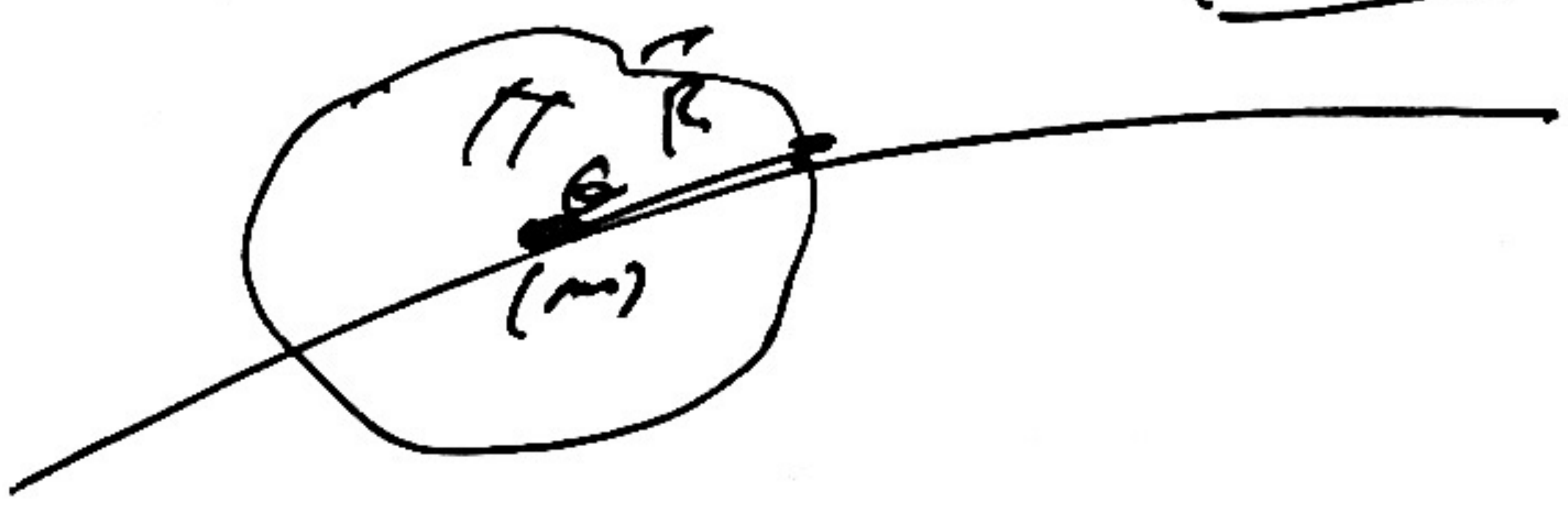
ATIVIDADE 28/11 PME 3100.

DINÂMICA DO CORPO RÍGIDO

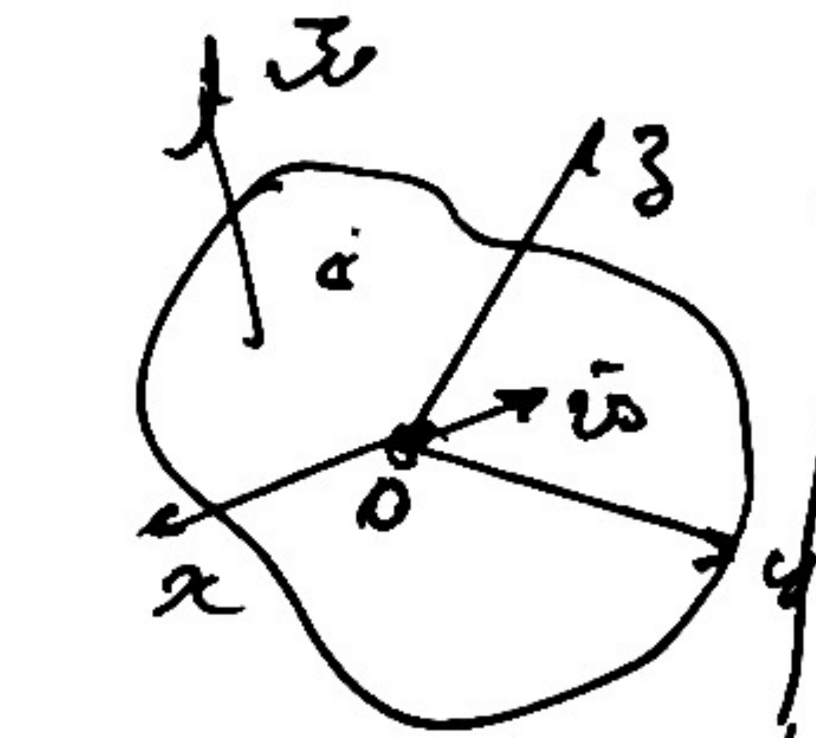
- TEO DA RESULTANTE (T.R) (T.M.B)

