where $\mathbf{y}_i$ is a $p \times 1$ vector of observed variables, $\boldsymbol{\mu}$ is a vector of intercepts, $\boldsymbol{\omega}_i = (\boldsymbol{\eta}_i^T, \boldsymbol{\xi}_i^T)^T$ is a vector of latent variables which is partitioned into a $q_1 \times 1$ vector of outcome latent variables $\boldsymbol{\eta}_i$ and a $q_2 \times 1$ vector of explanatory latent variables $\boldsymbol{\xi}_i$, $\boldsymbol{\epsilon}_i$ and $\boldsymbol{\delta}_i$ are residual errors, $\mathbf{d}_i$ is an $r \times 1$ vector of fixed covariates, $\boldsymbol{\Lambda}$, $\mathbf{B}$, $\boldsymbol{\Pi}$, and $\boldsymbol{\Gamma}$ are parameter matrices of unknown regression coefficients, and $\mathbf{F}(\cdot)$ is a given vector of differentiable functions of $\boldsymbol{\xi}_i$. Similarly to the model described in Chapter 2, the distributions of $\boldsymbol{\xi}_i$, $\boldsymbol{\epsilon}_i$, and $\boldsymbol{\delta}_i$ are $N[\mathbf{0}, \boldsymbol{\Phi}]$, $N[\mathbf{0}, \boldsymbol{\Psi}_\epsilon]$, and $N[\mathbf{0}, \boldsymbol{\Psi}_\delta]$, respectively; and the assumptions as given in Chapter 2 are satisfied. In this model, the unknown parameters are $\boldsymbol{\mu}$, $\boldsymbol{\Lambda}$, $\mathbf{B}$, $\boldsymbol{\Pi}$, and $\boldsymbol{\Gamma}$ which are related to the mean vectors of $\mathbf{y}_i$ and $\boldsymbol{\eta}_i$; and $\boldsymbol{\Phi}$, $\boldsymbol{\Psi}_\epsilon$, and $\boldsymbol{\Psi}_\delta$ which are the covariance matrices. Now consider the prior distributions of the parameters $\boldsymbol{\mu}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{\Psi}_\epsilon$ that are involved in the measurement equation. Let $\boldsymbol{\Lambda}_k^T$ be the kth row of $\boldsymbol{\Lambda}$, and $\psi_{\epsilon k}$ be the kth diagonal element of $\boldsymbol{\Psi}_\epsilon$. It can be shown (see Lee, 2007) that the conjugate type prior distributions of $\boldsymbol{\mu}$ and $(\boldsymbol{\Lambda}_k, \psi_{\epsilon k})$ are

$$
\begin{aligned}
&\psi_{\epsilon k} \stackrel{D}{=} \mathrm{IG}[\alpha_{0\epsilon k}, \beta_{0\epsilon k}] \text{ or equivalently } \psi_{\epsilon k}^{-1} \stackrel{D}{=} \mathrm{Gamma}[\alpha_{0\epsilon k}, \beta_{0\epsilon k}], \\
&\boldsymbol{\mu} \stackrel{D}{=} N[\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0], \text{ and } [\boldsymbol{\Lambda}_k | \psi_{\epsilon k}] \stackrel{D}{=} N\left[\boldsymbol{\Lambda}_{0k}, \psi_{\epsilon k}\mathbf{H}_{0yk}\right], \qquad (3.6)
\end{aligned}
$$

where $\mathrm{IG}[\cdot, \cdot]$ denotes the inverted gamma distribution, $\alpha_{0\epsilon k}$, $\beta_{0\epsilon k}$, and elements in $\boldsymbol{\mu}_0$, $\boldsymbol{\Lambda}_{0k}$, $\boldsymbol{\Sigma}_0$, and $\mathbf{H}_{0yk}$ are hyperparameters, and $\boldsymbol{\Sigma}_0$ and $\mathbf{H}_{0yk}$ are positive definite matrices. For simplicity of notation, we rewrite the structural equation (3.5) as

$$
\boldsymbol{\eta}_i = \boldsymbol{\Lambda}_\omega \mathbf{G}(\boldsymbol{\omega}_i) + \boldsymbol{\delta}_i, \qquad (3.7)
$$

where $\boldsymbol{\Lambda}_\omega = (\mathbf{B}, \boldsymbol{\Pi}, \boldsymbol{\Gamma})$ and $\mathbf{G}(\boldsymbol{\omega}_i) = (\mathbf{d}_i^T, \boldsymbol{\eta}_i^T, \mathbf{F}(\boldsymbol{\xi}_i)^T)^T$. Let $\boldsymbol{\Lambda}_{\omega k}^T$ be the kth row of $\boldsymbol{\Lambda}_\omega$, and $\psi_{\delta k}$ be the kth diagonal element of $\boldsymbol{\Psi}_\delta$. Based on reasoning similar to that used earlier, the conjugate type prior distributions of $\boldsymbol{\Phi}$ and $(\boldsymbol{\Lambda}_{\omega k}, \psi_{\delta k})$ are:

$$
\begin{aligned}
&\boldsymbol{\Phi} \stackrel{D}{=} \mathrm{IW}_{q_2}\left[\mathbf{R}_0^{-1}, \rho_0\right], \text{ or equivalently } \boldsymbol{\Phi}^{-1} \stackrel{D}{=} W_{q_2}[\mathbf{R}_0, \rho_0], \\
&\psi_{\delta k} \stackrel{D}{=} \mathrm{IG}[\alpha_{0\delta k}, \beta_{0\delta k}] \text{ or equivalently } \psi_{\delta k}^{-1} \stackrel{D}{=} \mathrm{Gamma}[\alpha_{0\delta k}, \beta_{0\delta k}], \\
&[\boldsymbol{\Lambda}_{\omega k} | \psi_{\delta k}] \stackrel{D}{=} N[\boldsymbol{\Lambda}_{0\omega k}, \psi_{\delta k}\mathbf{H}_{0\omega k}], \qquad (3.8)
\end{aligned}
$$

where $W_{q_2}[\mathbf{R}_0, \rho_0]$ is a q_2-dimensional Wishart distribution with hyperparameters ρ_0 and a positive definite matrix $\mathbf{R}_0$, $\mathrm{IW}_{q_2}[\mathbf{R}_0^{-1}, \rho_0]$ is a q_2-dimensional inverted Wishart distribution with hyperparameters ρ_0 and a positive definite matrix $\mathbf{R}_0^{-1}$, $\alpha_{0\delta k}$, $\beta_{0\delta k}$, and elements in $\boldsymbol{\Lambda}_{0\omega k}$ and $\mathbf{H}_{0\omega k}$ are hyperparameters, and $\mathbf{H}_{0\omega k}$ is a positive definite matrix. Note that the prior distribution of $\boldsymbol{\Phi}^{-1}$ (or $\boldsymbol{\Phi}$) is a multivariate extension of the prior distribution of $\psi_{\delta k}^{-1}$ (or $\psi_{\delta k}$).