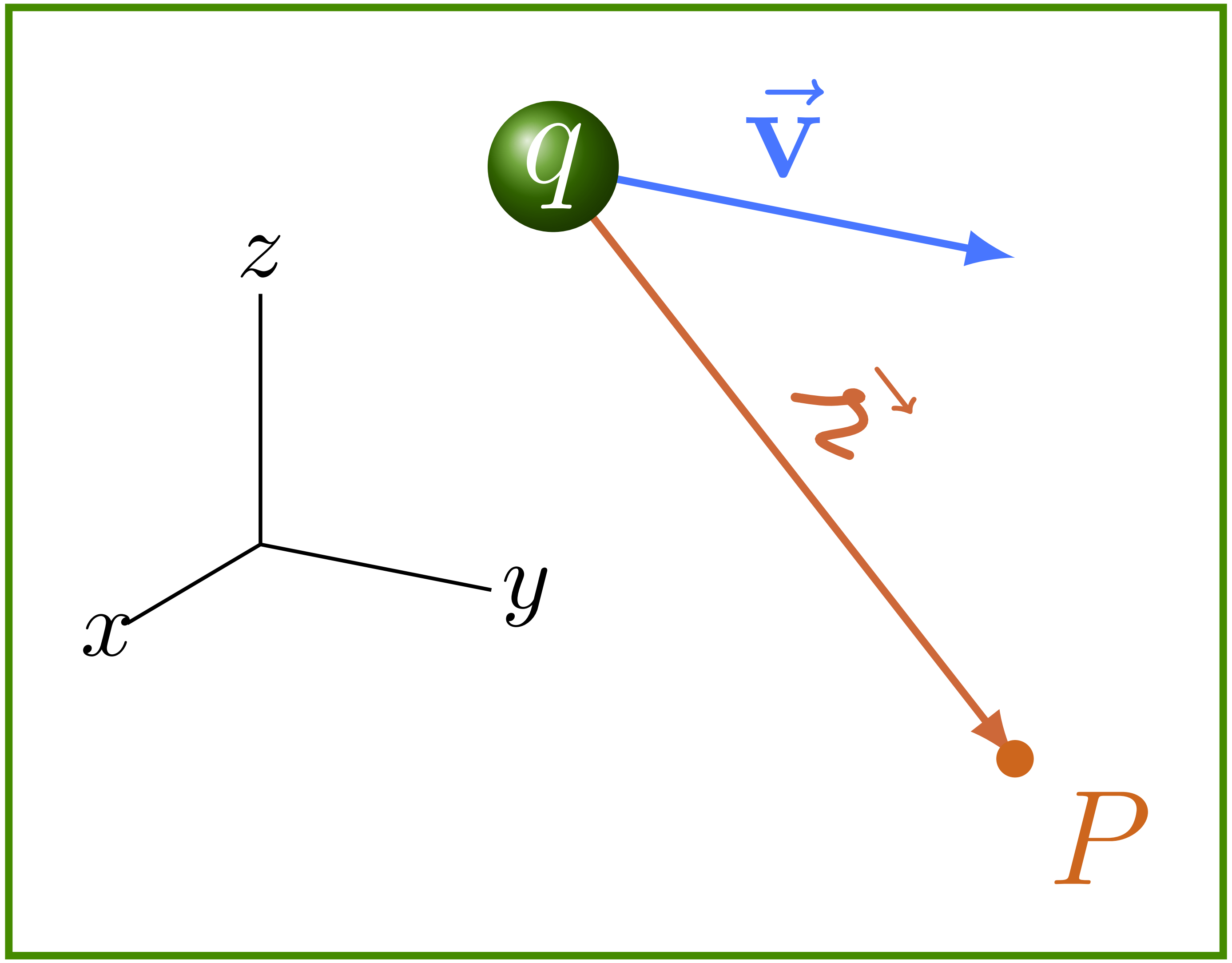
Eletromagnetismo Avançado

24 de novembro
Potenciais e campos

Potenciais de Liénard e Wiechert

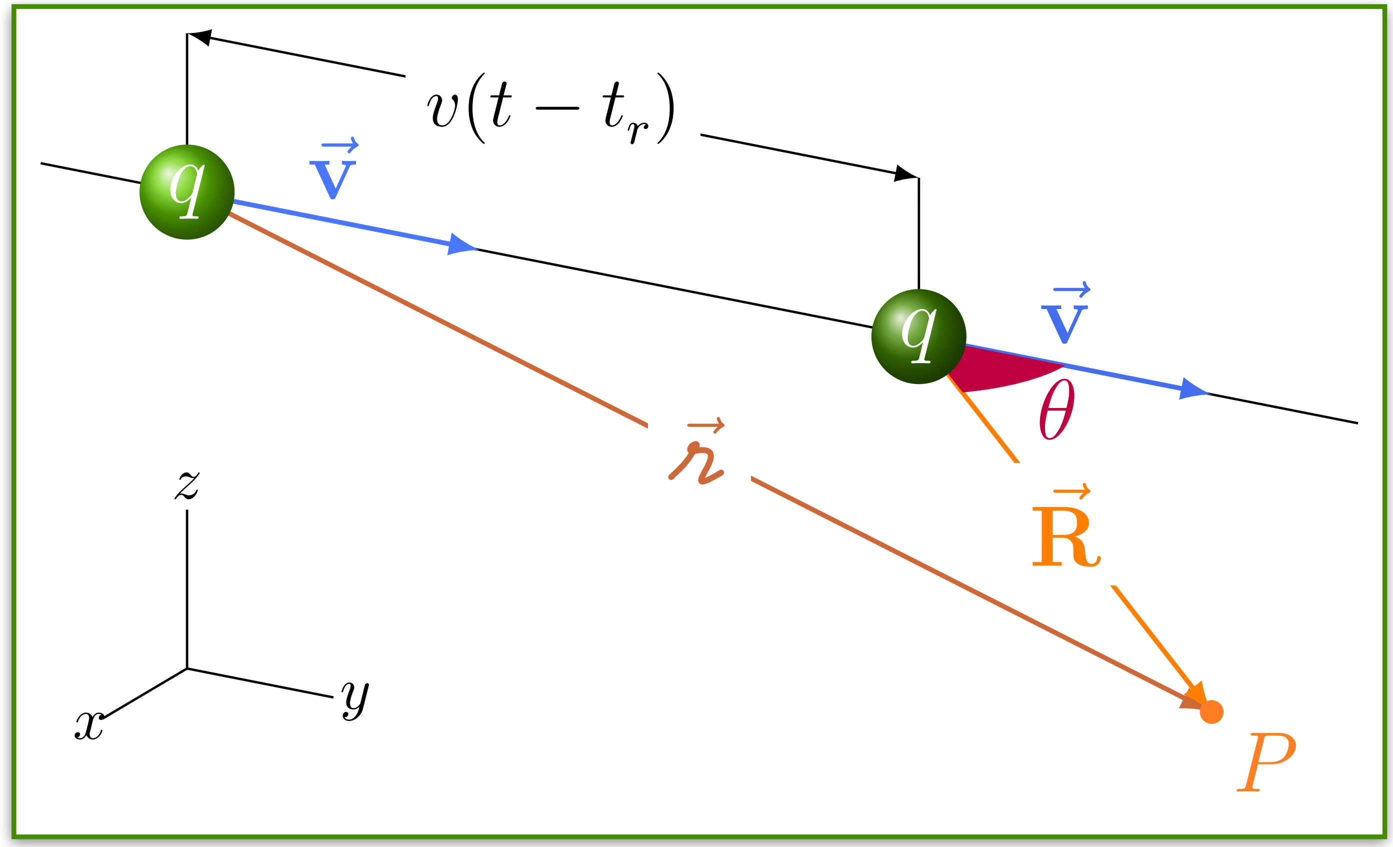
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \frac{q\vec{\mathbf{v}}}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