$$\boxed{m\vec{a}_G = \vec{R}^{EXT}}$$



- TEO DA ENERGIA CINÉTICA

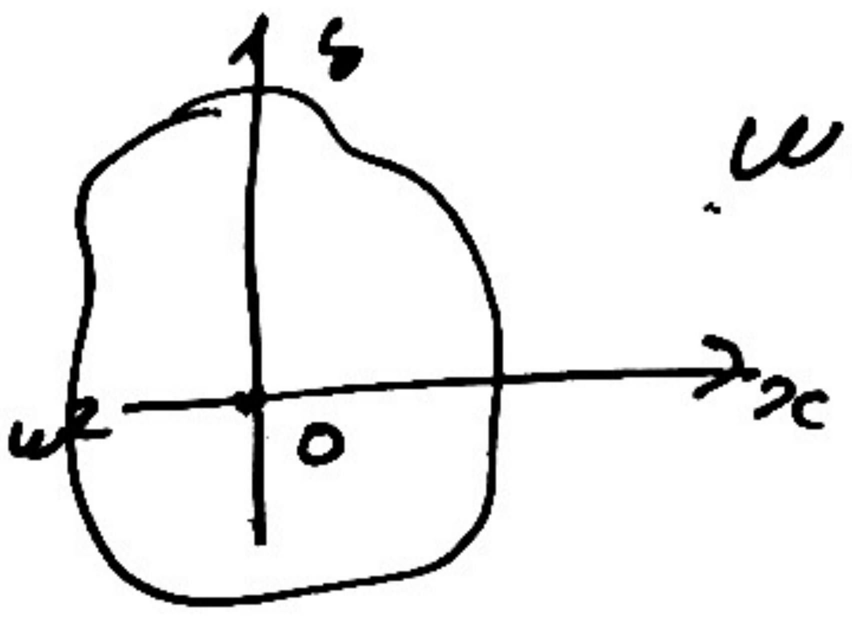
$$\boxed{\Delta T = \tau^{EXT}} \qquad \boxed{\Delta T = T - T_0}$$



$$\boxed{T = \frac{1}{2} m v_O^2 + m\vec{v}_O \cdot \vec{\omega} \wedge (G-O) + \frac{1}{2} J_x \omega_x^2 + \frac{1}{2} J_y \omega_y^2 + \frac{1}{2} J_z \omega_z^2 - J_{xy}\omega_x\omega_y - J_{xz}\omega_x\omega_z - J_{yz}\omega_y\omega_z}$$

$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$

MOVIMENTO PLANO



$\omega = \omega_z \vec{k}$

$\omega_x = \omega_y = 0$

$$T = \frac{1}{2} m v_O^2 + m\vec{v}_O \cdot \vec{\omega} \wedge (G-O) + \frac{1}{2} J_z \omega^2$$

