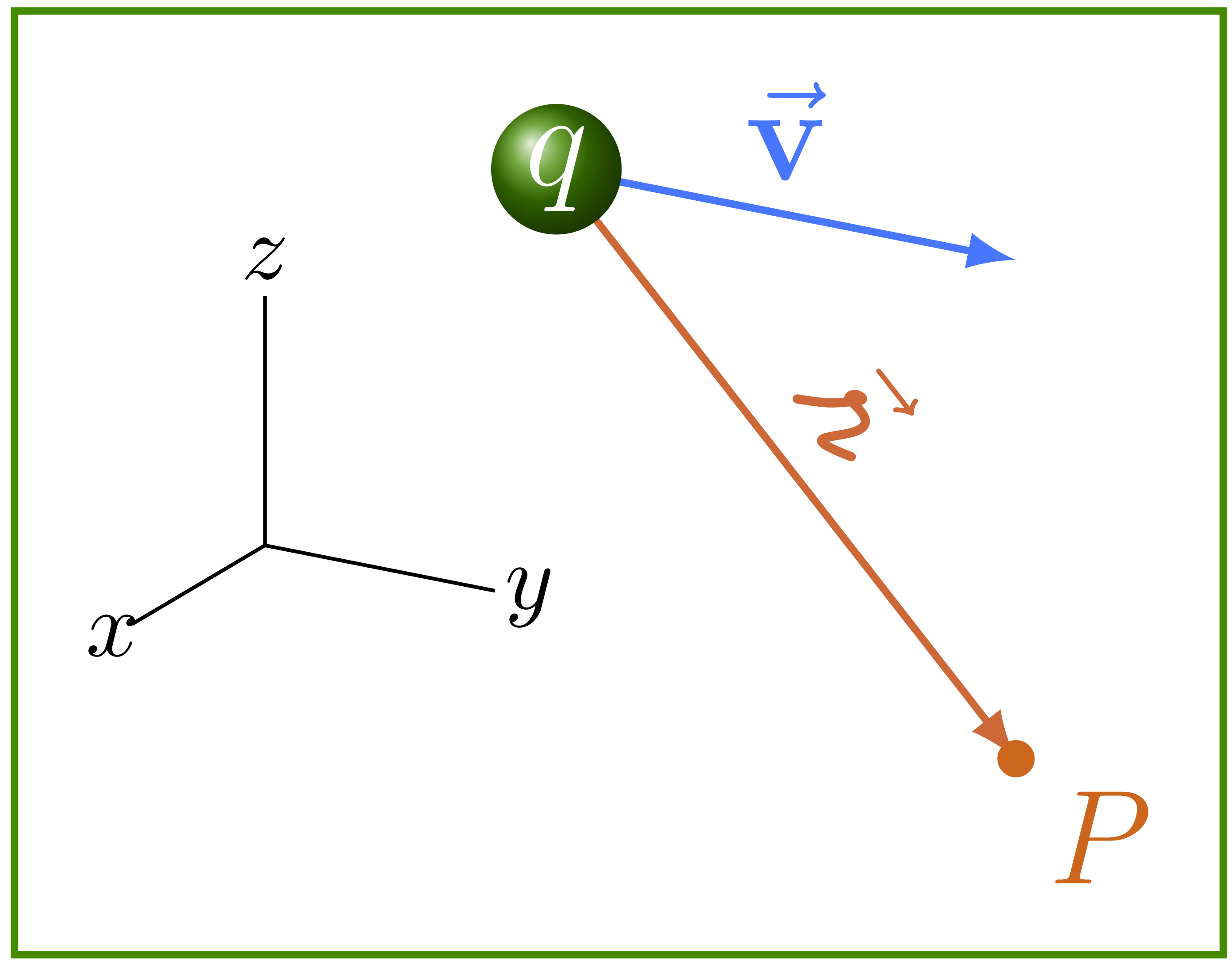
# Eletromagnetismo Avançado

*27 de novembro*
*Potenciais e campos*

# Potenciais de Liénard e Wiechert

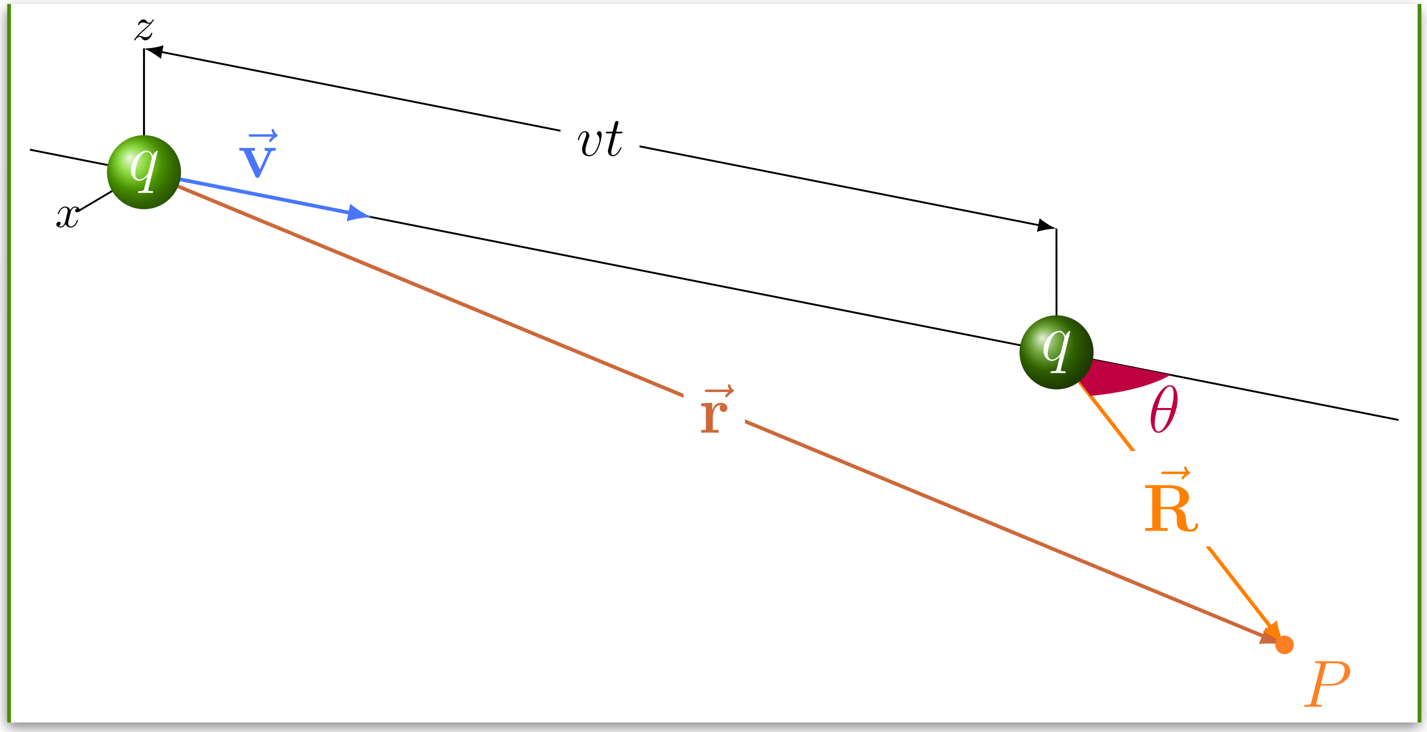
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \dfrac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \frac{q\vec{\mathbf{v}}}{\mathscr{r} - \dfrac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



# Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$



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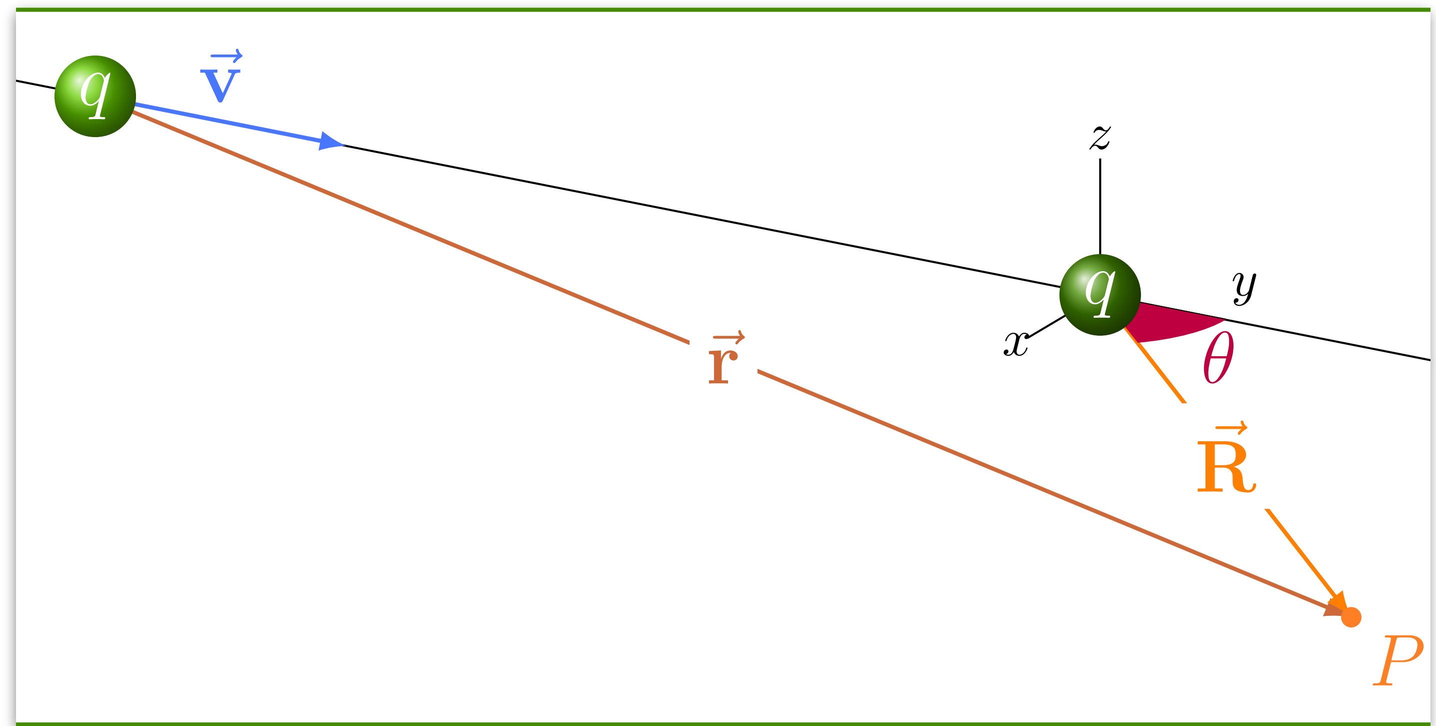
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \dfrac{v^2}{c^2}\sin^2\theta}}$$

$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \partial_t \vec{\mathbf{A}}$$



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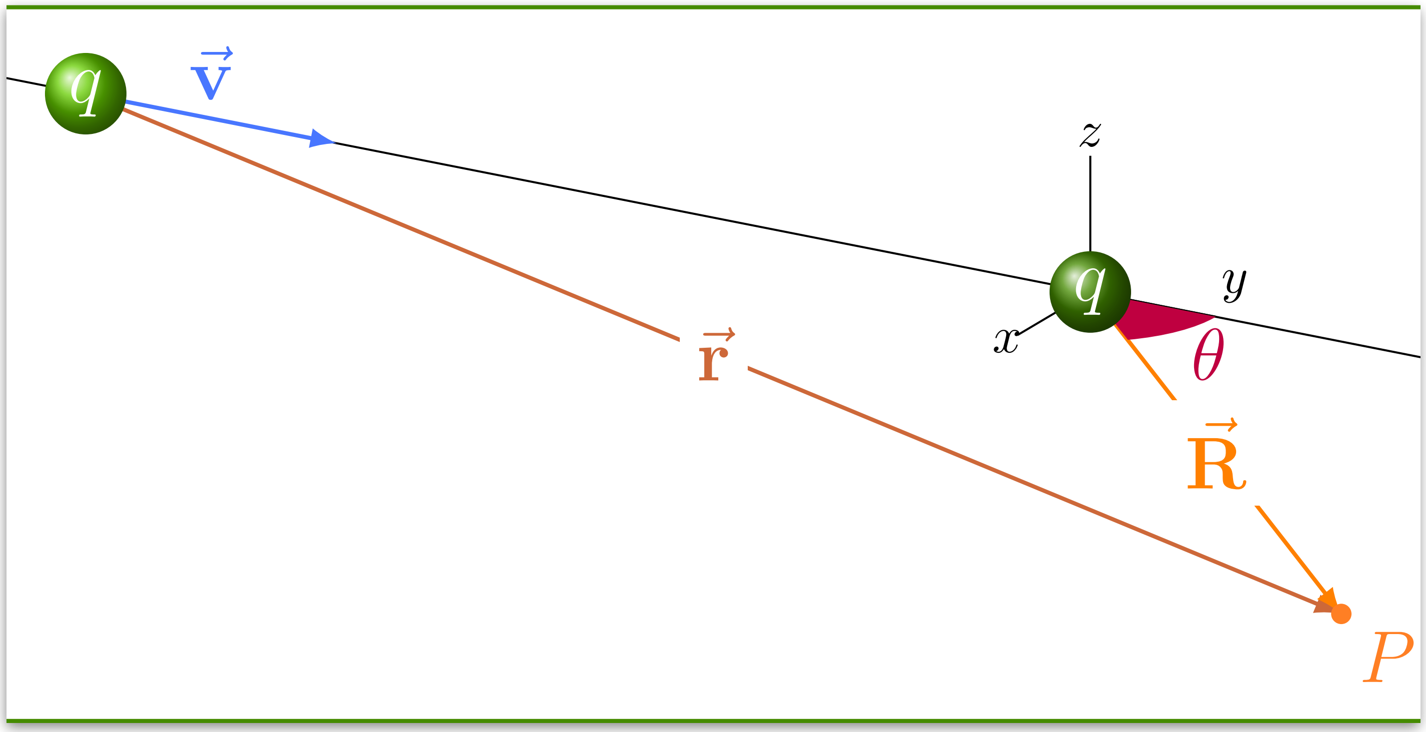



