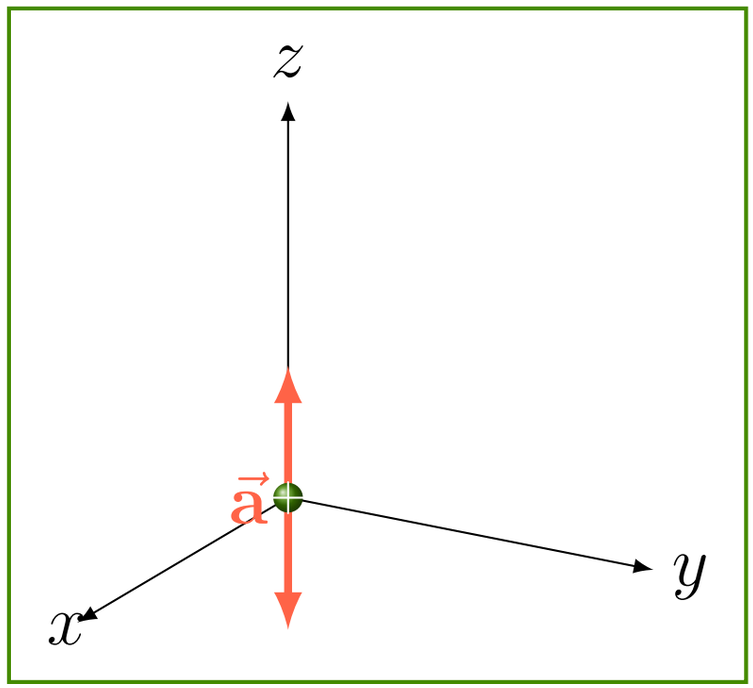
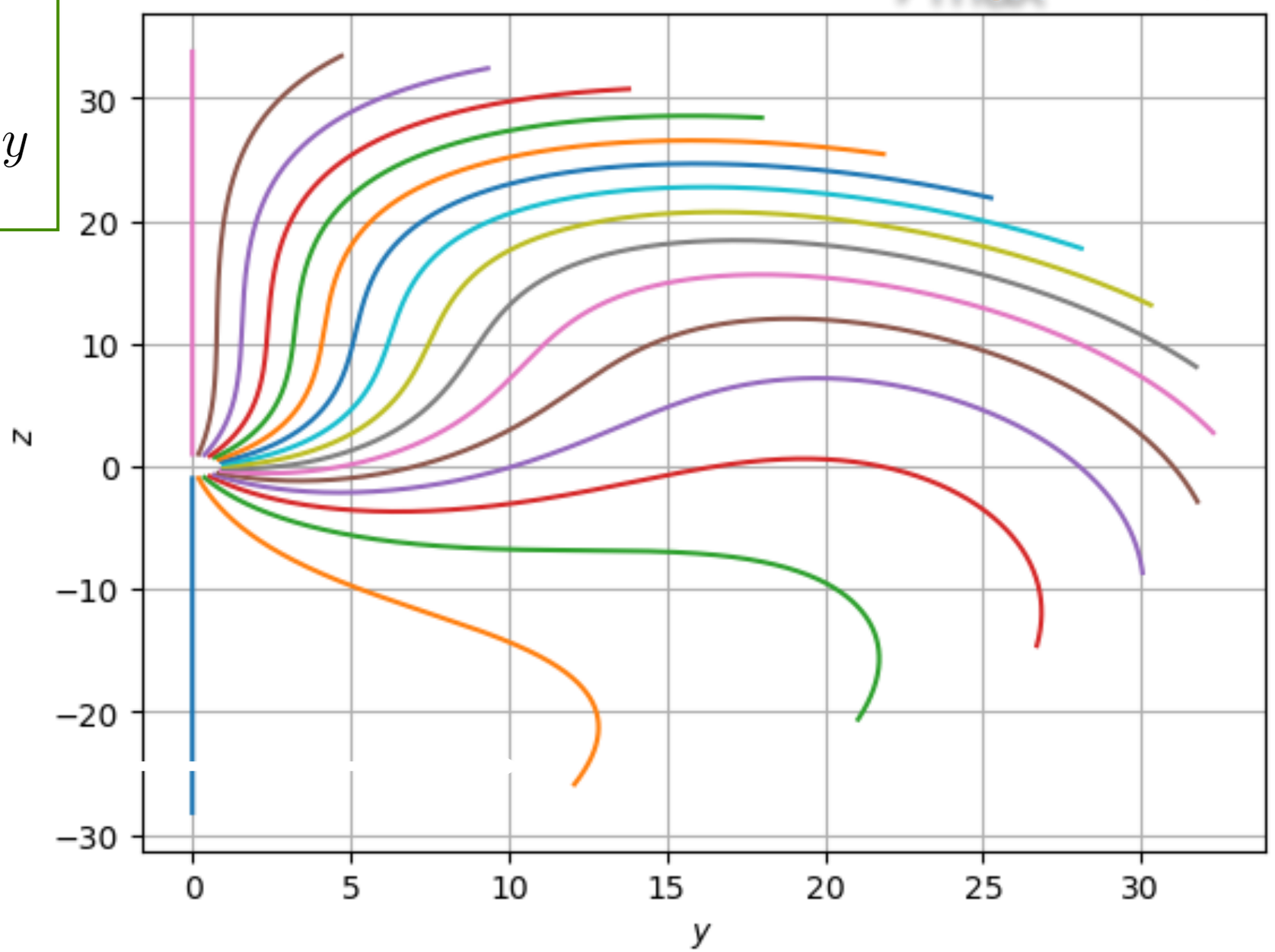
Eletromagnetismo Avançado

1º dezembro
Radiação

Campo elétrico de carga acelerada



$\beta_{max} = 0.157$



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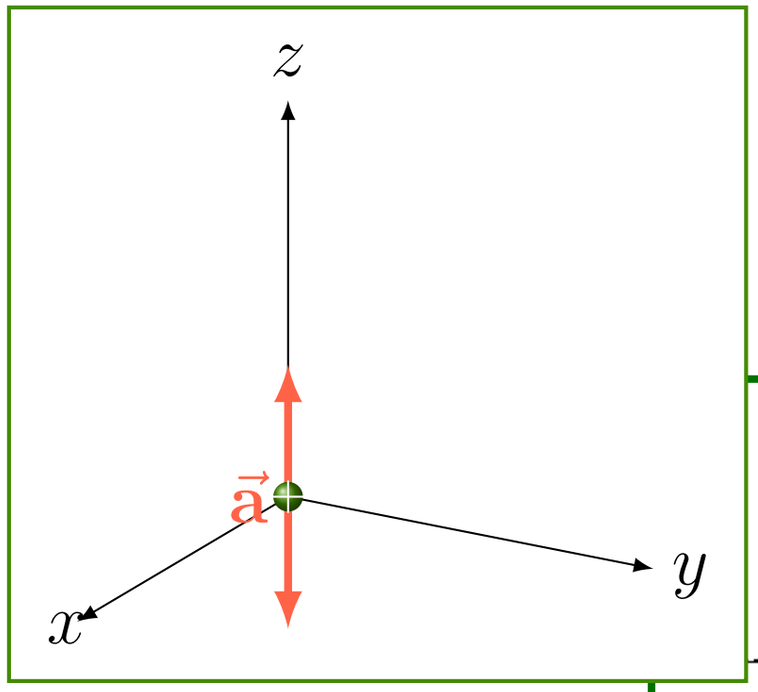
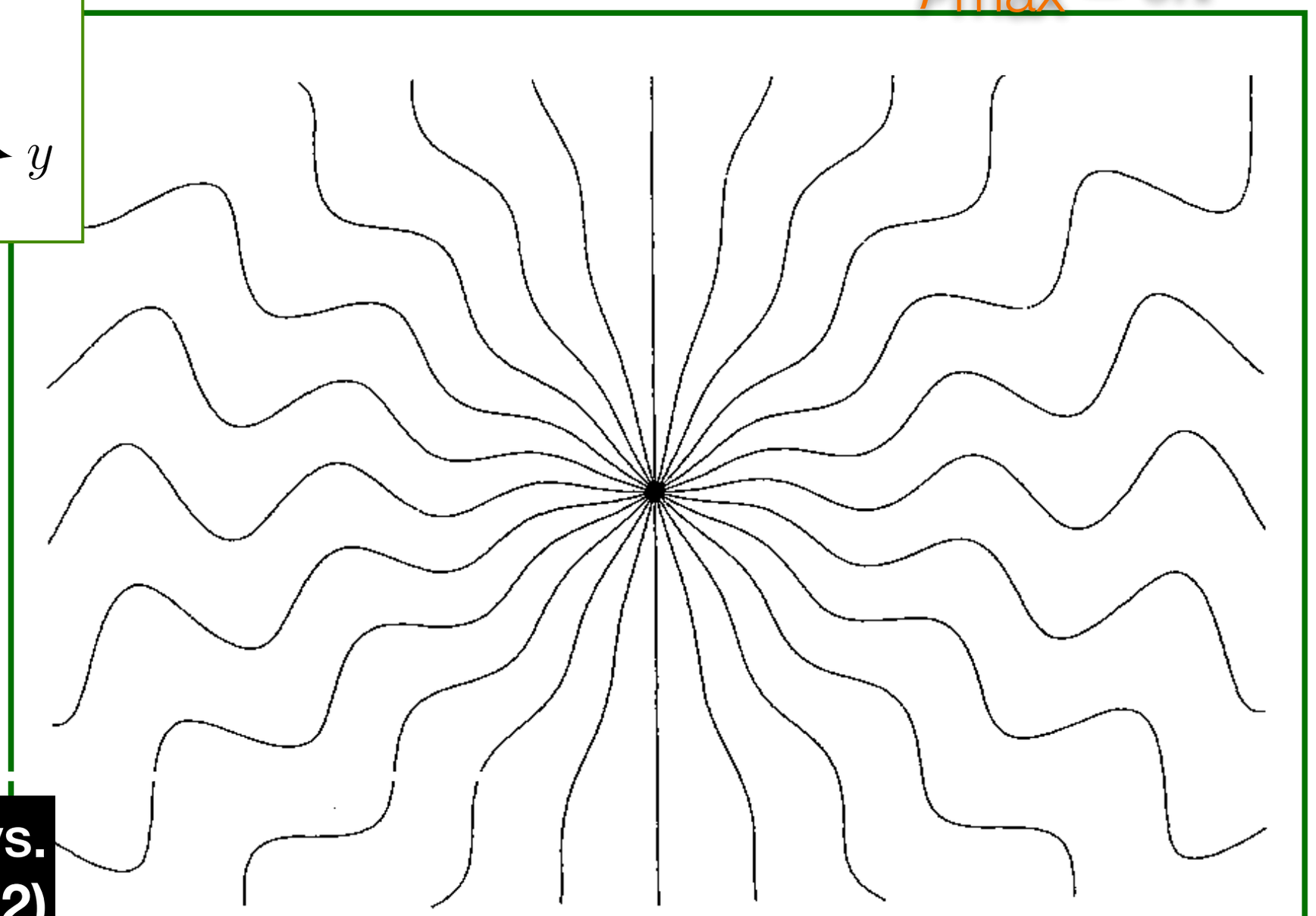
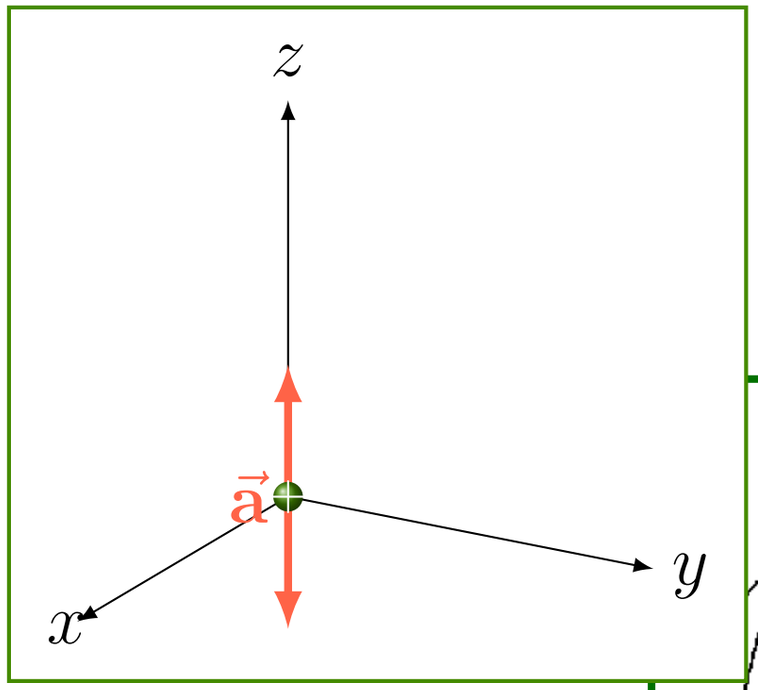