TMA

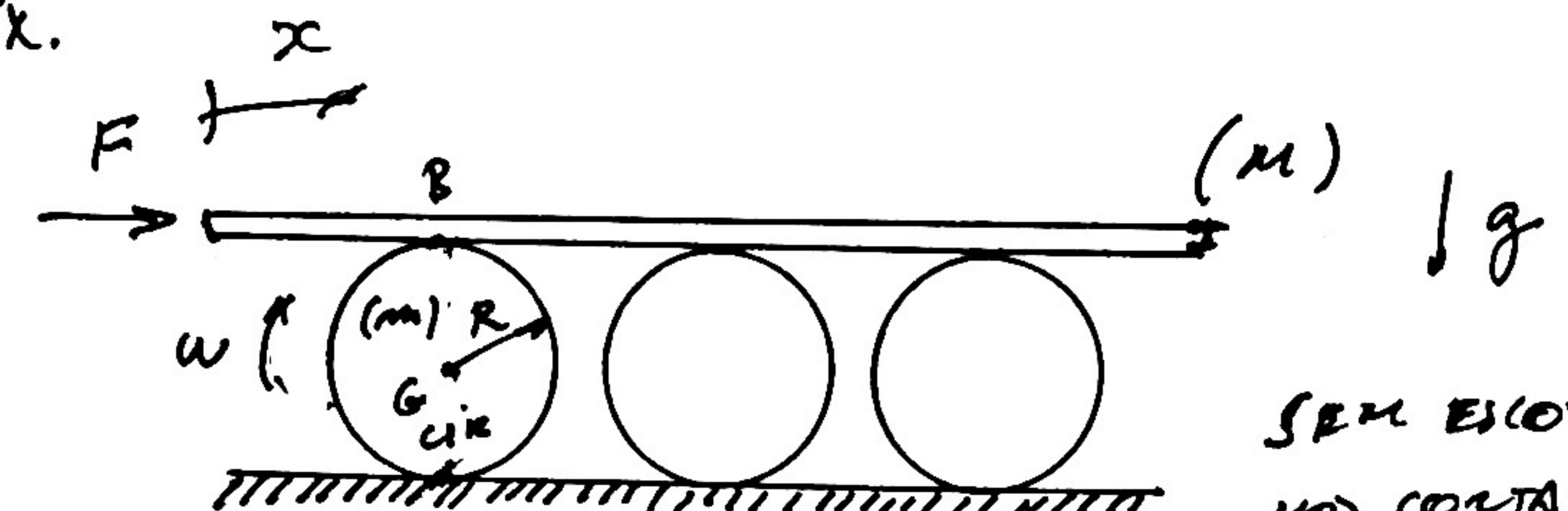
- TEO DA VARIAÇÃO DA QUANTIDADE DE MOVIMENTO ANGULAR ~~(TVQMA)~~
- TEO DO MOMENTO DA QUANTIDADE DE MOVIMENTO (T.M.QM)
- TEO DA QUANTIDADE DE MOVIMENTO ANGULAR (~~TMA~~) T.QMA

$\boxed{\dot{\vec{H}}_O = \vec{M}_O^{EXT}}$ P/ O FIXO OU O ≡ G

$$\boxed{\vec{H}_O = m(G-O)\wedge\vec{v}_O + (J_x\vec{i} - J_{xy}\vec{j} - J_{xz}\vec{k})\omega_x + (-J_{xy}\vec{i} + J_y\vec{j} - J_{yz}\vec{k})\omega_y + (-J_{xz}\vec{i} - J_{yz}\vec{j} + J_z\vec{k})\omega_z}$$

MOVIMENTO PLANO $\boxed{\vec{H}_O = m(G-O)\wedge\vec{v}_O + J_O\omega\vec{k}}$

Ex.

