Distribuição de Velocidades $f(v_x, v_y, v_z)$

$pV = nRT$ (lei dos Gases Ideais)

$p(z) = p_0 \exp\left(-g \frac{\rho_0}{p_0} z\right)$ (lei de Halley)

$E = \frac{m}{2} v^2 + mgh$ (cons. de energia mecânica)

Boltzmann (1876)

Boltzmann: $\frac{p(z)}{p(0)} = \frac{\rho(z)}{\rho(0)} = \exp\left[-g \frac{\rho(0)}{p(0)} \cdot z\right]$

$\rho \to$ dens. do gás (kg/m^3)

$p \to$ pressão ($Pa = N/m^2$)

$p \cdot V = nRT = M \cdot \frac{RT}{M_m}$

$n \to$ moles

$M_m \to$ massa molar, massa de 1 mol

molécula-grama

$M_m = N_0 \cdot m$

$m \to$ massa da molécula

$N_0 \to$ nº de Avogadro

$p = \left(\frac{M}{V}\right) \frac{RT}{M_m} = \rho \frac{R}{N_0} \cdot \frac{T}{m}$

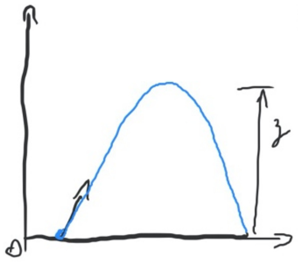
$p = \frac{\rho}{m} k_B T \Rightarrow \frac{\rho}{p} = \frac{\rho_0}{p_0} = \frac{m}{k_B T}$ → massa → Energia $\sim \boxed{\frac{1}{v^2}}$

$$\frac{\rho(z)}{\rho(0)} = \exp\left[-\frac{m}{k_B T} \cdot g z\right] \;; \rho(z) = N(z) \cdot m \quad \Big| \quad \frac{N(z)}{N(0)} = \exp\left(-\frac{m g z}{k_B T}\right)$$

$N(z)$ ↓ dens. molecular

Ignorar colisões

$E_p + E_k = cte$ → Calcular $f(v_z)$



$m g z = \frac{m v_z^2}{2}$ ↓ alt. máxima

$v_z = w$

Para deixar a notação mais leve

Velocidade w, $f_1(w)$; $\int_{-\infty}^{\infty} f_1(w)\, dw = 1$ → distribuição bem comportada

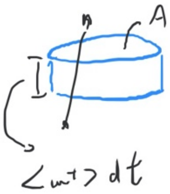
Moléculas ascendentes: $w > 0 \to w^+$

$$\boxed{\langle w^+ \rangle = \int_0^{\infty} f_1(w)\, w\, dw} = -\langle w^- \rangle$$

$$\langle w \rangle = \int_{-\infty}^{\infty} f_1(w)\, w\, dw = \int_{-\infty}^{0} f_1(w)\, w\, dw + \int_0^{\infty} f_1(w)\, w\, dw = 0$$

$$\langle w^- \rangle \quad + \quad \langle w^+ \rangle = 0$$

Número médio de moléculas cruzando uma seção de área 1



Quantas moléculas temos? $N(z)\cdot\underbrace{\langle w^+\rangle dt}_{\text{altura}}\cdot \underset{1}{A}$

$N(z)$ por unid. de tempo

$N(z)\langle w^+\rangle$ $[s^{-1}]$

Média cruzando a área A

Trabalhando por classe de velocidade

$n(z)\,\underbrace{f_1(w)dw}_{\text{Fração de moléculas}}\cdot w$

N° de moléculas, em função da velocidade

$z=0$ $N(0)\cdot \underbrace{f_1(w)\,dw}_{\text{transformação}}\cdot w$

$mgz=\frac{mw^2}{2}$ $mg\,dz=d\left(\frac{1}{2}mw^2\right)=mw\,dw$

Variação de densidade molecular

$[N(z)-N(z+dz)]\cdot\langle w^+\rangle=n_0\ f_1(w)\,dw\cdot w$

Dif. do fluxo de moléculas n° de moléculas que atravessam 1, mas não cruzam a região 2

$$N(z) - N(z+dz) = -\frac{\partial N(z)}{\partial z} \cdot dz$$

$$-\frac{\partial}{\partial z} N(z) \cdot \langle w^{-1} \rangle = N(0)\, f_1(w)\, w \frac{dw}{dz}$$

$$\frac{w}{g}$$

$$-\frac{\partial}{\partial z} N(z) = \frac{N_0}{\langle w^{-1} \rangle} \cdot f_1(w) \cdot g$$

↳ independ. de z

$$g\, dz = w\, dw$$

$$\langle w^2 \rangle = \int_{-\infty}^{\infty} \underbrace{f(w)}_{\text{distribuição}} w^2\, dw$$

$\to \propto T \to \overline{K}$ por molécula

$f(w)$ torna-se independente de z!

