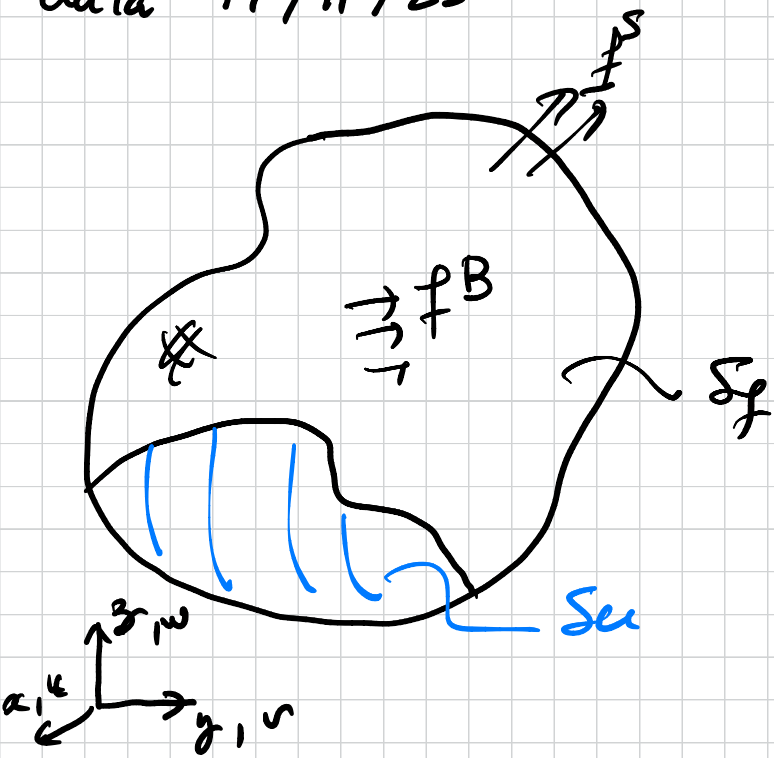
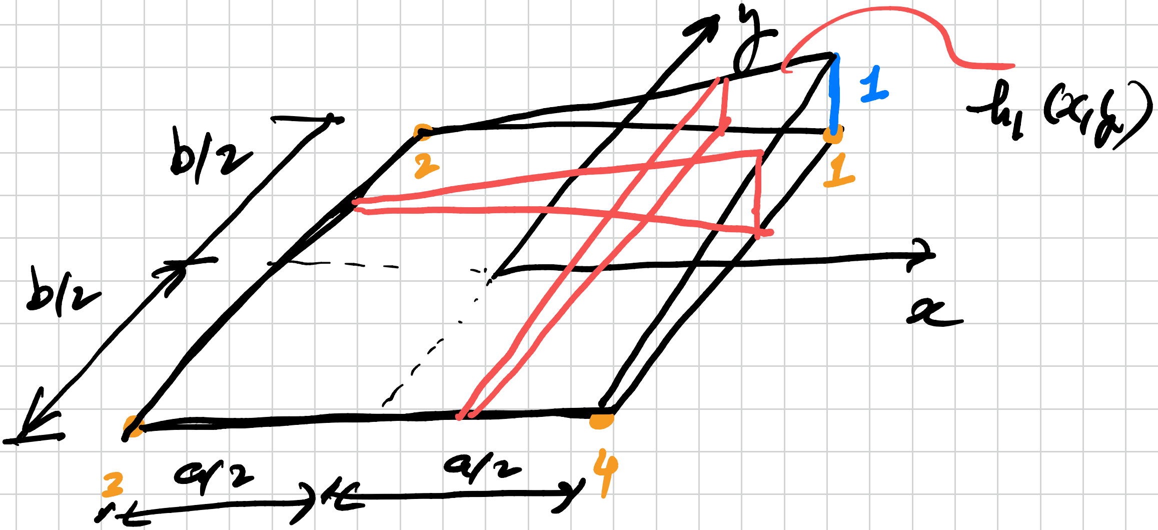
Aula 14/11/23



$\underline{\sigma}$

$\underline{\varepsilon}$



$$h_1(x,y) = \frac{1}{4}\left(1 + \frac{x}{(a/2)}\right)\left(1 + \frac{y}{(b/2)}\right)$$

nó 1 $\left(\frac{a}{2}, \frac{b}{2}\right)$ $\quad \left(1 + \frac{a/2}{a/2}\right)\left(1 + \frac{b/2}{b/2}\right)$

nó 2 $\left(-\frac{a}{2}, \frac{b}{2}\right)$ $\quad \cancelto{0}{\left(1 + \frac{(-a/2)}{(a/2)}\right)}$ $\quad 0$

nó 3 $\left(-a/2, -\frac{b}{2}\right)$ $\quad \cancelto{0}{\left(1 + \frac{(-a/2)}{a/2}\right)} \left(1 + \frac{-b/2}{b/2}\right)$

