

In specifying conjugate prior distributions, we assign values to their hyperparameters. These preassigned values (prior inputs) represent the available prior knowledge. In general, if we are confident of having good prior information about a parameter, then it is advantageous to select the corresponding prior distribution with a small variance; otherwise the prior distribution with a larger variance should be selected. We use the prior distributions given in (3.6) to illustrate this. If we have confidence that the true  $\mathbf{\Lambda}_k$  is not too far away from the preassigned hyperparameter value  $\mathbf{\Lambda}_{0k}$ , then  $\mathbf{H}_{0yk}$  should be taken as a matrix with small variances (such as  $0.5\mathbf{I}$ ). The choice of  $\alpha_{0\epsilon k}$  and  $\beta_{0\epsilon k}$  is based on the same general rationale and the nature of  $\psi_{\epsilon k}$  in the model. First, we note that the distribution of  $\epsilon_k$  is  $N[0, \psi_{\epsilon k}]$ . Hence, if we think that the variation of  $\epsilon_k$  is small (i.e.  $\mathbf{\Lambda}_k^T \boldsymbol{\omega}_i$  is a good predictor of  $y_{ik}$ ), then the prior distribution of  $\psi_{\epsilon k}$  should have a small mean as well as a small variance. Otherwise, the prior distribution of  $\psi_{\epsilon k}$  should have a large mean and/or a large variance. This gives some idea in choosing the hyperparameters  $\alpha_{0\epsilon k}$  and  $\beta_{0\epsilon k}$  in the inverted gamma distribution. Note that for the inverted gamma distribution, the mean is equal to  $\beta_{0\epsilon k}/(\alpha_{0\epsilon k} - 1)$ , and the variance is equal to  $\beta_{0\epsilon k}^2/[(\alpha_{0\epsilon k} - 1)^2(\alpha_{0\epsilon k} - 2)]$ . Hence, we may take  $\alpha_{0\epsilon k} = 9$  and  $\beta_{0\epsilon k} = 4$  for a situation where we have confidence that  $\mathbf{\Lambda}_k^T \boldsymbol{\omega}_i$  is a good predictor of  $y_{ik}$  in the measurement equation. Under this choice, the mean of  $\psi_{\epsilon k}$  is  $4/8 = 0.5$ , and the variance of  $\psi_{\epsilon k}$  is  $4^2/[(9 - 1)^2(9 - 2)] = 1/28$ . For a situation with little confidence, we may take  $\alpha_{0k} = 6$  and  $\beta_{0\epsilon k} = 10$ , so that the mean of  $\psi_{\epsilon k}$  is 2.0 and the variance is 1.0. The above ideas for choosing preassigned hyperparameter values can similarly be used in specifying  $\mathbf{\Lambda}_{0\omega k}$ ,  $\alpha_{0\delta k}$ , and  $\beta_{0\delta k}$  in the conjugate prior distributions of  $\mathbf{\Lambda}_{\omega k}$  and  $\psi_{\delta k}$ ; see (3.8). We now consider the choice of  $\mathbf{R}_0$  and  $\rho_0$  in the prior distribution of  $\boldsymbol{\Phi}$ . It follows from Muirhead (1982, p.97) that the mean of  $\boldsymbol{\Phi}$  is  $\mathbf{R}_0^{-1}/(\rho_0 - q_2 - 1)$ . Hence, if we have confidence that  $\boldsymbol{\Phi}$  is not too far away from a known matrix  $\boldsymbol{\Phi}_0$ , we can choose  $\mathbf{R}_0^{-1}$  and  $\rho_0$  such that  $\mathbf{R}_0^{-1} = (\rho_0 - q_2 - 1)\boldsymbol{\Phi}_0$ . Other values of  $\mathbf{R}_0^{-1}$  and  $\rho_0$  may be considered for situations without good prior information.

We now discuss some methods for obtaining  $\mathbf{\Lambda}_{0k}$ ,  $\mathbf{\Lambda}_{0\omega k}$ , and  $\boldsymbol{\Phi}_0$ . As mentioned before, these hyperparameter values may be obtained from subjective knowledge of experts in the field, and/or analysis of past or closely related data. If this kind of information is not available