$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \partial_t \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = \frac{q}{4\pi\epsilon_0} \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}} \frac{\hat{\mathbf{R}}}{R^2}$$

# Campo magnético

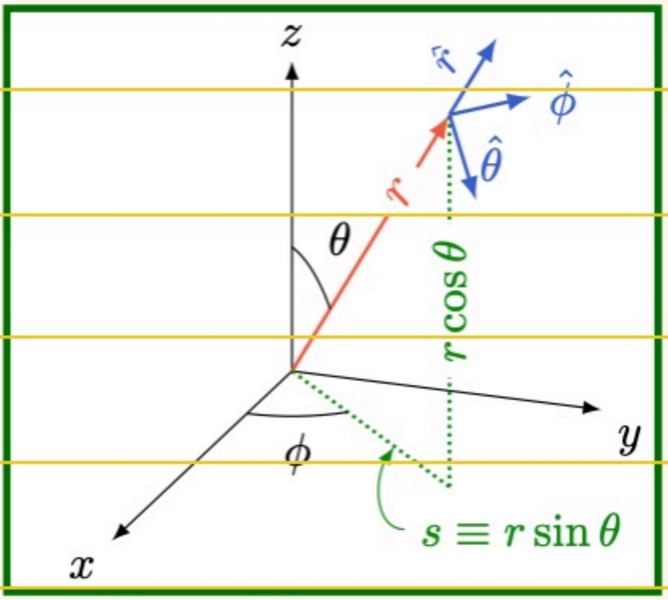
$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{qv}{4\pi\epsilon_0 c^2} \frac{\cos\theta\hat{\mathbf{R}} - \sin\theta\hat{\theta}}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$

# Campo magnético

## Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$




$$\vec{\nabla}t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

# Campo magnético

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{qv}{4\pi\epsilon_0 c^2} \frac{\cos\theta\hat{\mathbf{R}} - \sin\theta\hat{\theta}}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}} = \frac{1}{R}\left(\frac{\partial(Rv_\theta)}{\partial R} - \frac{\partial v_R}{\partial\theta}\right)\hat{\varphi}$$

# Campo magnético

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\frac{q}{R\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}$$

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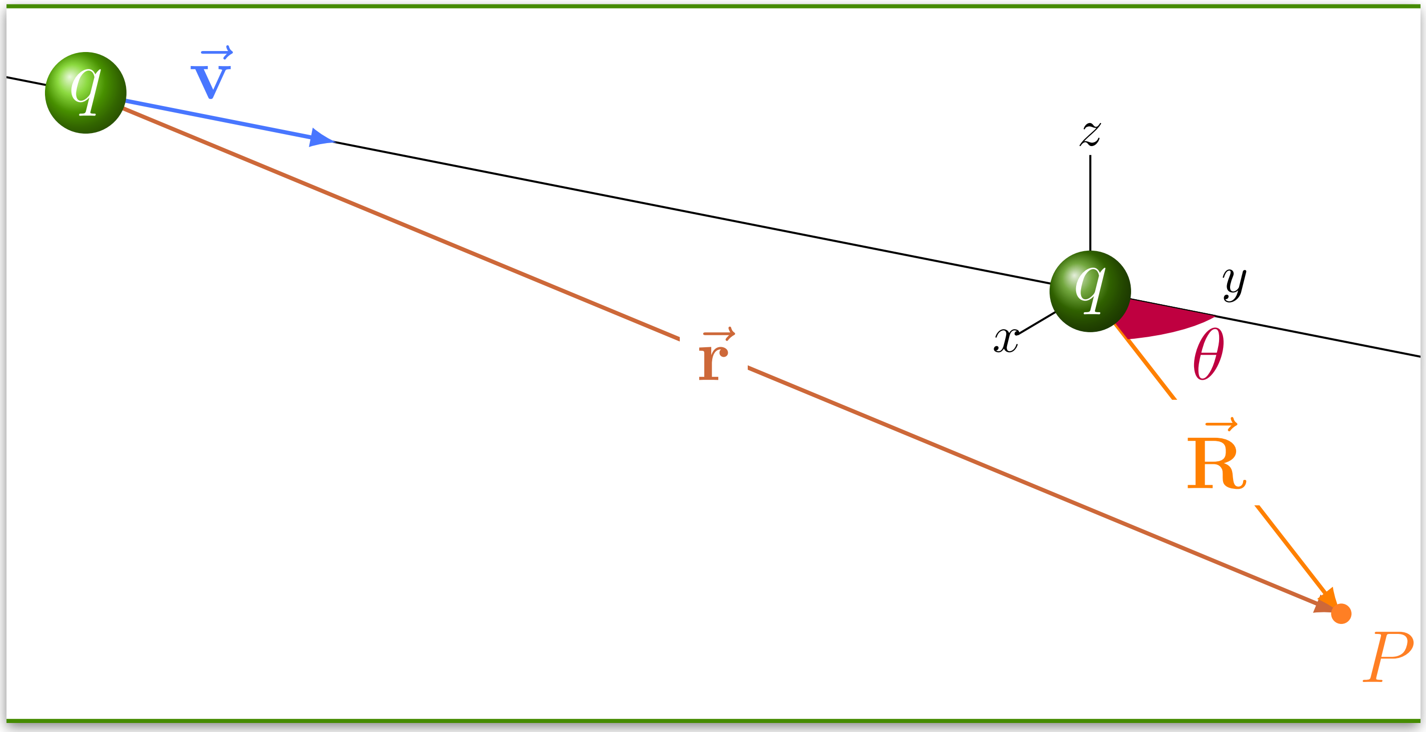
$$\vec{\mathbf{\nabla}} \times \vec{\mathbf{A}} = \frac{1}{R}\left(\frac{\partial(Rv_\theta)}{\partial R} - \frac{\partial v_R}{\partial\theta}\right)\hat{\varphi}$$

$$\vec{\mathbf{B}} = \frac{qv\sin\theta}{4\pi\epsilon_0 c^2}\frac{1-\frac{v^2}{c^2}}{R^2\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}}\hat{\varphi}$$

# Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$



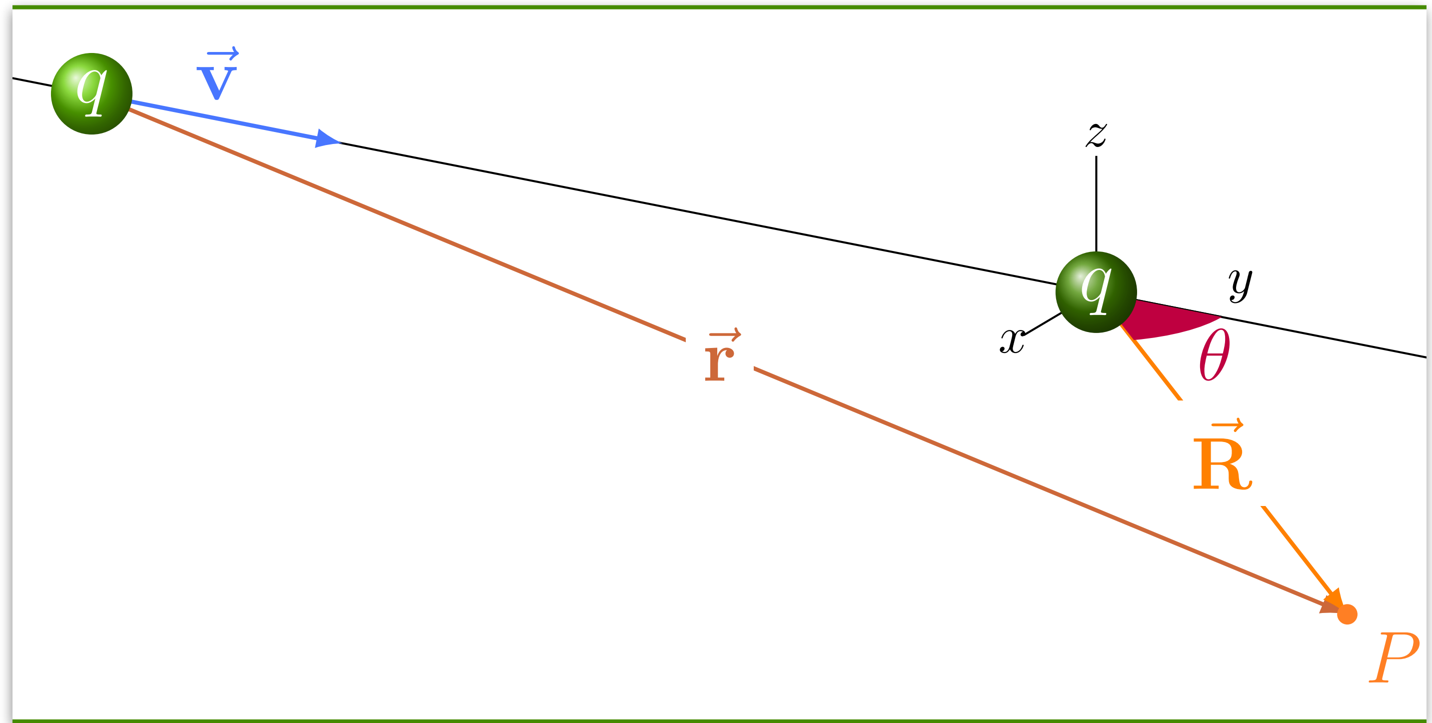

$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \partial_t \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = \frac{q}{4\pi\epsilon_0} \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}} \frac{\hat{\mathbf{R}}}{R^2}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 q}{4\pi} \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}} v \sin\theta \frac{\hat{\varphi}}{R^2}$$

# Movimento uniforme

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\frac{q}{R\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}$$




$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}}V - \partial_t\,\vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = \frac{q}{4\pi\epsilon_0}\frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}}\frac{\hat{\mathbf{R}}}{R^2}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 q}{4\pi}\frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}}v\sin\theta\frac{\hat{\varphi}}{R^2}$$

$$\vec{\mathbf{B}} = \frac{\vec{\mathbf{v}}}{c^2}\times\vec{\mathbf{E}}$$

# Movimento uniforme

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\frac{q}{R\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}$$

$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}}V - \partial_t\,\vec{\mathbf{A}}$$