Lei de Halley: $n(z) = n_0 \exp\left(-\frac{mg}{k_B T} \cdot z\right)$

$$\frac{\partial}{\partial z} n(z) = -\frac{mg}{k_B T} \cdot n_0 \exp\left(-\frac{mg}{k_B T} \cdot z\right)$$

$$\frac{d}{dz} n = -\frac{mg}{kT}\, n(0) \exp\left(-\frac{mgz}{kT}\right) \quad / \quad \frac{d}{dz} n = -n(0) \frac{g f_1(w)}{\langle w^{-1} \rangle}$$

Halley

$$\frac{f_1(w)}{\langle w^{-1} \rangle} = \frac{m}{kT} \exp\left(-\frac{mgz}{kT}\right) \quad \to \quad mgz = \frac{m w^2}{2}$$

$$\boxed{f_1(w) = \frac{m \langle w^{-1} \rangle}{kT} \exp\left(-\frac{1}{2} \frac{m w^2}{kT}\right)}$$

Calcular $\langle w^{-1} \rangle$

$\to$ Normalização de $f_1(w)$...

$$\int_{-\infty}^{\infty} f_1(u)\,du = 1$$

$$\int_{-\infty}^{\infty} \frac{m\langle u^1\rangle}{kT}\exp\left(-\frac{1}{2}\frac{mu^2}{kT}\right)du$$

$\to x^2 = \dfrac{mu^2}{2kT}$

$x = \sqrt{\dfrac{m}{2kT}}\cdot u$

$dx = \sqrt{\dfrac{m}{2kT}}\,du$

$$= \frac{m\langle u^1\rangle}{kT}\cdot\sqrt{\frac{2kT}{m}}\int_{-\infty}^{\infty}\exp(-x^2)\,dx$$

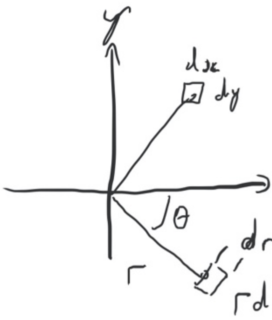
$$= \sqrt{\frac{2m}{kT}}\cdot\langle u^1\rangle\cdot\int_{-\infty}^{\infty}e^{-x^2}dx$$

$\to$ software

tabela de integração

Calculando explicitamente

$$\int_{-\infty}^{\infty} e^{-x^2}dx = A \Rightarrow A^2 = \int_{-\infty}^{\infty} e^{-x^2}dx \cdot \int_{-\infty}^{\infty} e^{-y^2}dy$$



$r^2 = x^2+y^2$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\,dy$$

$$= \int_0^{2\pi}\left[\int_0^{\infty} e^{-r^2}\,r\,dr\right]d\theta = 2\pi\int_0^{\infty} e^{-r^2}r\,dr$$

$$\frac{d}{dr}e^{-r^2} = -2re^{-r^2}$$

$$= -\pi\left[e^{-r^2}\right]_0^{\infty}$$

$$= \pi$$

$$\therefore \boxed{A = \sqrt{\pi}}$$

Portanto $\sqrt{\frac{2m}{kT}} \cdot \langle u^+ \rangle \cdot \sqrt{\pi} = 1 \Rightarrow \langle u^+ \rangle = \sqrt{\frac{kT}{2m\pi}}$

$$\boxed{f_1(u) = \sqrt{\frac{m}{2\pi k_B T}} \exp\left(-\frac{1}{2}\frac{m u^2}{k_B T}\right)}$$