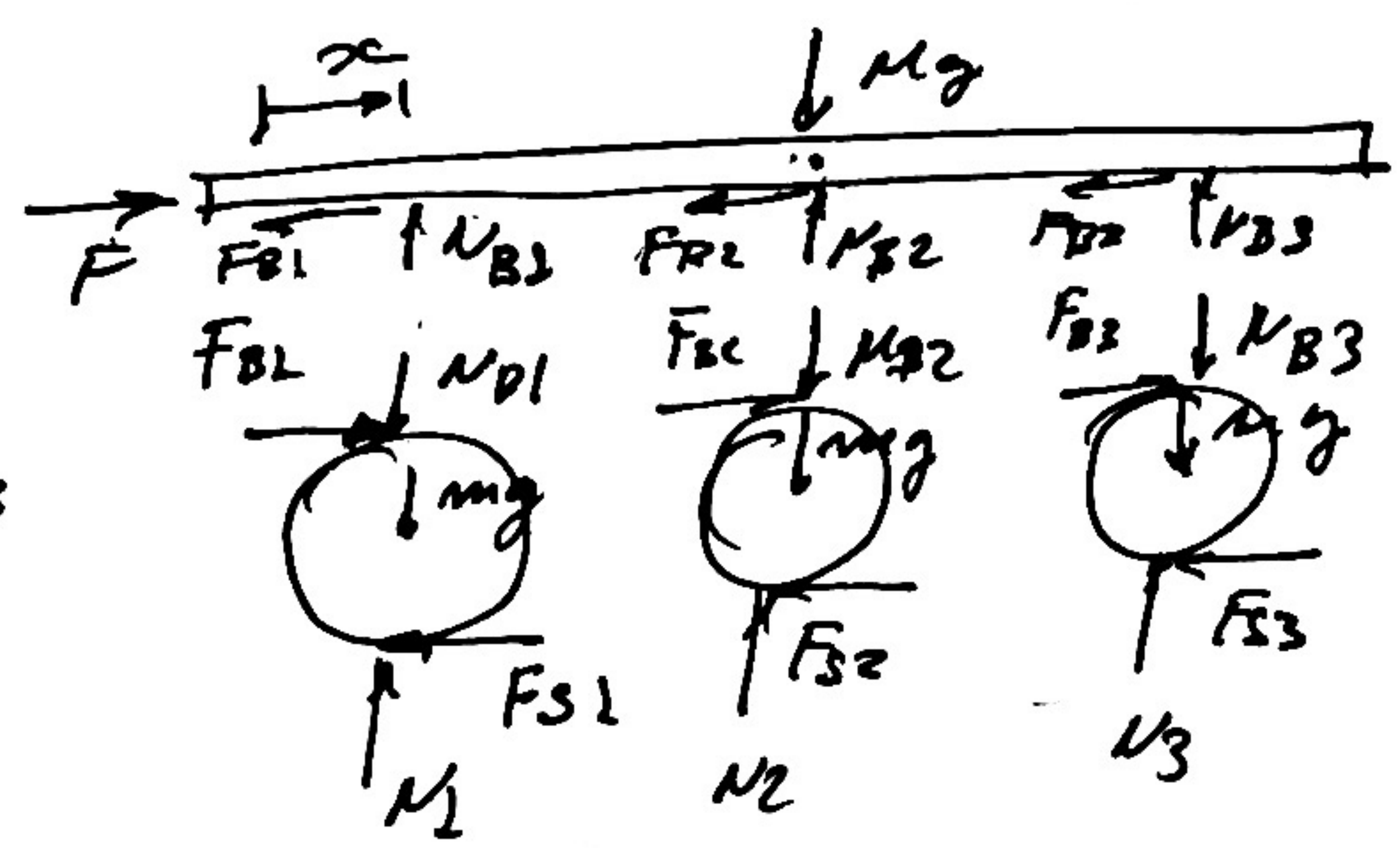


SEM ESCORREGAMENTO NOS CONTATOS

$\boxed{\text{ACELERAÇÃO DA BARRA } a_B \text{ ?}}$

1. DIAGRAMA DE CORPO LIVRE
2. A EXPRESSÃO DA ENERGIA CINÉTICA DO SISTEMA.
3. ACELERAÇÃO DA BARRA.

1) D.C.L.



$\dot{x} = v_B$

2) $$T = T_B + 3T_D = \frac{1}{2} M v_B^2 + 3\left(\frac{1}{2} m v_G^2 + \frac{1}{2} J_G \omega^2\right)$$

$\boxed{\begin{aligned} v_B &= 2\omega R \\ a_B &= 2\dot{\omega} R \end{aligned}}$ $\boxed{v_G = \omega R}$ $\boxed{J_G = \frac{1}{2} m R^2}$

$$T = \frac{1}{2} M \, 4\omega^2 R^2 + 3\left(\frac{1}{2} m \omega^2 R^2 + \frac{1}{4} m R^2 \omega^2\right)$$

$$T = \frac{8}{4} M R^2 \omega^2 + \frac{9}{4} m R^2 \omega^2 = \left(\frac{8M + 9m}{4}\right) \omega^2 R^2$$

