Eletromagnetismo Avançado

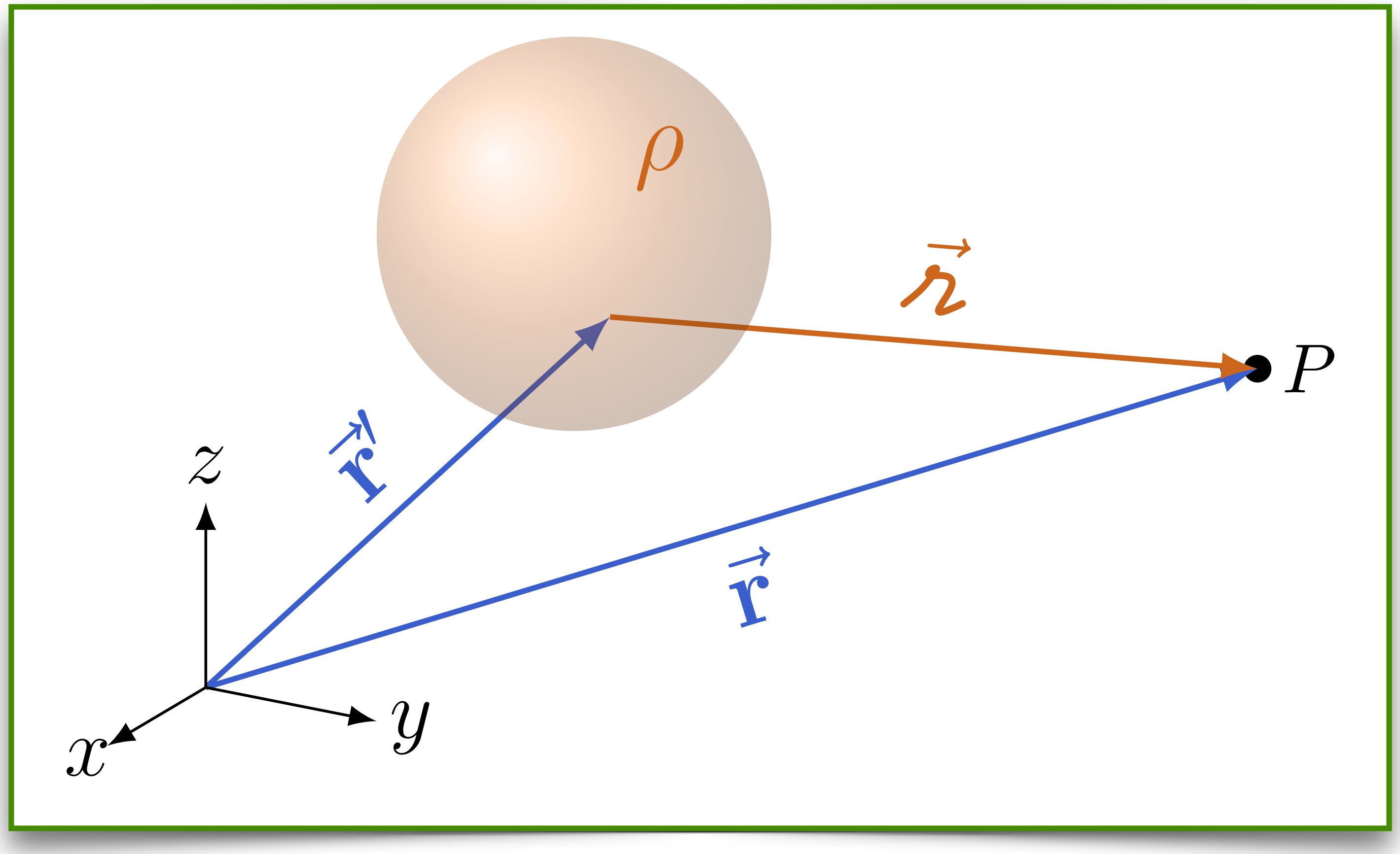
17 de novembro

Potenciais e campos

Equações diferenciais para os potenciais

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

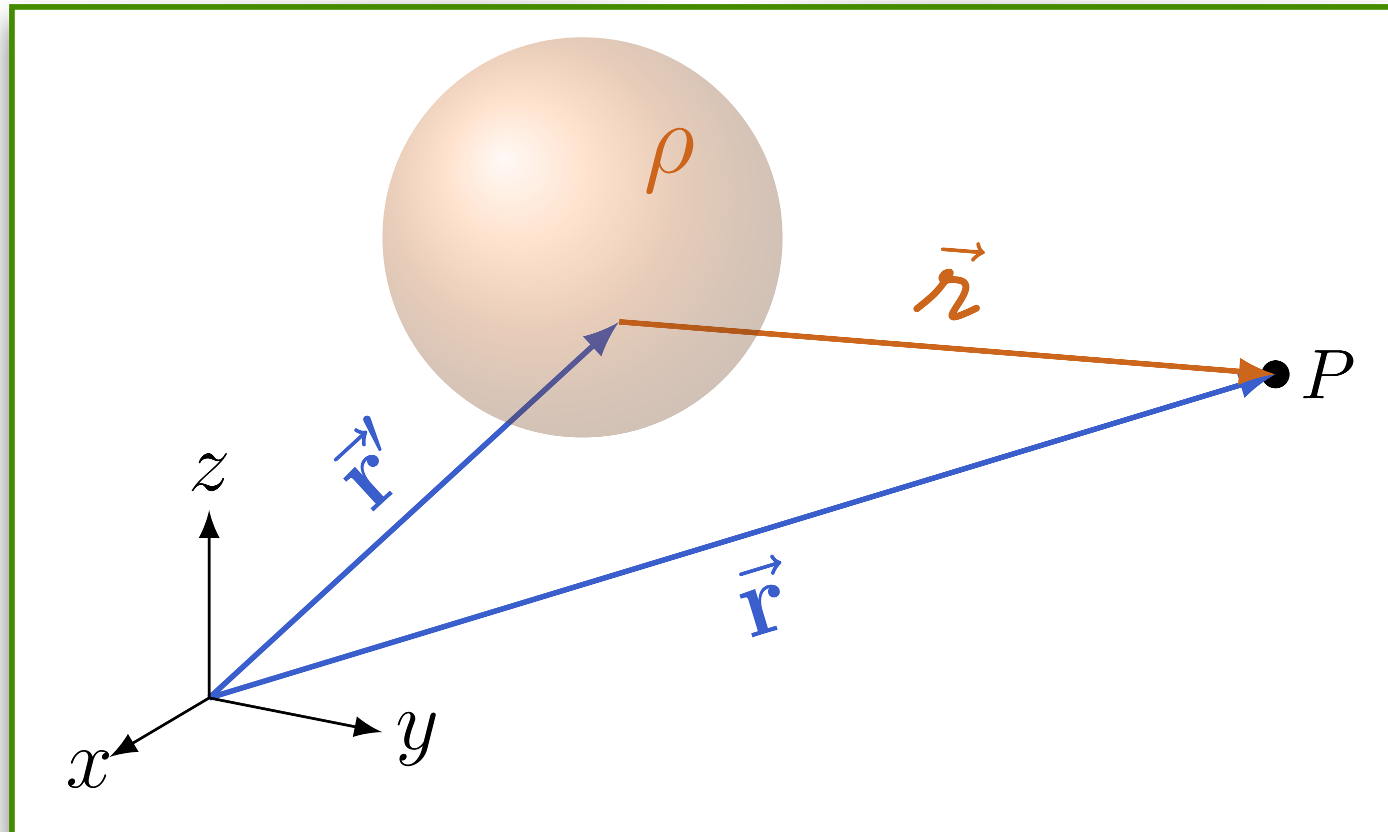
$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{A}}}{\partial t^2} = -\mu_0 \vec{\mathbf{J}}$$



Potenciais de distribuição contínua

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{A}}}{\partial t^2} = -\mu_0 \vec{\mathbf{J}}$$



$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{\mathbf{r}}', t_{\text{ret}})}{\mathscr{r}}\, d\tau'$$

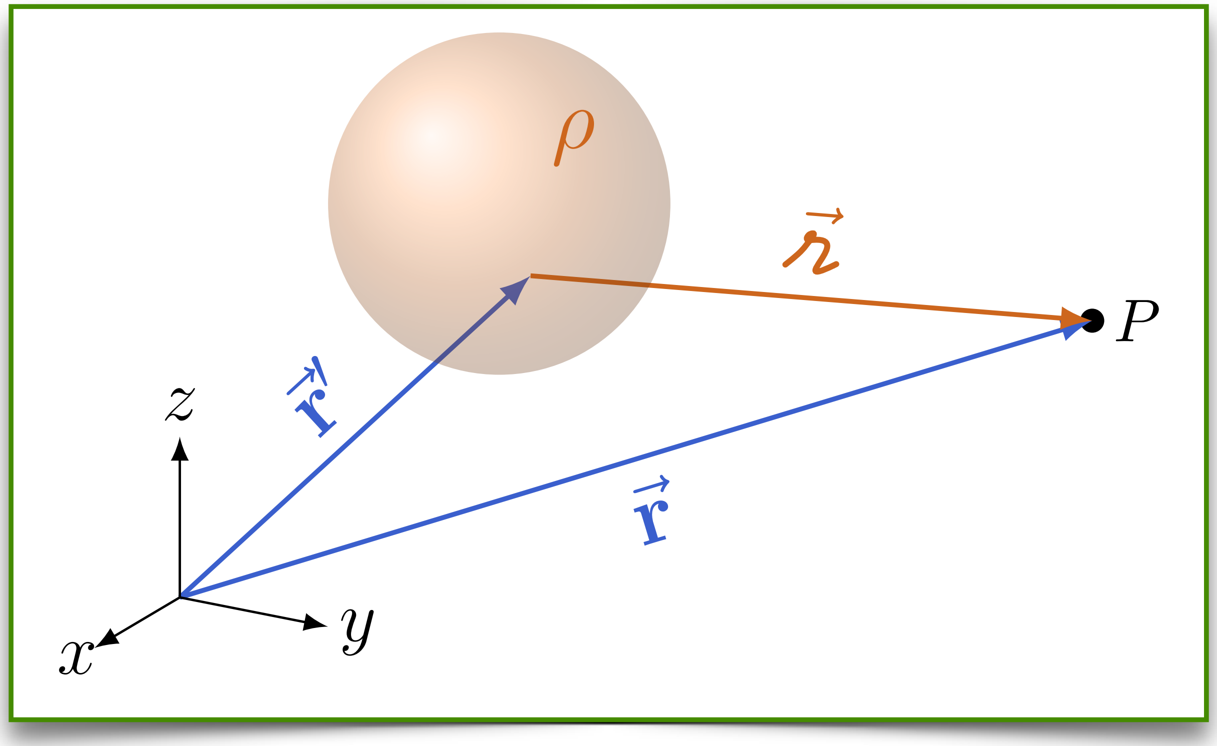
$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}', t_{\text{ret}})}{\mathscr{r}}\, d\tau'$$

$$t_r \equiv t - \frac{\mathscr{r}}{c}$$

Potenciais de distribuição contínua

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}', t_r)}{\mathscr{r}} \, \mathrm{d}\tau'$$

