

Lista de Exercícios de Integrais

1. Calcule a área do conjunto A :

a) $A = \{(x, y) \in \mathbb{R}^2; 0 \leq y \leq x^3, 1 \leq x \leq 3\}$

b) $A = \{(x, y) \in \mathbb{R}^2; x^2 - 1 \leq y \leq 0\}$

c) $A = \{(x, y) \in \mathbb{R}^2; 0 \leq y \leq 4 - x^2\}$

d) $A = \{(x, y) \in \mathbb{R}^2; x^2 + 1 \leq y \leq x + 1\}$

e) $A = \{(x, y) \in \mathbb{R}^2; x \geq 0, x^3 - x \leq y \leq -x^2 + 5x\}$

f) $A = \{(x, y) \in \mathbb{R}^2; x > 0, \frac{1}{x^2} \leq y \leq 5 - x^2\}$

2. Calcule

a) $\int \sqrt{1 - 4x^2} dx$ b) $\int \frac{1}{\sqrt{4 - x^2}} dx$

c) $\int \frac{1}{\sqrt{4 + x^2}} dx$ d) $\int x^2 \sqrt{1 - x^2} dx$

e) $\int \frac{1}{x\sqrt{1 + x^2}} dx$ f) $\int \sqrt{9 - (x - 1)^2} dx$

3. Calcule

a) $\int \cos^2(5x) dx$ b) $\int \sin^2(7x) dx$

c) $\int \sin(6x) \sin(9x) dx$ d) $\int \sin(8x) \cos(2x) dx$

e) $\int \cos(3x) \cos(4x) dx$ f) $\int \cos x \sin^4 x dx$

4. Calcule o volume do sólido obtido pela rotação em torno do eixo x , do conjunto:

a) $A = \{(x, y) \in \mathbb{R}^2; 1 \leq x \leq e, 0 \leq y \leq \ln x\}$

b) $A = \{(x, y) \in \mathbb{R}^2; 0 \leq x \leq 8, 0 \leq y \leq \sqrt[3]{x}\}$

c) $A = \{(x, y) \in \mathbb{R}^2; 1 \leq x \leq 2, 0 \leq y \leq x^2 + 1\}$

5. Verifique se as integrais impróprias são convergentes:

a) $\int_1^{+\infty} \frac{1}{x} dx$ b) $\int_1^{\infty} \frac{1}{x^4} dx$ c) $\int_{-\infty}^{-1} \frac{1}{x^3} dx$ d) $\int_{-\infty}^0 e^x dx$

e) $\int_{-\infty}^{+\infty} \frac{1}{1+x^2} dx$ f) $\int_{-\infty}^{+\infty} \frac{x}{1+x^2} dx$

6. Verifique se as integrais impróprias são convergentes:

a) $\int_0^1 \frac{1}{\sqrt{x}} dx$ b) $\int_{-1}^1 \frac{1}{x^2-1} dx$ c) $\int_{-2}^2 \frac{x}{x^2-4} dx$

d) $\int_{-3}^3 \frac{1}{\sqrt{9-x^2}} dx$ e) $\int_0^1 \frac{1}{x \ln x} dx$