$\beta_{max} = 0.1$

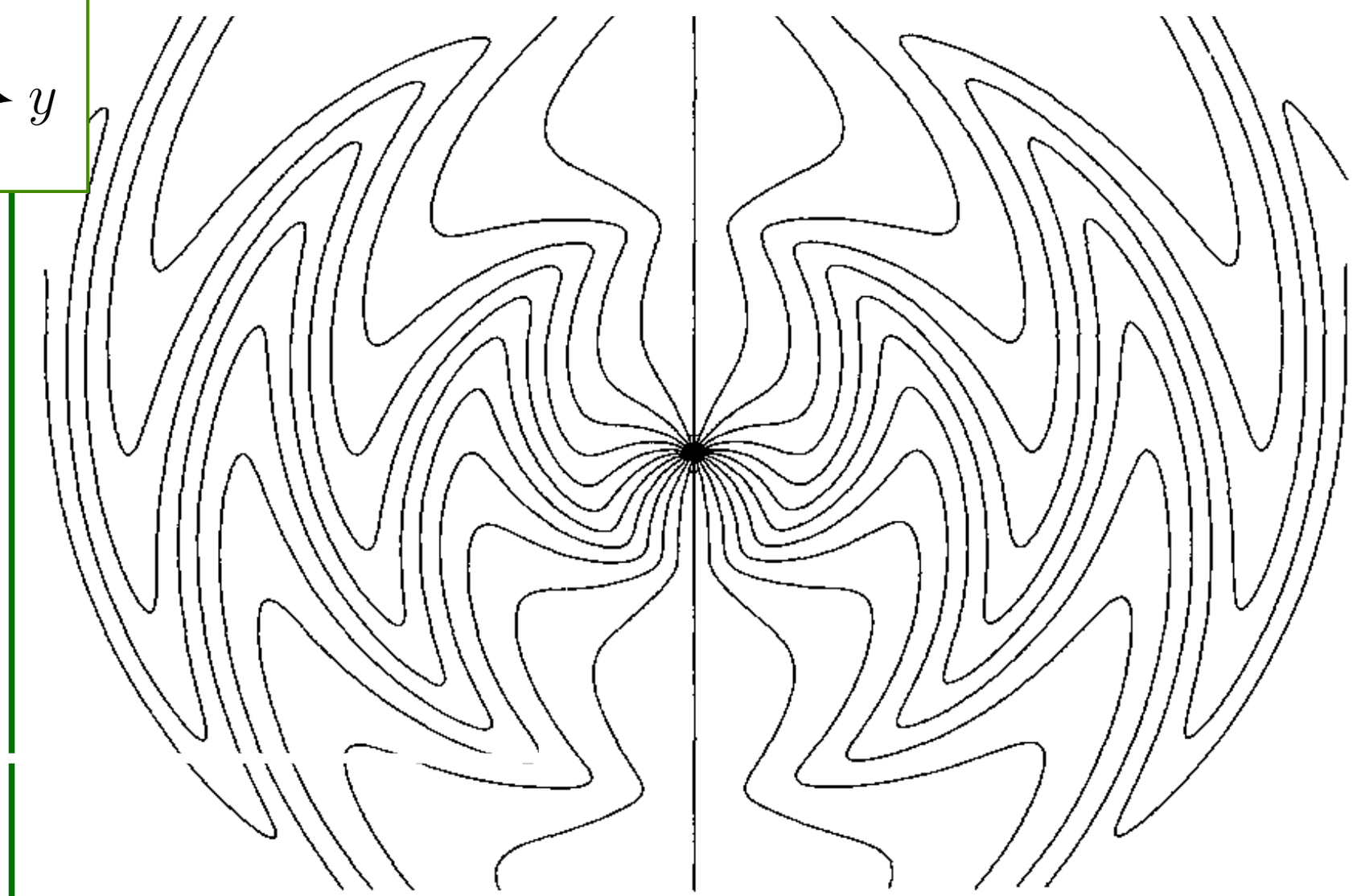


R. Y. Tsien, Am.J. Phys. 40, 46 (1972)

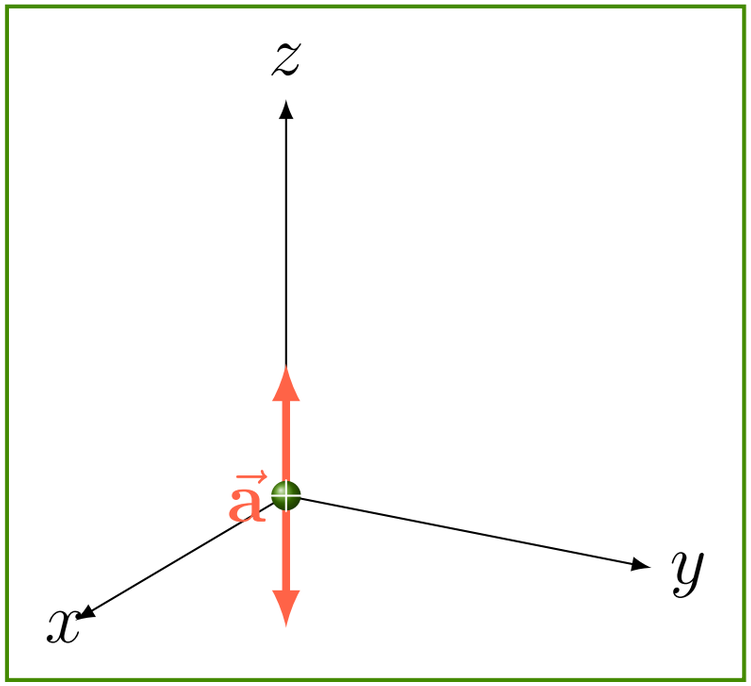
Campo elétrico de carga acelerada



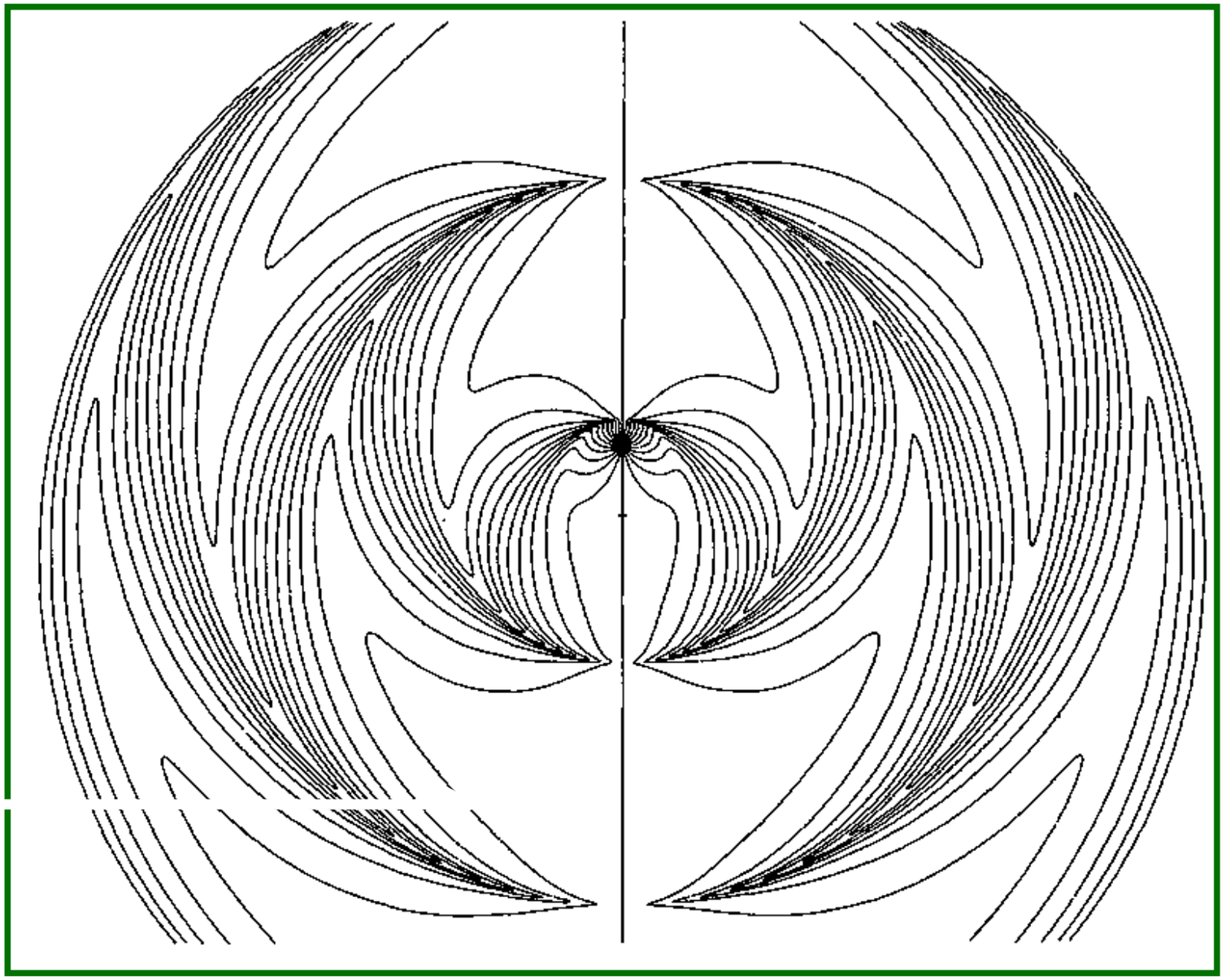
$\beta_{max} = 0.5$



Campo elétrico de carga acelerada



$\beta_{max} = 0.9$



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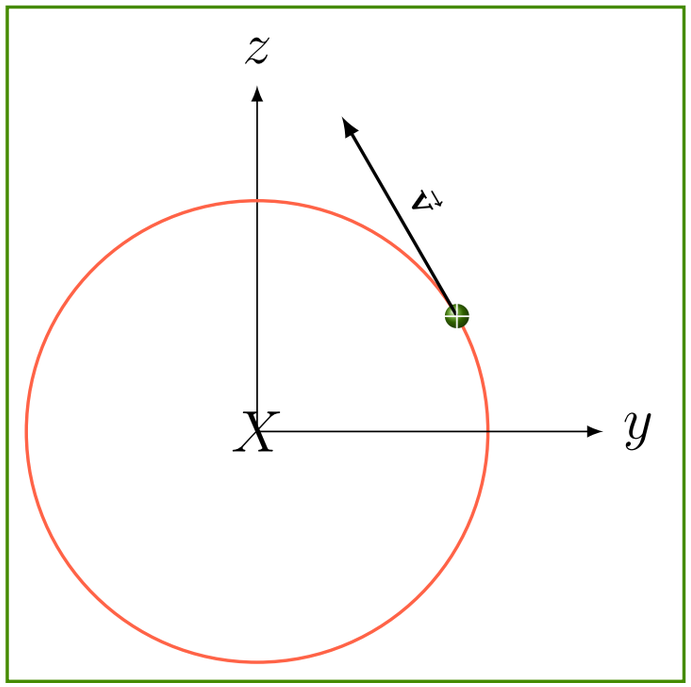