3) T.E.C $\quad \Delta T = \tau^{EXT} \quad \Delta T = T - \cancel{T_0}^{\,0}$

$$\tau^{EXT} = Fx$$

$$\left(\frac{8M + 9m}{4}\right) \omega^2 R^2 = F . x$$

Obs: $v_B^2 = 4\omega^2 R^2 \Rightarrow \omega^2 R^2 = \frac{v_B^2}{4}$

$$\left(\frac{8M+9m}{4}\right)\frac{v_B^2}{4} = Fx$$

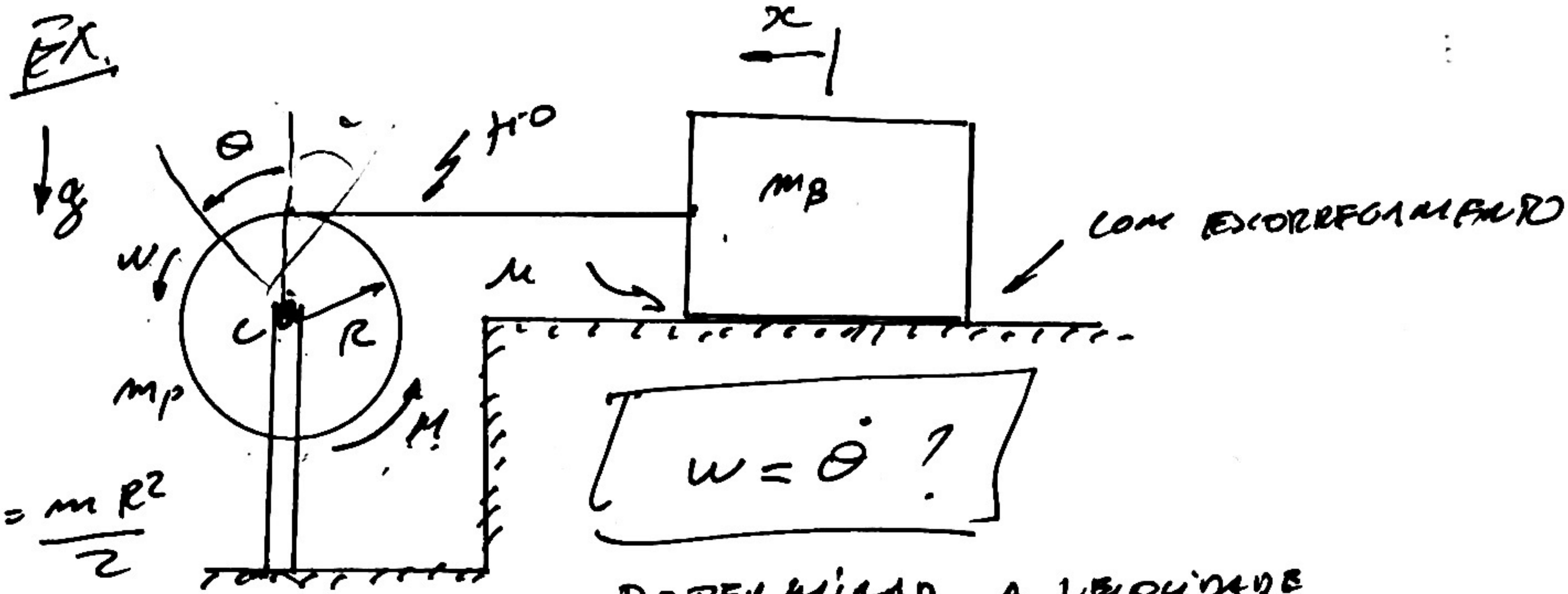
$$v_B^2 = \frac{16Fx}{(8M+9m)}$$

DERIVANDO EM RELAÇÃO AO TEMPO:

$$\cancel{2v_B}\cdot a_B = \frac{16F}{(8M+9m)}\cancel{\dot{x}} \qquad \dot{x} = v_B$$

$$\boxed{a_B = \frac{8F}{(8M+9m)}}$$

4) EX.



$J_C = J_O = \frac{m R^2}{2}$

$$\boxed{\omega = \dot{\theta}\ ?}$$

DETERMINAR A VELOCIDADE ANGULAR DA POLIA.