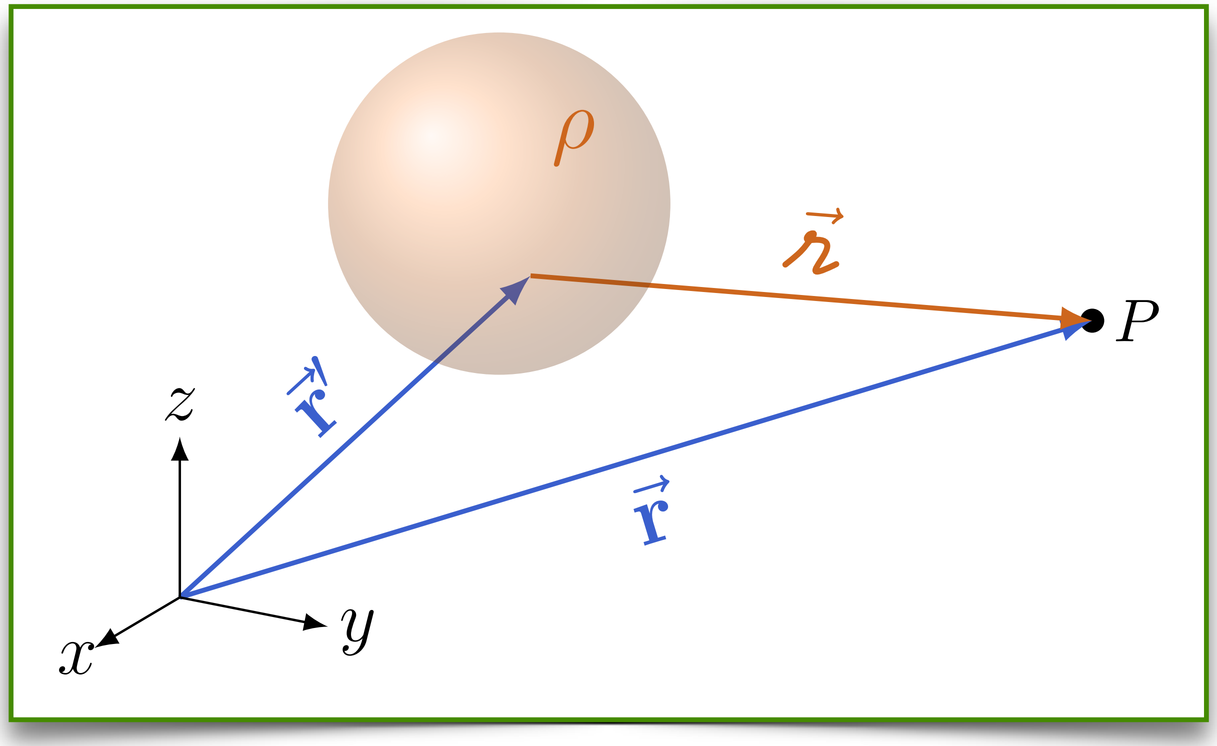
$$t_r \equiv t - \frac{\mathscr{r}}{c}$$



Potenciais de distribuição contínua

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}', t_r)}{\mathscr{r}} \, \mathrm{d}\tau'$$

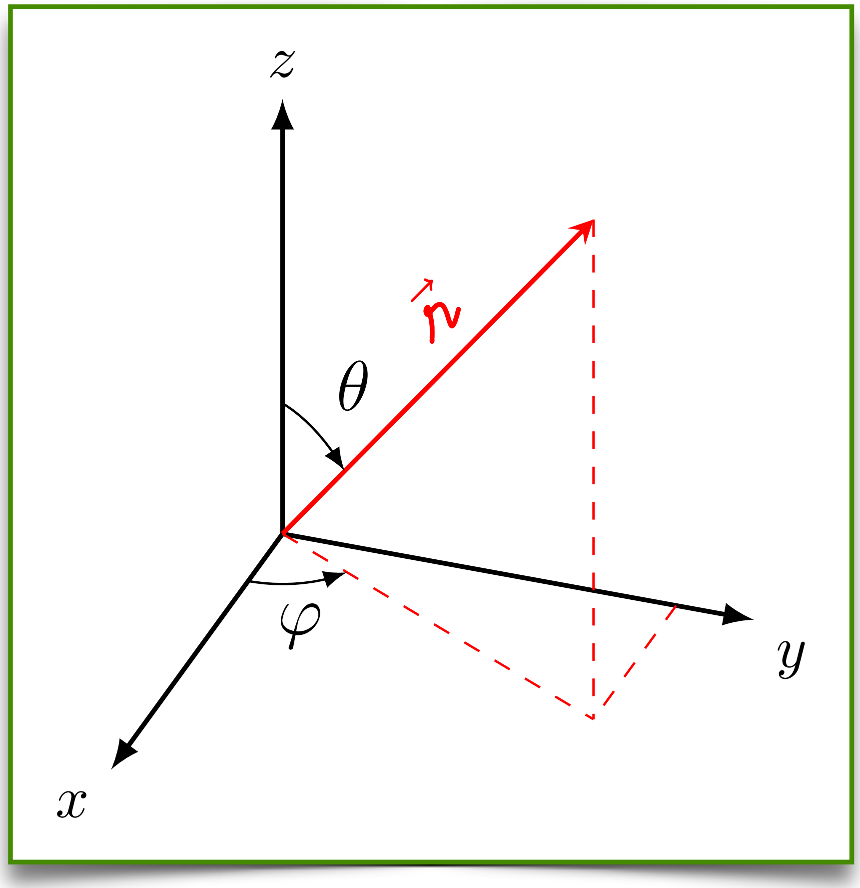
$$t_r \equiv t - \frac{\mathscr{r}}{c}$$



$$\vec{\boldsymbol{\nabla}} V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \left(\int \frac{1}{\mathscr{r}} \vec{\boldsymbol{\nabla}} \rho(\vec{\mathbf{r}}', t_r) \, \mathrm{d}\tau' + \int \rho(\vec{\mathbf{r}}', t_r) \vec{\boldsymbol{\nabla}} \left(\frac{1}{\mathscr{r}}\right) \mathrm{d}\tau' \right)$$

Potenciais de distribuição contínua

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}', t_r)}{\mathscr{r}} \, d\tau'$$



$$t_r \equiv t - \frac{\mathscr{r}}{c}$$

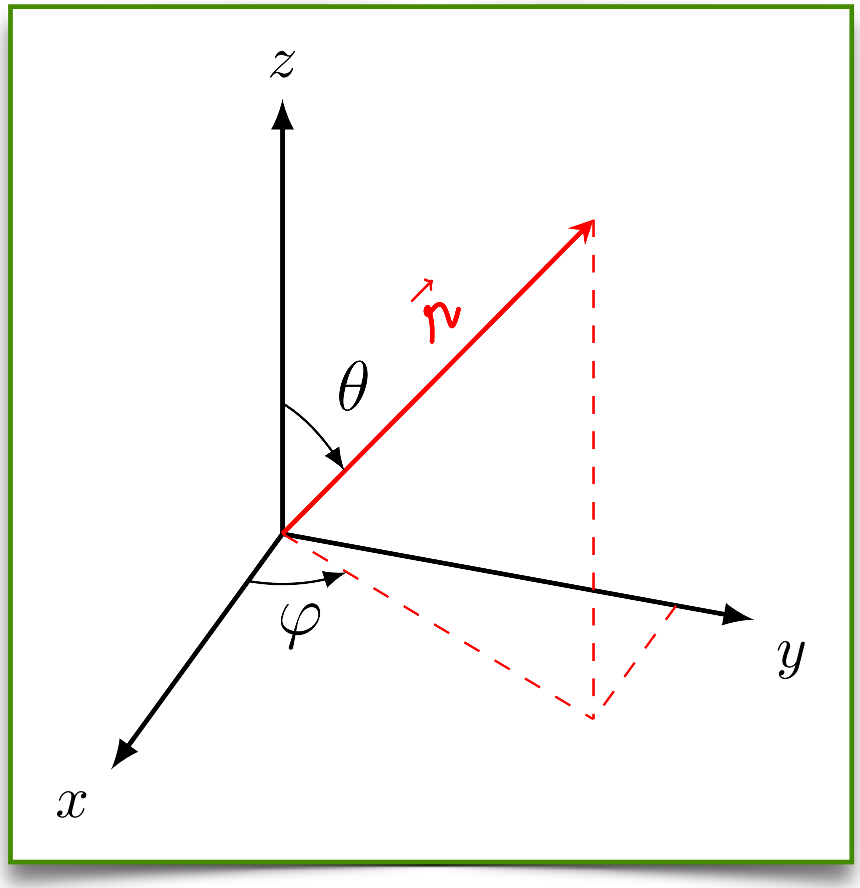
$$\vec{\nabla} V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \Big(\int \frac{1}{\mathscr{r}} \vec{\nabla} \rho(\vec{\mathbf{r}}', t_r) \, d\tau' + \int \rho(\vec{\mathbf{r}}', t_r) \vec{\nabla} \Big(\frac{1}{\mathscr{r}}\Big) \, d\tau' \Big)$$

$$\vec{\nabla} \mathscr{r} = \hat{\mathscr{r}}$$

Potenciais de distribuição contínua

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{\mathbf{r}}', t_r)}{\mathscr{r}}\, \mathrm{d}\tau'$$

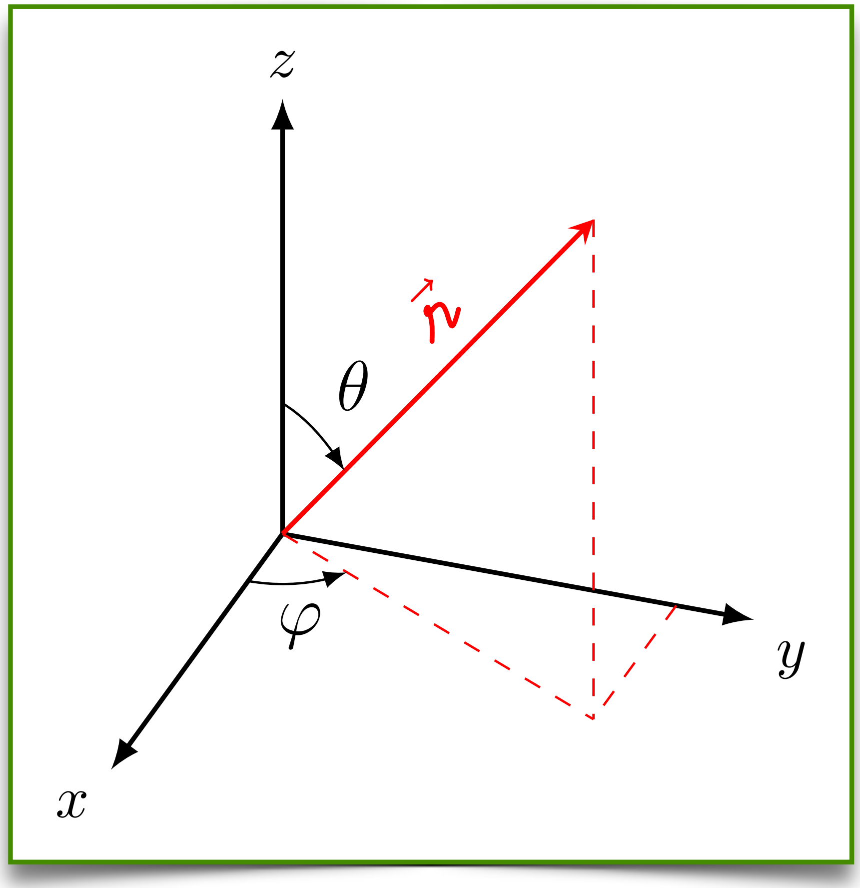
$$t_r \equiv t - \frac{\mathscr{r}}{c}$$



$$\vec{\boldsymbol{\nabla}} V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\left(\int \frac{1}{\mathscr{r}} \vec{\boldsymbol{\nabla}} \rho(\vec{\mathbf{r}}', t_r)\, \mathrm{d}\tau' + \int \rho(\vec{\mathbf{r}}', t_r) \vec{\boldsymbol{\nabla}}\left(\frac{1}{\mathscr{r}}\right) \mathrm{d}\tau' \right)$$

$$\vec{\boldsymbol{\nabla}} \mathscr{r} = \hat{\mathscr{r}} \quad \Rightarrow \quad \vec{\boldsymbol{\nabla}} t_r = -\frac{1}{c}\hat{\mathscr{r}}$$