→ Gaussiana

$$\boxed{e^{i\pi} + 1 = 0}$$

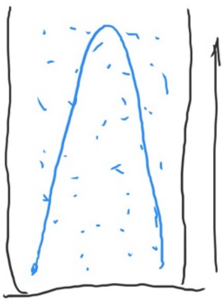
$$\begin{cases} pV = nRT \\ p(z) = p_0 e^{-\left(g \frac{\rho_0}{p_0} z\right)} \\ m g z = \frac{m w^2}{2} \end{cases}$$

alt. máx

↳ comp. z da velocidade em $z=0$

Tratamento de Boltzmann → Gaussiana

Distr. de Maxwell →



v_z, v_x, v_y → distribuição isotrópica em $\vec{v}$

g Foi instrumento de cálculo

z → mudança na densidade

Hipótese → sujeito à confirmação experimental

$$\underbrace{f(v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z}_{\text{probabilidade } \in (\vec{v},\, \vec{v}+d\vec{v})} = f_1(v_x)\, dv_x \cdot f_1(v_y)\, dv_y\, f_1(v_z)\, dv_z$$

$$f(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot \exp\left(-\frac{1}{2}\frac{m(v_x^2+v_y^2+v_z^2)}{k_B T}\right)$$

$$= \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot \exp\left[-\frac{1}{2}\frac{m v^2}{k_B T}\right]$$

$v^2 = v_x^2 + v_y^2 + v_z^2$

$f_1(v_x)\,dv_x \rightarrow$ Integração marginal

$$\underbrace{f_1(v_x)\,dv_x}_{\text{prob. 1D}} = dv_x \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{f(\vec{v})\,dv_y\,dv_z}_{\text{integração nas outras 2 dim.}}$$

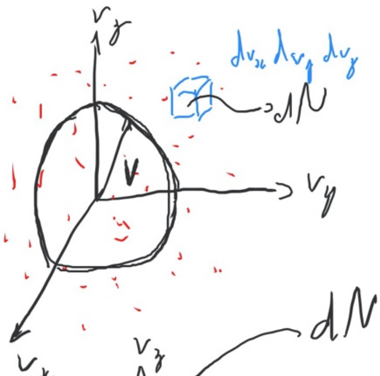
$\frac{mv^2}{2} = K$ en. cinética

$$f(\vec{v}) = \sqrt[3]{\frac{m}{2\pi k_B T}} \cdot \exp\left(-\frac{K}{k_B T}\right)$$

↳ distr. de Maxwell Boltzmann

energia → relacionada com temperatura

Espaço de velocidades



Fração de Moléculas

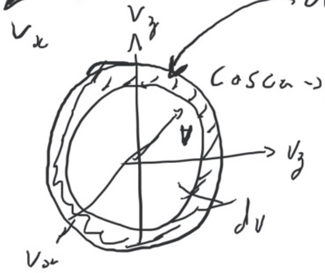
$$\frac{dN(v_x,v_y,v_z)}{N} = \underbrace{f(\vec v)}_{f(v)}\ \underbrace{dv_x\, dv_y\, dv_z}_{\hookrightarrow 3\ \text{coord}}$$

$$dN(v_x,v_y,v_z) = N\cdot F(v)\,dv = N f(\vec v)\, \overbrace{4\pi v^2 dv}$$

distr. do módulo da velocidade

1 dim

$$= N\cdot 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot \exp\left(\frac{-mv^2}{2k_B T}\right) v^2 dv$$



casca → volume $4\pi v^2 \cdot dv$

$$F(v) = 4\pi \cdot v^2 f(\vec v)$$

$$= 4\pi\, v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot \exp\left(-\frac{mv^2}{2 k_B T}\right)$$