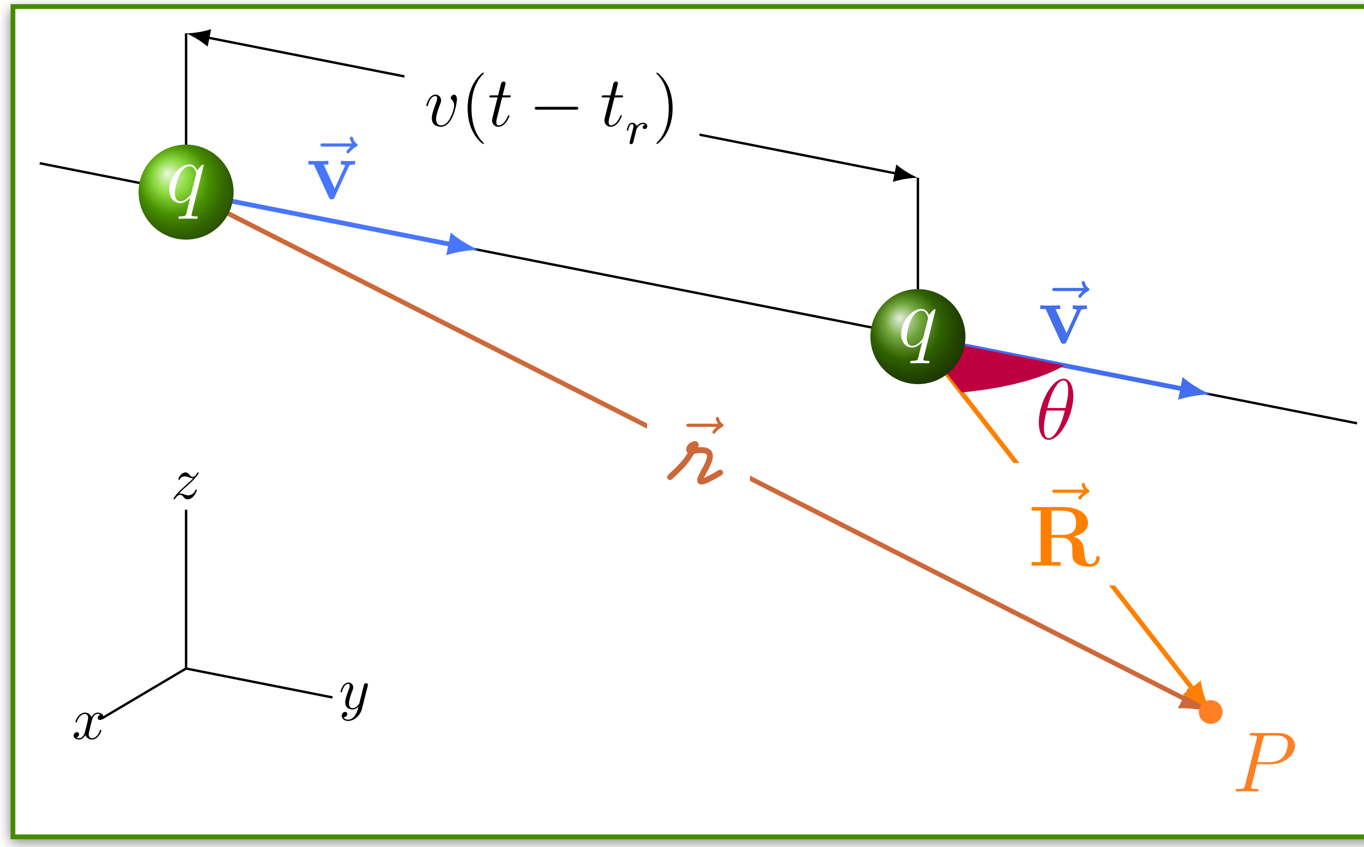
Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



Movimento uniforme

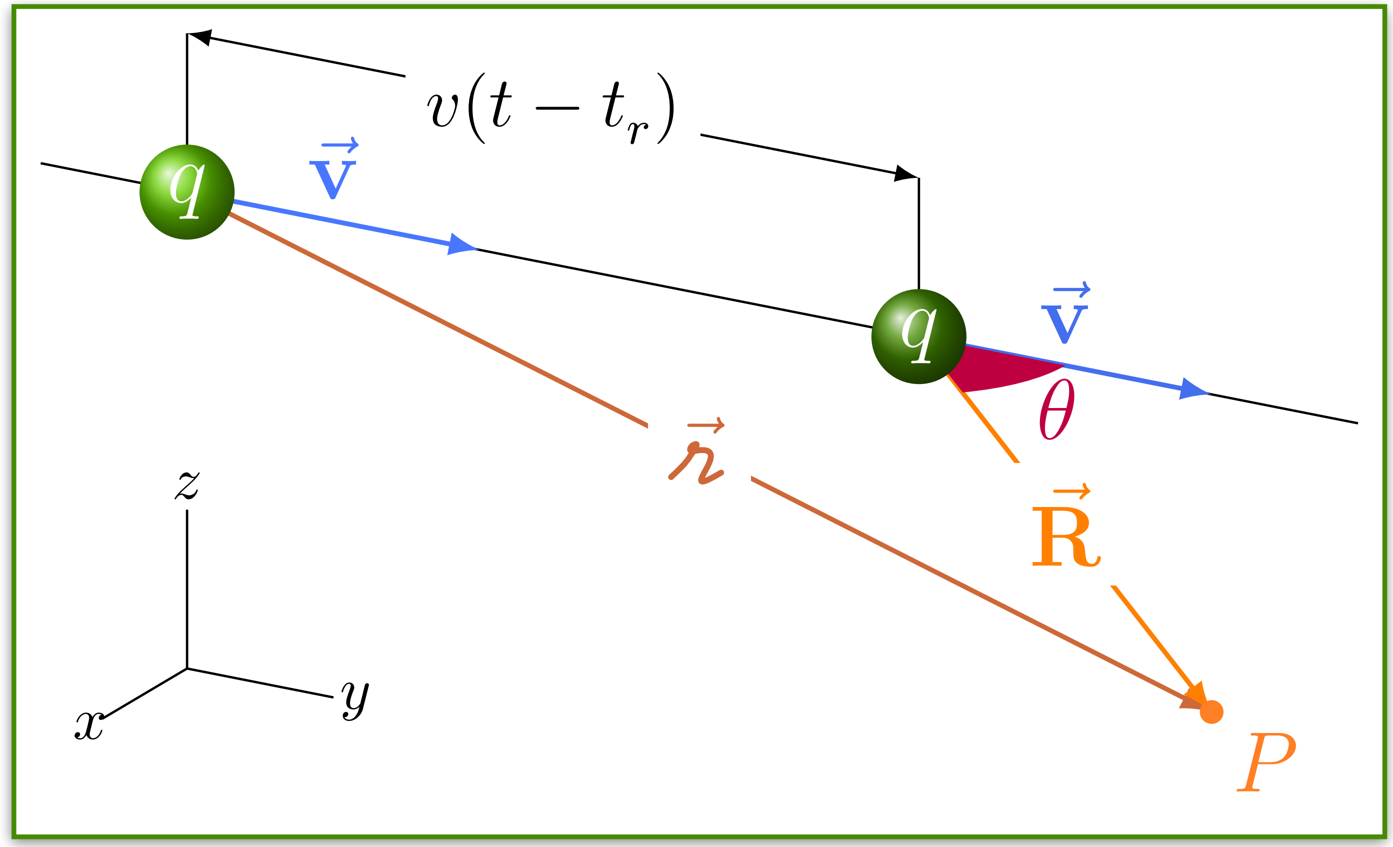
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \dfrac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



$$\vec{\mathscr{r}} = \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \vec{\mathbf{R}}$$

Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$

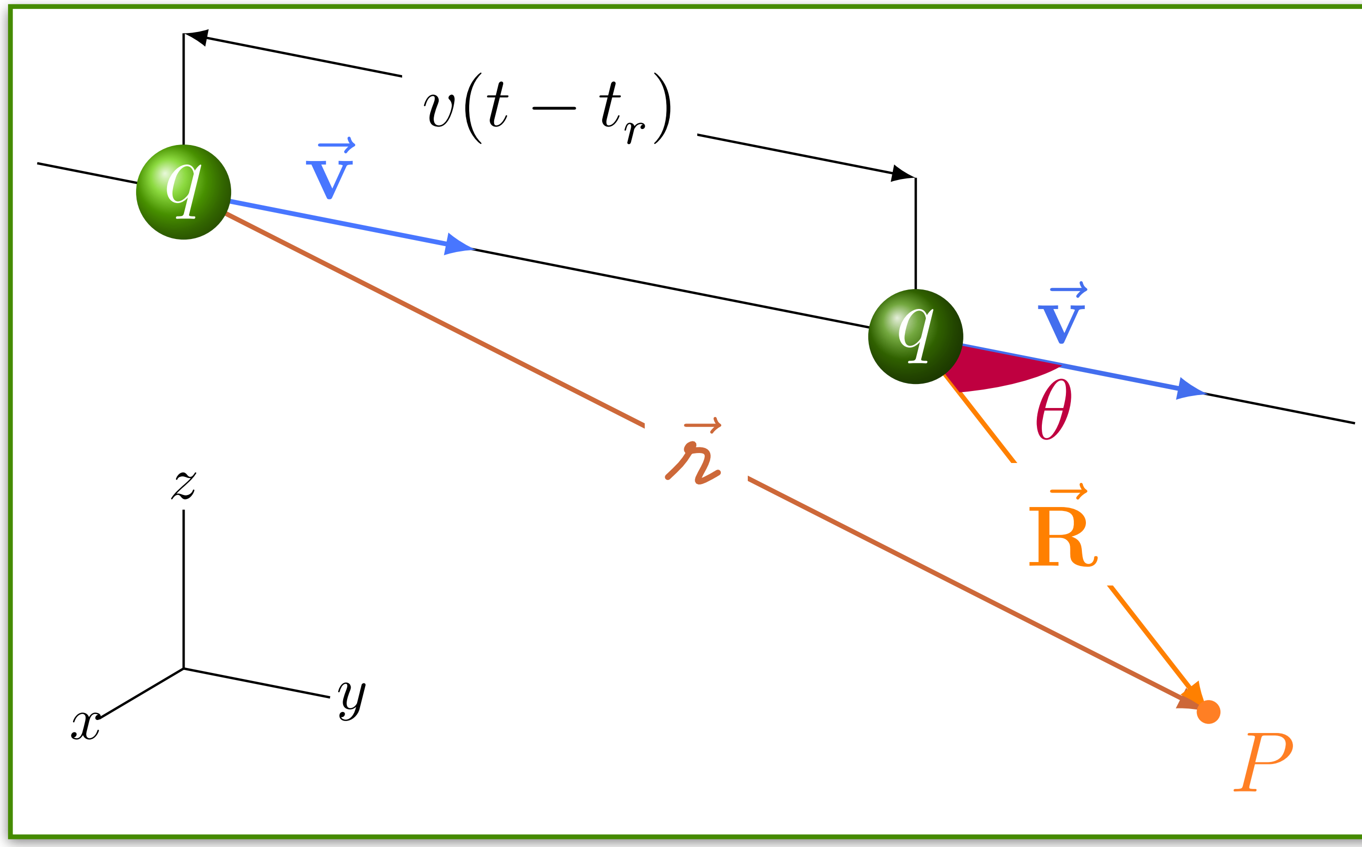


$$\vec{\mathscr{r}} = \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \vec{\mathbf{R}}$$

$$\hat{\mathscr{r}} \cdot \vec{\mathscr{r}} = \hat{\mathscr{r}} \cdot \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \hat{\mathscr{r}} \cdot \vec{\mathbf{R}}$$

Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



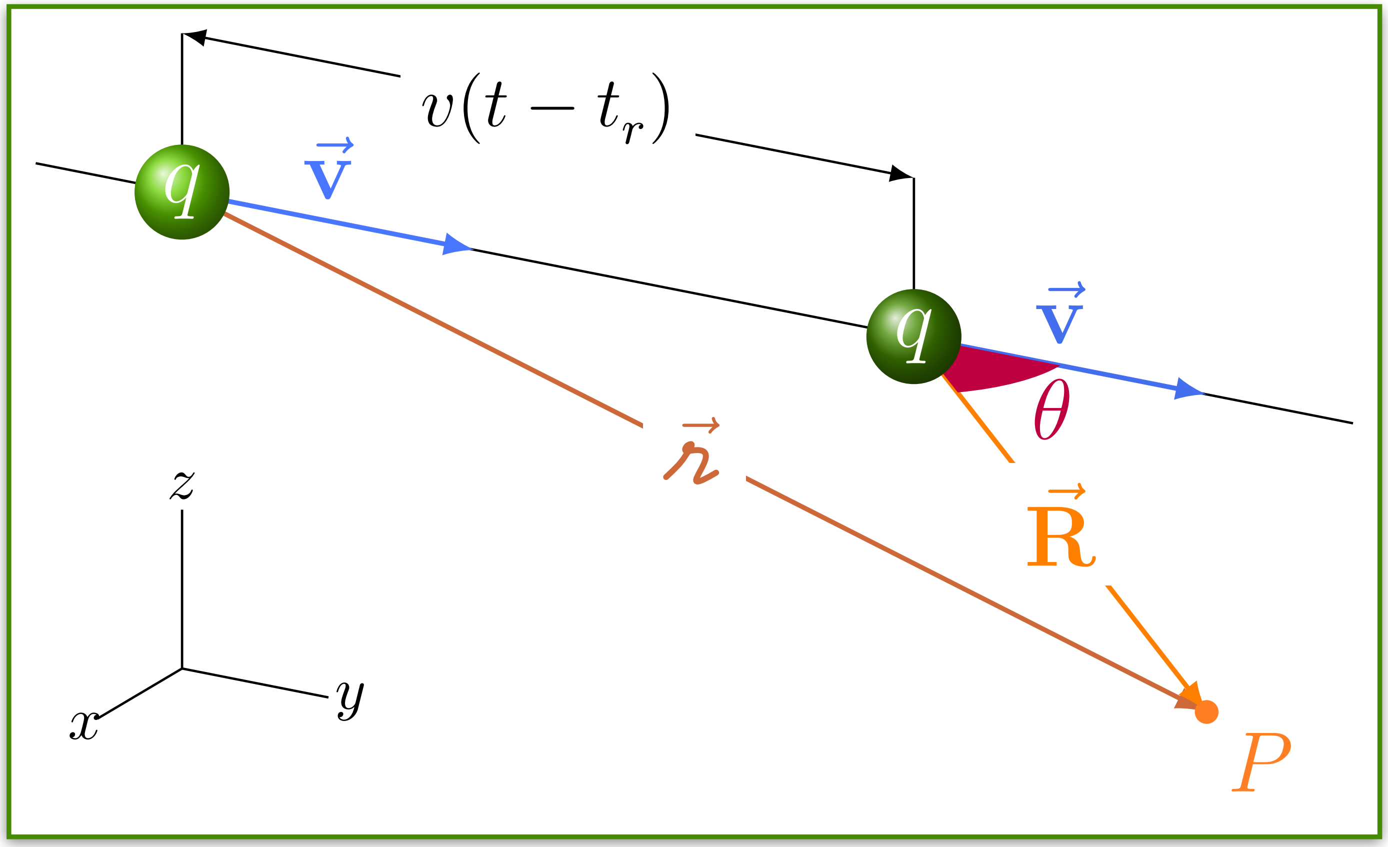
$$\vec{\mathscr{r}} = \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \vec{\mathbf{R}}$$

$$\hat{\mathscr{r}} \cdot \vec{\mathscr{r}} = \hat{\mathscr{r}} \cdot \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \hat{\mathscr{r}} \cdot \vec{\mathbf{R}}$$