Potenciais de distribuição contínua



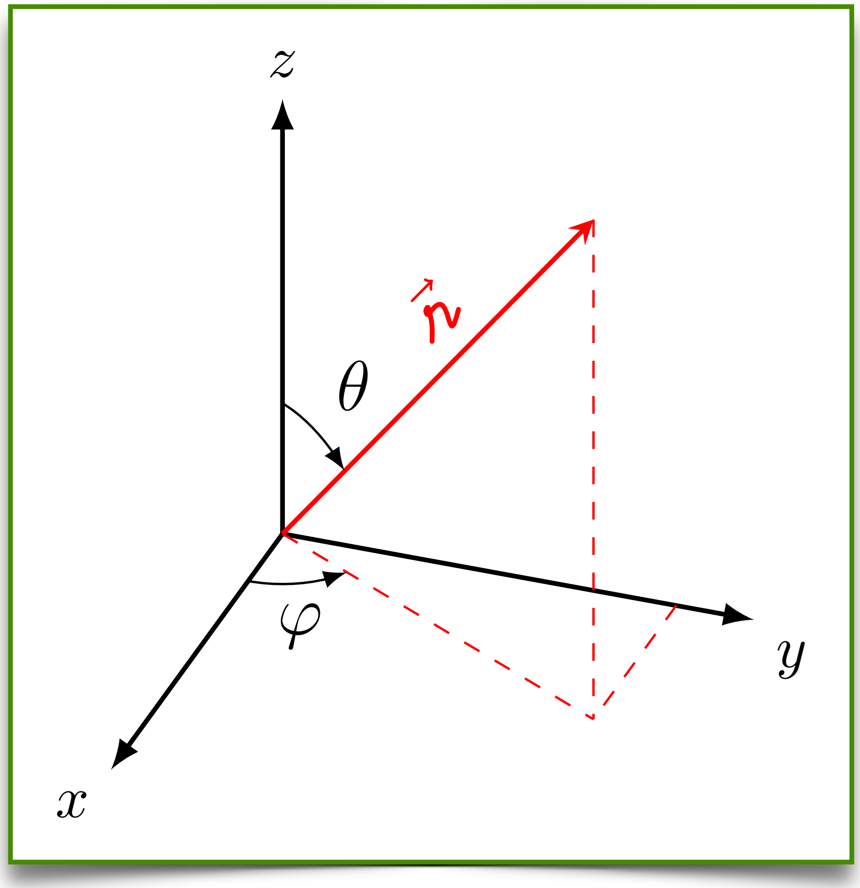
$$\vec{\nabla} V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(\int \frac{1}{\mathscr{r}}\vec{\nabla}\rho(\vec{\mathbf{r}}', t_r)\,\mathrm{d}\tau' + \int \rho(\vec{\mathbf{r}}', t_r)\vec{\nabla}\big(\frac{1}{\mathscr{r}}\big)\,\mathrm{d}\tau'\Big)$$

$$\vec{\nabla}\mathscr{r} = \hat{\mathscr{r}} \quad \Rightarrow \quad \vec{\nabla} t_r = -\frac{1}{c}\hat{\mathscr{r}}$$

$$\vec{\nabla} V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(-\frac{1}{c}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}}\partial_t\,\rho(\vec{\mathbf{r}}', t_r)\,\mathrm{d}\tau' + \int \rho(\vec{\mathbf{r}}', t_r)\vec{\nabla}\big(\frac{1}{\mathscr{r}}\big)\,\mathrm{d}\tau'\Big)$$

Potenciais de distribuição contínua

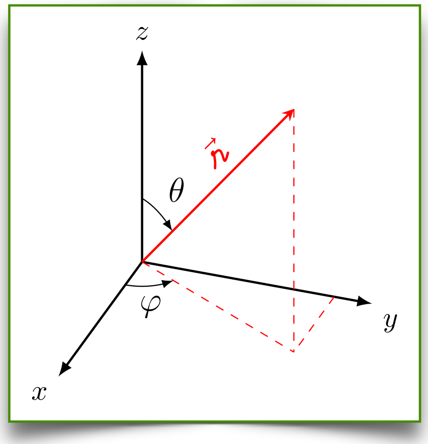
$$\vec{\nabla}\,\mathscr{r} = \hat{\mathscr{r}} \;\Rightarrow\; \vec{\nabla} t_r = -\frac{1}{c}\hat{\mathscr{r}}$$



$$\vec{\nabla} V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\left(-\frac{1}{c}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}}\partial_t\,\rho(\vec{\mathbf{r}}',t_r)\,\mathrm{d}\tau' + \int \rho(\vec{\mathbf{r}}',t_r)\vec{\nabla}\Big(\frac{1}{\mathscr{r}}\Big)\,\mathrm{d}\tau'\right)$$

$$\nabla^2 V(\vec{\mathbf{r}},t) \equiv \vec{\nabla}\cdot\vec{\nabla} V(\vec{\mathbf{r}},t) = ?$$

Potenciais de distribuição contínua



$$\vec{\nabla} \mathscr{r} = \hat{\mathscr{r}} \quad \Rightarrow \quad \vec{\nabla} t_r = -\frac{1}{c}\hat{\mathscr{r}}$$

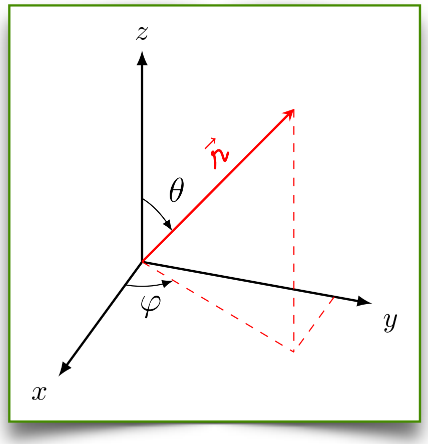
$$\vec{\nabla} V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(-\frac{1}{c}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}} \partial_t \rho(\vec{\mathbf{r}}', t_r)\, \mathrm{d}\tau' + \int \rho(\vec{\mathbf{r}}', t_r) \vec{\nabla}\big(\frac{1}{\mathscr{r}}\big)\, \mathrm{d}\tau' \Big)$$

$$\nabla^2 V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(-\frac{1}{c}\int \vec{\nabla}\cdot\big(\frac{\hat{\mathscr{r}}}{\mathscr{r}}\big)\partial_t \rho(\vec{\mathbf{r}}', t_r)\, \mathrm{d}\tau' - \frac{1}{c}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}}\cdot\vec{\nabla}\partial_t \rho(\vec{\mathbf{r}}', t_r)\, \mathrm{d}\tau'$$

$$+ \int \vec{\nabla}\rho(\vec{\mathbf{r}}', t_r)\cdot\vec{\nabla}\big(\frac{1}{\mathscr{r}}\big)\, \mathrm{d}\tau' + \int \rho(\vec{\mathbf{r}}', t_r)\nabla^2\big(\frac{1}{\mathscr{r}}\big)\, \mathrm{d}\tau' \Big)$$

Potenciais de distribuição contínua

$$\vec{\nabla}\mathscr{r} = \hat{\mathscr{r}} \quad \Rightarrow \quad \vec{\nabla} t_r = -\frac{1}{c}\hat{\mathscr{r}}$$



$$\nabla^2 V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(-\frac{1}{c}\int \vec{\nabla}\cdot\big(\frac{\hat{\mathscr{r}}}{\mathscr{r}}\big)\partial_t\,\rho(\vec{\mathbf{r}}', t_r)\,\mathrm{d}\tau' - \frac{1}{c}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}}\cdot\vec{\nabla}\partial_t\,\rho(\vec{\mathbf{r}}', t_r)\,\mathrm{d}\tau'$$

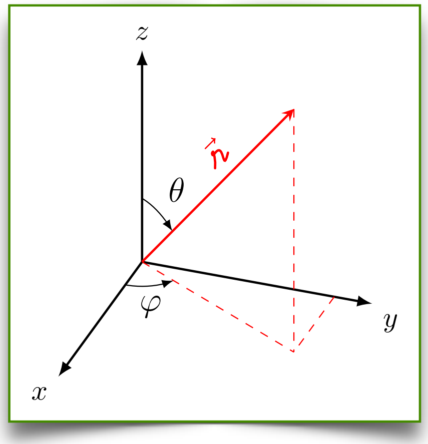
$$+ \int \vec{\nabla}\rho(\vec{\mathbf{r}}', t_r)\cdot\vec{\nabla}\big(\frac{1}{\mathscr{r}}\big)\,\mathrm{d}\tau' + \int \rho(\vec{\mathbf{r}}', t_r)\nabla^2\big(\frac{1}{\mathscr{r}}\big)\,\mathrm{d}\tau'\Big)$$

$$\vec{\nabla}\cdot\big(\frac{\hat{\mathscr{r}}}{\mathscr{r}}\big) = \frac{1}{\mathscr{r}^2}\frac{\partial\big(\mathscr{r}^2\frac{1}{\mathscr{r}}\big)}{\partial\mathscr{r}}$$

$$\vec{\nabla}\rho(\vec{\mathbf{r}}', t_r) = \partial_t\,\rho(\vec{\mathbf{r}}', t_r)\vec{\nabla}t_r$$

$$\vec{\nabla}\big(\frac{1}{\mathscr{r}}\big) = -\frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}$$