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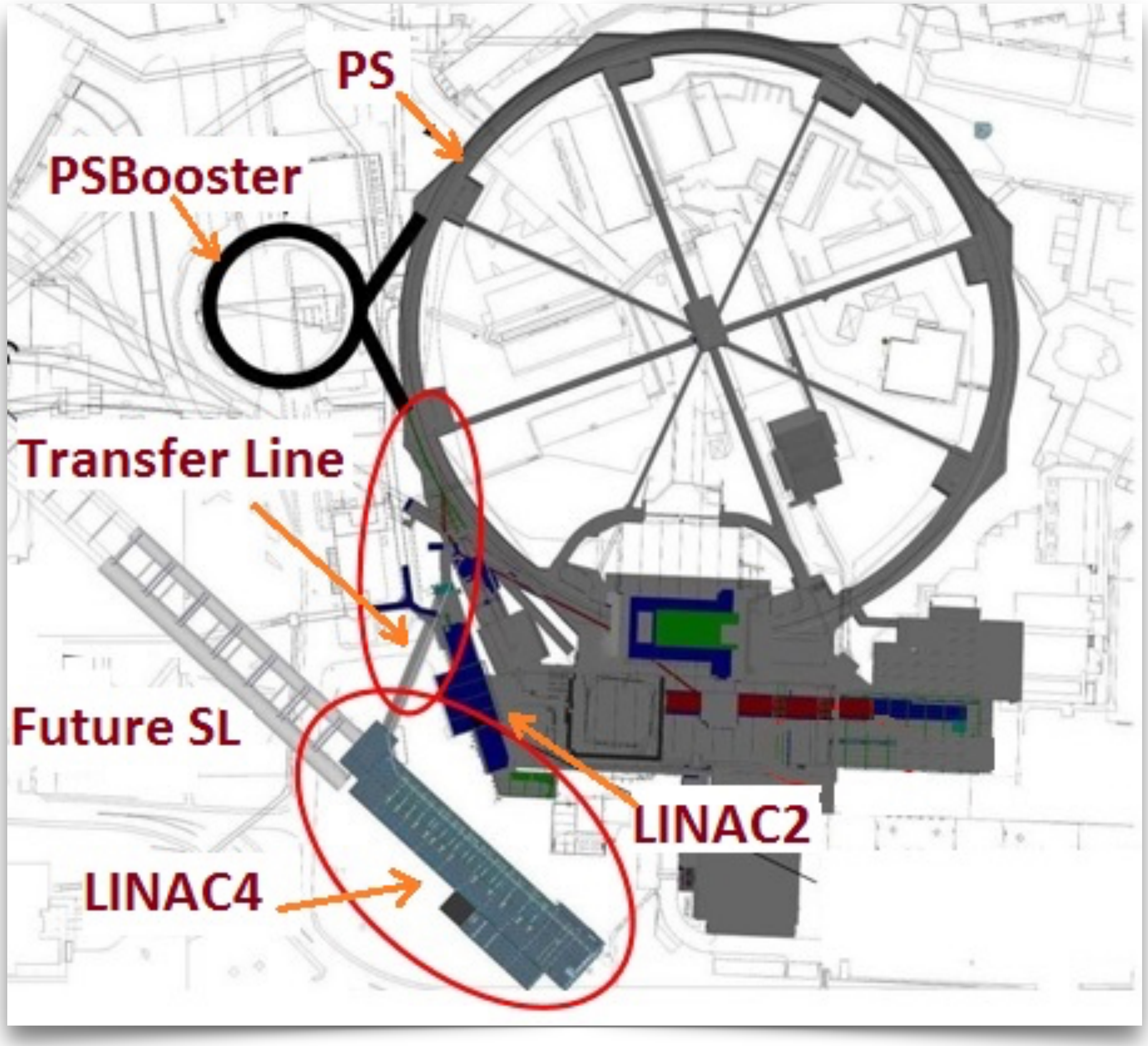
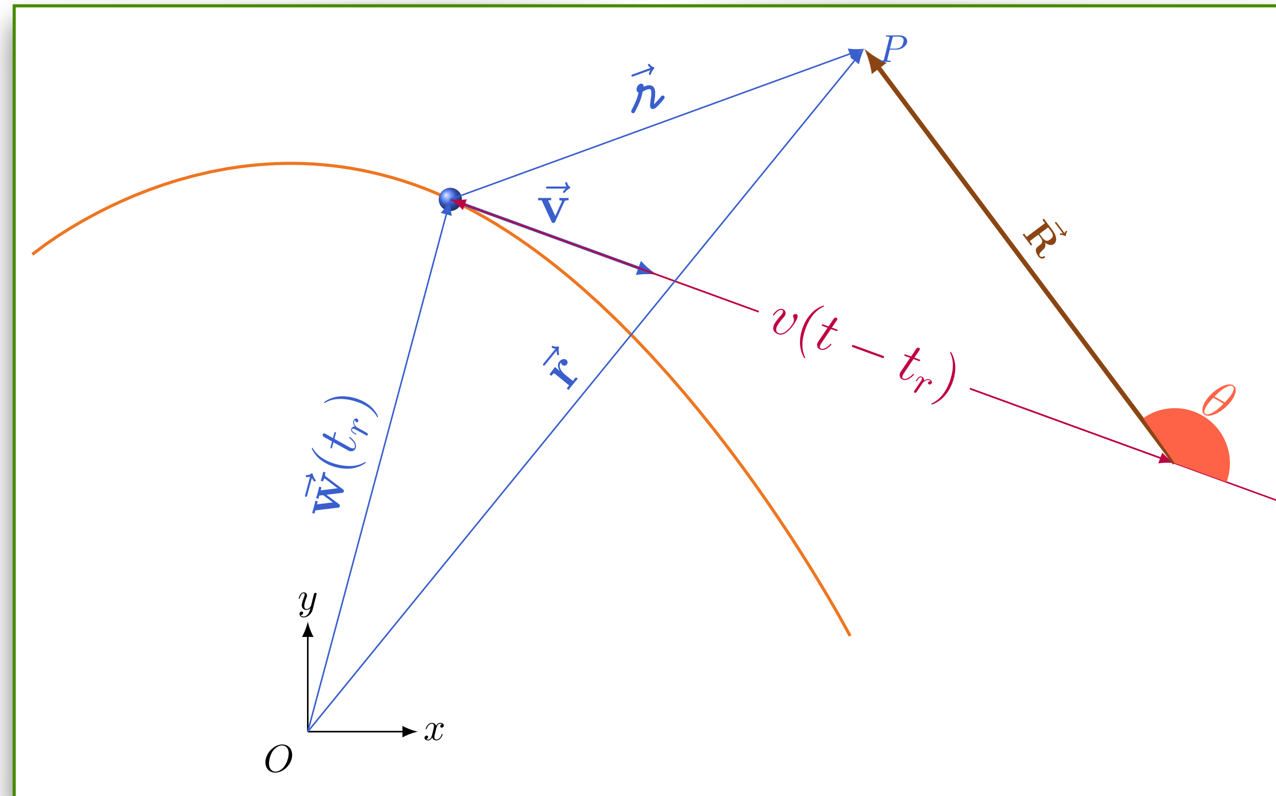


# Movimento acelerado

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{q}{4\pi\epsilon_0 c^2} \frac{\vec{\mathbf{v}}}{R(1 - \frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}}$$




$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \partial_t \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = \frac{q}{4\pi\epsilon_0} \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}} \frac{\hat{\mathbf{R}}}{R^2} - \left(\frac{\partial \vec{\mathbf{A}}}{\partial t}\right)_{\vec{\mathbf{v}}}$$

# Movimento acelerado

$$V(\vec{\mathbf{r}},t)=\frac{1}{4\pi\epsilon_0}\frac{q}{R\sqrt{1-\dfrac{v^2}{c^2}\sin^2\theta}}$$

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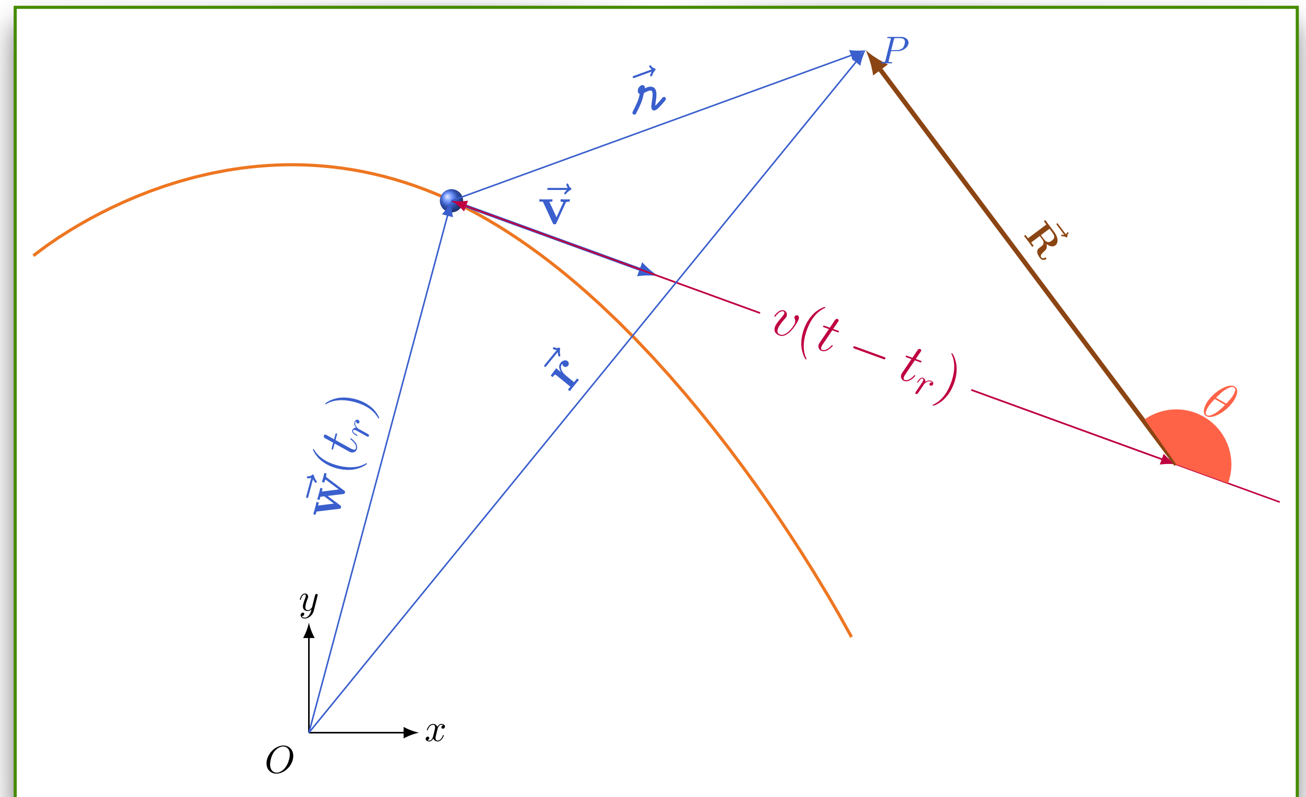