$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R\cos\alpha$$

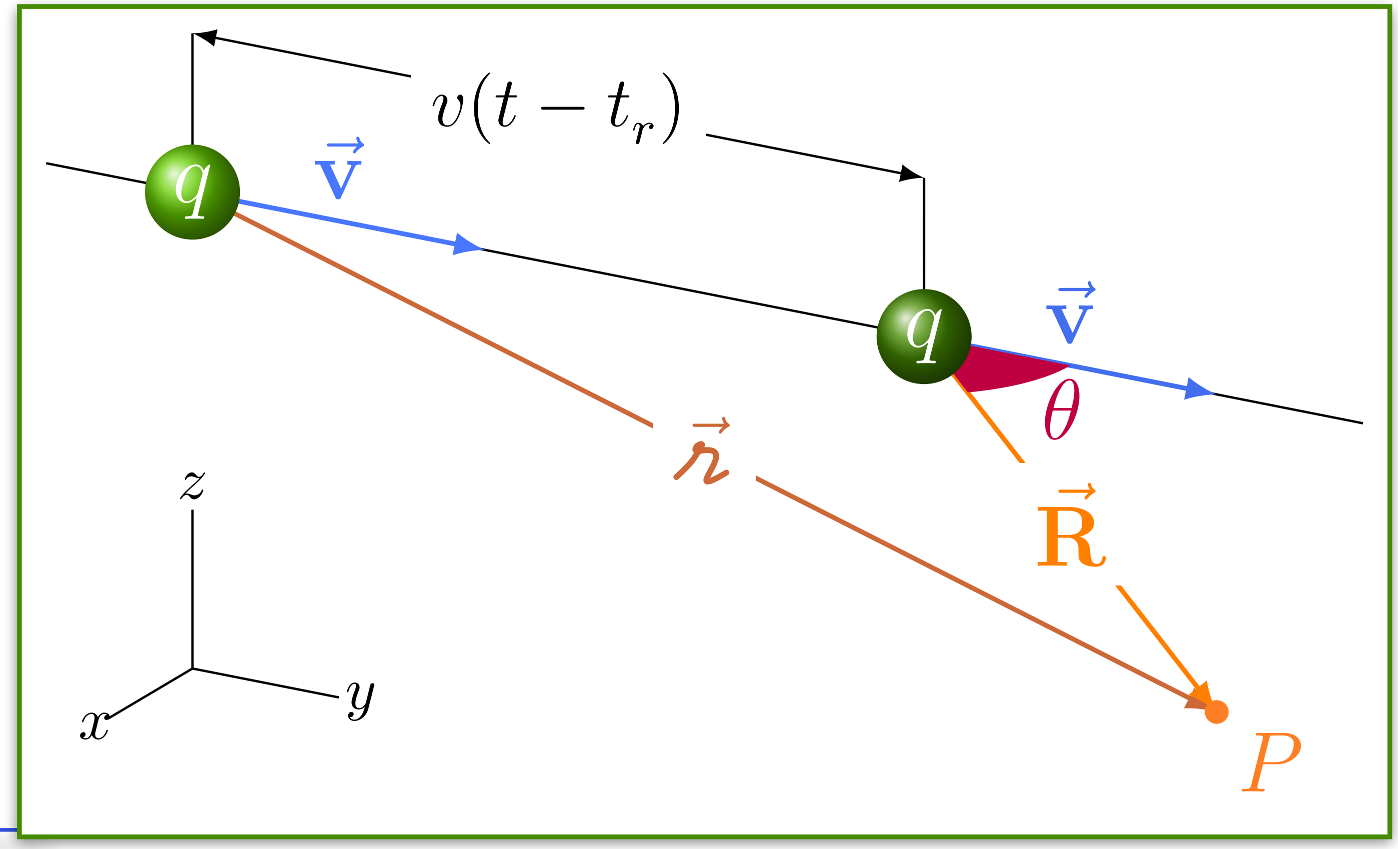
Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R\cos\alpha$$

Movimento uniforme



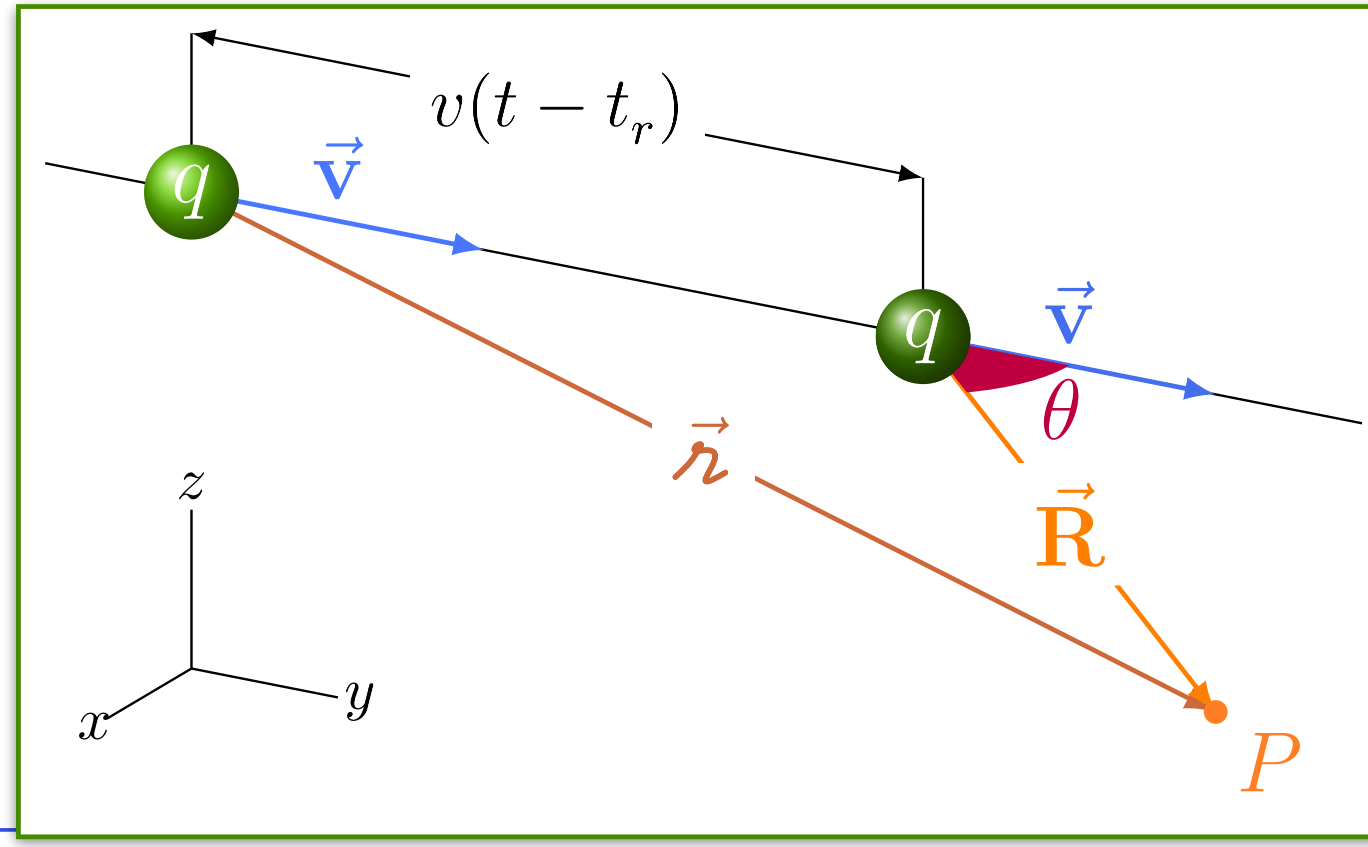
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$