Potenciais de distribuição contínua



$$\vec{\nabla} \mathscr{r} = \hat{\mathscr{r}} \;\Rightarrow\; \vec{\nabla} t_r = -\frac{1}{c}\hat{\mathscr{r}}$$

$$\nabla^2 V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(-\frac{1}{c}\int \frac{1}{\mathscr{r}^2}\partial_t\, \rho(\vec{\mathbf{r}}', t_r)\, \mathrm{d}\tau' - \frac{1}{c}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}} \cdot \partial_t\, \vec{\nabla}\rho(\vec{\mathbf{r}}', t_r)\, \mathrm{d}\tau'$$

$$+ \frac{1}{c}\int \partial_t\, \rho(\vec{\mathbf{r}}', t_r)\hat{\mathscr{r}} \cdot \Big(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\Big)\, \mathrm{d}\tau' - 4\pi \int \rho(\vec{\mathbf{r}}', t_r)\delta(\vec{\mathscr{r}})\, \mathrm{d}\tau' \Big)$$

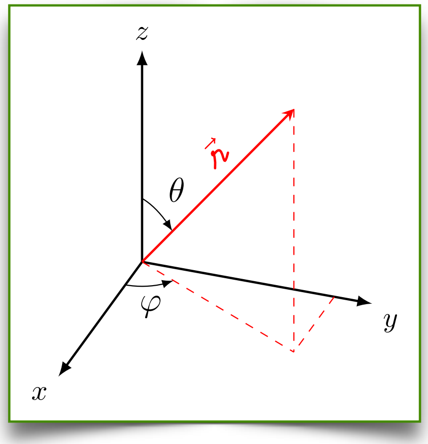
$$\vec{\nabla} \cdot \Big(\frac{\hat{\mathscr{r}}}{\mathscr{r}}\Big) = \frac{1}{\mathscr{r}^2}\frac{\partial\Big(\mathscr{r}^2 \frac{1}{\mathscr{r}}\Big)}{\partial \mathscr{r}}$$

$$\vec{\nabla}\rho(\vec{\mathbf{r}}', t_r) = \partial_t\, \rho(\vec{\mathbf{r}}', t_r)\vec{\nabla} t_r$$

$$\vec{\nabla}\Big(\frac{1}{\mathscr{r}}\Big) = -\frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}$$

Potenciais de distribuição contínua

$$\vec{\nabla}\mathscr{r} = \hat{\mathscr{r}} \quad \Rightarrow \quad \vec{\nabla} t_r = -\frac{1}{c}\hat{\mathscr{r}}$$



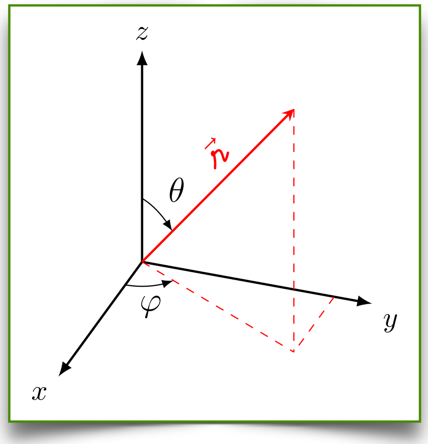
$$\nabla^2 V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(-\frac{1}{c}\int \frac{1}{\mathscr{r}^2}\partial_t\, \rho(\vec{\mathbf{r}}', t_r)\, \mathrm{d}\tau' - \frac{1}{c}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}} \cdot \partial_t\, \vec{\nabla}\rho(\vec{\mathbf{r}}', t_r)\, \mathrm{d}\tau'$$

$$+ \frac{1}{c}\int \partial_t\, \rho(\vec{\mathbf{r}}', t_r)\hat{\mathscr{r}} \cdot \big(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\big)\, \mathrm{d}\tau' - 4\pi \int \rho(\vec{\mathbf{r}}', t_r)\delta(\vec{\mathscr{r}})\, \mathrm{d}\tau' \Big)$$

$$\nabla^2 V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(\frac{1}{c^2}\int \frac{1}{\mathscr{r}}\partial_t^2 \rho(\vec{\mathbf{r}}', t_r)\, \mathrm{d}\tau' - 4\pi \int \rho(\vec{\mathbf{r}}', t_r)\delta(\vec{\mathscr{r}})\, \mathrm{d}\tau' \Big)$$

Potenciais de distribuição contínua

$$\vec{\nabla}\mathscr{r} = \hat{\mathscr{r}} \quad \Rightarrow \quad \vec{\nabla}t_r = -\frac{1}{c}\hat{\mathscr{r}}$$



$$\nabla^2 V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\Big(-\frac{1}{c}\int \frac{1}{\mathscr{r}^2}\partial_t\,\rho(\vec{\mathbf{r}}',t_r)\,\mathrm{d}\tau' - \frac{1}{c}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}}\cdot\partial_t\,\vec{\nabla}\rho(\vec{\mathbf{r}}',t_r)\,\mathrm{d}\tau'$$

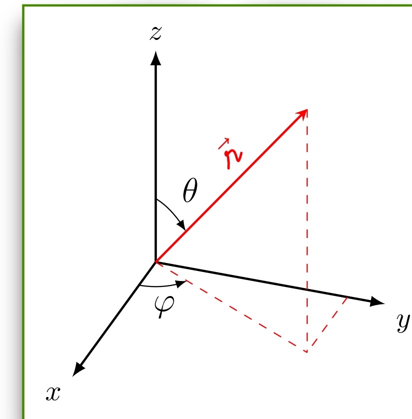
$$+\frac{1}{c}\int \partial_t\,\rho(\vec{\mathbf{r}}',t_r)\hat{\mathscr{r}}\cdot\Big(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\Big)\,\mathrm{d}\tau' - 4\pi\int\rho(\vec{\mathbf{r}}',t_r)\delta(\vec{\mathscr{r}})\,\mathrm{d}\tau'\Big)$$

$$\nabla^2 V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\Big(\frac{1}{c^2}\int\frac{1}{\mathscr{r}}\partial_t^2\rho(\vec{\mathbf{r}}',t_r)\,\mathrm{d}\tau' - 4\pi\int\rho(\vec{\mathbf{r}}',t_r)\delta(\vec{\mathscr{r}})\,\mathrm{d}\tau'\Big)$$

$$\nabla^2 V(\vec{\mathbf{r}},t) = \frac{1}{c^2}\partial_t^2 V - \frac{\rho(\vec{\mathbf{r}},t)}{\epsilon_0}$$

Potenciais de distribuição contínua

$$\vec{\nabla} \mathscr{r} = \hat{\mathscr{r}} \;\Rightarrow\; \vec{\nabla} t_r = -\frac{1}{c}\hat{\mathscr{r}}$$



$$\nabla^2 V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(-\frac{1}{c}\int \frac{1}{\mathscr{r}^2}\partial_t\,\rho(\vec{\mathbf{r}}', t_r)\,\mathrm{d}\tau' - \frac{1}{c}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}}\cdot\partial_t\,\vec{\nabla}\rho(\vec{\mathbf{r}}', t_r)\,\mathrm{d}\tau'$$

$$+\frac{1}{c}\int \partial_t\,\rho(\vec{\mathbf{r}}', t_r)\hat{\mathscr{r}}\cdot\Big(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\Big)\,\mathrm{d}\tau' - 4\pi\int \rho(\vec{\mathbf{r}}', t_r)\delta(\vec{\mathscr{r}})\,\mathrm{d}\tau'\Big)$$

$$\nabla^2 V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\Big(\frac{1}{c^2}\int \frac{1}{\mathscr{r}}\partial_t^2\rho(\vec{\mathbf{r}}', t_r)\,\mathrm{d}\tau' - 4\pi\int \rho(\vec{\mathbf{r}}', t_r)\delta(\vec{\mathscr{r}})\,\mathrm{d}\tau'\Big)$$

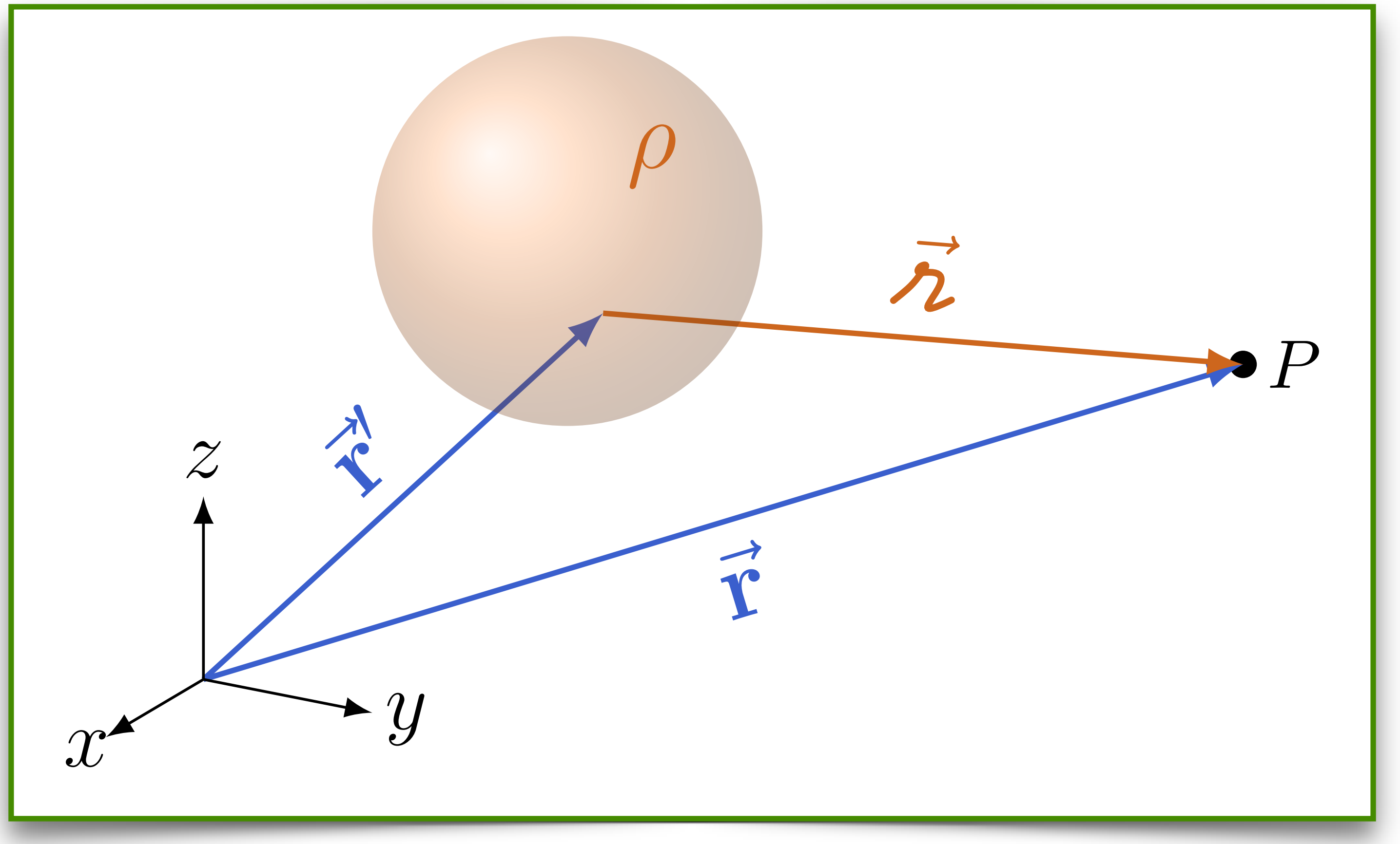
$$\nabla^2 V(\vec{\mathbf{r}}, t) = \frac{1}{c^2}\partial_t^2 V - \frac{\rho(\vec{\mathbf{r}}, t)}{\epsilon_0}$$

$$\boxed{\nabla^2 V(\vec{\mathbf{r}}, t) - \frac{1}{c^2}\partial_t^2 V = -\frac{\rho(\vec{\mathbf{r}}, t)}{\epsilon_0}}$$