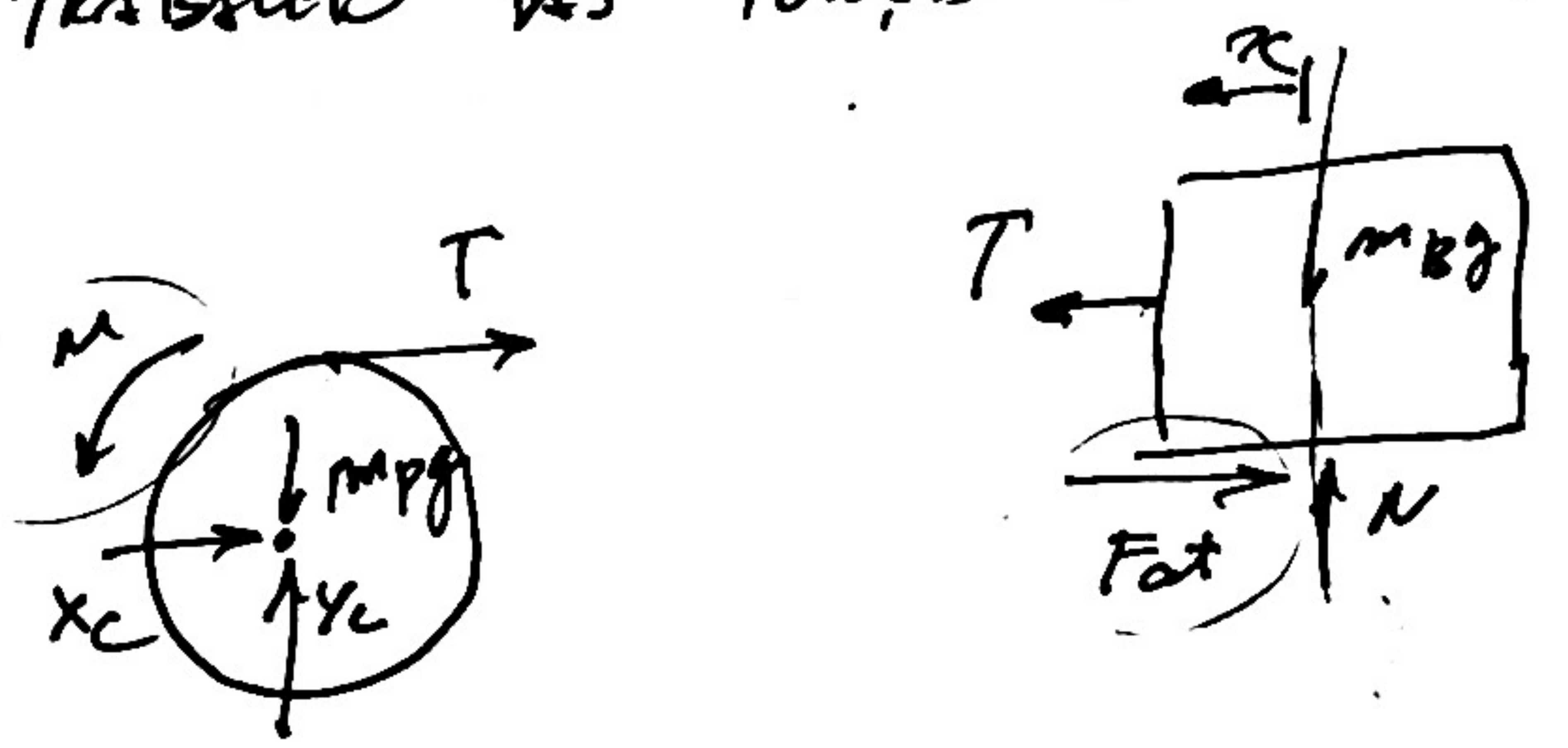
ENERGIA CINÉTICA DO SISTEMA:

$$T = T_D + T_B = \frac{1}{2} J_C \omega^2 + \frac{1}{2} m_B v_B^2 \qquad (v_B = \omega R)$$

$$T = \frac{1}{2} \cdot \frac{1}{2} m_p R^2 \omega^2 + \frac{1}{2} m_B \omega^2 R^2$$

$$\boxed{T = \frac{R^2 \omega^2}{4} (m_p + 2 m_B)}$$

TRABALHO DAS FORÇAS EXTERNAS:



$F_{at} = \mu N = \mu m_B g$

ESCORREGAMENTO

$\tau_M = M\theta$

$\tau_{F_{at}} = -F_{at} \cdot x = -x \mu m_B g$

$\tau_{F_{at}} = -\mu m_B g R \theta$

$\tau^{EXT} = M\theta - \mu m_B g R \theta$

TEC. $\Delta T = \tau^{EXT}$ $\qquad \Delta T = T - \cancel{T_0} \to 0$

$$\frac{R^2\omega^2}{4}(m_p+2m_B) = M\theta - \mu m_B g R\theta$$

$$\omega^2 = \frac{4(M\theta - \mu m_B g R\theta)}{R^2(m_p+2m_B)}$$

$$\boxed{\omega = \frac{2}{R}\sqrt{\frac{(M\theta - \mu m_B g R\theta)}{(m_p+2m_B)}}}$$

