Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecânica

PME-3554 – Introdução às Estruturas Aeronáuticas

Aula #19

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Idealização Estrutural [1-20]

1. Fundamentos da idealização estrutural [1-20.1]

2. Idealização de um painel [1-20.2]

3. Exemplo de aplicação

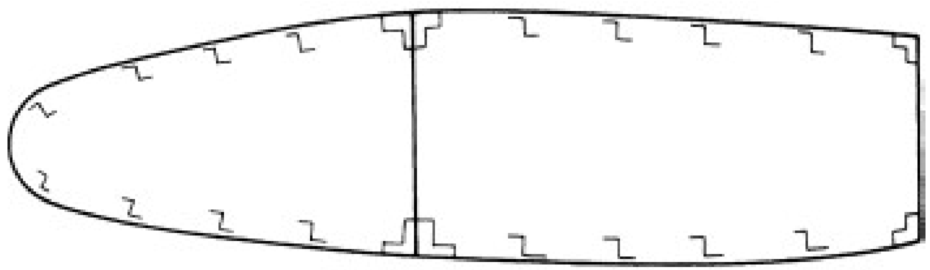
4. Efeito da idealização na análise de vigas sob flexão pura [1-20.3.1]

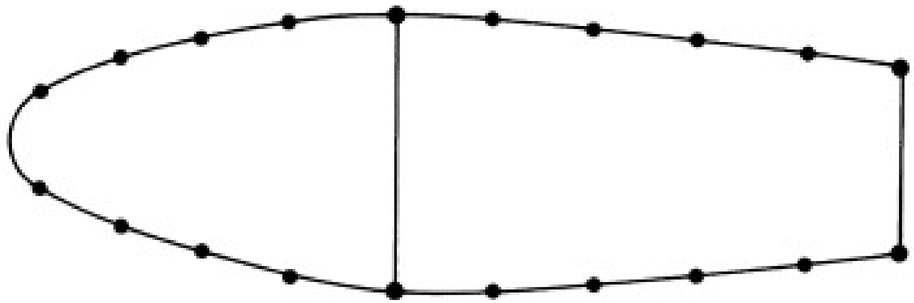
5. Efeito da idealização nos fluxos de cisalhamento de vigas com seções transversais abertas [1-20.3.2]

6. Exemplo de aplicação

1. Fundamentos da idealização estrutural [1-20.1]

Complex structural sections may be idealized into simpler "mechanical model" forms, which behave, under given loading conditions, in the same, or very nearly the same, way as the actual structure. We shall see, however, that different models of the same structure are required to simulate actual behavior under different systems of loading.





2. Idealização de um painel [1-20.2]

Suppose we wish to idealize the panel of Fig. 20.3(a) into a combination of direct stress carrying booms and shear stress only carrying skin, as shown in Fig. 20.3(b). In Fig. 20.3(a), the direct stress carrying thickness t_D of the skin is equal to its actual thickness t, while, in Fig. 20.3(b), $t_D = 0$. Suppose also that the direct stress distribution in the actual panel varies linearly from an unknown value σ_1 to an unknown value σ_2. Clearly, the analysis should predict the extremes of stress σ_1 and σ_2, although the distribution of direct stress is obviously lost. Since the loading producing the direct stresses in the actual and idealized panels must be the same, we can equate moments to obtain expressions for the boom areas B_1 and B_2. Thus, taking moments about the right-hand edge of each panel,