$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R \cos\alpha$$

$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R\sqrt{1 - \sin^2\alpha}$$

Movimento uniforme

$$V(\vec{\mathbf{r}},t)=\frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}-\frac{\vec{\mathbf{v}}}{c}\cdot\vec{\mathscr{r}}}$$



$$\mathscr{r}-\vec{\mathscr{r}}\cdot\frac{\vec{\mathbf{v}}}{c}=R\cos\alpha$$

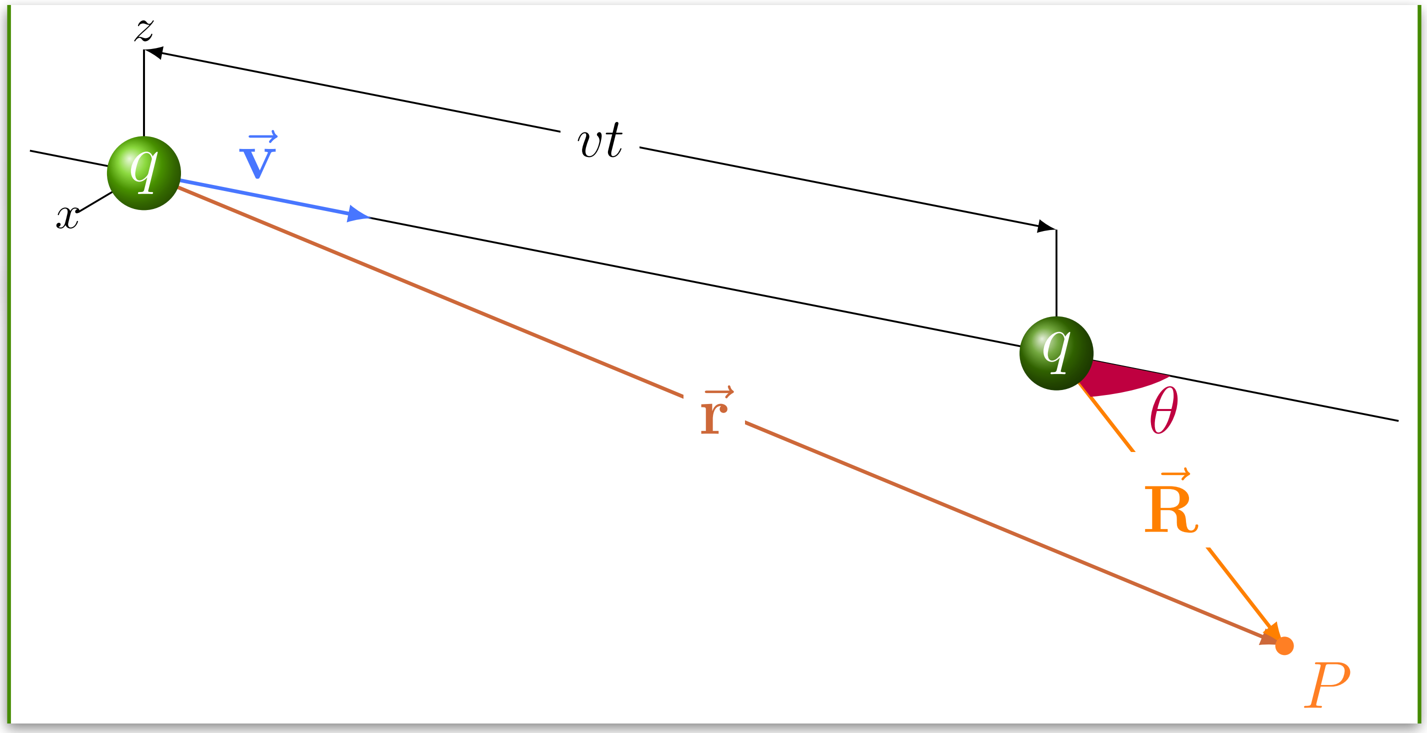
$$\mathscr{r}-\vec{\mathscr{r}}\cdot\frac{\vec{\mathbf{v}}}{c}=R\sqrt{1-\sin^2\alpha}$$

$$\boxed{\frac{\sin\alpha}{v\frac{\mathscr{r}}{c}}=\frac{\sin\theta}{\mathscr{r}}}$$

$$V(\vec{\mathbf{r}},t)=\frac{1}{4\pi\epsilon_0}\frac{q}{R\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}$$

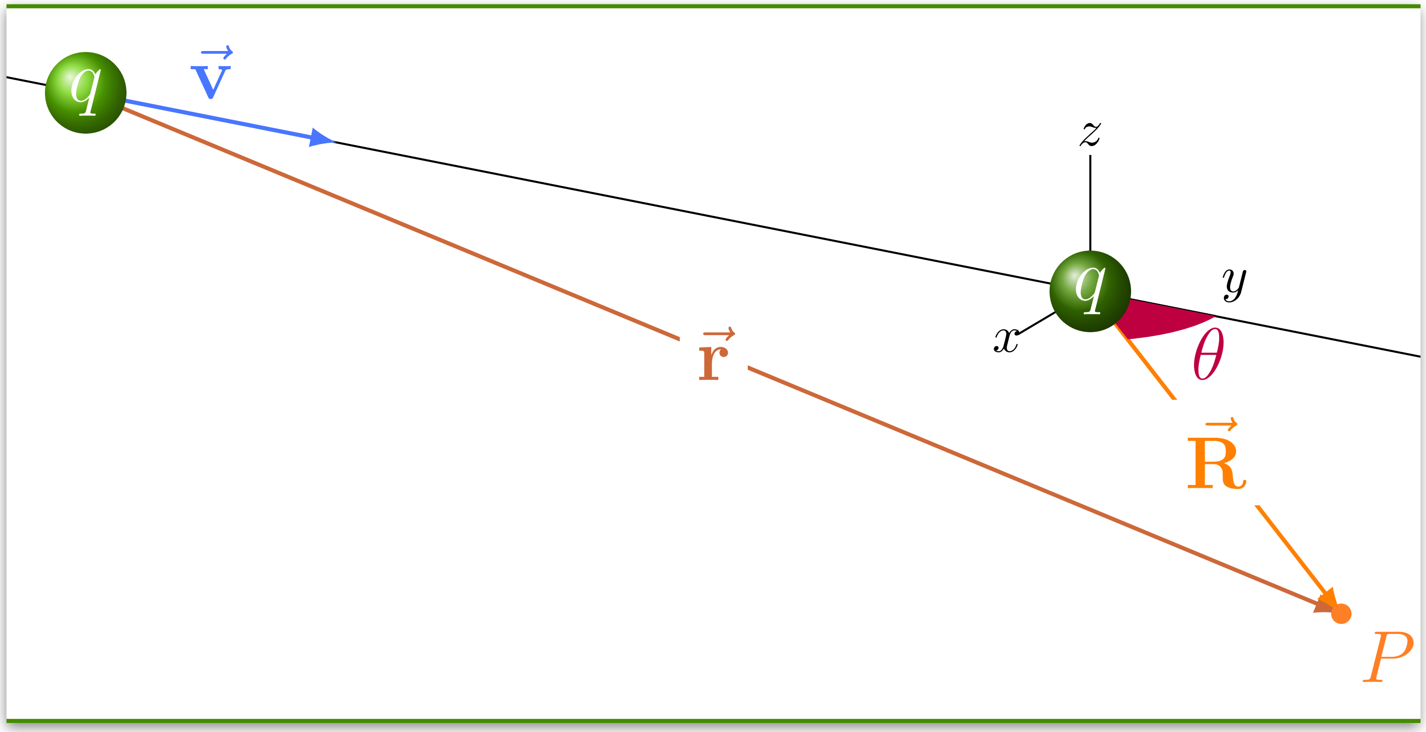
Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$