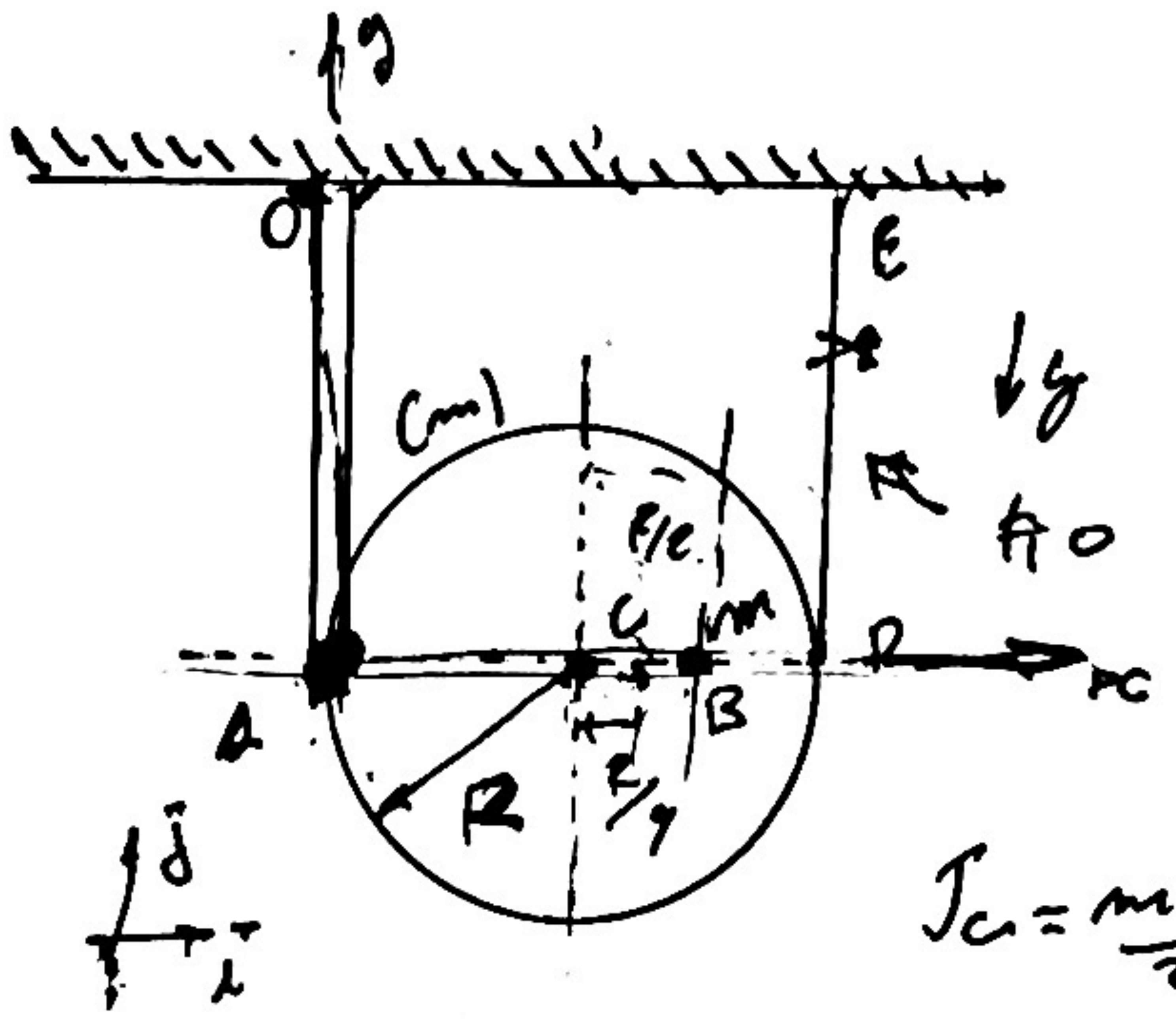
$\dot\omega = ?$ DERIVANDO ω^2

$$\cancel{2}\cancel{\omega}\dot\omega = \frac{\cancel{4}^{\,2}(M\cancel{\dot\theta} - \mu m_B g R\cancel{\dot\theta})}{R^2(m_p+2m_B)} \qquad \dot\theta = \omega$$

$$\boxed{\dot\omega = \frac{2M - 2\mu m_B g R}{R^2(m_p+2m_B)}}$$

' $\tau = F.d$ ''
$(N.m)$

$T. \quad M.\theta$
$(N.m)$



PARA O INSTANTE DE ROMPIMENTO DO FIO.

- $\dot{\omega}$ DO DISCO
- REAÇÕES VINCULARES EM A

$J_C = \frac{mR^2}{2}$

a) COORDENADAS DO CENTRO DE MASSA. (BARICENTRO).

b) J_A (DISCO + MASSA)