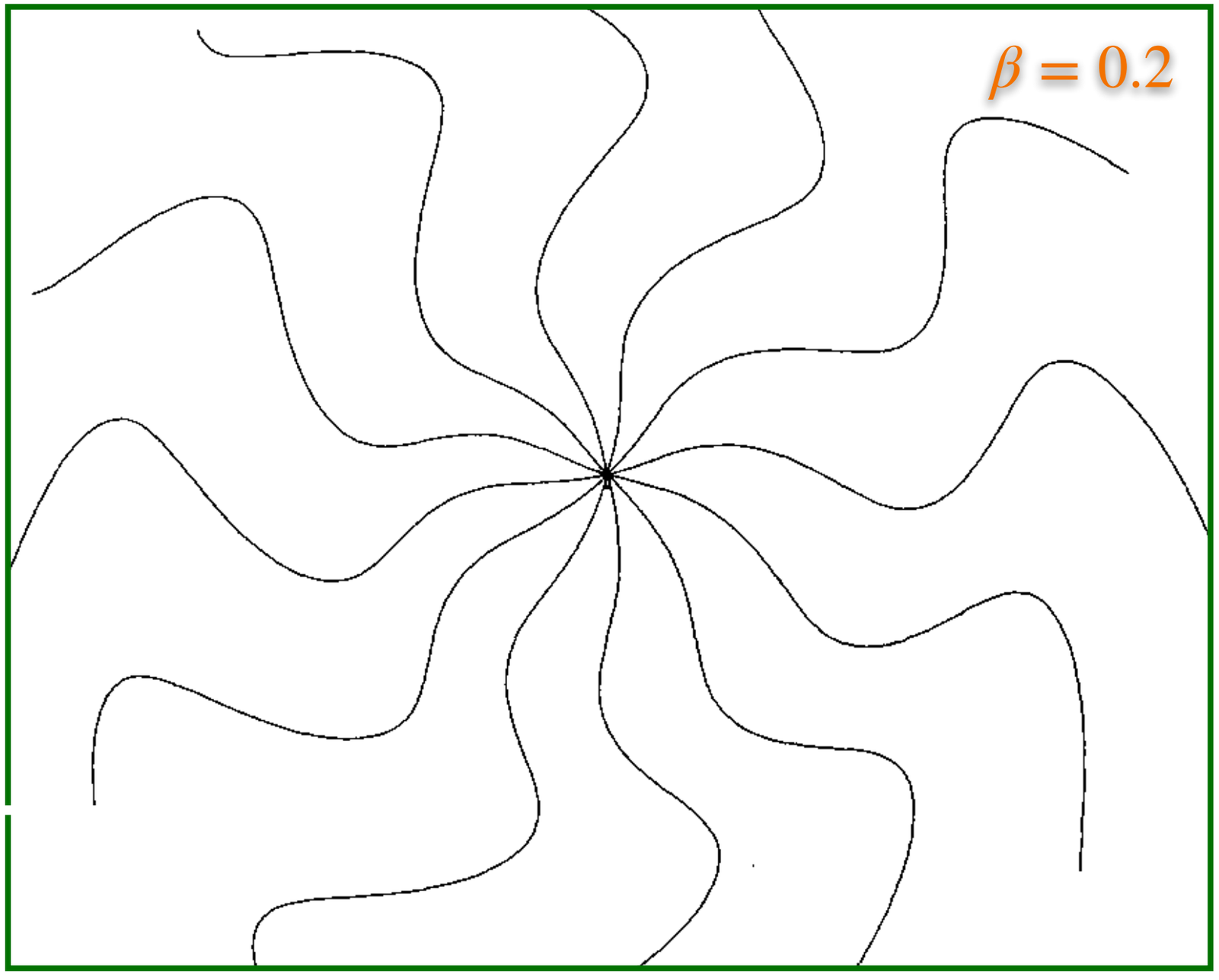

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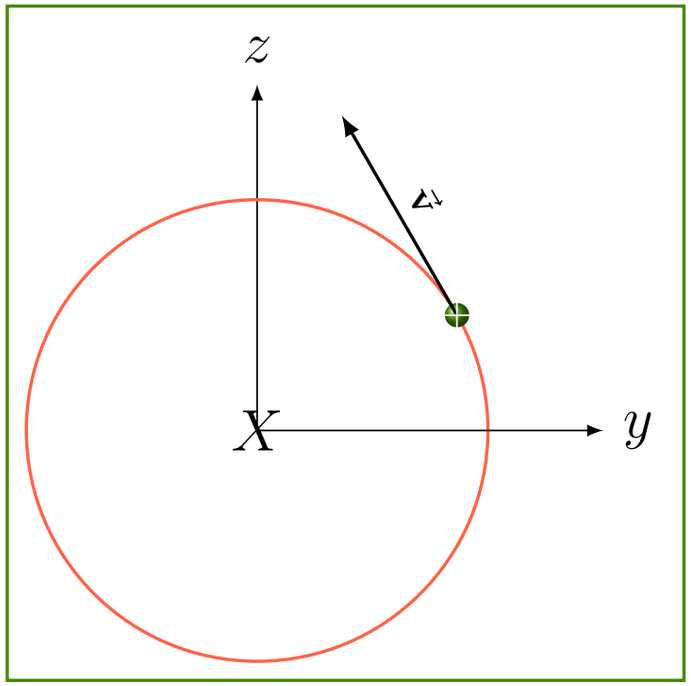

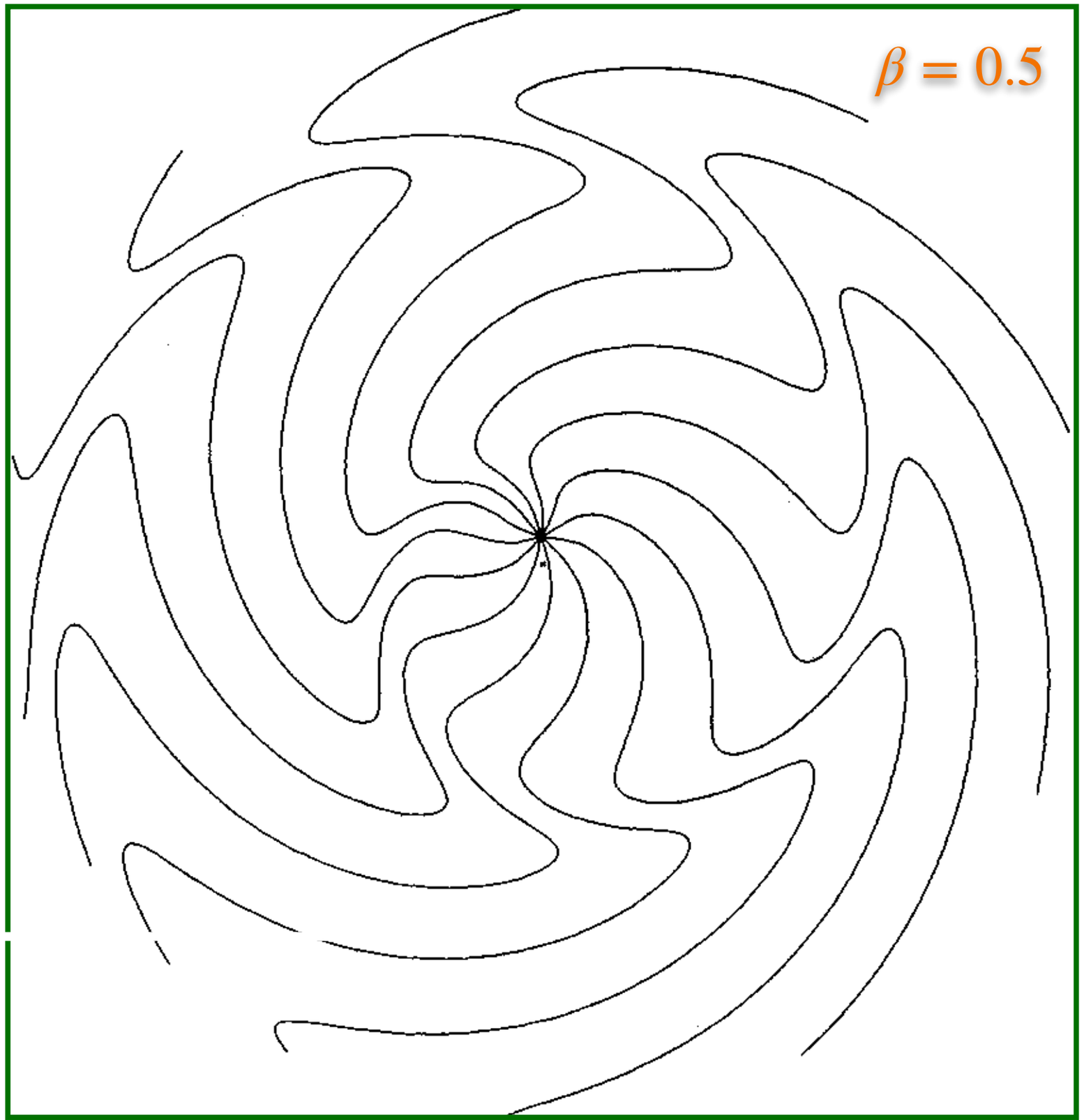

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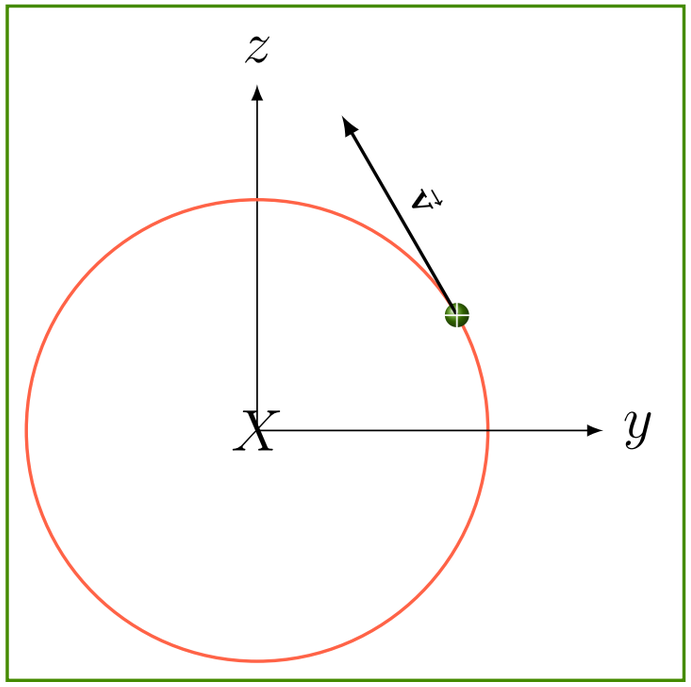
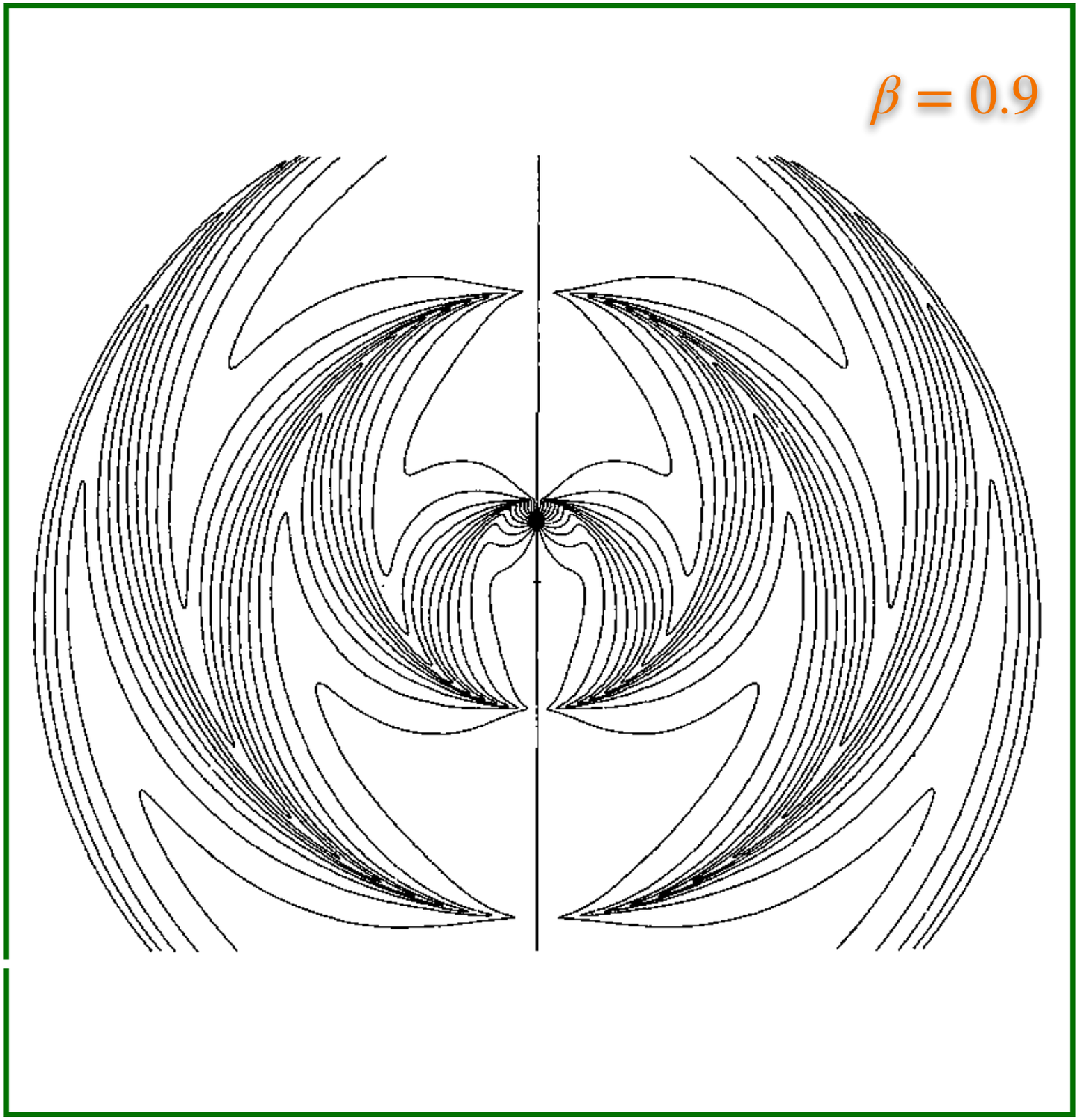