$$\int_0^\infty F(v)\,dv = 1$$

v → valor absoluto da velocidade

v_{qm}^2, v mais provável, $\langle |v| \rangle$

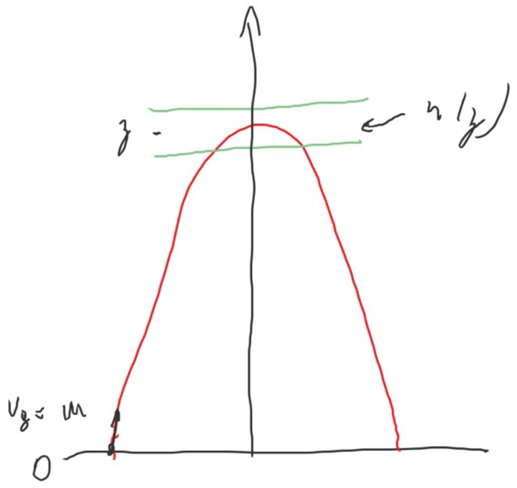
Distr. de Maxwell

Lei de Halley

$n(z) = n(0)\,\exp\left(-\frac{mgz}{kT}\right)$

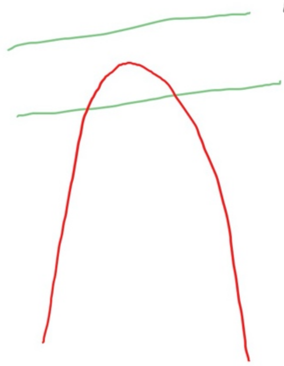
↓

dens. atômica



$\frac{mu^2}{2} = mg.z$

Boltzman



moléculas que atingem z e não saem por $z+dz$

$[n(z) - n(z+dz)]\langle w^+\rangle = n_0 f_1(u)\,du\,u$

$\uparrow \Delta z$ moléculas que atingem a ascensão máxima em z

$\uparrow \Delta z$ moléculas que partiram de $z=0$ com velocidade u

$\lim_{\Delta z \to 0} \Rightarrow -\frac{\partial n}{\partial z}(z).\langle w^+\rangle = n(0)\,f_1(u)\frac{du}{dz}\,u$

$$\frac{m}{2} u^2 = m g z$$

$$\frac{d}{dz} n(z) \langle u_1 \rangle = n_0 f_1(u) \cdot g$$

$$\frac{1}{2} \frac{d}{dz} (u^2) = \frac{2}{2} u \frac{du}{dz} = g$$

u

$$\frac{dn(z)}{dz} = -\frac{mg}{kT} \cdot n_0 \exp\left(-\frac{mg}{kT} z\right)$$

$$f_1(u) = \frac{m \langle u_1 \rangle}{kT} \exp\left(-\frac{1}{2} \frac{m u^2}{kT}\right)$$

↳ gaussiana

$$\int_{-\infty}^{\infty} f_1(u)\, du = 1$$

$$f_1(u) = \sqrt{\frac{m}{2\pi k T}} \exp\left(-\frac{1}{2} \frac{m u^2}{kT}\right)$$

$$f(v_x, v_y, v_z) = \left(\frac{m}{2\pi k T}\right)^{3/2} \cdot \exp\left[-\frac{1}{2} \frac{m(v_x^2 + v_y^2 + v_z^2)}{kT}\right]$$

↳ densidade de probabilidade em $\vec{v}$

$$\int_{-\infty}^{\infty} f(\vec{v})\, dv_x\, dv_y\, dv_z = 1$$ → isotropia

$$F(v)$$

$$\int_0^{\infty} F(v)\,dv = 0 \qquad v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$F(v) = 4\pi \cdot \left(\frac{m}{2\pi k T}\right)^{1/2} \cdot \frac{1}{\pi} \cdot \left(\frac{m v^2}{2kT}\right) \cdot \exp\left(-\frac{m v^2}{2kT}\right)$$

$$\boxed{= 4 \cdot \left(\frac{m}{2\pi k T}\right)^{1/2} \cdot \left(\frac{m v^2}{2kT}\right) \exp\left(-\frac{m v^2}{2kT}\right)}$$

$$\frac{m v^2}{2kT} = x^2 \qquad \underline{F(x) \propto x^2 e^{-x^2}}$$

<u>Calcular</u> velocidades característicos → Distr. Maxwell-Boltzmann

$$V_{qm}^2 = \langle v^2 \rangle = \int_0^{\infty} v^2 F(v)\,dv$$

$$F(v) = \frac{4\pi}{\pi^{3/2}} \cdot \left(\frac{m}{2kT}\right)^{3/2} v^2 \exp\left(-\frac{m v^2}{2kT}\right) = \frac{4}{\sqrt{\pi}} \cdot \lambda^{3/2} v^2 \exp(-v^2 \lambda)$$

$$\lambda = m/2kT$$

Normalização

$$\int_0^{\infty} F(v)\,dv = \frac{4}{\sqrt{\pi}} \lambda^{3/2} \int_0^{\infty} v^2 e^{-\lambda v^2}\,dv = 1$$