$$\vec{\mathbf{E}}=-\vec{\boldsymbol{\nabla}}V-\partial_t\,\vec{\mathbf{A}}$$

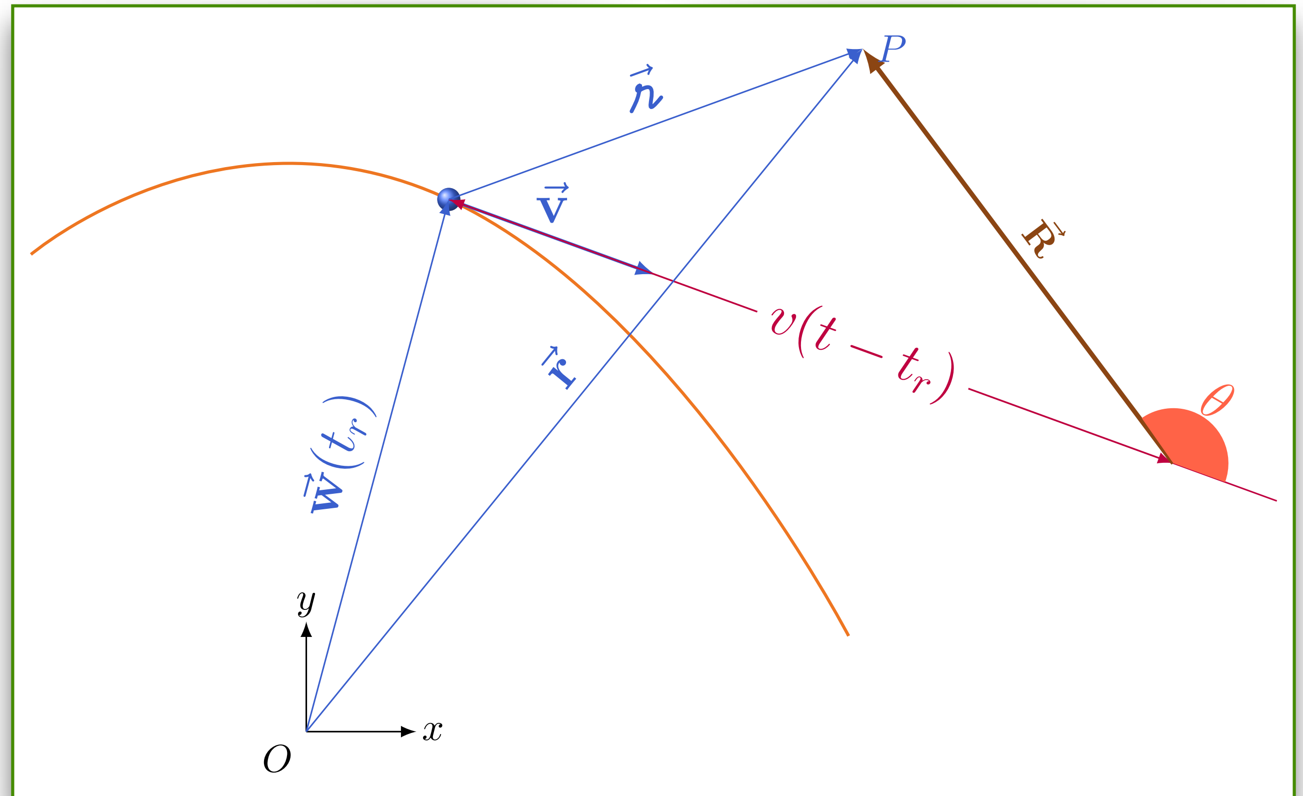
$$\vec{\mathbf{E}}=\frac{q}{4\pi\epsilon_0}\frac{1-\dfrac{v^2}{c^2}}{\left(1-\dfrac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}}\frac{\hat{\mathbf{R}}}{R^2}-\left(\frac{\partial\vec{\mathbf{A}}}{\partial t}\right)_{\vec{\mathbf{v}}}$$

$$\frac{\partial\vec{\mathbf{A}}}{\partial t}_{v}=\frac{q}{4\pi\epsilon_0 c^2}\left(\frac{\vec{\mathbf{a}}}{R(1-\dfrac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}}+\frac{\vec{\mathbf{v}}\dfrac{v\dot{v}}{c^2}\sin^2\theta}{(1-\dfrac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}R}\right)$$

# Movimento acelerado

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\frac{q}{R\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{q}{4\pi\epsilon_0 c^2}\frac{\vec{\mathbf{v}}}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}}$$




$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}}V - \partial_t\,\vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = \frac{q}{4\pi\epsilon_0}\frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}}\frac{\hat{\mathbf{R}}}{R^2} - \left(\frac{\partial\vec{\mathbf{A}}}{\partial t}\right)_{\vec{\mathbf{v}}}$$

$$\frac{\partial\vec{\mathbf{A}}}{\partial t}_v = \frac{q}{4\pi\epsilon_0 c^2}\left(\frac{\vec{\mathbf{a}}(1-\frac{v^2}{c^2}\sin^2\theta)}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}} + \frac{\vec{\mathbf{v}}\frac{v\dot{v}}{c^2}\sin^2\theta}{(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}R}\right)$$