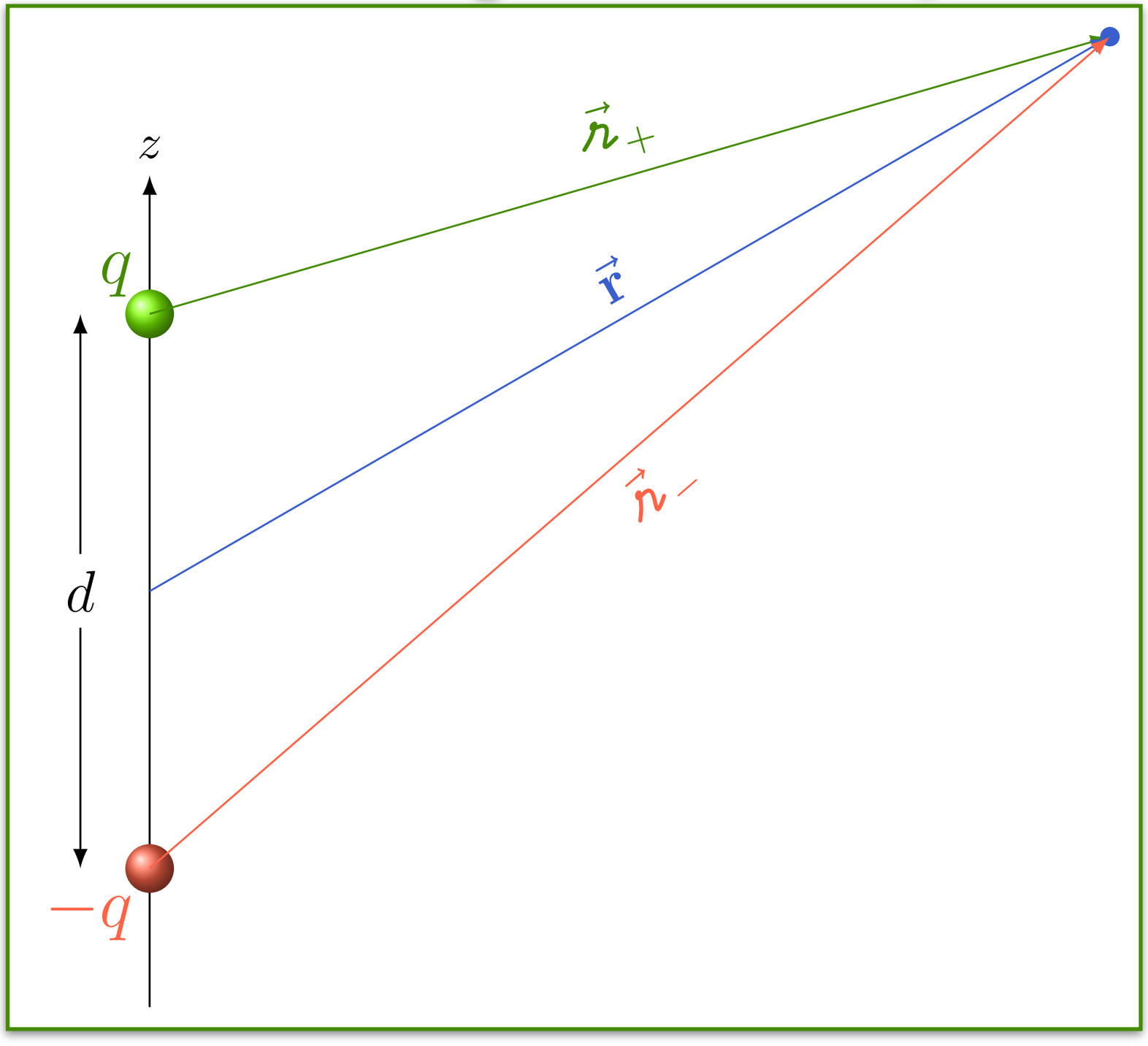


Radiação de dipolo

$$q(t) = q_0 \cos(\omega t)$$

$$r \gg \lambda \gg d$$

$$\lambda = \frac{2\pi c}{\omega}$$



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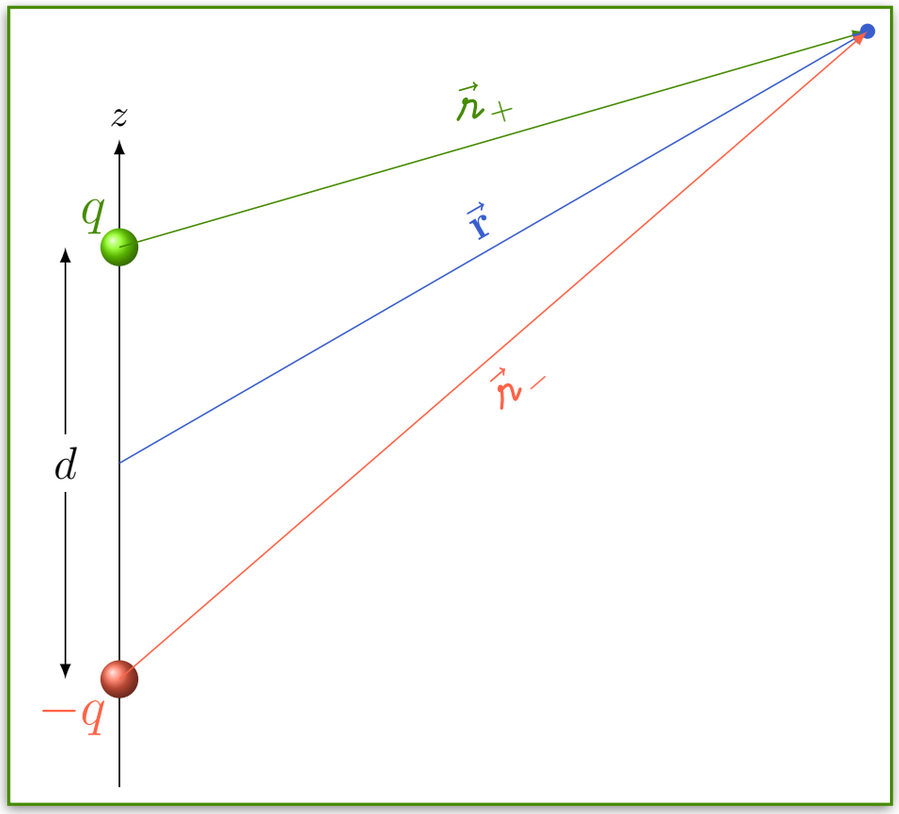

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_0 \cos(\omega t_r)}{r_+} - \frac{q_0 \cos(\omega t_r)}{r_-}\right)$$

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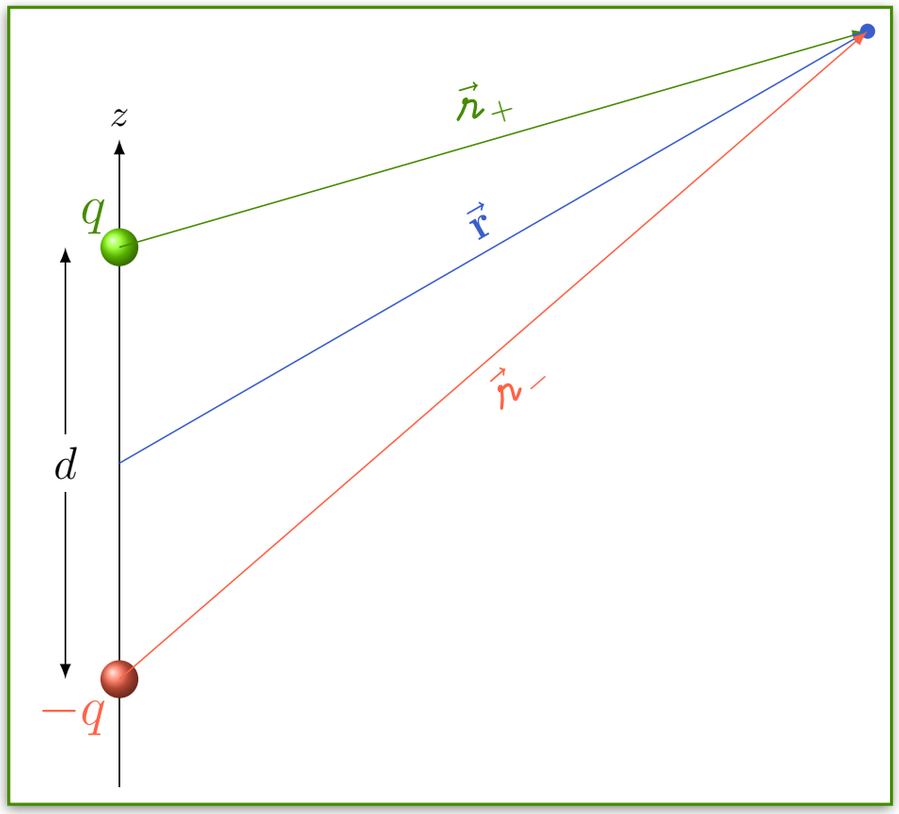

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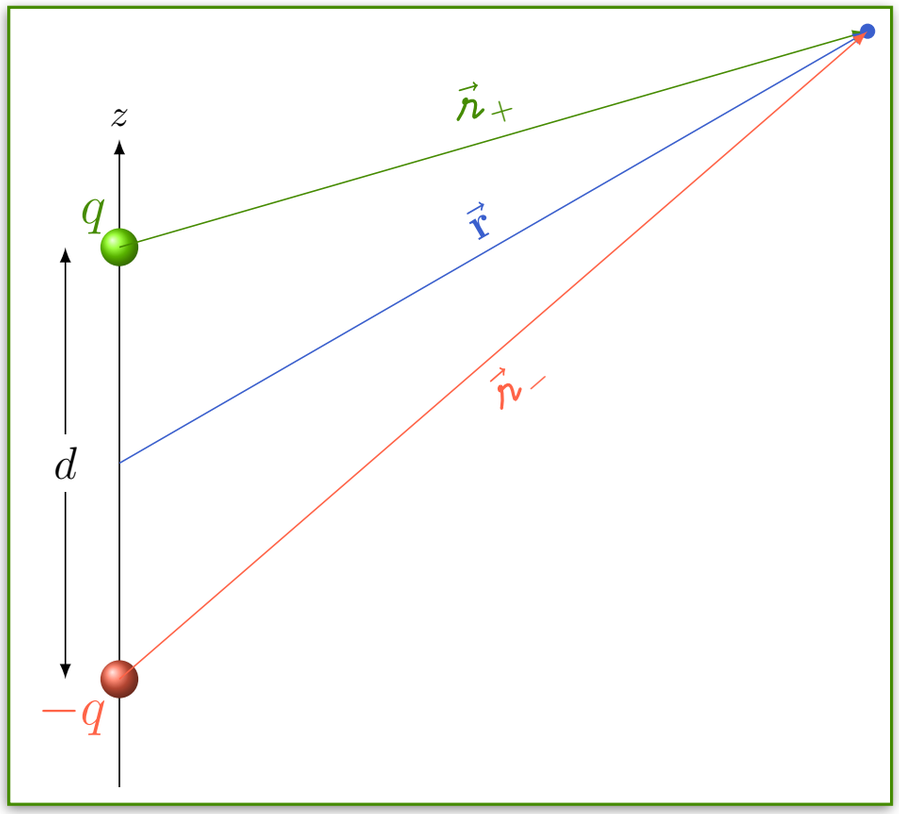

