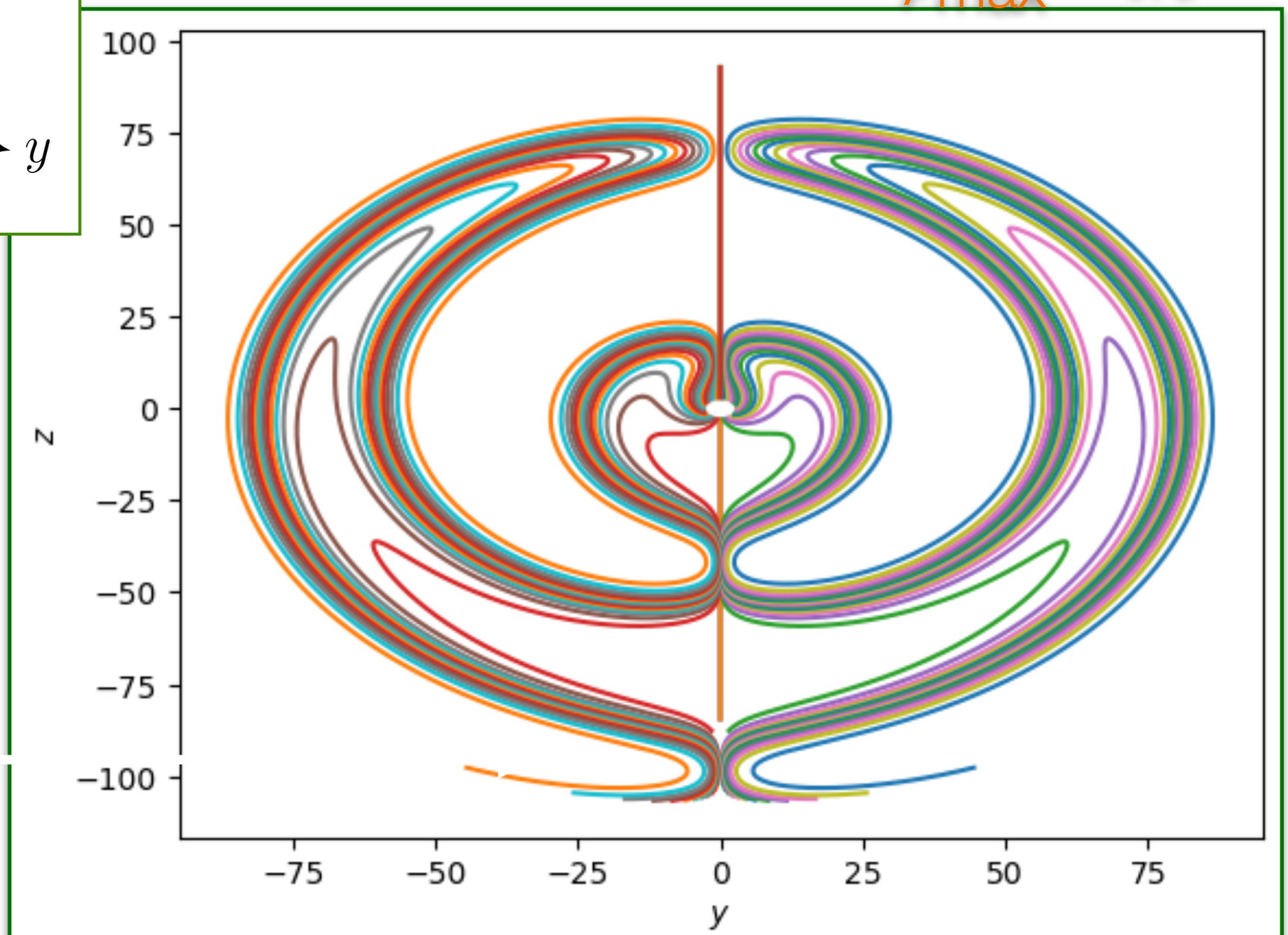
Eletromagnetismo Avançado

4 dezembro
Radiação

Campo elétrico de carga acelerada



$\beta_{max} = 1/3$

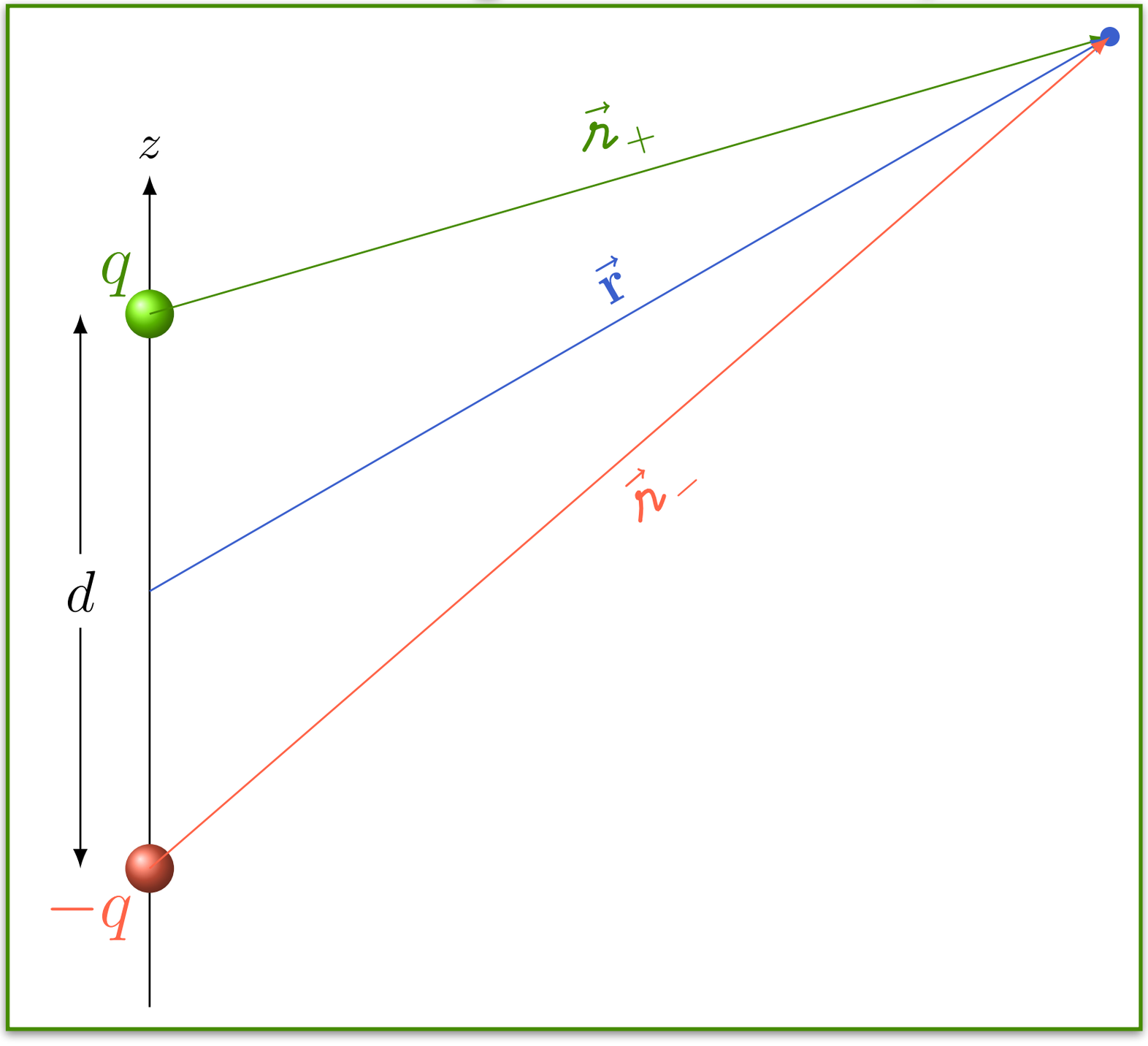


Radiação de dipolo

$$q(t) = q_0 \cos(\omega t)$$

$$r \gg \lambda \gg d$$

$$\lambda = \frac{2\pi c}{\omega}$$



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$$V(\vec{\mathbf{r}}, t) = -\frac{p_0 \omega}{4\pi\epsilon_0 c} \frac{\cos\theta}{r} \sin\omega(t - \frac{r}{c})$$