Potenciais de distribuição contínua

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \vec{\mathbf{A}} - \frac{1}{c^2}\frac{\partial^2 \vec{\mathbf{A}}}{\partial t^2} = -\mu_0 \vec{\mathbf{J}}$$



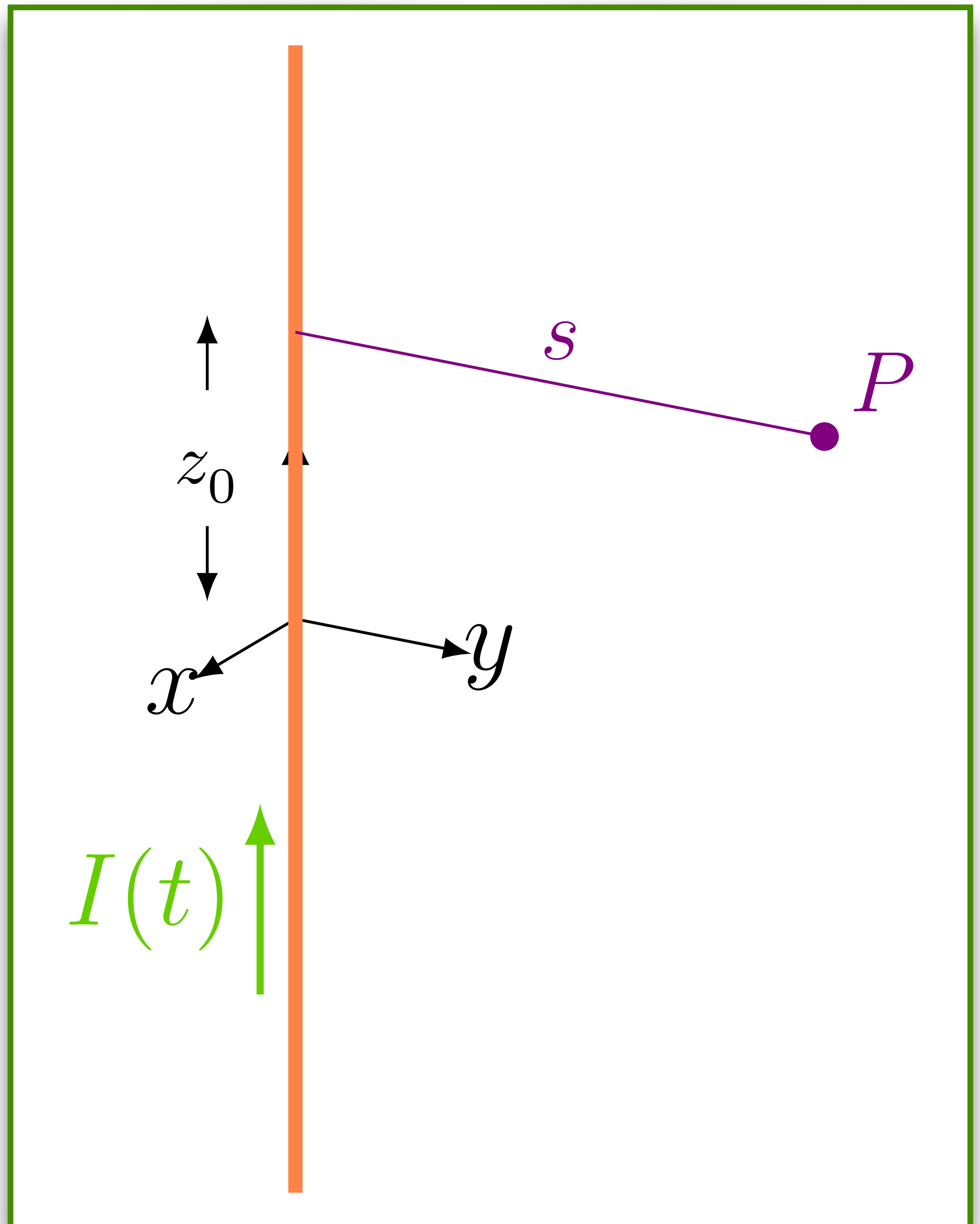
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{\mathbf{r}}', t_{\mathrm{ret}})}{\mathscr{r}}\, d\tau'$$

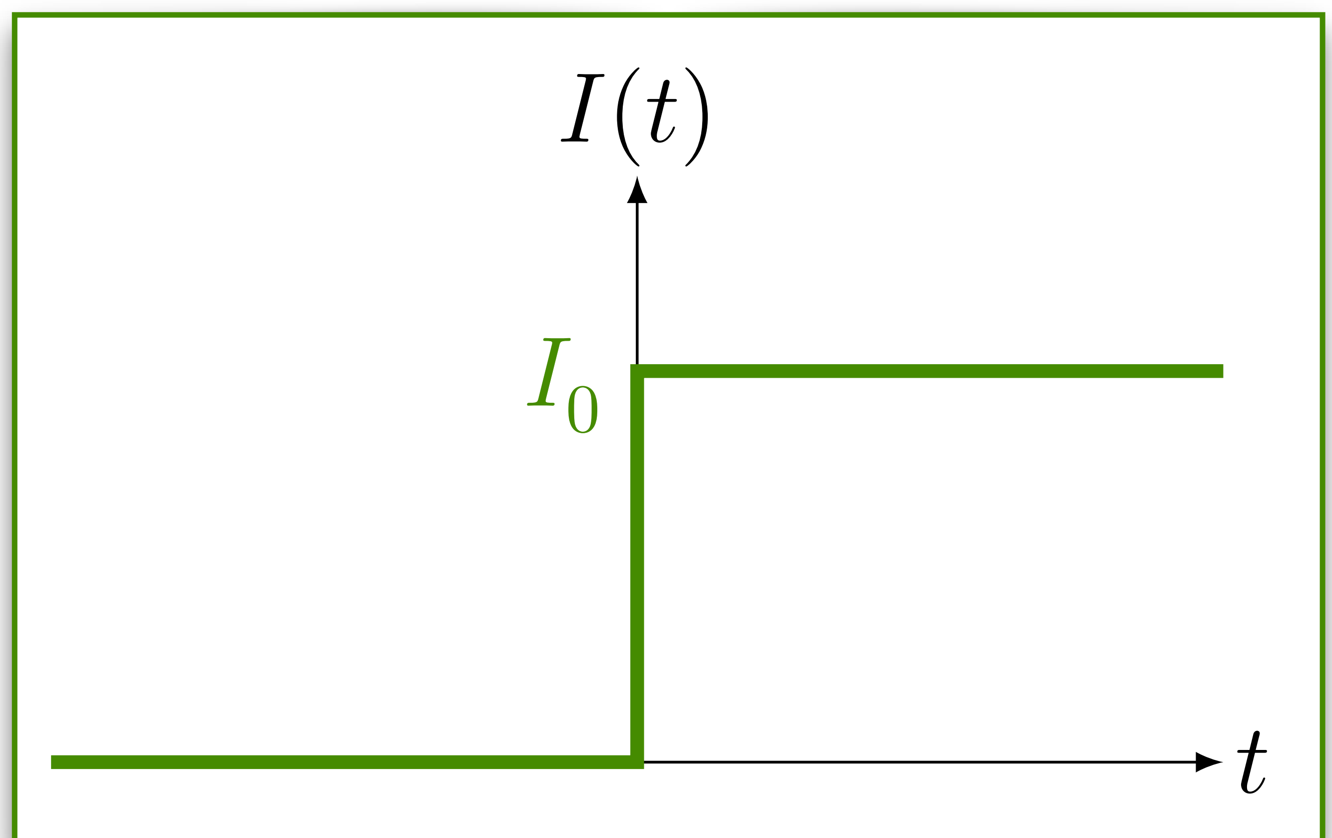
$$t_r \equiv t - \frac{\mathscr{r}}{c}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}', t_{\mathrm{ret}})}{\mathscr{r}}\, d\tau'$$

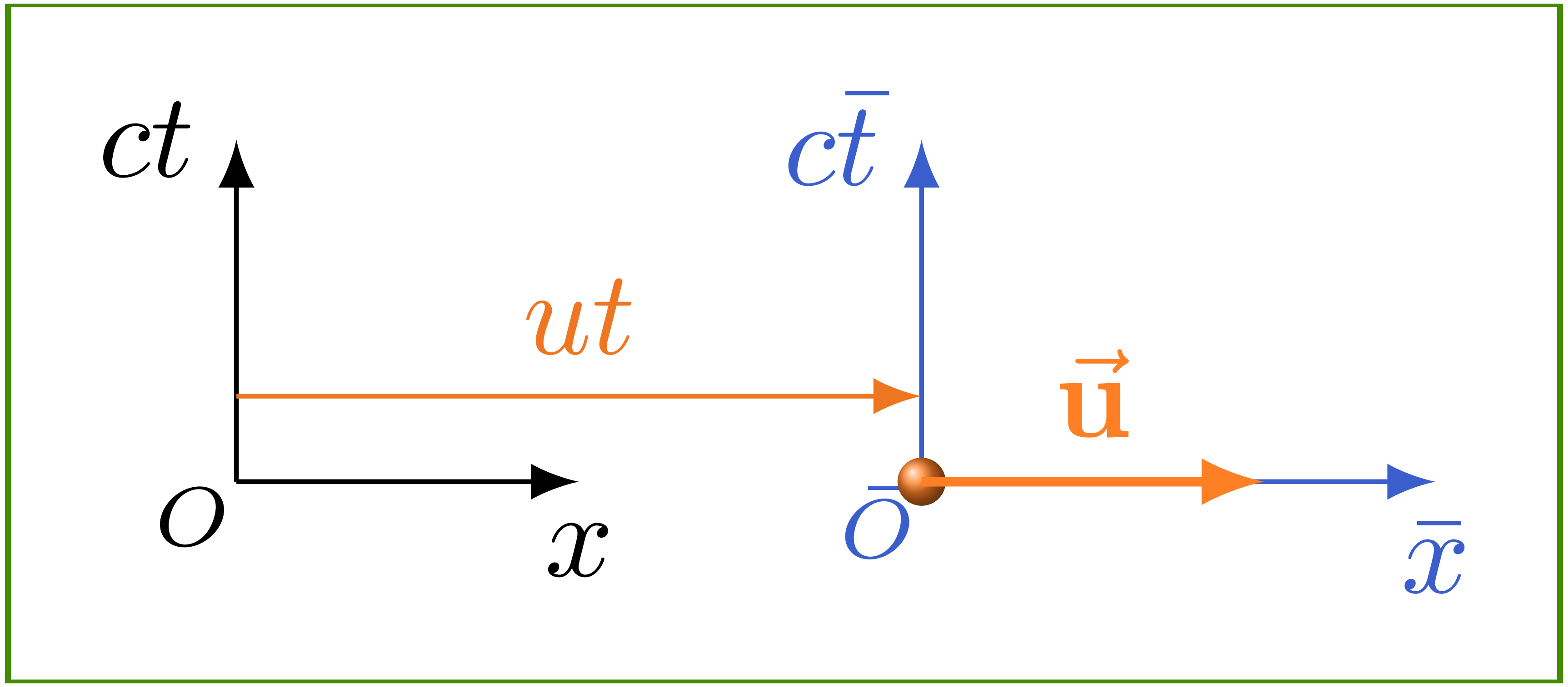
Pratique o que aprendeu

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}}',t_r)}{\mathscr{r}}\,\mathrm{d}\tau'$$