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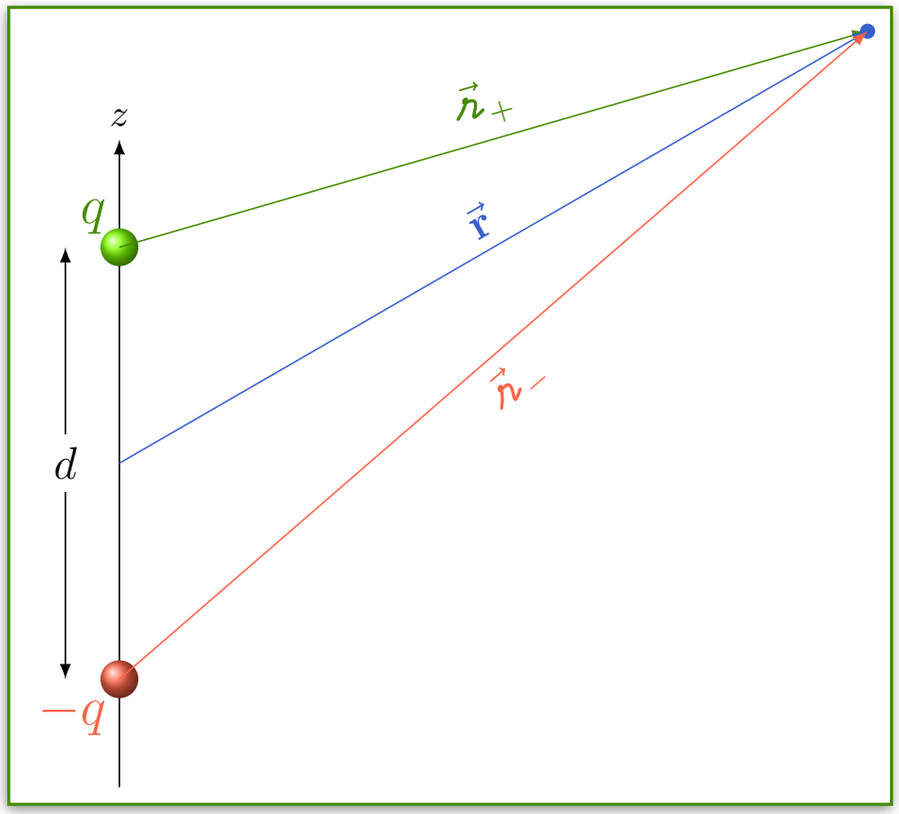

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$$V(\vec{\mathbf{r}}, t) = \frac{q_0}{4\pi\epsilon_0 r}\left(\cos\omega(t - \frac{r_+}{c}) - \cos\omega(t - \frac{r_-}{c})\right)$$

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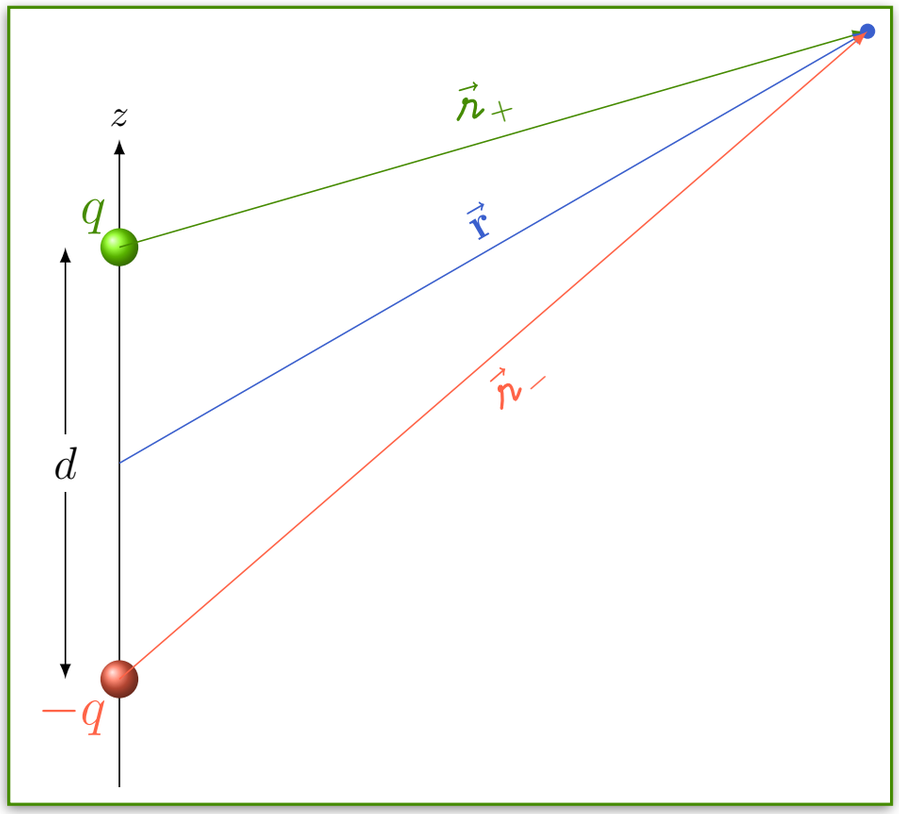

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$$V(\vec{\mathbf{r}},t) = \frac{q_0}{2\pi\epsilon_0 r}\sin\omega(t-\frac{r}{c})\sin\frac{\omega}{2c}(r_+ - r_-)$$

Radiação de dipolo

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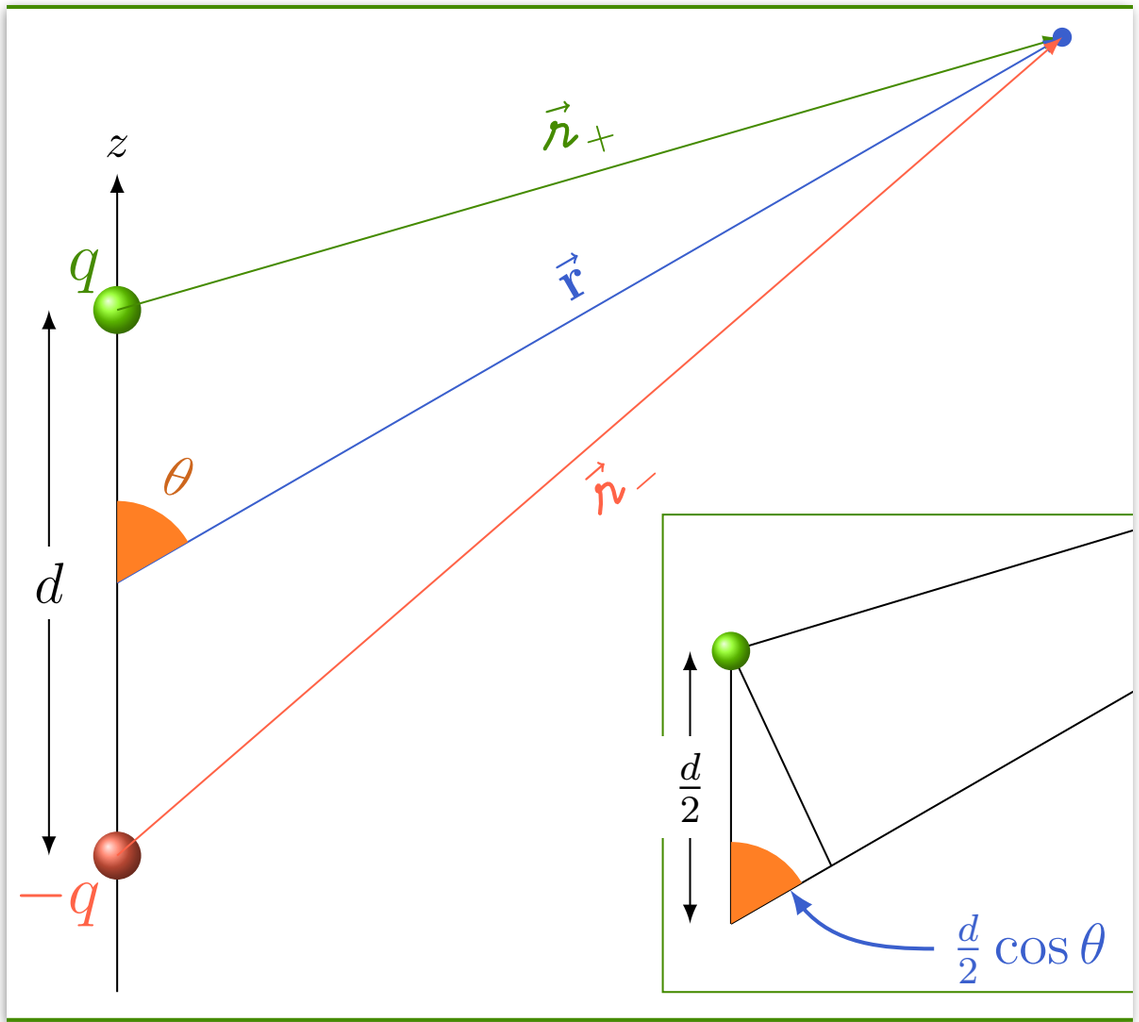

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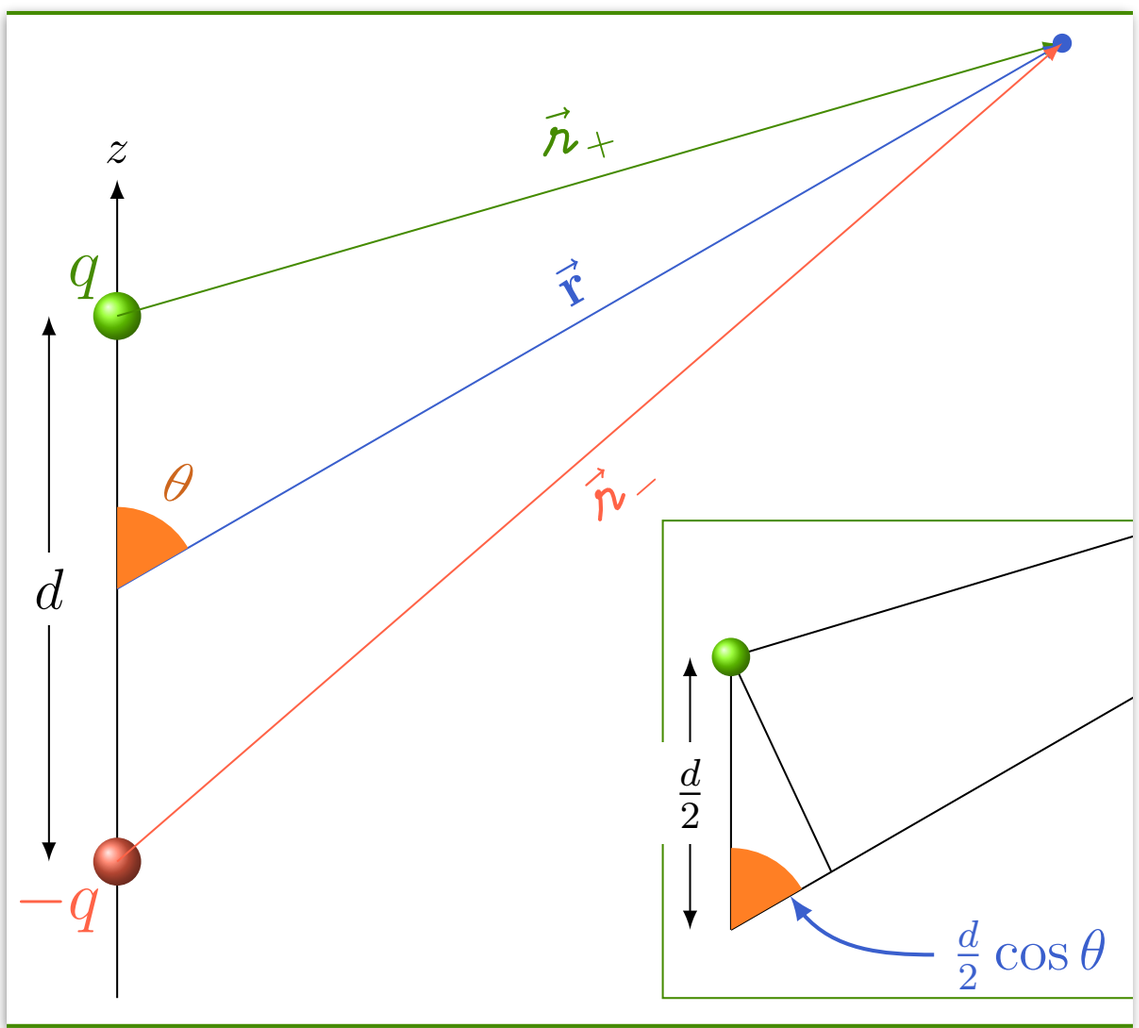
$$r_+ = r - \frac{d}{2}\cos\theta$$