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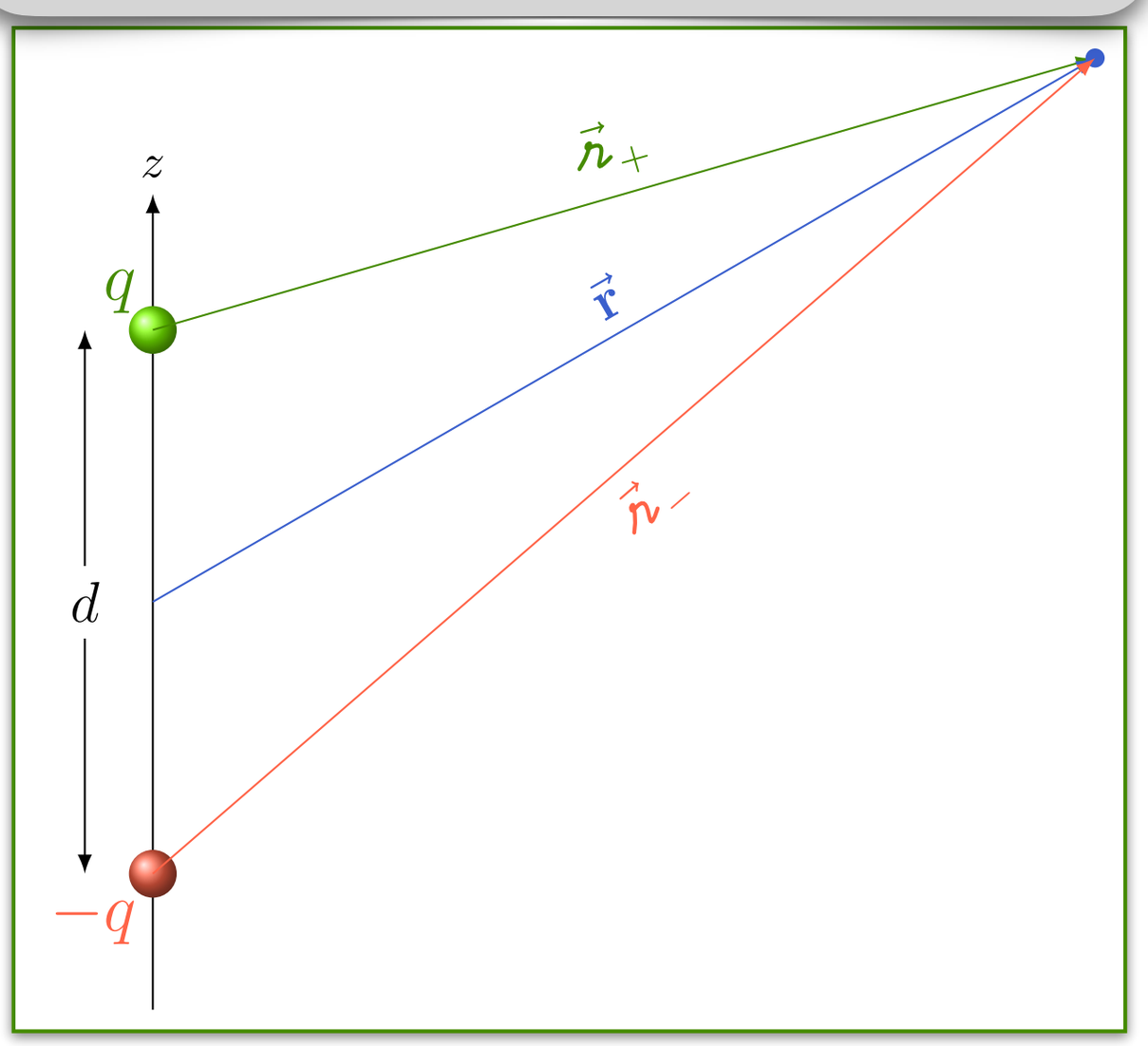

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$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \partial_t \vec{\mathbf{A}}$$

$$\vec{\boldsymbol{\nabla}} V(r,\theta) = \partial_r V + \frac{1}{r}\partial_\theta V$$

$$V(\vec{\mathbf{r}},t) = -\frac{p_0\omega}{4\pi\epsilon_0 c}\frac{\cos\theta}{r}\sin\omega(t-\frac{r}{c})$$