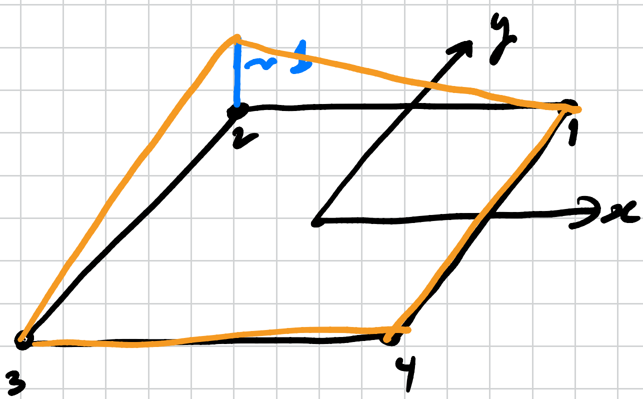
nó 4 $\left(a/2, -b/2\right)$ $\quad \cancelto{0}{\left(1 + \frac{(-b/2)}{b/2}\right)}$

$$h_1(x,y)\Big|_{\text{nó } 1} = h_1(x_1, y_1) = h_1\left(\frac{a}{2}, \frac{b}{2}\right) = 1$$

$$h_1(x,y)\Big|_{\text{nó } 2} = h_1(x_2, y_2) = h_1\left(-\frac{a}{2}, \frac{b}{2}\right) = 0$$

$$h_1(x,y)\Big|_{\text{nó } 3} = h_1(x_3, y_3) = h_1\left(-\frac{a}{2}, -\frac{b}{2}\right) = 0$$

$$h_1(x,y)\Big|_{\text{nó } 4} = h_1(x_4, y_4) = h_1\left(\frac{a}{2}, -\frac{b}{2}\right) = 0$$



$$h_2(x,y) = \frac{1}{4}\left(1 - \frac{x}{(a/2)}\right)\left(1 + \frac{y}{(b/2)}\right)$$

$$h_3(x,y) = \frac{1}{4}\left(1 - \frac{x}{(a/2)}\right)\left(1 - \frac{y}{(b/2)}\right)$$

$$h_4(x,y) = \frac{1}{4}\left(1 + \frac{x}{(a/2)}\right)\left(1 - \frac{y}{(b/2)}\right)$$

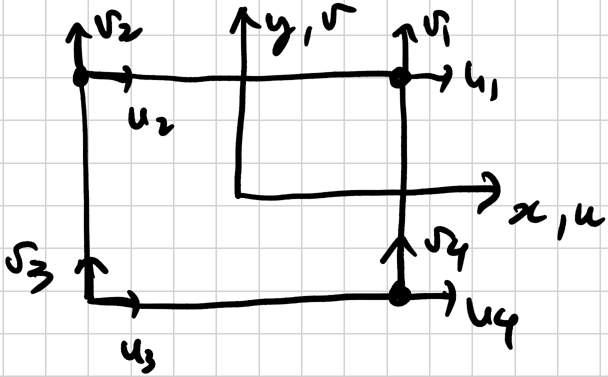
$$\boxed{u(x,y) = h_1(x,y)\,u_1 + h_2(x,y)\,u_2 + h_3(x,y)\,u_3 + h_4(x,y)\,u_4}$$

$$u(x_1,y_1) = 1.u_1 + 0.u_2 + 0.u_3 + 0.u_4 = u_1$$

$$u(x_2,y_2) = 0.u_1 + 1.u_2 + 0.u_3 + 0.u_4 = u_2$$

$$u(x_3,y_3) = u_3 \quad ; \quad u(x_4,y_4) = u_4$$

$$\boxed{v(x,y) = h_1(x,y)\,v_1 + h_2(x,y)\,v_2 + h_3(x,y)\,v_3 + h_4(x,y)\,v_4}$$