O pulo do gato $\quad \frac{d}{d\lambda}\left(\lambda^{3/2}\int_0^\infty v^2 \exp^{-\lambda v^2} dv\right) = \frac{d}{d\lambda}(1) = 0$

$$\frac{3}{2}\cdot\lambda^{1/2}\cdot\int_0^\infty v^2 e^{-\lambda v^2} dv + \lambda^{3/2}\cdot\int_0^\infty v^2\cdot\frac{d}{d\lambda}\left(e^{-\lambda v^2}\right) dv = 0$$

$$\frac{3}{2}\lambda^{1/2}\underline{\int_0^\infty v^2 e^{-\lambda v^2} dv} - \lambda^{3/2}\int_0^\infty v^2\cdot v^2 e^{-\lambda v^2} dv = 0$$

$$\boxed{\frac{3}{2}\cancel{\lambda^{1/2}}\cdot\frac{\sqrt{\pi}}{4}\cdot\frac{1}{\lambda^{\cancel{3/2}}} = \lambda^{3/2}\int_0^\infty v^4 e^{-\lambda v^2} dv}$$

$$v_{qm}^2 = \int_0^\infty v^2 F(v) dv = \frac{4}{\sqrt{\pi}}\cdot\int_0^\infty \lambda^{3/2} v^4 e^{-\lambda v^2} dv$$

$$= \frac{4}{\sqrt{\pi}}\cdot\frac{3}{2}\frac{\sqrt{\pi}}{4}\cdot\frac{1}{\lambda} = \frac{3}{2}\cdot\frac{1}{\lambda} = \frac{3}{2}\cdot\frac{2kT}{m}$$

$$\therefore\ v_{qm} = \sqrt{\frac{3kT}{m}}$$

$\rightarrow$ equipartição de energia $\quad \frac{1}{2} m v_{qm}^2 = 3\,\frac{kT}{2}$

teoria cinética dos gases

$\frac{1}{2} m v_{qm}^2$ ↓ en. cinética total

$3\,\frac{kT}{2}$ ↓ 3. en. de 1 grau de liberdade

$$V_{qm}^2 = \int_{-\infty}^{\infty} (v_x^2 + v_y^2 + v_z^2) \cdot f(v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z =$$

$$= \int_{-\infty}^{\infty} v_x^2 f_1(v_x)\, dv_x \cdot \int_{-\infty}^{\infty} \cancelto{1}{f_1(v_y)\, dv_y} \int_{-\infty}^{\infty} \cancelto{1}{f_1(v_z)\, dv_z}$$

$$+ \int_{-\infty}^{\infty} v_y^2 f_1(v_y)\, dv_y \int_{-\infty}^{\infty} \cancelto{1}{f_1(v_x)\, dv_x} \int_{-\infty}^{\infty} \cancelto{1}{f_1(v_z)\, dv_z}$$

$$+ \int_{-\infty}^{\infty} \underbrace{v_z^2 f_1(v_z)\, dv_z}_{\frac{1}{2} kT} \int_{-\infty}^{\infty} \cancelto{1}{f_1(v_x)\, dv_x} \int_{-\infty}^{\infty} \cancelto{1}{f_1(v_y)\, dv_y} = \frac{3}{2} kT$$

<u>Velocidade média</u> $\langle v \rangle = \int_{-\infty}^{\infty} f(v_x, v_y, v_z) \cdot v\, dv_x\, dv_y\, dv_z = 0$

Para $F(v)$

$$\langle v \rangle = \int_0^{\infty} v \cdot F(v)\, dv = \int_0^{\infty} v \cdot \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} v^2 \exp\left(-\underbrace{\frac{mv^2}{2kT}}\right) dv$$

$$x = \frac{mv^2}{2kT}$$

$$\frac{dx}{dv} = \frac{m \cdot v}{kT}$$

$$= \frac{4}{\sqrt{\pi}} \cdot \int_0^{\infty} \left(\frac{mv^2}{2kT}\right) \cdot \exp\left(-\frac{mv^2}{2kT}\right) \sqrt{\frac{m}{2kT}} \cdot v\, dv$$

$$\frac{4}{\sqrt{\pi}} \cdot \sqrt{\frac{kT}{2m}} \int_0^{\infty} x e^{-x}\, dx$$