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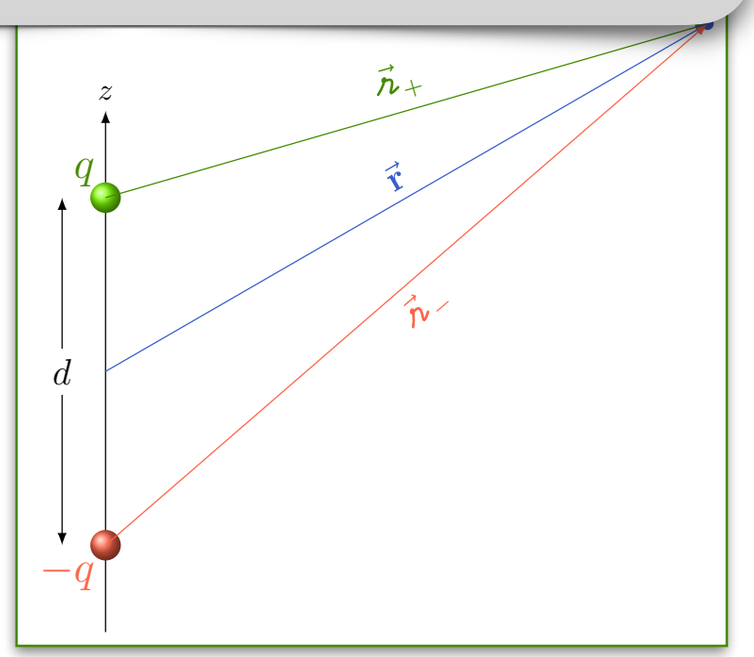

$$\vec{\boldsymbol{\nabla}} V = \frac{p_0\omega}{4\pi\epsilon_0 cr}\cos\theta\left(\frac{1}{r}\sin\omega(t-\frac{r}{c}) + \frac{\omega}{c}\cos\left(t-\frac{r}{c}\right)\right)\hat{\mathbf{r}}$$

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$$\partial_t \vec{\mathbf{A}} = -\frac{p_0 \omega^2}{4\pi\epsilon_0 c^2 r} \cos\omega(t - \frac{r}{c})(\cos\theta\, \hat{\mathbf{r}} - \sin\theta\, \hat{\theta})$$

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Pequenos

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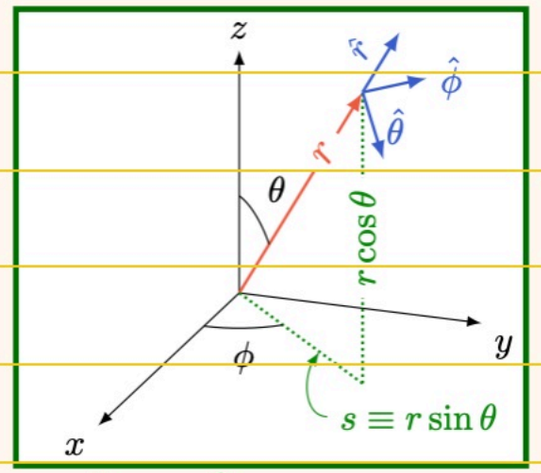
$$\vec{\mathbf{B}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}$$

Radiação de dipolo

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla}t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$



$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

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$$\vec{\mathbf{B}} = \frac{\mu_0 p_0 \omega}{4\pi r}\left(-\frac{\omega}{c}\cos\omega(t - \frac{r}{c})\sin\theta + \frac{1}{r}\sin\theta\right)\hat{\varphi}$$