Movimento uniforme

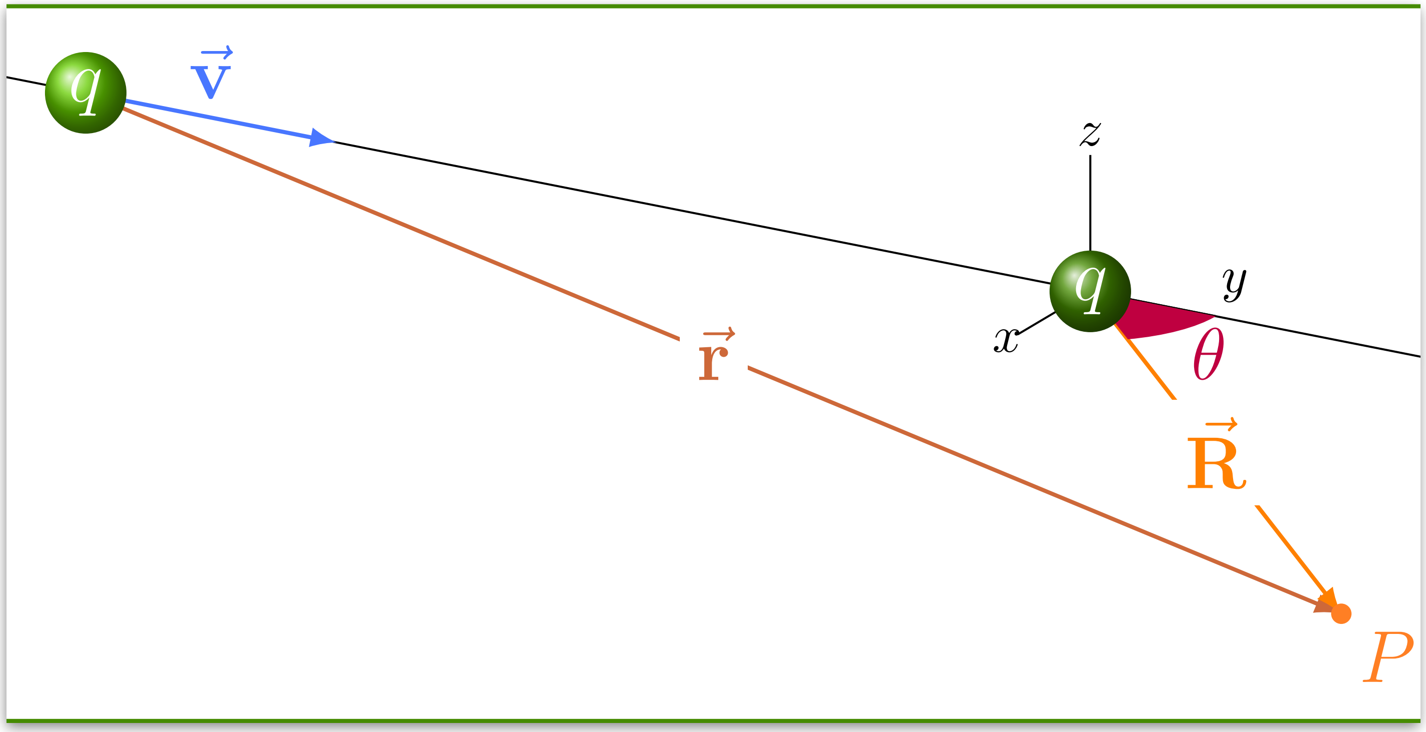
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$



$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \partial_t \vec{\mathbf{A}}$$

Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$

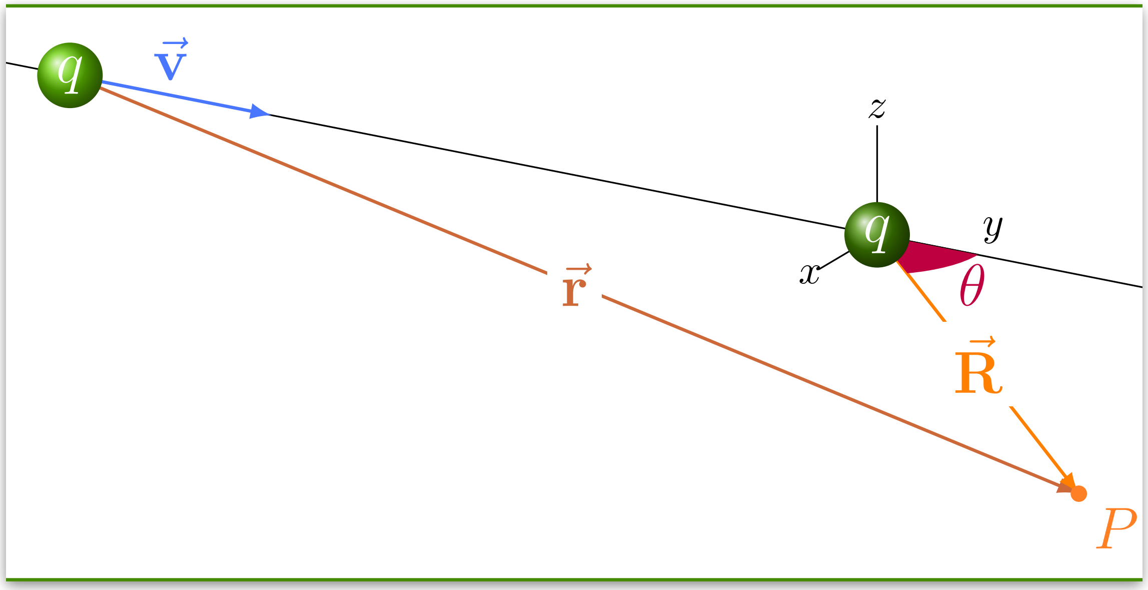


$$\vec{\mathbf{E}} = -\vec{\nabla} V - \partial_t \vec{\mathbf{A}}$$

$$\vec{\nabla} V = \frac{\partial V}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial V}{\partial \theta}\hat{\theta}$$

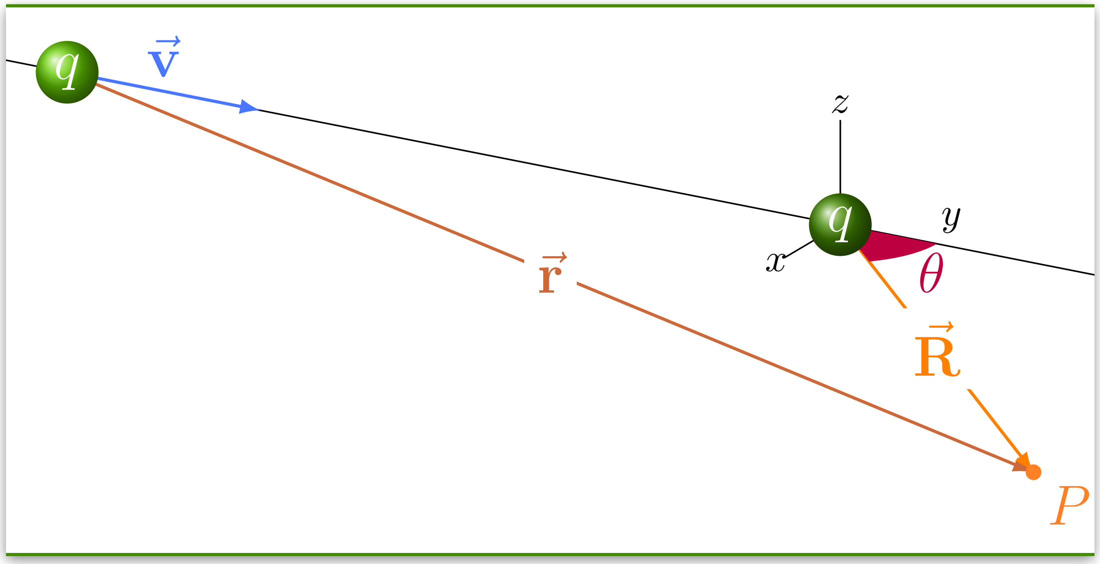
Movimento uniforme

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\frac{q}{R\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}$$



Movimento uniforme

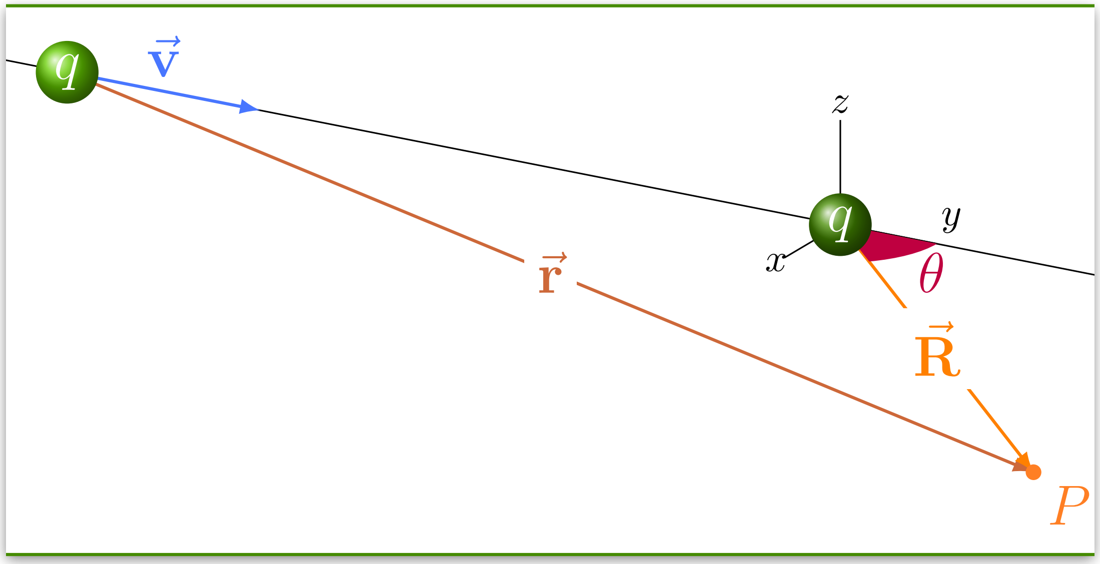
$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\frac{q}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}}$$