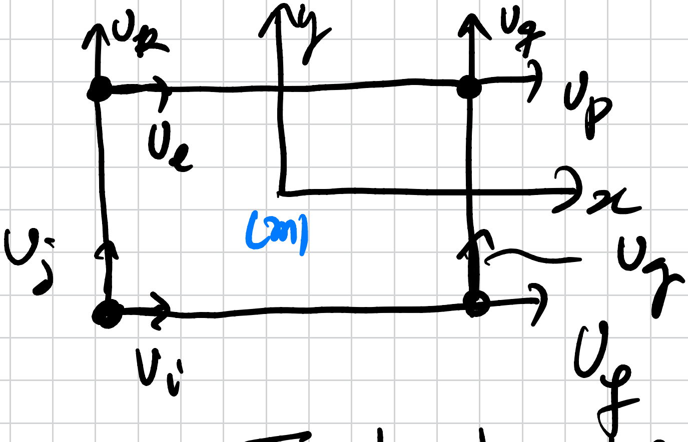
$$\{\hat{u}\} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

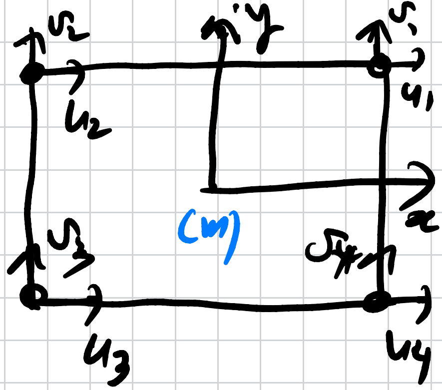
$$\{u^{(m)}\} = \begin{Bmatrix} u^{(m)} \\ v^{(m)} \end{Bmatrix}_{2\times 1} = [H]\{\hat{u}\}$$

$$= \begin{bmatrix} h_1 & 0 & h_2 & 0 & h_3 & 0 & h_4 & 0 \\ \hline 0 & h_1 & 0 & h_2 & 0 & h_3 & 0 & h_4 \end{bmatrix}_{2\times 8} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}_{8\times 1}$$

$$\{u^{(m)}\} = \begin{Bmatrix} u^{(m)} \\ v^{(m)} \end{Bmatrix} = [H^{(m)}]_{2\times N}\{U\}_{N\times 1}$$

$$\{U\}^T = \{U_1 \; U_2 \cdots\cdots U_i \; U_j \text{ ----- } U_{N-1} \; U_N\}$$





$$\begin{Bmatrix} u^{(m)} \\ v^{(m)} \end{Bmatrix} = \left[\begin{array}{cccccccccccccc} 0 & 0 & \cdots & h_4 & 0 & \cdots & 0 & h_3 & 0 & 0 & h_2 & 0 & h_1 & 0 \\ & & & & h_4 & & & & h_3 & & & h_2 & & h_1 \end{array}\right] \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ \vdots \\ U_N \end{Bmatrix}$$

$$\begin{array}{cccccccccccccc} & & & \uparrow f & \uparrow g & & & i & j & & \uparrow l & \uparrow k & \uparrow p & \uparrow q \end{array}$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\sum_{i=1}^{4} h_i u_i\right)$$

$$= \sum_{i=1}^{4} \frac{\partial h_i}{\partial x} u_i$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \sum_{i=1}^{4} \frac{\partial h_i}{\partial y} v_i$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \sum_{i=1}^{4} \frac{\partial h_i}{\partial y} u_i + \sum_{i=1}^{4} \frac{\partial h_i}{\partial x} v_i$$

$$\{\varepsilon^{(m)}\} = \begin{Bmatrix} \varepsilon_{xx}^{(m)} \\ \varepsilon_{yy}^{(m)} \\ \gamma_{xy}^{(m)} \end{Bmatrix}_{3\times 1} = [B]_{3\times 8}\{\hat{u}\}_{8\times 1}$$

$$\begin{Bmatrix} \varepsilon_{xx}^{(m)} \\ \varepsilon_{yy}^{(m)} \\ \gamma_{xy}^{(m)} \end{Bmatrix} = \begin{bmatrix} \frac{\partial h_1}{\partial x} & 0 & \frac{\partial h_2}{\partial x} & 0 & \frac{\partial h_3}{\partial x} & 0 & \frac{\partial h_4}{\partial x} & 0 \\ 0 & \frac{\partial h_1}{\partial y} & 0 & \frac{\partial h_2}{\partial y} & 0 & \frac{\partial h_3}{\partial y} & 0 & \frac{\partial h_4}{\partial y} \\ \frac{\partial h_1}{\partial y} & \frac{\partial h_1}{\partial x} & \frac{\partial h_2}{\partial y} & \frac{\partial h_2}{\partial x} & \frac{\partial h_3}{\partial y} & \frac{\partial h_3}{\partial x} & \frac{\partial h_4}{\partial y} & \frac{\partial h_4}{\partial x} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