c) $\dot{\omega}$

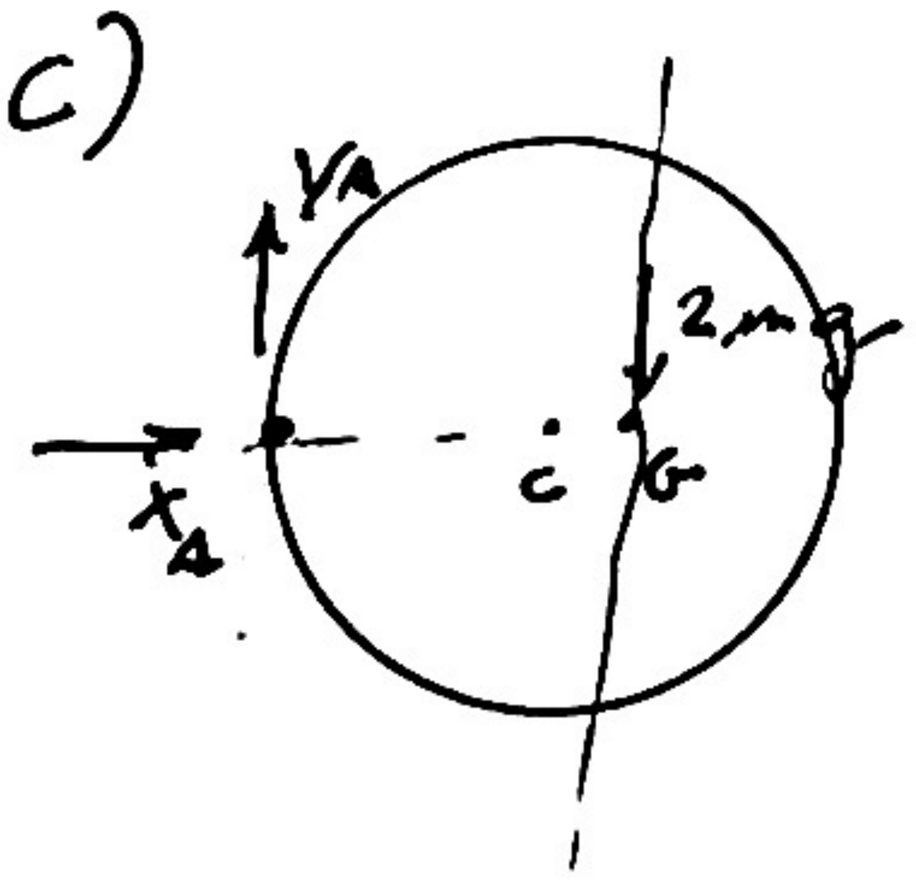
d) REAÇÕES VINCULARES

SOLUÇÃO

a) $\boxed{y_G = 0}$ $\quad \boxed{x_G = R + \frac{R}{4} = \frac{5R}{4}}$

b) $J_A = \underbrace{\frac{mR^2}{2} + mR^2}_{\text{DISCO}} + \underbrace{m\left(\frac{3R}{2}\right)^2}_{\text{MASSA CONC.}} = \boxed{\frac{15}{4} mR^2}$

c)



T.Q.M.A.

$\dot{\vec{H}}_A = \vec{M}_A^{EXT.}$

$J_A \dot{\omega} (-\vec{k}) = 2mg \frac{5}{4} R (-\vec{k})$

$\frac{\cancel{15}^{3}}{\cancel{4}} \cancel{m} R^{\cancel{2}} \dot{\omega} = 2\cancel{m}g \frac{\cancel{5}}{\cancel{4}} \cancel{R}$

$\boxed{\dot{\omega} = \frac{2g}{3R}}$ $\quad \dot{\vec{\omega}} = -\frac{2g}{3R} \vec{k}$

T.R. (T.M.B) $\boxed{2m\vec{a}_G = \vec{R}^{EXT}}$

$$\vec{a}_G = \vec{a}_A^{\,0} + \dot{\vec{\omega}} \wedge (G-A) + \cancel{\vec{\omega} \wedge [\vec{\omega} \wedge (G-A)]}$$

$$\vec{a}_A = \vec{0} \qquad \dot{\vec{\omega}} = -\frac{2g}{3R}\vec{k} \qquad (G-A) = \frac{5}{4}R\vec{i} \qquad \vec{\omega} = \vec{0}$$

$$\vec{a}_G = -\frac{2g}{3R}\vec{k} \wedge \frac{5}{4}R\vec{i} = -\frac{5}{6}g\vec{j}$$

$$\vec{R}^{EXT} = X_A\vec{i} + (Y_A - 2mg)\vec{j}$$

$$2m\left(-\frac{5}{6}g\vec{j}\right) = X_A\vec{i} + (Y_A - 2mg)\vec{j}$$

$$-\frac{5}{3}mg\vec{j} = X_A\vec{i} + (Y_A - 2mg)\vec{j}$$

$\vec{i}$) $\boxed{X_A = 0}$

$\vec{j}$) $-\frac{5}{3}mg = Y_A + 2mg$

$$\boxed{Y_A = \frac{mg}{3}}$$