$$\int_0^\infty x e^{-x}\,dx = \cancel{x\cdot(-e^{-x})\Big|_0^\infty}^{\,0} - \int_0^\infty 1\cdot e^{-x}\,dx = \int_0^\infty e^{-x}\,dx = -e^{-x}\Big|_0^\infty = 1$$

$$\underset{F}{x}\ \underset{g}{e^{-x}} \qquad F \quad G \qquad f \quad G$$

$$\langle v \rangle = \frac{4}{\sqrt{\pi}}\sqrt{\frac{kT}{2m}} = \sqrt{\frac{8}{\pi}\frac{kT}{m}} = \sqrt{\frac{8}{3\pi}}\cdot v_{qm} \approx 0{,}92\, v_{qm}$$

Qual a velocidade mais provável?

$$v_p \;/\; \frac{d}{dv}\left[F(v)\right] = 0 \qquad \frac{d}{dv}F = \frac{d}{dx}\left[\frac{4}{\sqrt{\pi}}\cdot\sqrt{\frac{2kT}{m}}\,x^2 e^{-x^2}\right]\cdot \underset{\neq 0}{\frac{dx}{dv}} = 0$$

$$\frac{d}{dx}\left[x^2 e^{-x^2}\right] = \cancel{2x}\,\cancel{e^{-x^2}} - x^2\cdot \cancel{2x}\,\cancel{e^{-x^2}} = 0$$

$$\rightarrow x = 1 \qquad x = \frac{mv^2}{2kT}$$

$$v_p = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2}{3}}\,v_{qm} \approx 0{,}82\, v_{qm}$$

$$v_p < \langle v \rangle < v_{qm}$$

$\underbrace{F(v)\,dv} \to \quad v_p \Rightarrow \underbrace{F(v_p)\,dv}_{\text{máxima}} \to$ unidade probabilidade

---

$n(z) = n(0)\, e^{\left(-\frac{mgz}{kT}\right)}$ $\quad \to$ $\to V_{cub}$

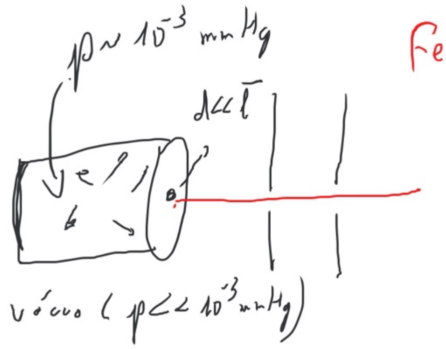
$\exp\left(\frac{-mv^2}{2kT}\right)$

Função geral: $F(x, y, z, v_x, v_y, v_z) = C \exp -\left[\frac{mgz + \frac{mv^2}{2}}{kT}\right]$

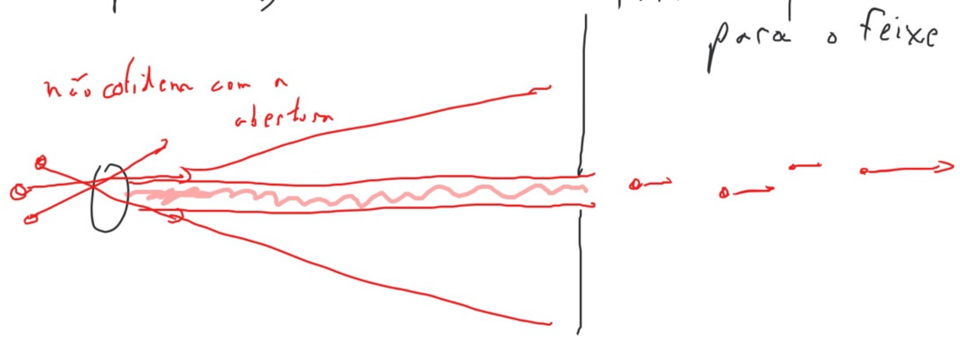
$= C \exp\left(-\frac{E}{\underbrace{kT}_{\to \text{energia}}}\right)$ Distr. de Boltzmann