Potenciais de Liénard e Wiechert



$$V = \frac{q}{4\pi\epsilon_0\left(r - \frac{\vec{\mathbf{u}}}{c}\cdot\vec{\mathbf{r}}\right)}$$

$$\bar{V} = \frac{q}{4\pi\epsilon_0 r}$$

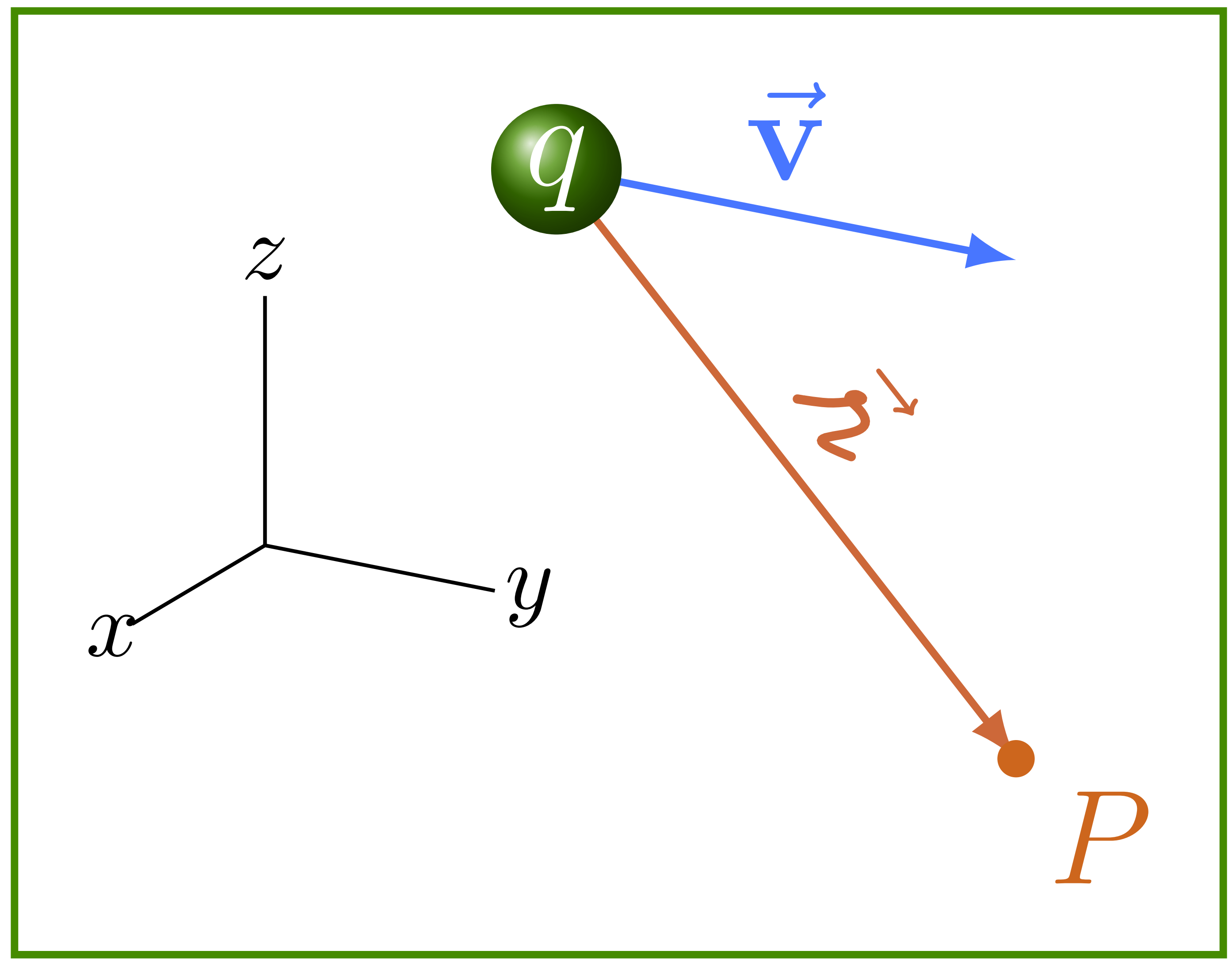
$$\vec{\mathbf{A}} = \mu_0 \frac{q\vec{\mathbf{u}}}{4\pi\left(r - \frac{\vec{\mathbf{u}}}{c}\cdot\vec{\mathbf{r}}\right)}$$

$$\bar{\vec{\mathbf{A}}} = 0$$

Potenciais de Liénard e Wiechert

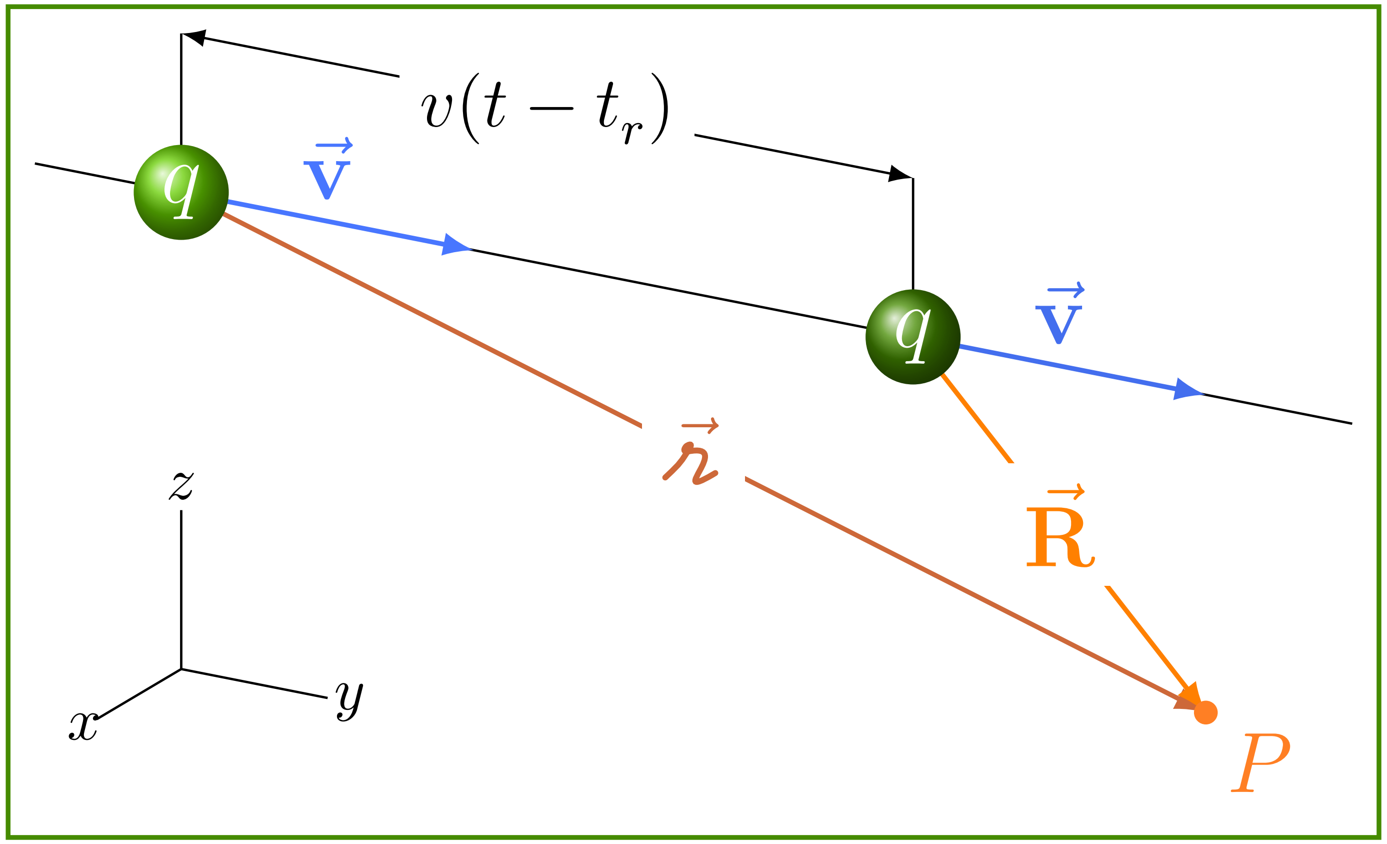
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \dfrac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi} \frac{q\vec{\mathbf{v}}}{\mathscr{r} - \dfrac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



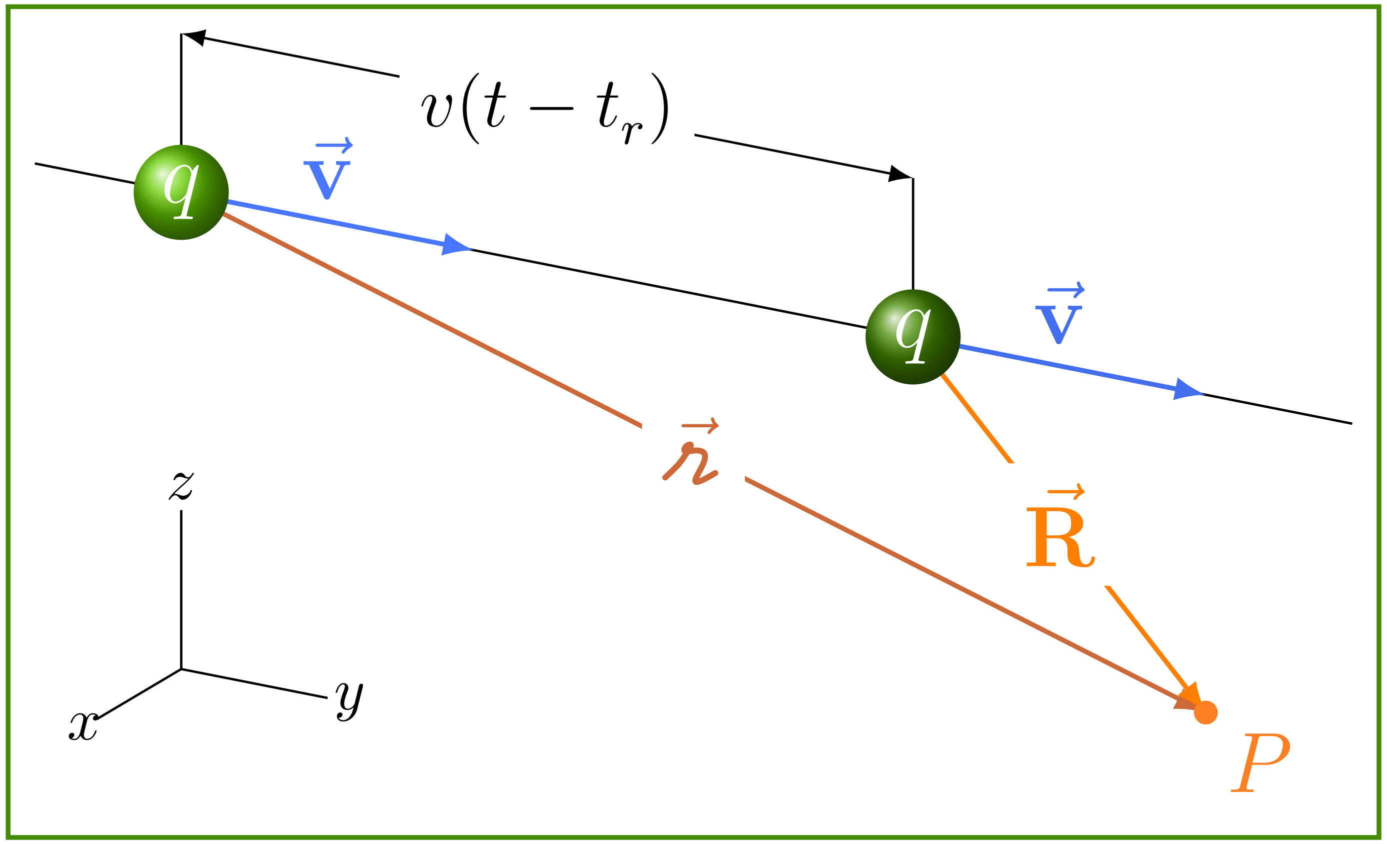
Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