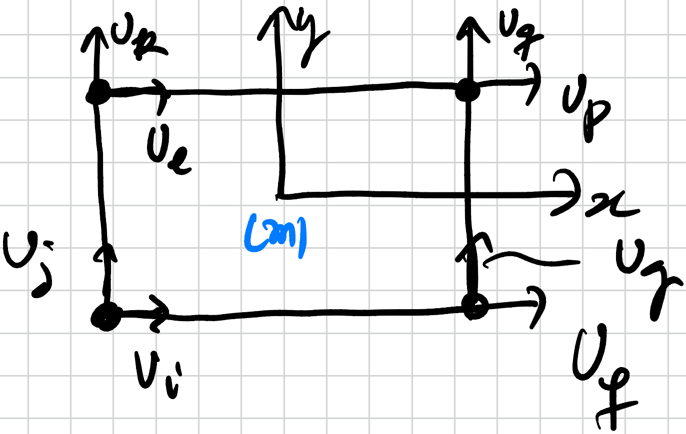
$$\varepsilon_{xx}^{(m)} = \sum_{i=1}^{4} \frac{\partial h_i}{\partial x} u_i = \frac{\partial h_1}{\partial x} u_1 + \frac{\partial h_2}{\partial x} u_2 + \frac{\partial h_3}{\partial x} u_3 + \frac{\partial h_4}{\partial x} u_4$$

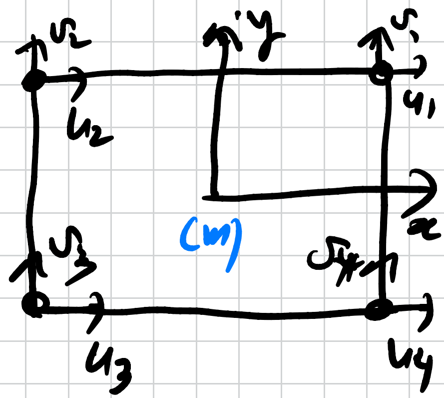
$$\varepsilon_{yy}^{(m)} = \sum_{i=1}^{4} \frac{\partial h_i}{\partial y} v_i$$

$$\gamma_{xy}^{(m)} = \sum_{i=1}^{4} \frac{\partial h_i}{\partial y} u_i + \sum_{i=1}^{4} \frac{\partial h_i}{\partial x} v_i$$

$$\frac{\partial h_1}{\partial y} u_1 + \frac{\partial h_2}{\partial y} u_2 + \frac{\partial h_3}{\partial y} u_3 + \frac{\partial h_4}{\partial y} u_4 \qquad \frac{\partial h_1}{\partial x} v_1 + \frac{\partial h_2}{\partial x} v_2 + \frac{\partial h_3}{\partial x} v_3 + \frac{\partial h_4}{\partial x} v_4$$

$$\{\varepsilon^{(m)}\} = [B]\{\hat{u}\}$$
$$= [B^{(m)}]\{U\}$$





$$\begin{Bmatrix} \varepsilon_{xx}^{(m)} \\ \varepsilon_{yy}^{(m)} \\ \gamma_{xy}^{(m)} \end{Bmatrix} = \begin{array}{c} \left[\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c|c}
0 & \cdots & \frac{\partial h_4}{\partial x} & 0 & 0 & \cdots & \frac{\partial h_3}{\partial x} & 0 & \cdots & 0 & \frac{\partial h_2}{\partial x} & \frac{\partial h_1}{\partial x} & 0 \\
0 & & 0 & \frac{\partial h_4}{\partial y} & 0 & & 0 & \frac{\partial h_3}{\partial y} & & \frac{\partial h_2}{\partial y} & 0 & 0 & \frac{\partial h_1}{\partial y} \\
0 & & \frac{\partial h_4}{\partial y} & \partial h_4/\partial x & 0 & & \frac{\partial h_3}{\partial y} & \partial h_3/\partial x & & \partial h_2/\partial x & \partial h_2/\partial y & \frac{\partial h_1}{\partial y} & \partial h_1/\partial x
\end{array}\right] \\
\begin{array}{ccccccccccccc} & & \uparrow f & \uparrow g & & & i & j & & \uparrow k & \uparrow \ell & \uparrow p & \uparrow q \end{array}
\end{array}
\begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ \vdots \\ v_N \end{Bmatrix}$$