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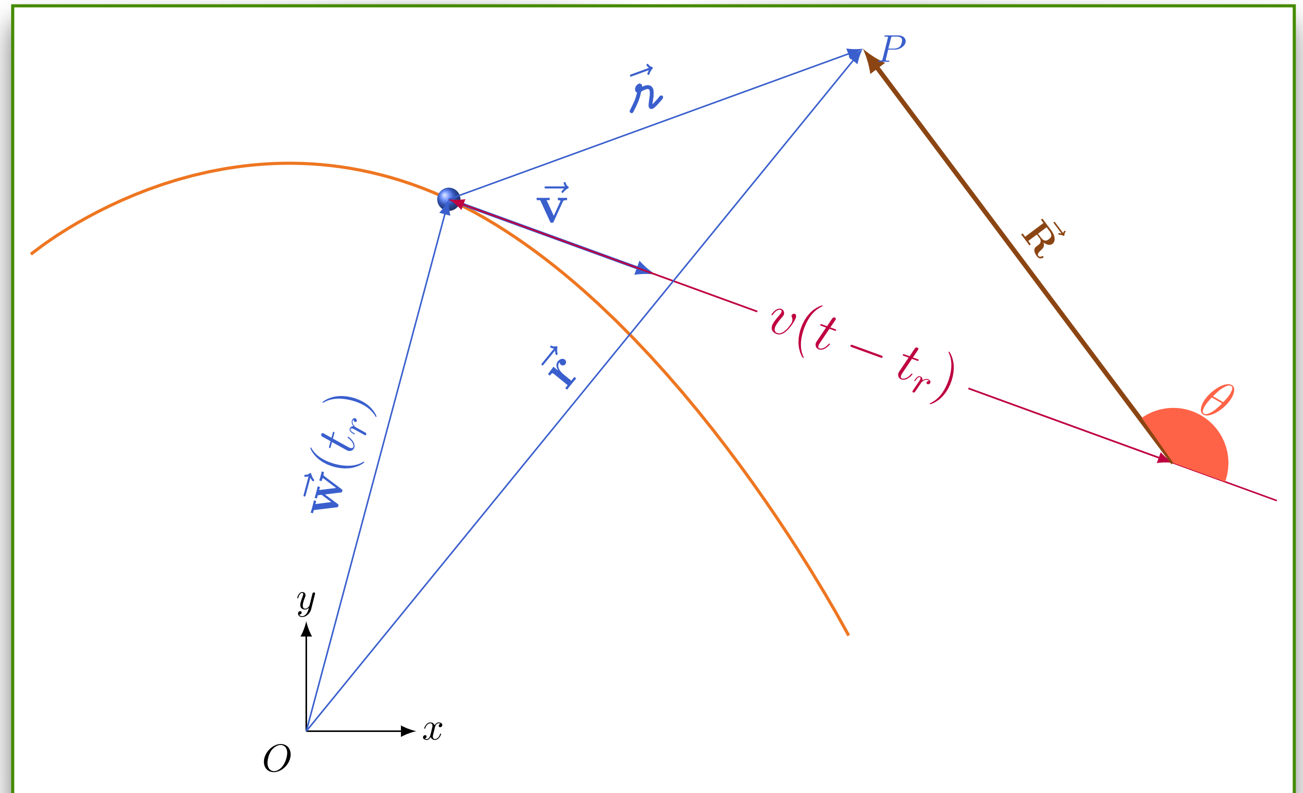


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$$\frac{\partial\vec{\mathbf{A}}}{\partial t}_{v} = \frac{q}{4\pi\epsilon_0 c^2}\left(\frac{\vec{\mathbf{a}}}{R(1-\dfrac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}} + \frac{\dfrac{\vec{\mathbf{v}}v\dot{v}-\vec{\mathbf{a}}v^2}{c^2}\sin^2\theta}{(1-\dfrac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}R}\right)$$

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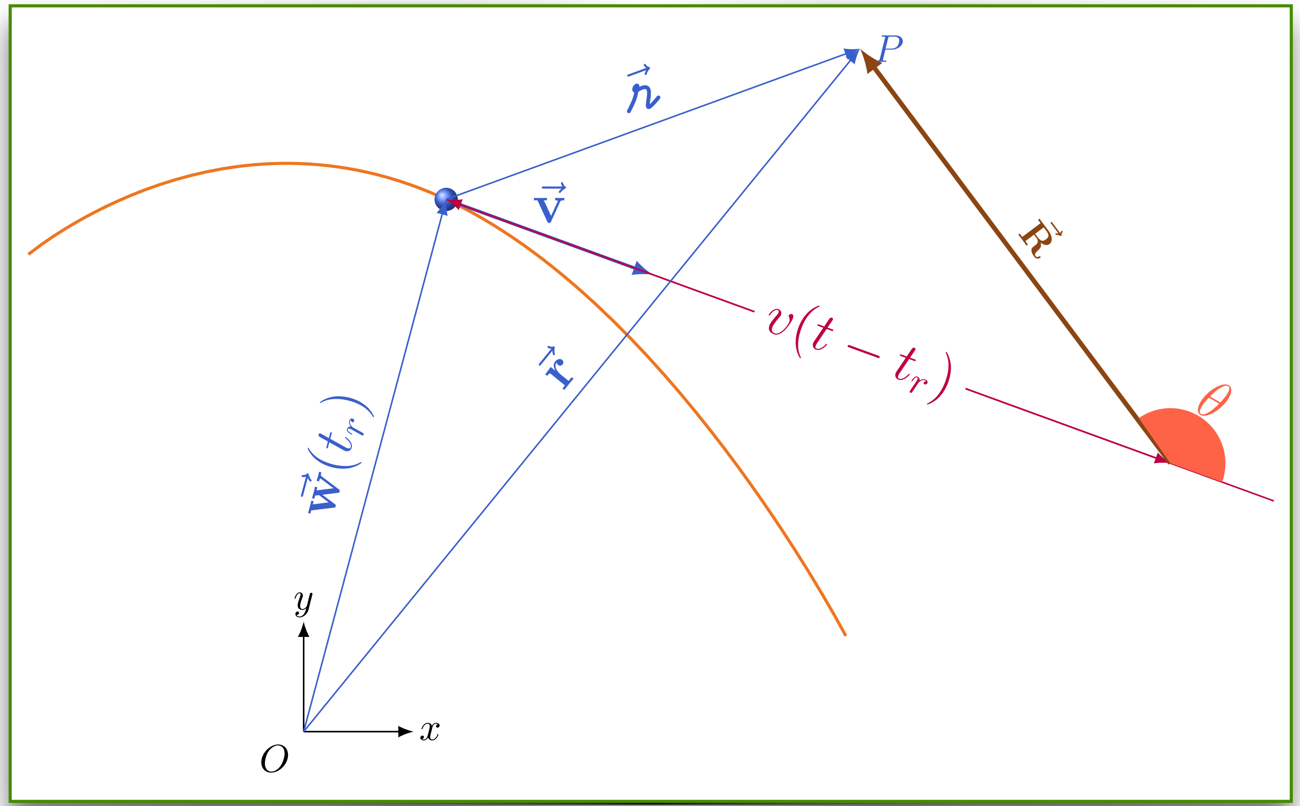


$$\vec{\mathbf{v}} = v(\cos\theta\hat{\mathbf{R}} - \sin\theta\hat{\boldsymbol{\theta}})$$

$$\vec{\mathbf{a}} = \dot{v}(\cos\theta\hat{\mathbf{R}} - \sin\theta\hat{\boldsymbol{\theta}}) - v\dot{\theta}(\sin\theta\hat{\mathbf{R}} + \cos\theta\hat{\boldsymbol{\theta}})$$