Movimento uniforme

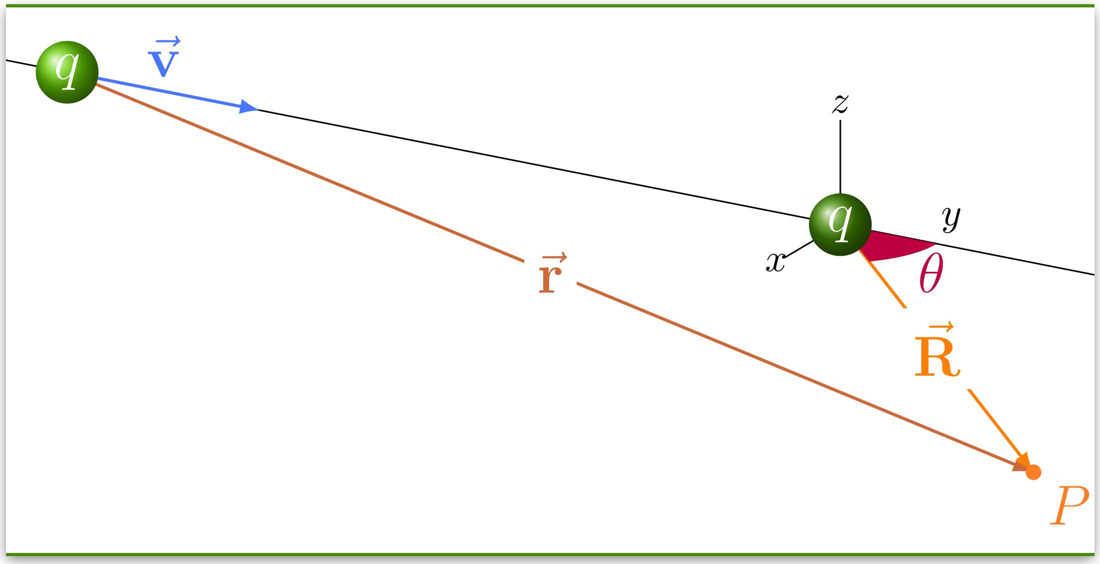
$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2} \frac{q}{R(1 - \frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}}$$

$$\partial_t \vec{\mathbf{A}} = \frac{\partial \vec{\mathbf{A}}}{\partial R}\dot{R} + \frac{\partial \vec{\mathbf{A}}}{\partial \theta}\dot{\theta}$$



Movimento uniforme

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\,\frac{q}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}}$$

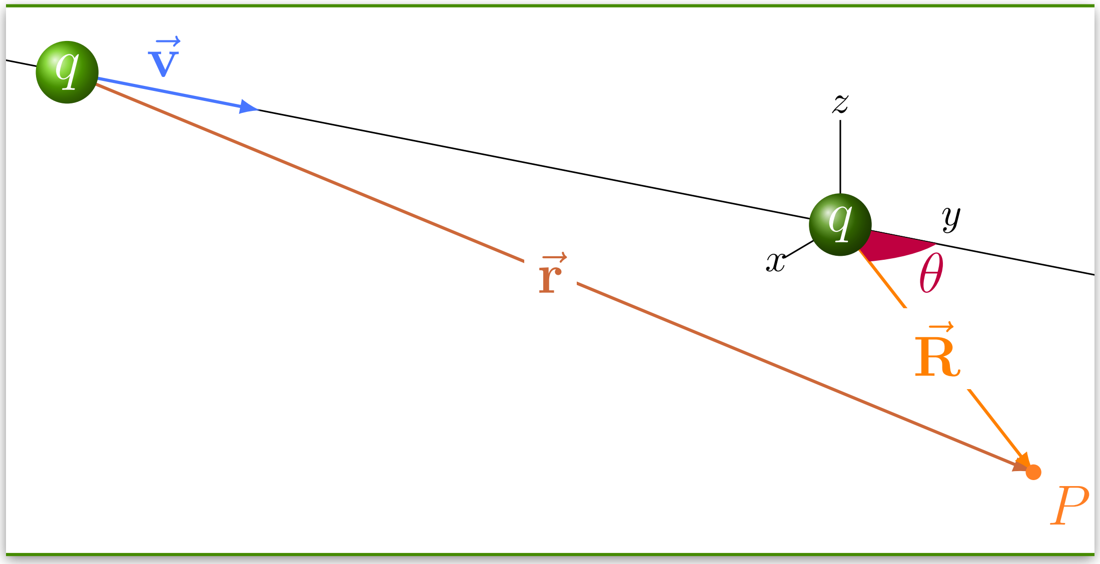


$$\partial_t\,\vec{\mathbf{A}} = \frac{\partial\vec{\mathbf{A}}}{\partial R}\dot{R} + \frac{\partial\vec{\mathbf{A}}}{\partial\theta}\dot{\theta}$$

$$-\partial_t\,\vec{\mathbf{A}} = \frac{q\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\left(\frac{\dot{R}}{R^2(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}} - \frac{\frac{v^2}{c^2}\sin\theta\cos\theta\,\dot{\theta}}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}}\right)$$

Movimento uniforme

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\frac{q}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}}$$



$$\partial_t\,\vec{\mathbf{A}} = \frac{\partial\vec{\mathbf{A}}}{\partial R}\dot{R} + \frac{\partial\vec{\mathbf{A}}}{\partial\theta}\dot{\theta}$$

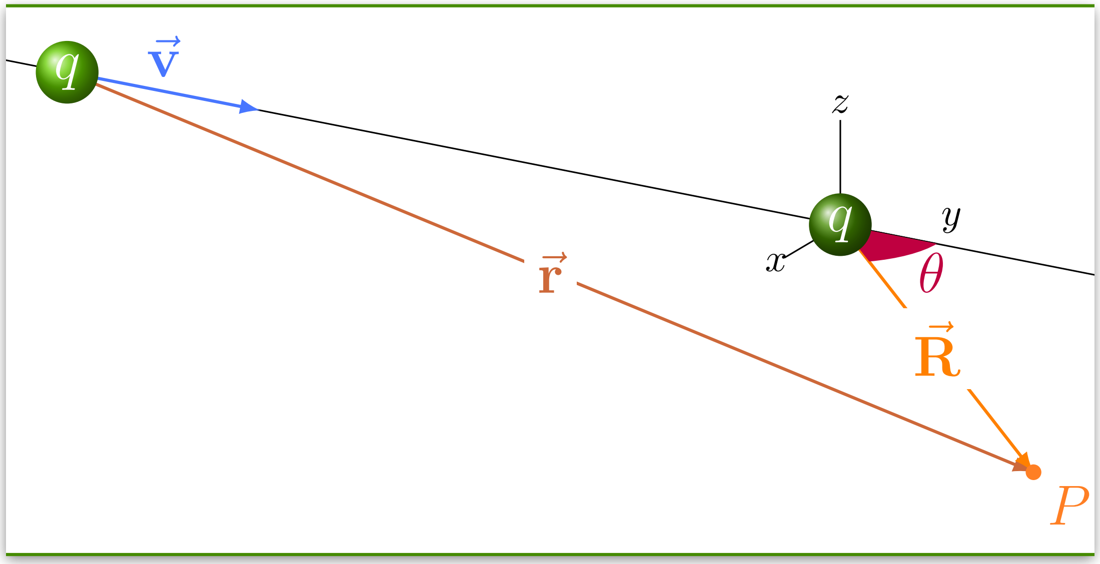
$$-\partial_t\,\vec{\mathbf{A}} = \frac{q\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\left(\frac{\dot{R}}{R^2(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}} - \frac{\frac{v^2}{c^2}\sin\theta\cos\theta\,\dot{\theta}}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}}\right)$$

$$\vec{\mathbf{v}} = v\big(\cos\theta\hat{\mathbf{R}} - \sin\theta\hat{\theta}\big)$$

$$\vec{\mathbf{v}} = -\dot{\vec{\mathbf{R}}}$$

Movimento uniforme

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\frac{q}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}}$$



$$\partial_t\,\vec{\mathbf{A}} = \frac{\partial\vec{\mathbf{A}}}{\partial R}\dot{R} + \frac{\partial\vec{\mathbf{A}}}{\partial\theta}\dot{\theta}$$