Princípio dos Trabalhos Virtuais

$$\int_A \{\delta \varepsilon\}^T \{\sigma\}\, dA = \int_A \{\delta u\}^T \{f^B\}\, dA + \int_{L_f} \{\delta u\}^T \{f^S\}\, dL$$

$$\sum_{m=1}^{n_e} \int_{A^{(m)}} \{\delta \varepsilon^{(m)}\}^T \{\sigma^{(m)}\}\, dA^{(m)} = \sum_{m=1}^{n_e} \int_{A^{(m)}} \{\delta u^{(m)}\}^T \{f^{B(m)}\}\, dA^{(m)} + \sum_{m=1}^{n_e} \int_{L_1, L_2, \ldots L_f} \{\delta u^{(m)}\}^T \{f^{S(m)}\}\, dL^{(m)}$$

$$\{\sigma^{(m)}\} = [C^{(m)}] \{\varepsilon^{(m)}\}$$

$$\{\varepsilon^{(m)}\} = [B^{(m)}] \{U\}$$

$$\{u^{(m)}\} = [H^{(m)}] \{U\}$$

$$\{\delta \varepsilon^{(m)}\} = [B^{(m)}] \{\delta U\}$$

$$\{\delta u^{(m)}\} = [H^{(m)}] \{\delta U\}$$

$$\sum_{m=1}^{n_e} \int_{A^{(m)}} \underbrace{\{\delta U\}^T [B^{(m)}]^T}_{\{\delta \varepsilon^{(m)}\}^T} \underbrace{[C^{(m)}] \underbrace{[B^{(m)}] \{U\}}_{\{\varepsilon^{(m)}\}}}_{\{\sigma^{(m)}\}} \, dA^{(m)}$$

$$\{\delta U\}^T \left(\sum_{m=1}^{n_e} \int_{A^{(m)}} \underbrace{\underset{N\times 3}{[B^{(m)}]^T} \underset{3\times 3}{[C^{(m)}]} \underset{3\times N}{[B^{(m)}]} \, dA^{(m)}}_{N\times N} \right) \{U\}$$

$$\underset{N\times N}{[K^{(m)}]} = \int_{A^{(m)}} [B^{(m)}]^T [C^{(m)}] [B^{(m)}] \, dA^{(m)}$$

$$\underset{N\times N}{[K]} = \sum_{m=1}^{n_e} [K^{(m)}]$$

$$= \{\delta U\}^T [K] \{U\}$$

$$\sum_{m=1}^{n_e} \int_{A^{(m)}} \underbrace{\{\delta U\}^T [H^{(m)}]^T}_{\{\delta u^{(m)}\}^T} \{f^{B^{(m)}}\} \, dA^{(m)}$$

$$+ \sum_{m=1}^{n_e} \int_{L_1^{(m)} \ldots L_q^{(m)}} \underbrace{\{\delta U\}^T [H^{(m)}]^T}_{\{\delta u^{(m)}\}^T} \{f^{S^{(m)}}\} \, dL^{(m)}$$

$$\{\delta U\}^T \left(\sum_{m=1}^{n_e} \int_{A^{(m)}} [H^{(m)}]^T_{N\times 2} \{f^{B^{(m)}}\}_{2\times 1} \, dA^{(m)} \right)$$

$$+ \left(\sum_{m=1}^{n_e} \int_{L_1^{(m)} \ldots L_q^{(m)}} [H^{(m)}]^T_{N\times 2} \{f^{S^{(m)}}\}_{N\times 1} \, dL^{(m)} \right)$$

$$\{R_B^{(m)}\}_{N\times 1} = \int_{A^{(m)}} [H^{(m)}]^T_{N\times 2} \{f^{B^{(m)}}\}_{2\times 1} \, dA^{(m)}$$

$$\{R_B\} = \sum_{m=1}^{n_e} \{R_B^{(m)}\}$$

$$\{R_s^{(m)}\}_{N\times 1} = \int_{L_1^{(m)} \ldots L_g^{(m)}} [H^{(m)}]^T_{N\times 2} \{f^{s(m)}\}_{2\times 1} \, dL^{(m)}$$

$$\{R_s\} = \sum_{m=1}^{ne} \{R_s^{(m)}\}$$

$$\{\delta U^T\} (\{R_B\} + \{R_s\})$$

P.T.V

$$\{\delta U\}^T [K] \{U\} = \{\delta U\}^T (\{R_B\} + \{R_s\})$$

$$\underbrace{\{\delta U\}^T}_{1\times N} \underbrace{([K]\{U\} - (\{R_B\} + \{R_s\}))}_{N\times 1} = 0$$

$$[K]\{U\} - (\{R_B\} + \{R_s\}) = 0$$

$$[K]\{U\} = \underbrace{\{R_B\} + \{R_s\}}_{\{R\}}$$