Movimento uniforme

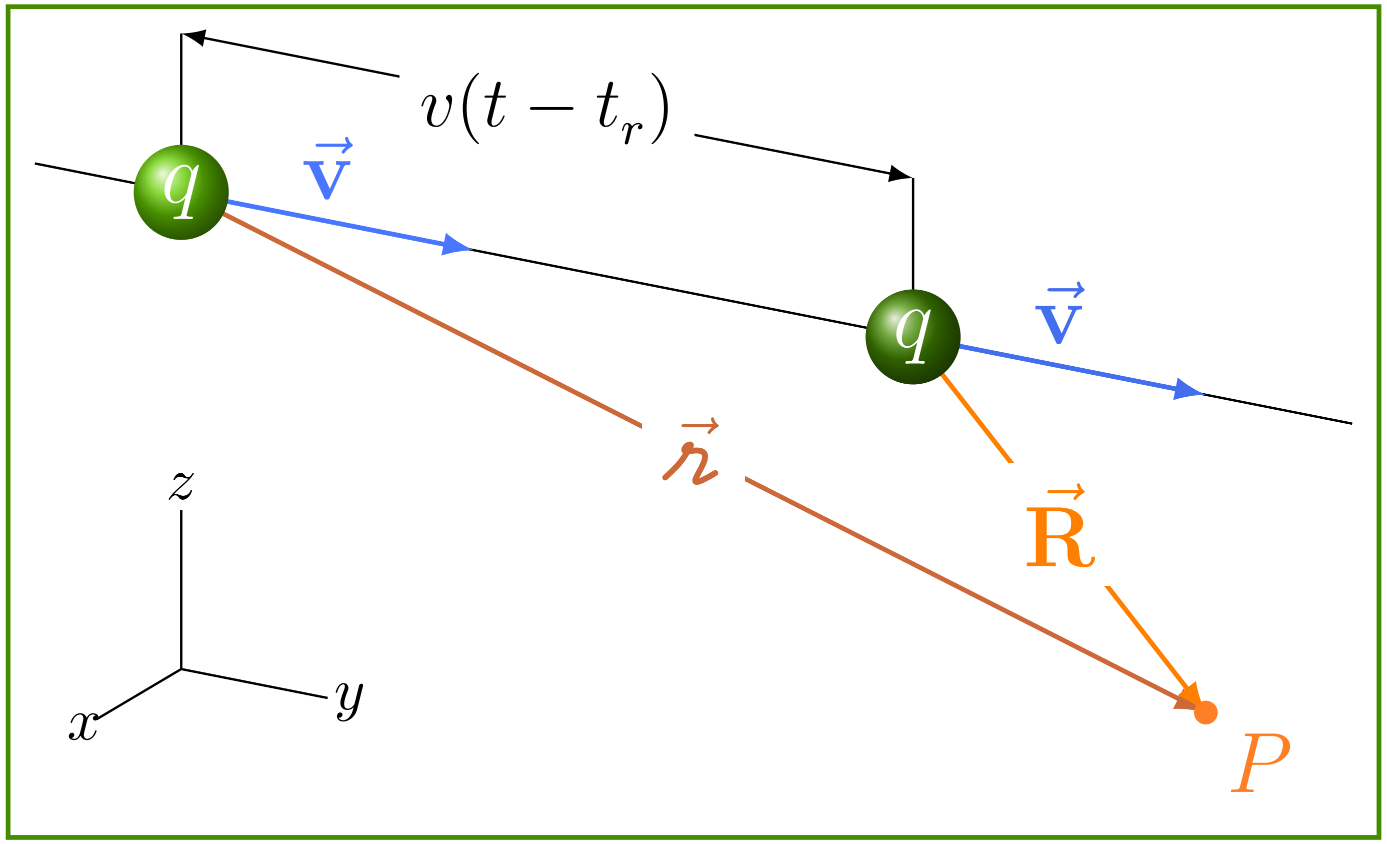
$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



$$\vec{\mathscr{r}} = \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \vec{\mathbf{R}}$$

Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$

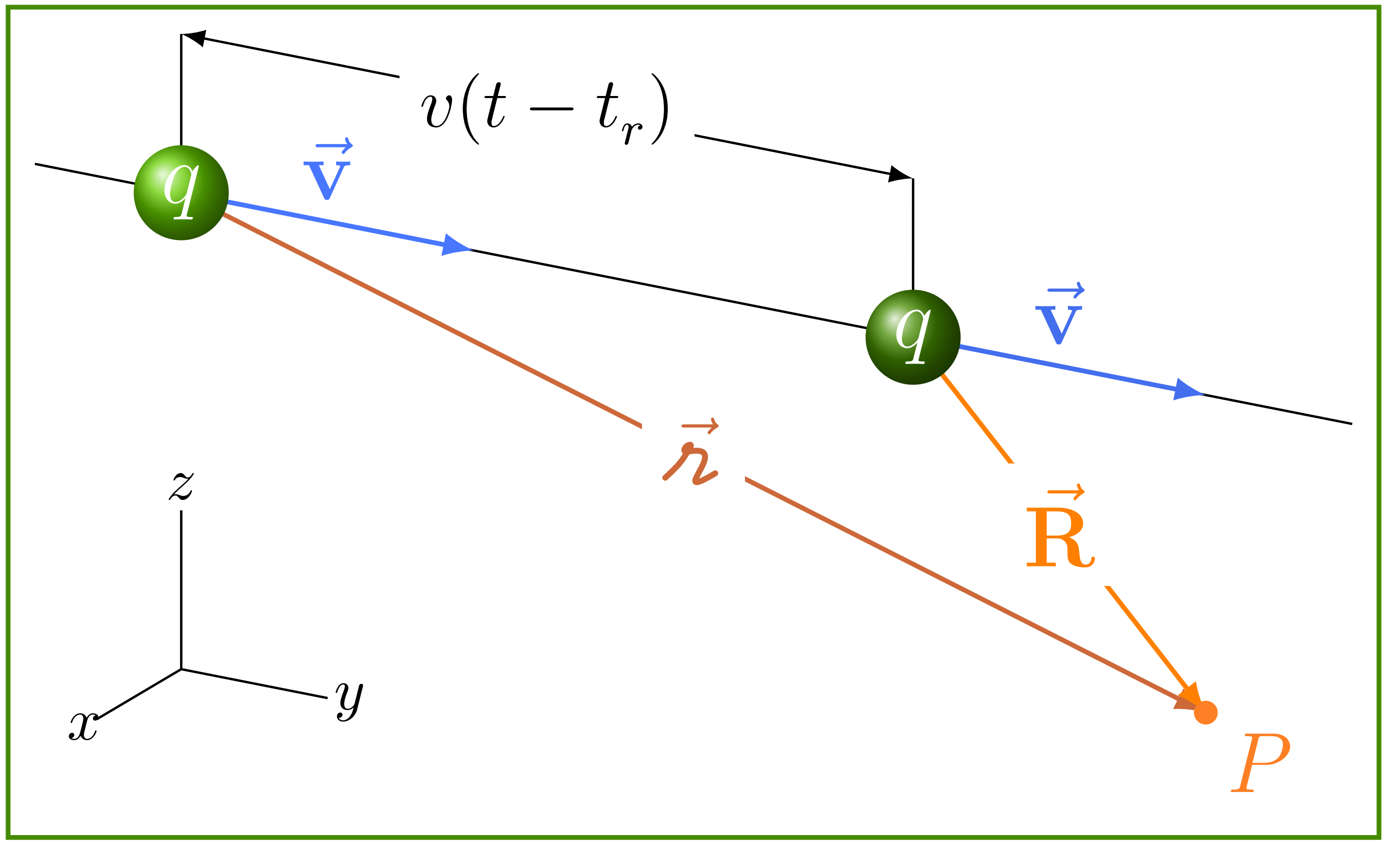


$$\vec{\mathscr{r}} = \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \vec{\mathbf{R}}$$

$$\hat{\mathscr{r}} \cdot \vec{\mathscr{r}} = \hat{\mathscr{r}} \cdot \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \hat{\mathscr{r}} \cdot \vec{\mathbf{R}}$$

Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



$$\vec{\mathscr{r}} = \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \vec{\mathbf{R}}$$