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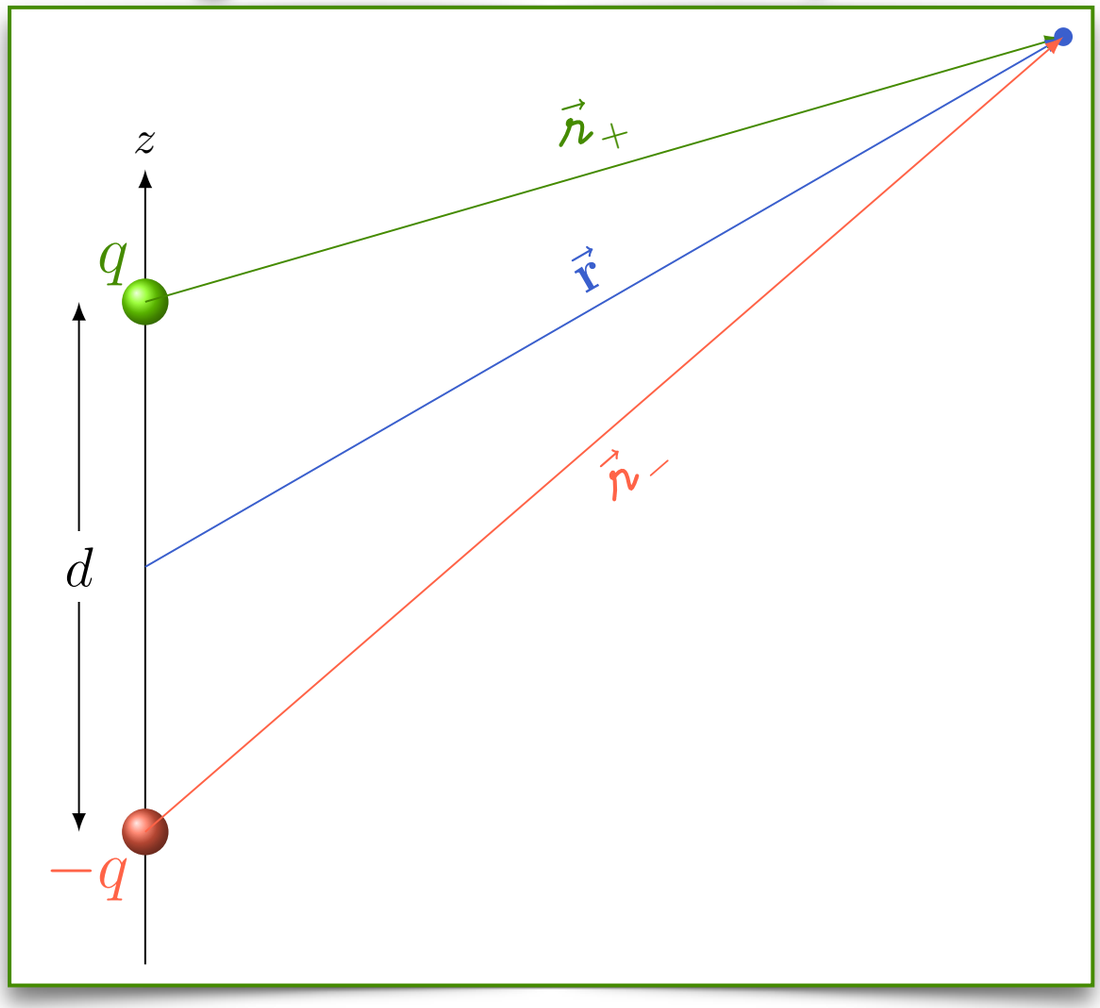

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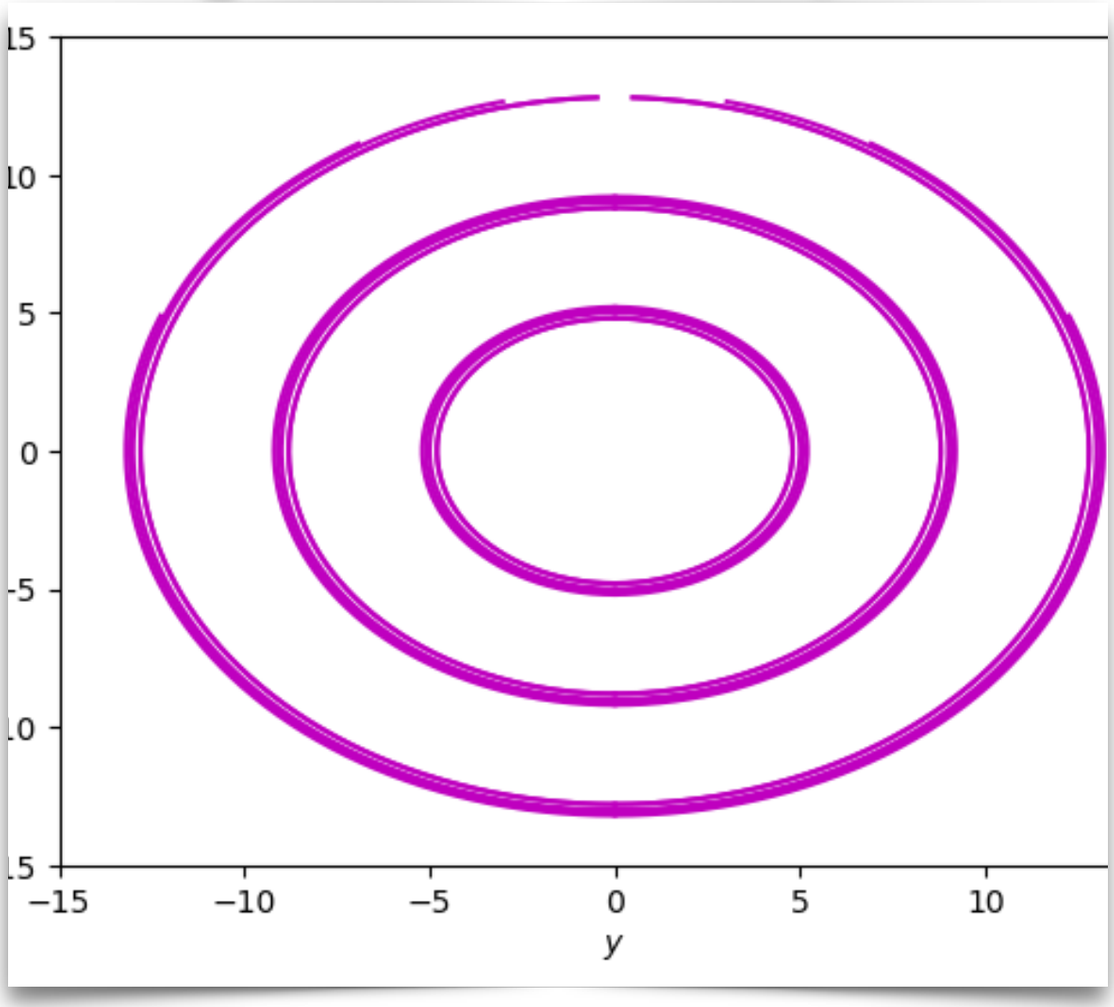