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$$r_+ = r - \frac{d}{2}\cos\theta$$

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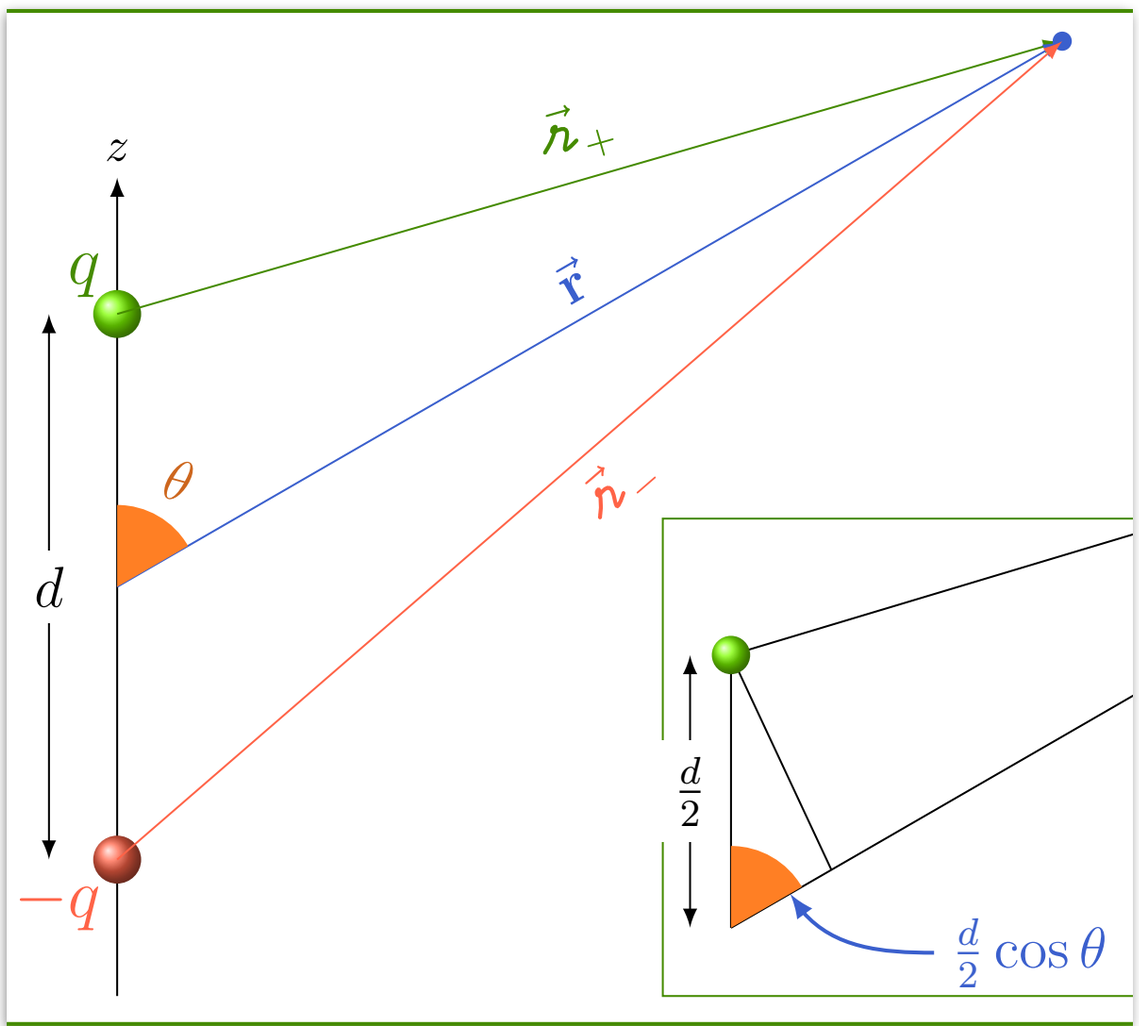

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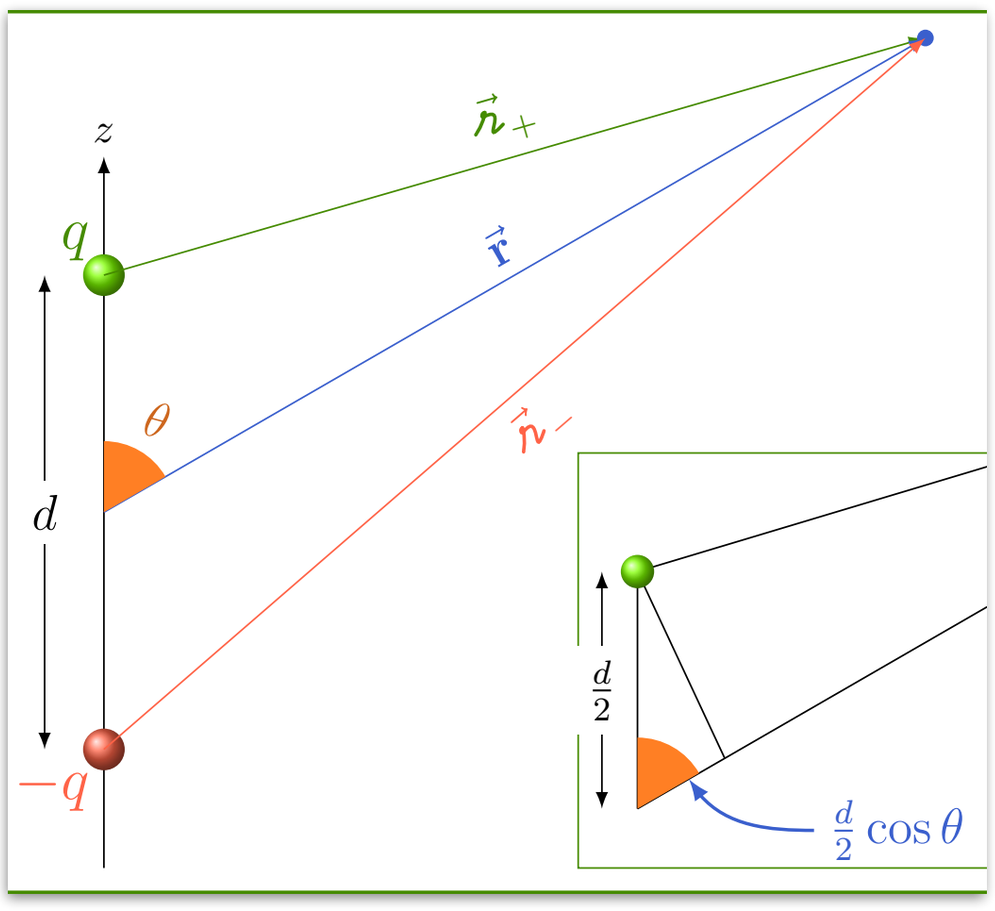
$$V(\vec{\mathbf{r}}, t) = -\frac{q_0}{2\pi\epsilon_0 r} \sin\omega(t - \frac{r}{c}) \sin\left(\frac{\omega}{2c} d\cos\theta\right)$$



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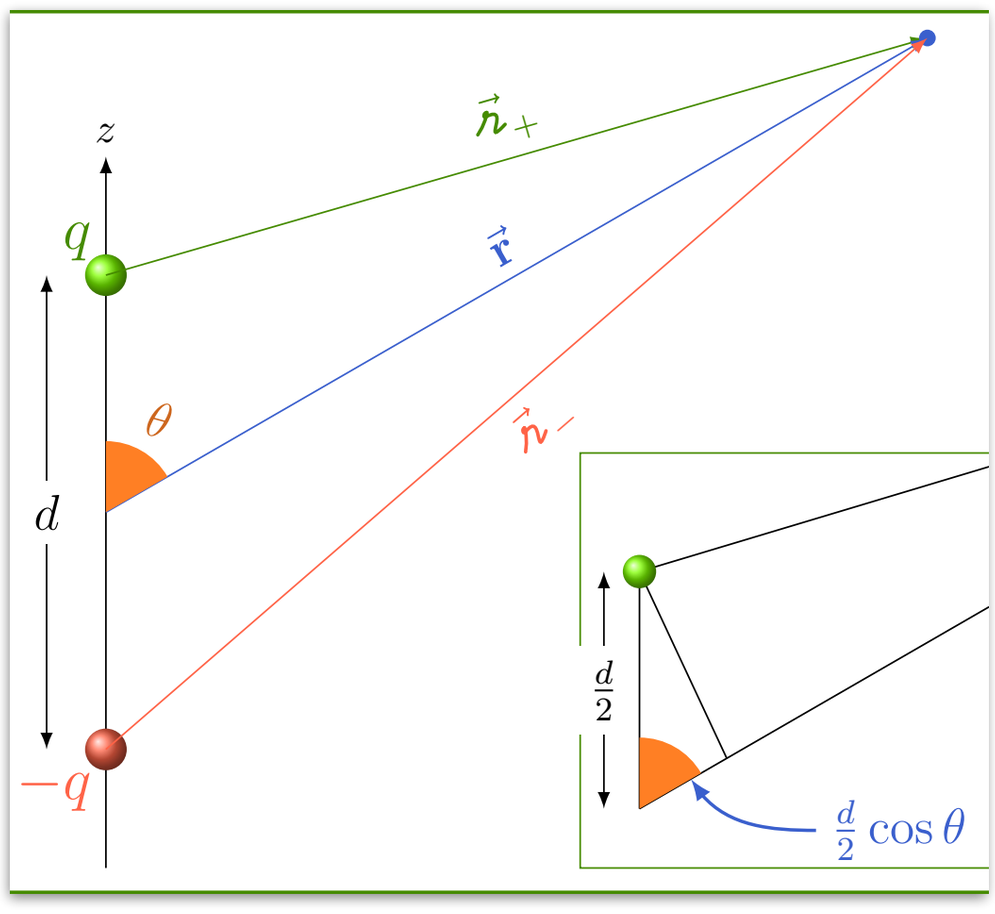
$$V(\vec{\mathbf{r}}, t) = -\frac{p_0\omega}{4\pi\epsilon_0 c}\frac{\cos\theta}{r} \sin\omega(t - \frac{r}{c})$$



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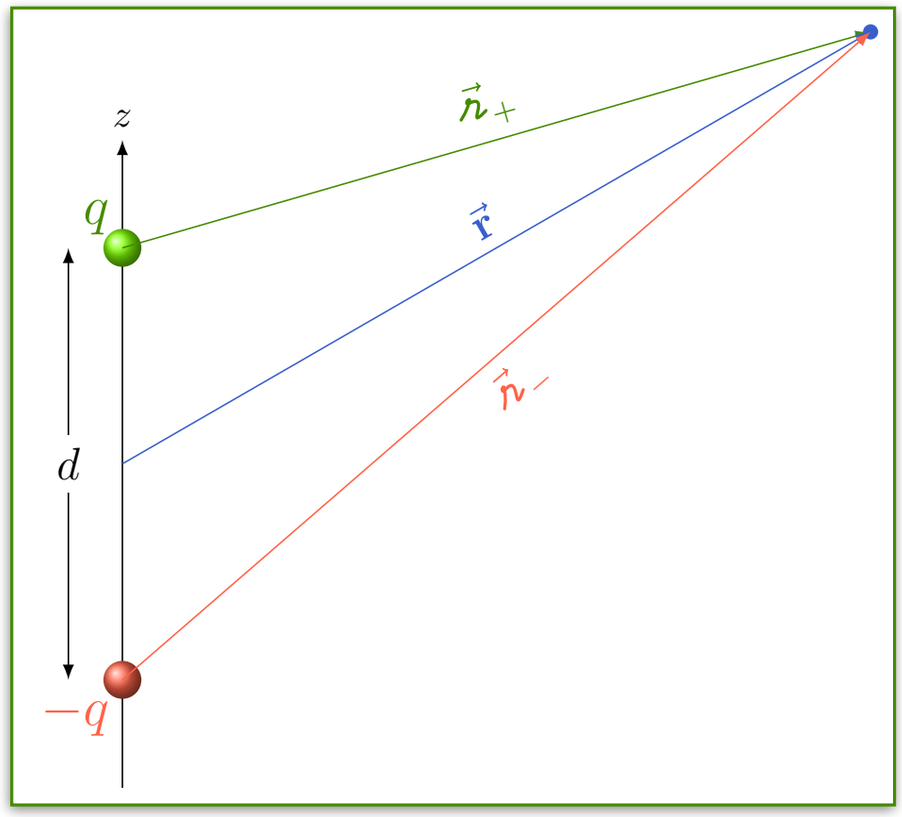