$$\Rightarrow \quad \vec{\mathbf{v}}\cdot\vec{\mathbf{a}} = v\dot{v}$$

# Movimento acelerado

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$

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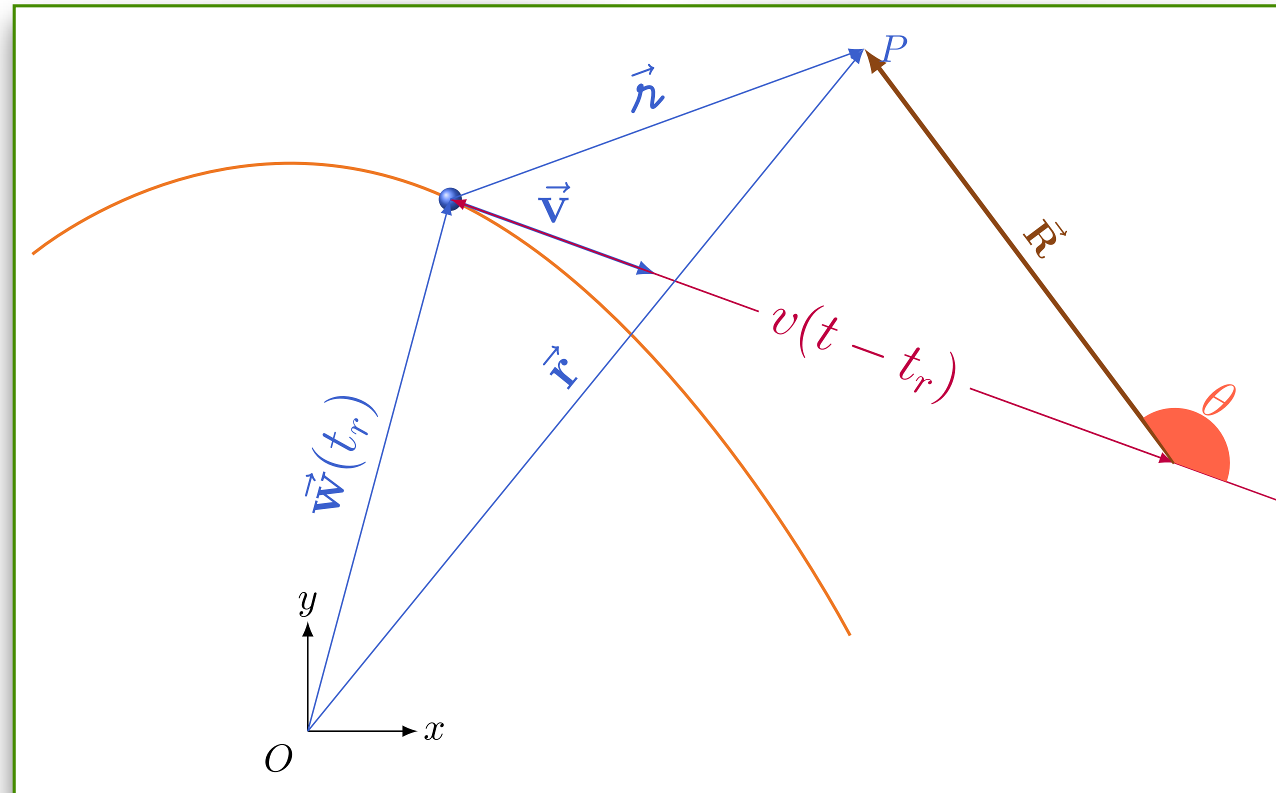


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$$\frac{\partial \vec{\mathbf{A}}}{\partial t}_v = \frac{q}{4\pi\epsilon_0 c^2}\left(\frac{\vec{\mathbf{a}}}{R(1 - \frac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}} + \frac{\frac{(\vec{\mathbf{v}}\cdot\vec{\mathbf{a}})\vec{\mathbf{v}} - v^2\vec{\mathbf{a}}}{c^2}\sin^2\theta}{(1 - \frac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}} R}\right)$$

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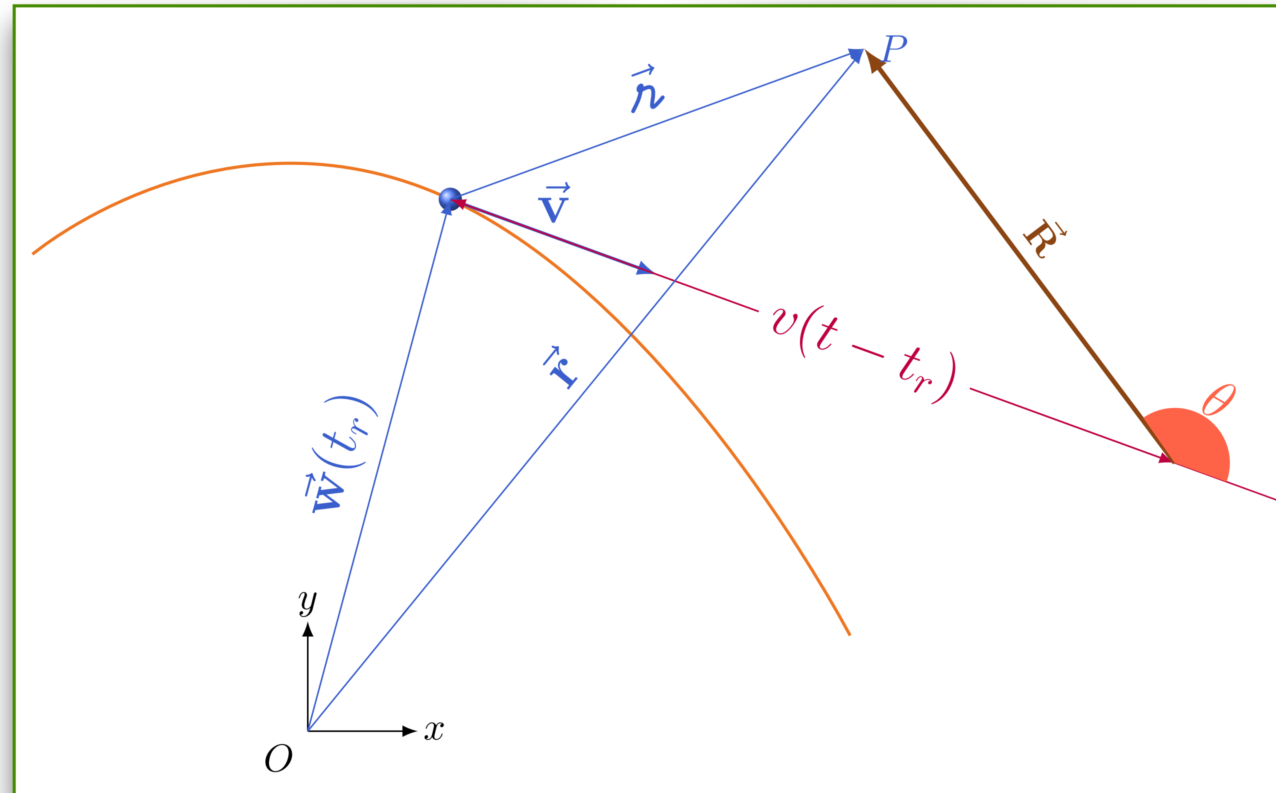


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$$\frac{\partial\vec{\mathbf{A}}}{\partial t}_v = \frac{q}{4\pi\epsilon_0 c^2}\frac{1}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}}\left(\vec{\mathbf{a}}+\vec{\mathbf{v}}\times(\vec{\mathbf{v}}\times\vec{\mathbf{a}})\frac{\sin^2\theta}{c^2}\right)$$