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Pratique o que aprendeu

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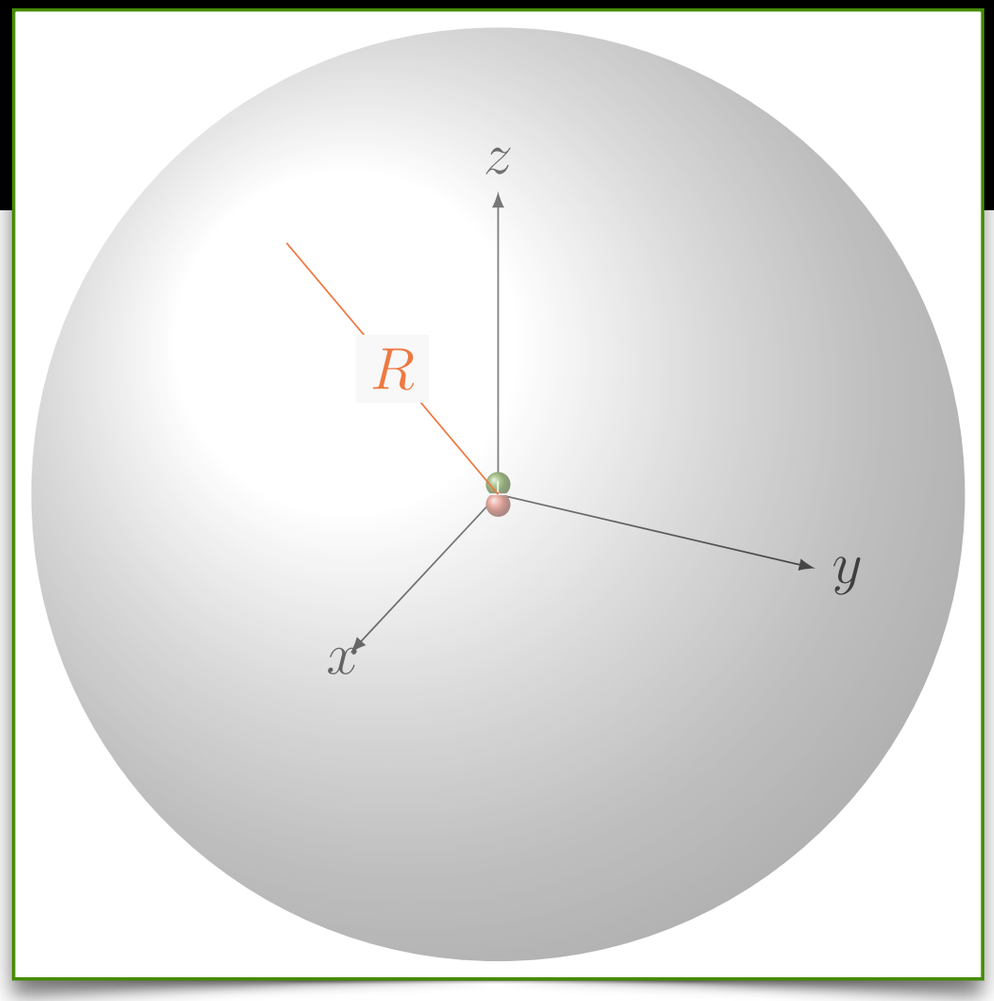

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$$\langle P \rangle = R_{\text{rad}} I^2 \Rightarrow R_{\text{rad}} = ?$$