Radiação de dipolo

$$q(t) = q_0 \cos(\omega t)$$

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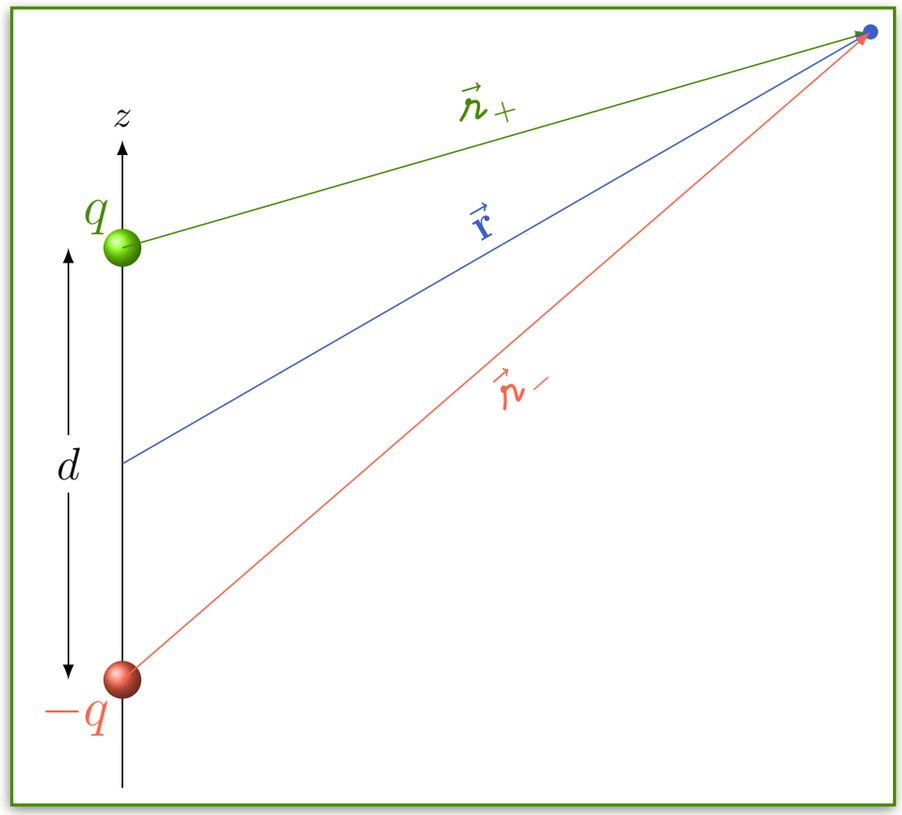
$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \int_{-d/2}^{d/2} \frac{I(t_r)}{r} \, \mathrm{d}z' \hat{\mathbf{z}}$$

$$V(\vec{\mathbf{r}}, t) = -\frac{p_0 \omega}{4\pi\epsilon_0 c} \frac{\cos\theta}{r} \sin\omega(t - \frac{r}{c})$$



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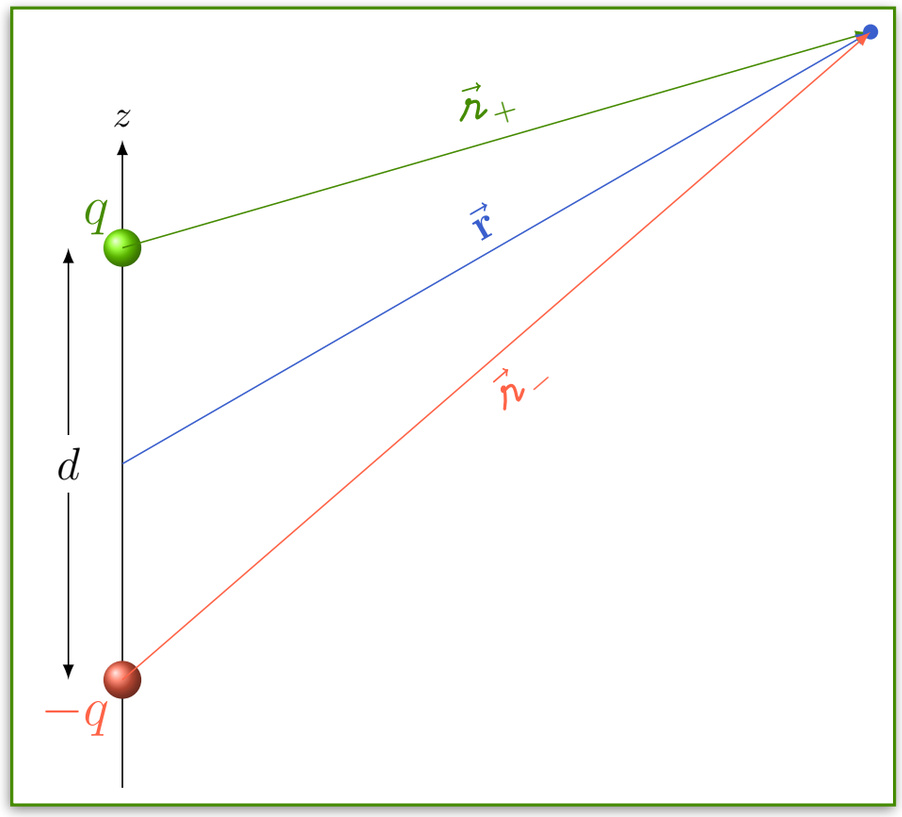
$$I(t) = \frac{\mathrm{d}q}{\mathrm{d}t}$$

$$V(\vec{\mathbf{r}}, t) = -\frac{p_0 \omega}{4\pi\epsilon_0 c} \frac{\cos\theta}{r} \sin\omega(t - \frac{r}{c})$$



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$$I(t) = \frac{\mathrm{d}q}{\mathrm{d}t}$$

$$I(t) = -\omega q_0 \sin(\omega t)$$

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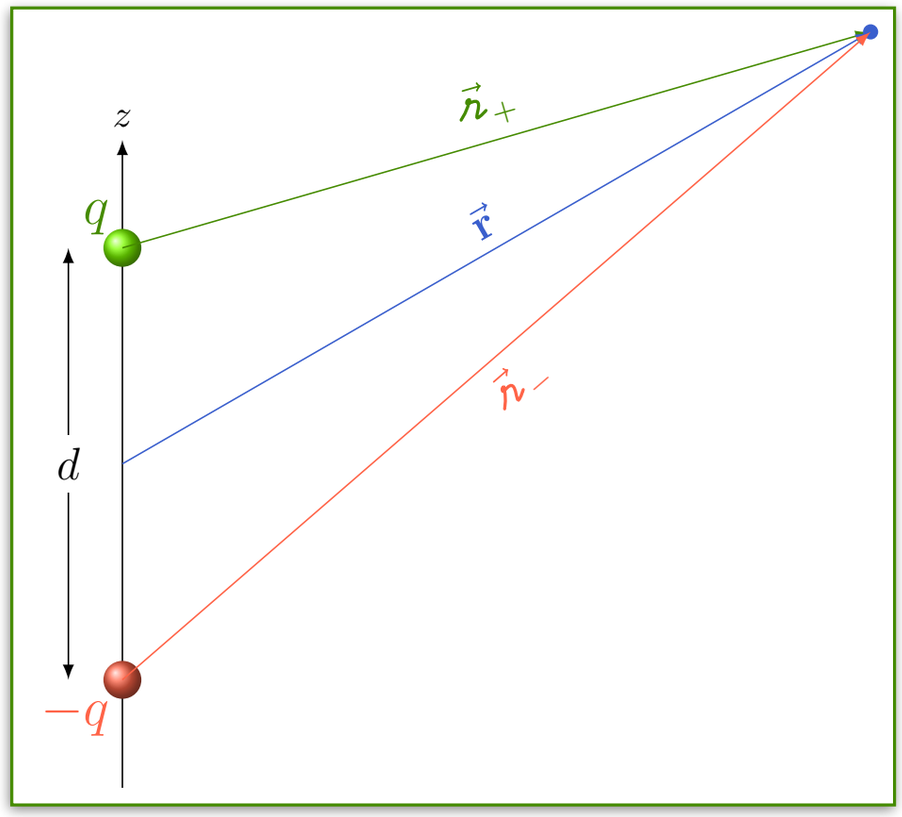

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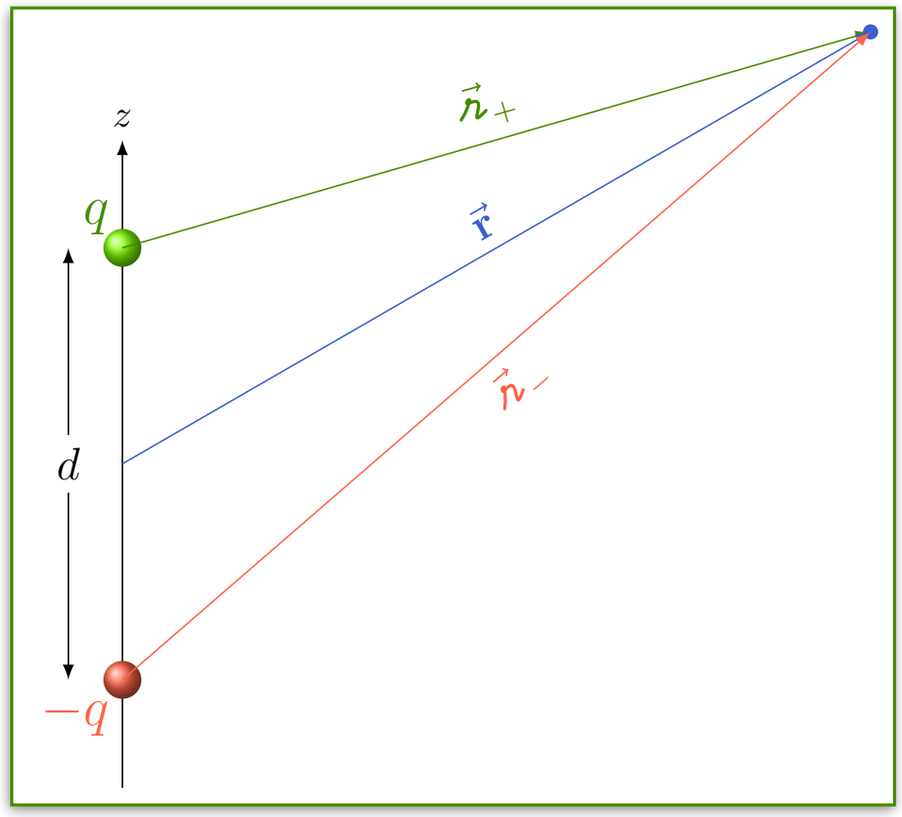
$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) \approx \frac{\mu_0}{4\pi} \frac{d}{r} I(t - \frac{r}{c}) \hat{\mathbf{z}}$$

$$V(\vec{\mathbf{r}}, t) = -\frac{p_0 \omega}{4\pi\epsilon_0 c} \frac{\cos\theta}{r} \sin\omega(t - \frac{r}{c})$$



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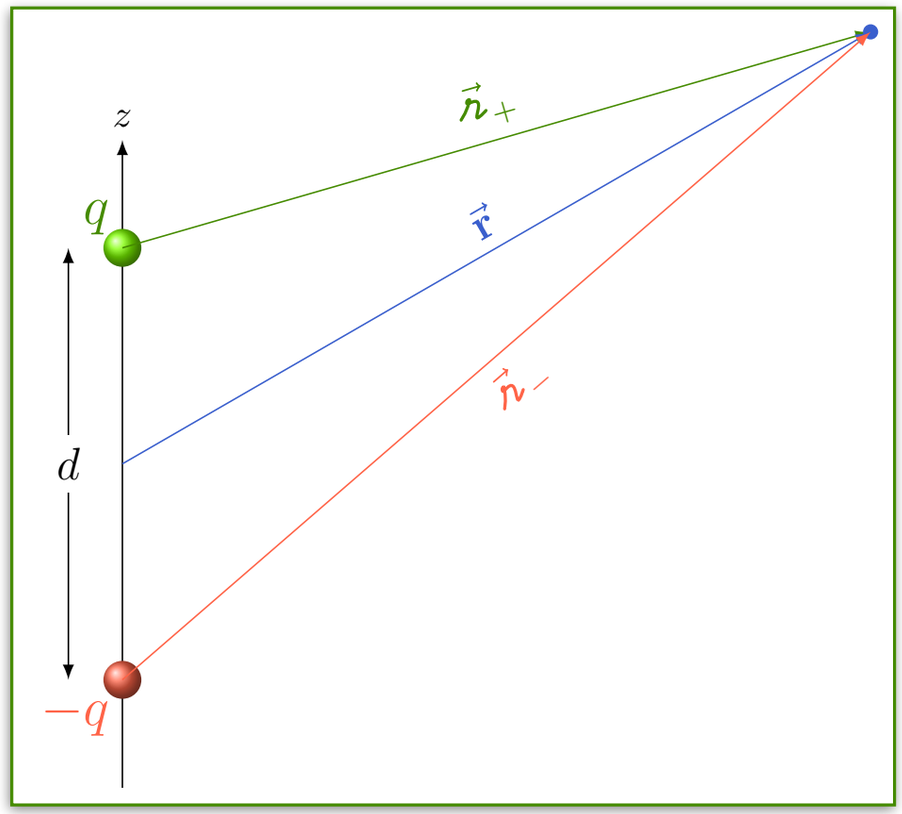