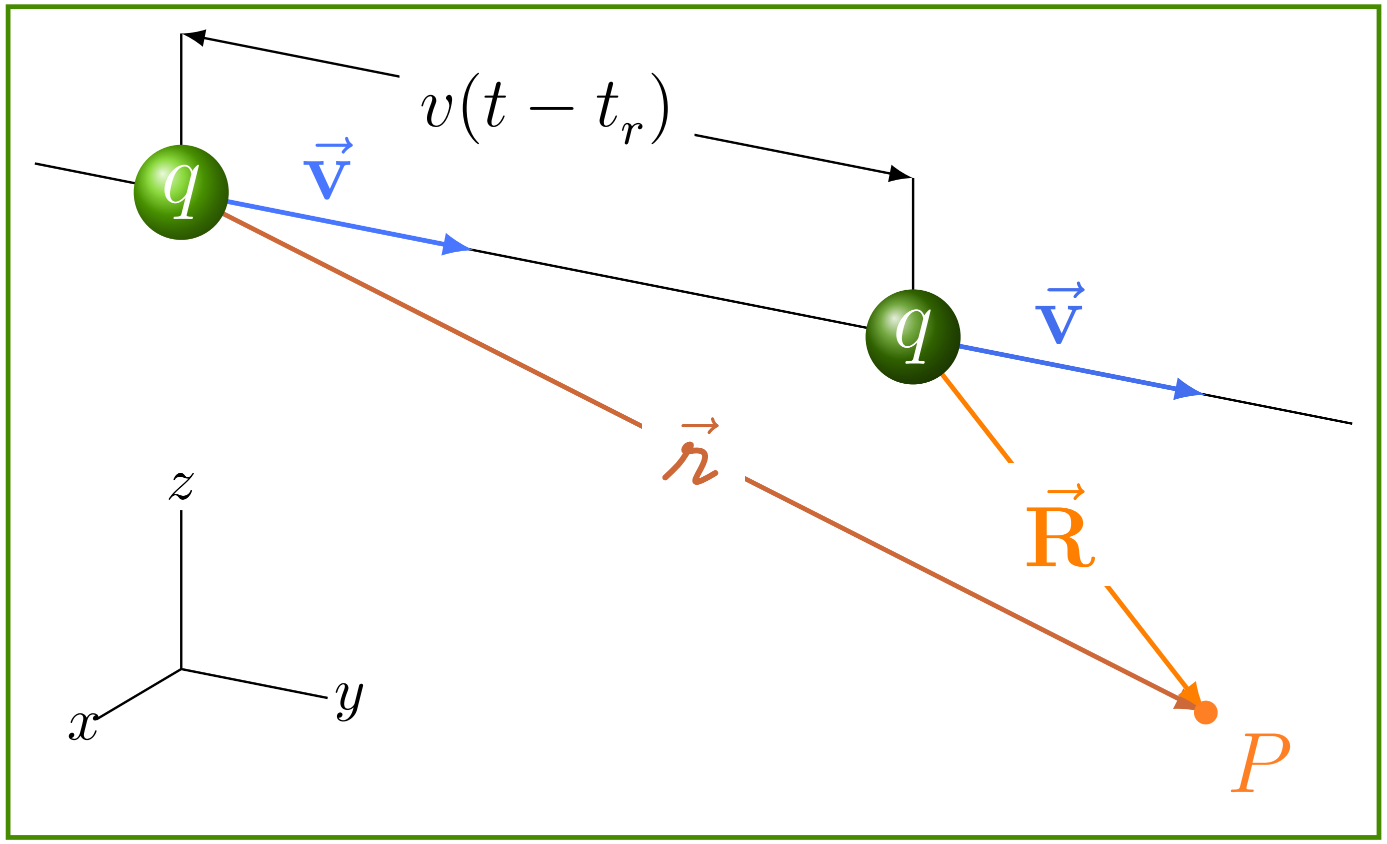
$$\hat{\mathscr{r}} \cdot \vec{\mathscr{r}} = \hat{\mathscr{r}} \cdot \frac{\mathscr{r}}{c}\vec{\mathbf{v}} + \hat{\mathscr{r}} \cdot \vec{\mathbf{R}}$$

$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R\cos\alpha$$

Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$

$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R \cos \alpha$$

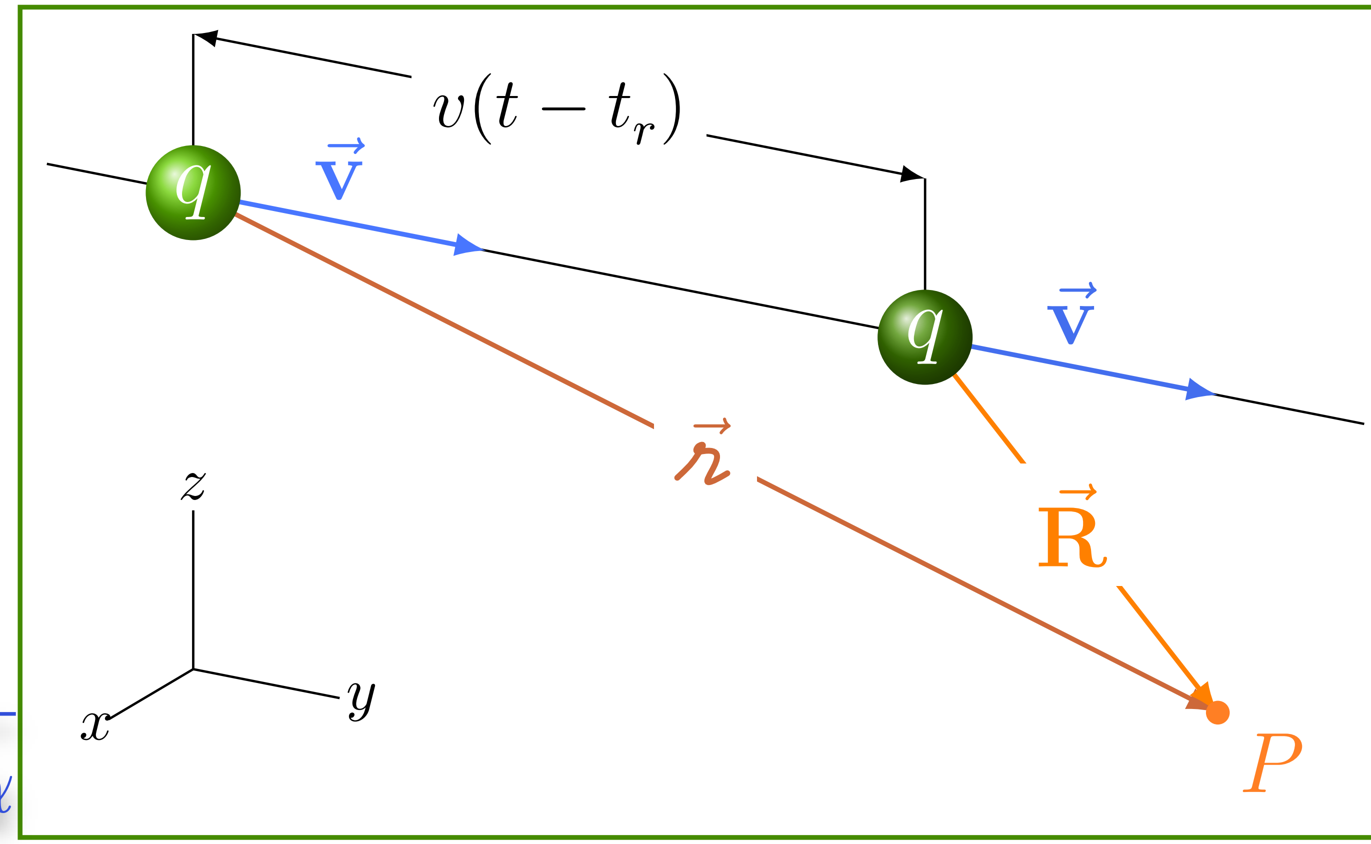


Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$

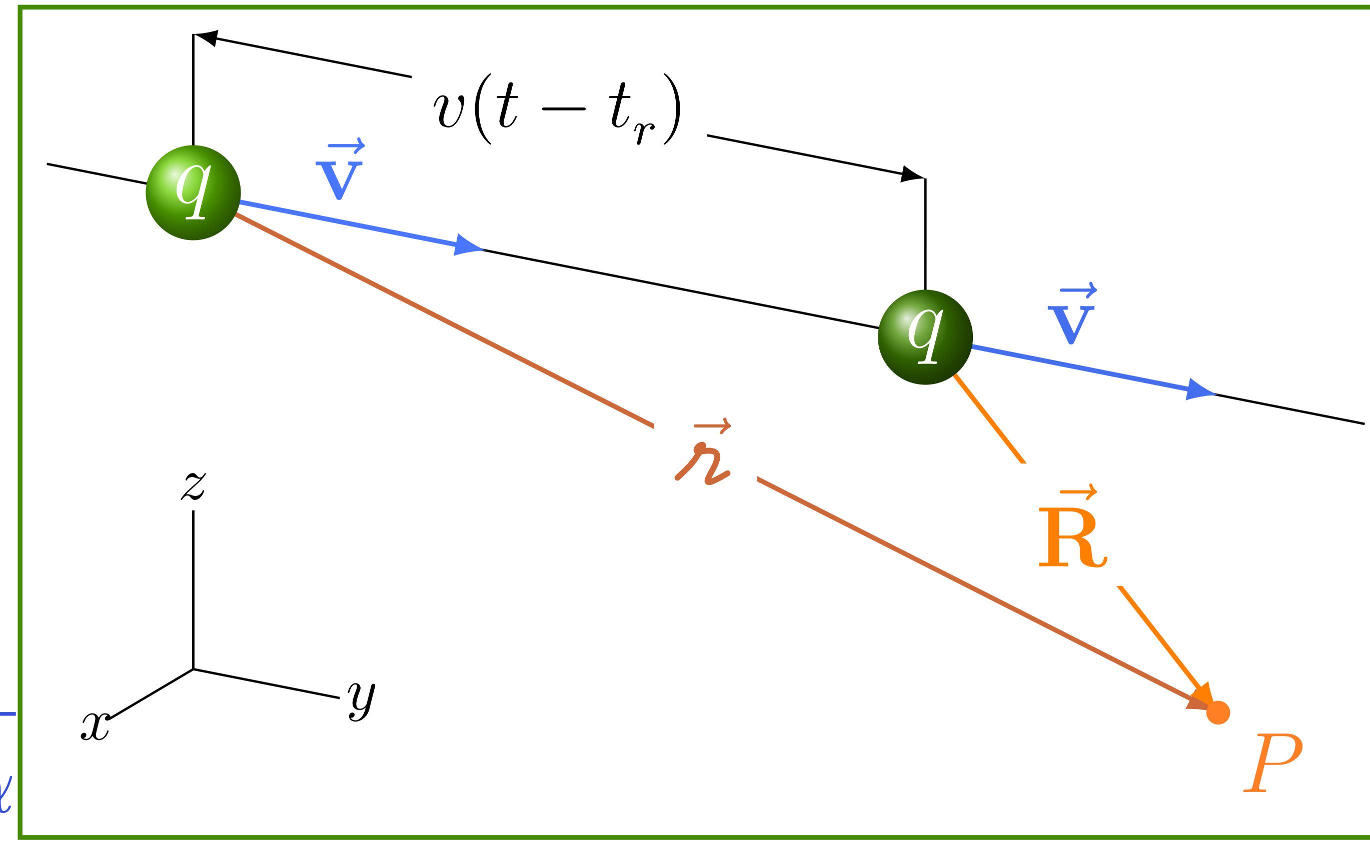
$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R \cos\alpha$$

$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R\sqrt{1 - \sin^2\alpha}$$



Movimento uniforme

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\mathscr{r} - \frac{\vec{\mathbf{v}}}{c} \cdot \vec{\mathscr{r}}}$$



$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R \cos\alpha$$

$$\mathscr{r} - \vec{\mathscr{r}} \cdot \frac{\vec{\mathbf{v}}}{c} = R\sqrt{1 - \sin^2\alpha}$$

$$\boxed{\frac{\sin\alpha}{v\frac{\mathscr{r}}{c}} = \frac{\sin\theta}{\mathscr{r}}}$$

$$V(\vec{\mathbf{r}}, t) = \frac{1}{4\pi\epsilon_0} \frac{q}{R\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$