$C \Rightarrow \int F\, d\vec{r}\, d\vec{v} = 1$

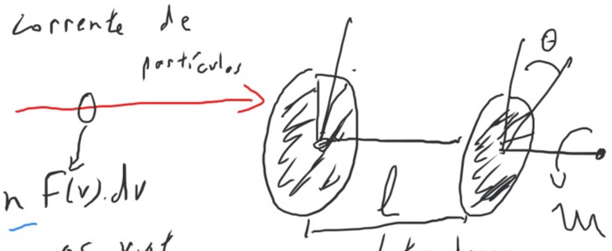
Feixe atômico



filtro espacial para o feixe



corrente de partículas



obturadores

D Contatos

$n\,F(v).dv$

$\Delta S \cdot v.\Delta t$

área — comprimento

nº de partículas

$\theta = \omega.\Delta t$

$\omega.\frac{l}{v} \rightarrow v = \frac{\omega l}{\theta}$

Densidade de corrente. $j(v) \propto F(v).n.v \propto v^3 \exp\left(\frac{-mv^2}{2kT}\right)$

Movimento Browniano: (1827)

pólen
partículas inorgânicas

$\varnothing \simeq 1\mu m$

partícula → macromolécula

$\frac{3}{2}kT = \frac{m}{2}v_{qm}^2 = \frac{M}{2}\langle V^2 \rangle$

$\frac{V_{qm}}{v_{qm}} = \sqrt{\frac{m}{M}}$ $\quad \frac{m}{M} \sim 10^{-8}$

$\sqrt{\frac{m}{M}} \sim 10^{-4}$

$V \sim 10^{-4} v_{qm} \sim 10^{-4} \cdot 400\,m/s \sim 4\,cm/s$ → rápido

Deslocamento ~ 0,1 mm/minuto. Por quê?

Movimento não é balístico → Deslocamento aleatório

Longo, $v \sim v_{qm}$, aleatório → choques

Passeio do bêbado

$x_0 = 0$

passo à frente $+l$ $\quad -l$ passo atrás

$x_1 = \pm l$

$\langle x_1 \rangle = 0$

$\langle x_1^2 \rangle = l^2$

$x_2 = x_1 \pm l$

$\langle x_2 \rangle = \langle x_1 \pm l \rangle = \underbrace{\cancel{\langle x_1 \rangle}}_{0} \pm l \underbrace{\cancel{\langle \pm 1 \rangle}}_{0}$

$\langle x_2^2 \rangle = \langle x_1^2 + l^2 \pm 2 x_1 l \rangle =$

$= \langle x_1^2 \rangle + l^2 + 2 \overbrace{\cancel{\langle x_1 \rangle}}^{0} \overbrace{\cancel{\langle \pm 1 \rangle}}^{0} \cdot l = 2 l^2$

$\langle x_n \rangle = 0$

$\langle x_n^2 \rangle = n \cdot l^2$

$x_{qm} = \sqrt{n} \cdot l$

↳ Tabuleiro de Galton

<u>Processo difusivo</u>

$\boxed{1D}$

Processo difusivo → passos → $l \sim v \cdot \tau$ → $\frac{\bar{l}}{\tau} \sim \bar{v}$

<u>Molécula</u> $\bar{l}$ → livre caminho médio

$\bar{v} \sim v_{qm}$ $\tau \sim$ intervalo entre colisões

$$\langle x^2 \rangle = N \cdot \bar{l}^2$$

$\bar{v} \sim 5 \cdot 10^2\, m/s$

$$= \frac{t}{\tau} \cdot \bar{l}^2 = \bar{l} \cdot \bar{v} \cdot t = 2D \cdot t$$

↳ $\sim 10^{-5}\, cm$ difusividade (m^2/s)

Difusão de um gás $D \sim 1\, cm^2/s$

Em 3 Dim

$$\boxed{\begin{array}{c} 2D \simeq \bar{l} \cdot \bar{v} \\ D \simeq \frac{\bar{l}\,\bar{v}}{2} \end{array}}$$

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle = \frac{1}{3} \langle r^2 \rangle$$

$$\langle r^2 \rangle = 3 \cdot \langle x^2 \rangle = 6 \cdot D \cdot t$$

$$r_{qm} = \sqrt{6D} \cdot \sqrt{t}$$

$$\boxed{\text{Crescimento com } \sqrt{N}}$$

→ partículas pequenas → "macro" moléculas

↳ grãos $\sim 1\,\mu m$? → 1905 Einstein

Forças aleatórias