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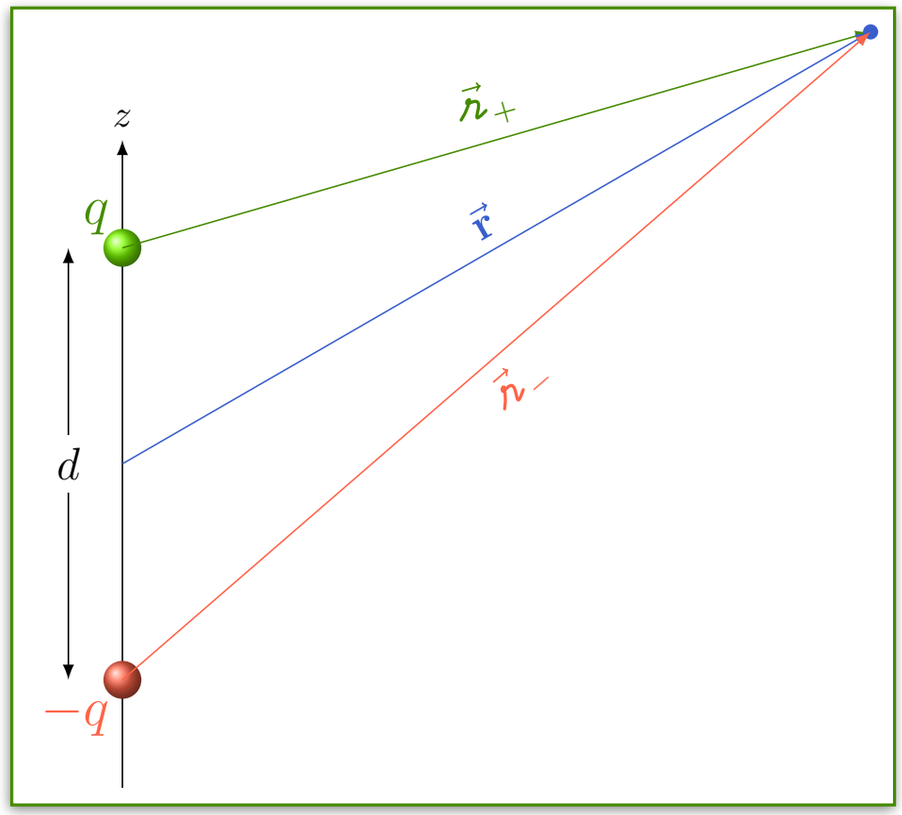

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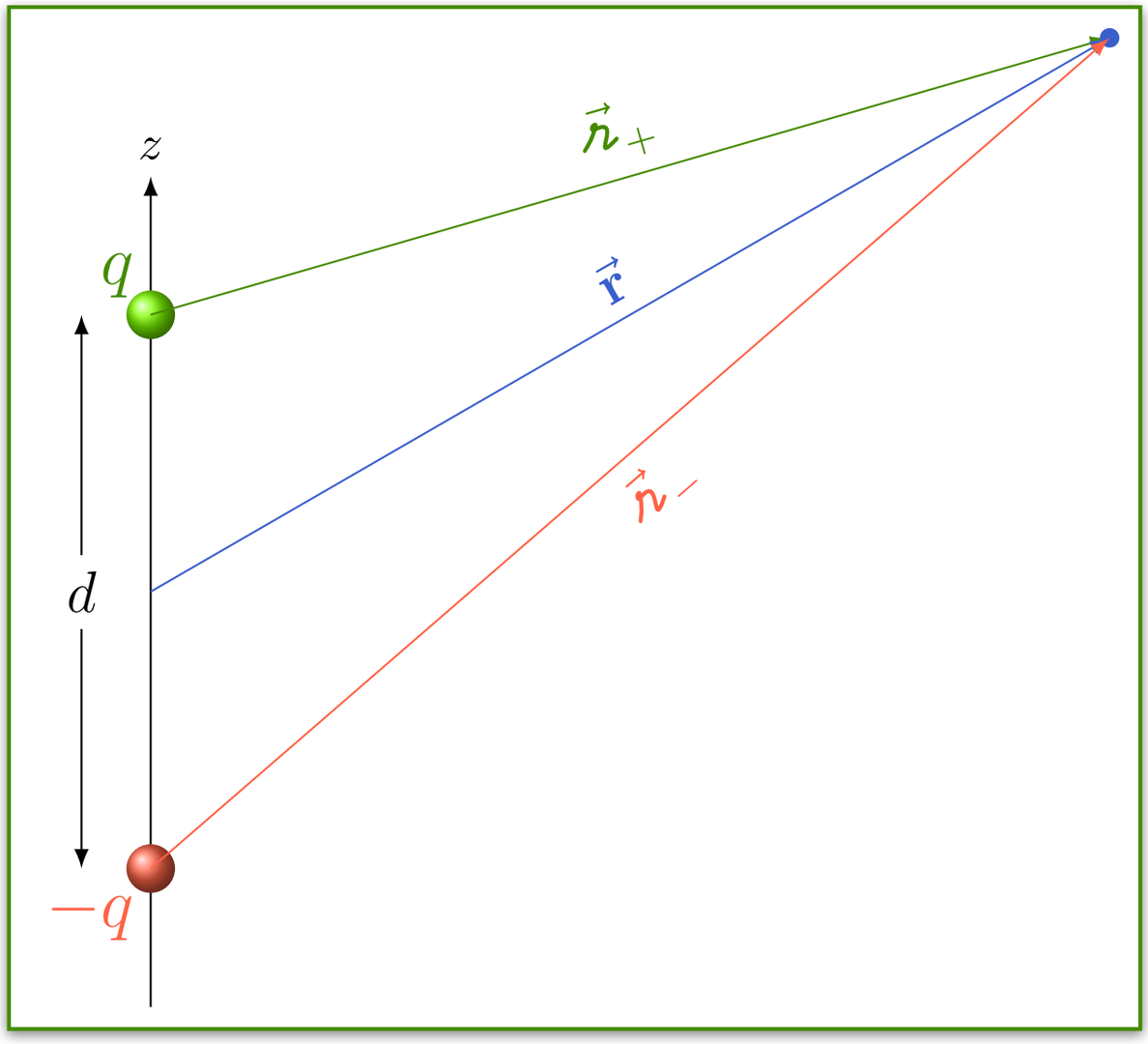
$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = -\frac{\mu_0 p_0 \omega}{4\pi r} \sin\omega(t - \frac{r}{c})\hat{\mathbf{z}}$$



Radiação de dipolo

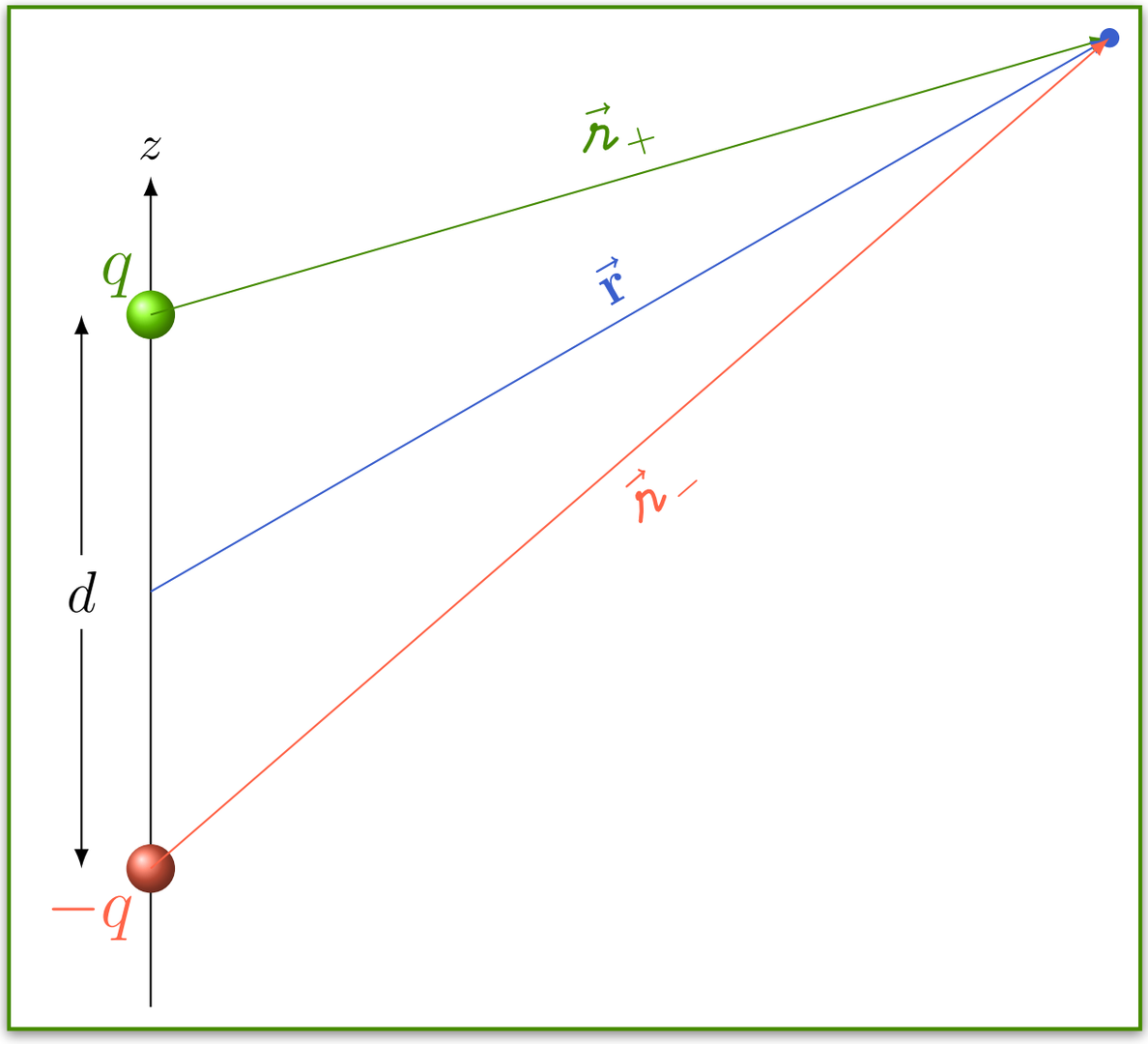
$$q(t) = q_0 \cos(\omega t)$$

$$r \gg \lambda \gg d$$

$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \partial_t \vec{\mathbf{A}}$$

$$V(\vec{\mathbf{r}}, t) = -\frac{p_0 \omega}{4\pi\epsilon_0 c} \frac{\cos\theta}{r} \sin\omega(t - \frac{r}{c})$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = -\frac{\mu_0 p_0 \omega}{4\pi r} \sin\omega(t - \frac{r}{c})\hat{\mathbf{z}}$$

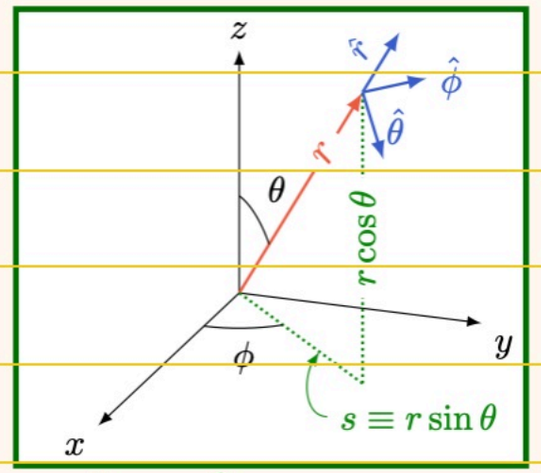


Radiação de dipolo

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla}t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$



$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$