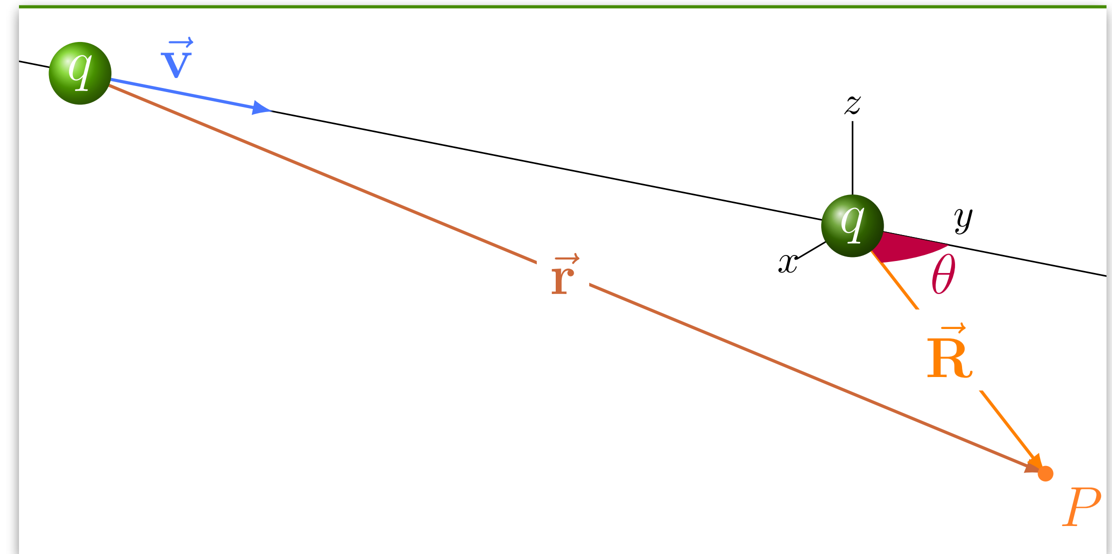
$$-\partial_t\,\vec{\mathbf{A}} = \frac{q\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\left(\frac{\dot{R}}{R^2(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}} - \frac{\frac{v^2}{c^2}\sin\theta\cos\theta\,\dot{\theta}}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}}\right)$$

$$\vec{\mathbf{v}} = v\big(\cos\theta\hat{\mathbf{R}} - \sin\theta\hat{\theta}\big)$$

$$\vec{\mathbf{v}} = -\dot{\vec{\mathbf{R}}} = -\dot{R}\hat{\mathbf{R}} - R\dot{\theta}\hat{\theta}$$

Movimento uniforme

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\frac{q}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}}$$



$$\partial_t\,\vec{\mathbf{A}} = \frac{\partial\vec{\mathbf{A}}}{\partial R}\dot{R} + \frac{\partial\vec{\mathbf{A}}}{\partial\theta}\dot{\theta}$$

$$-\partial_t\,\vec{\mathbf{A}} = \frac{q\vec{\mathbf{v}}}{4\pi\epsilon_0 c^2}\left(\frac{\dot{R}}{R^2(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{1}{2}}} - \frac{\frac{v^2}{c^2}\sin\theta\cos\theta\,\dot{\theta}}{R(1-\frac{v^2}{c^2}\sin^2\theta)^{\frac{3}{2}}}\right)$$

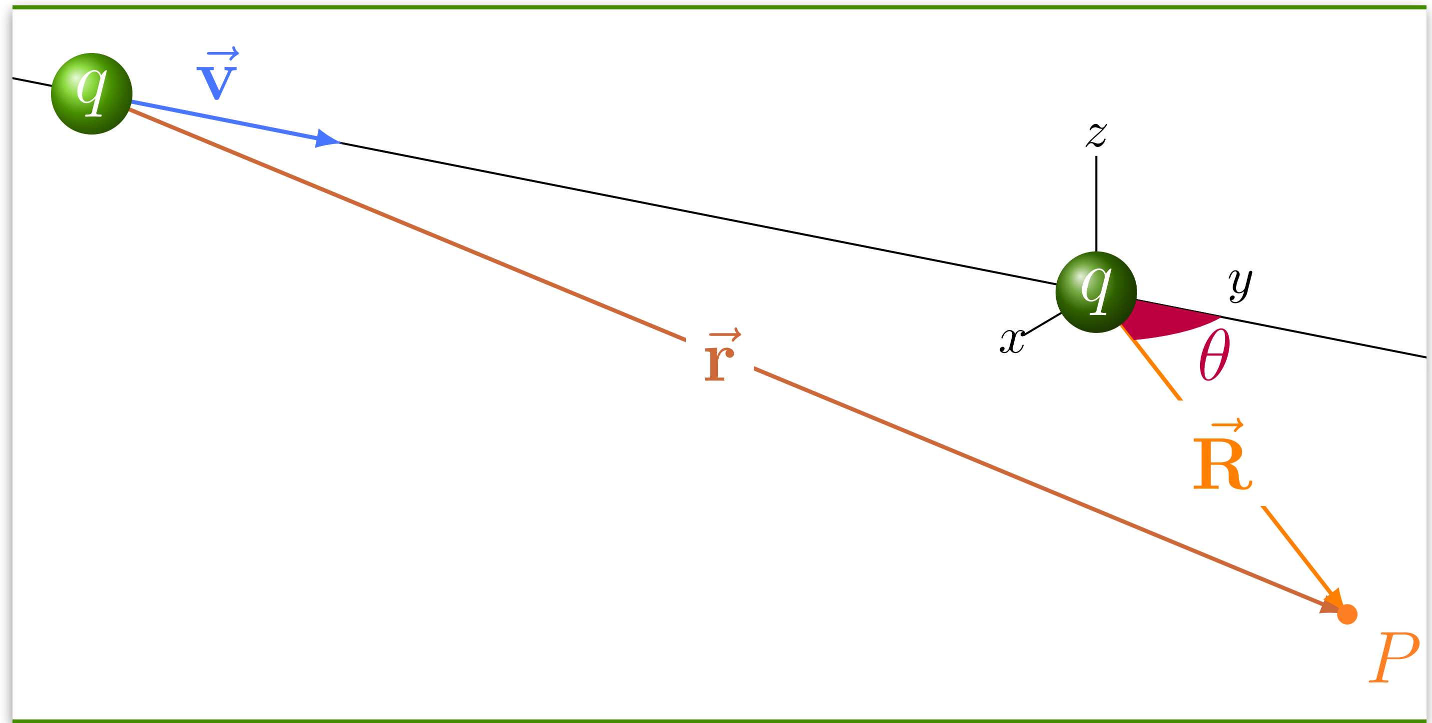
$$\vec{\mathbf{v}} = v\big(\cos\theta\hat{\mathbf{R}} - \sin\theta\hat{\theta}\big)$$

$$\vec{\mathbf{v}} = -\dot{\vec{\mathbf{R}}} = -\dot{R}\hat{\mathbf{R}} - R\dot{\theta}\hat{\theta}$$

$$\Rightarrow \begin{cases}\dot{R} &= -v\cos\theta\\ \dot{\theta} &= \frac{v}{R}\sin\theta\end{cases}$$

Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$



$$\vec{\mathbf{E}} = -\vec{\boldsymbol{\nabla}} V - \partial_t \vec{\mathbf{A}}$$

$$\vec{\mathbf{E}} = \frac{q}{4\pi\epsilon_0} \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^{\frac{3}{2}}} \frac{\hat{\mathbf{R}}}{R^2}$$