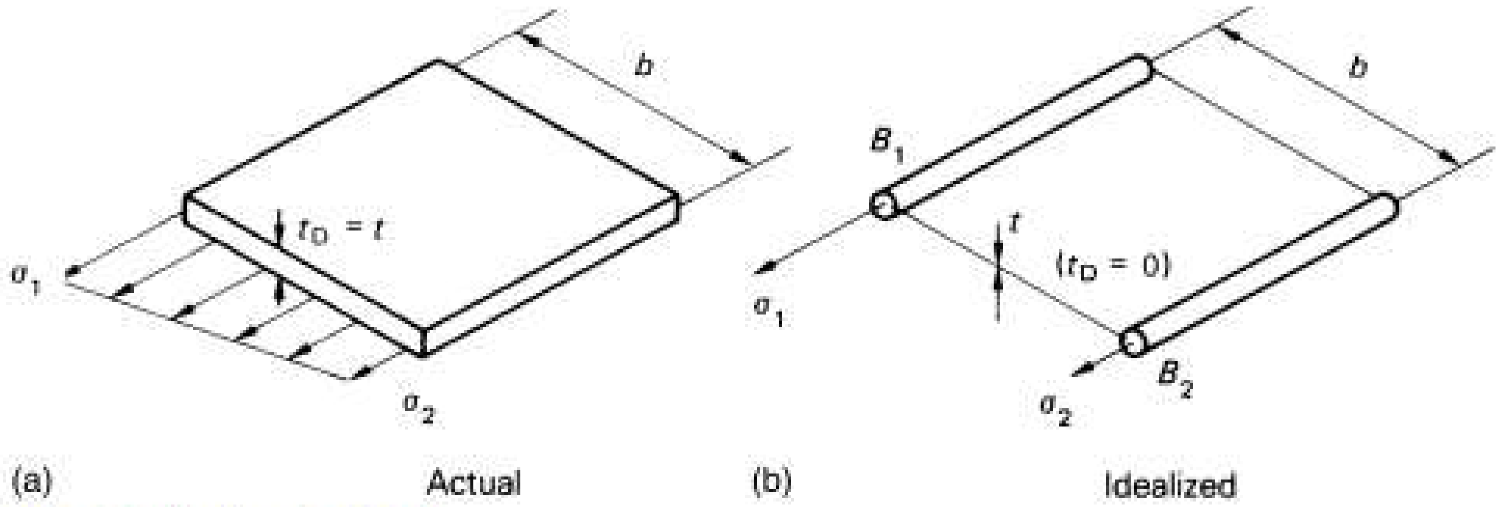


FIGURE 20.3 Idealization of a Panel

$$\sigma_2 t_D \frac{b^2}{2} + \frac{1}{2}(\sigma_1 - \sigma_2) t_D b \frac{2}{3} b = \sigma_1 B_1 b$$

from which

$$B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) \tag{20.1}$$

Similarly,

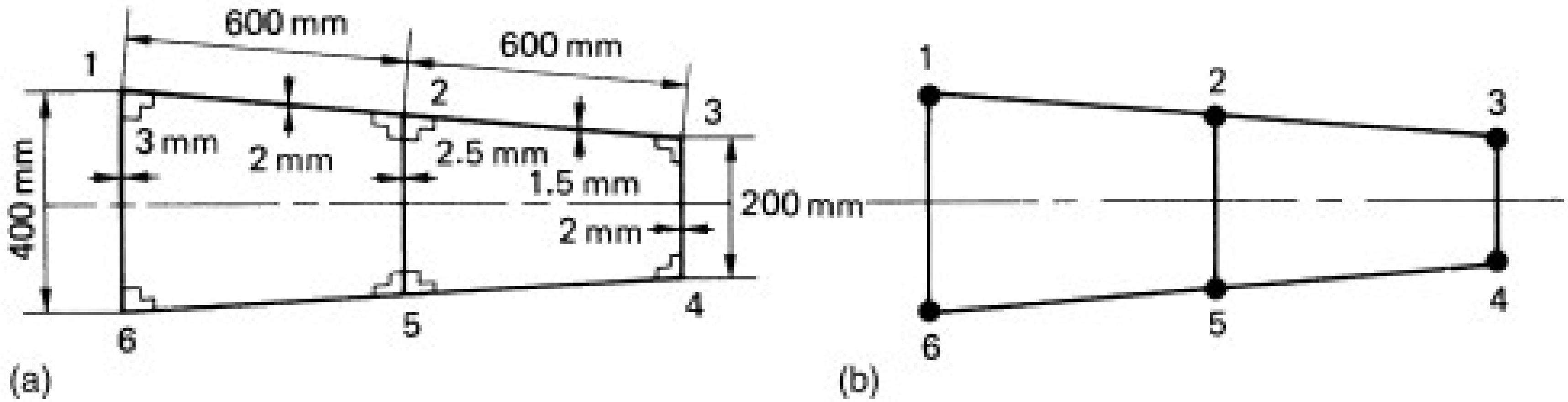
$$B_2 = \frac{t_D b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right) \tag{20.2}$$

In Eqs. (20.1) and (20.2), the ratio of σ_1 to σ_2, if not known, may frequently be assumed.

The direct stress distribution in Fig. 20.3(a) is caused by a combination of axial load and bending moment. For axial load only, $\sigma_1/\sigma_2 = 1$ and $B_1 = B_2 = t_D b/2$; for a pure bending moment, $\sigma_1/\sigma_2 = -1$ and $B_1 = B_2 = t_D b/6$. Thus, different idealizations of the same structure are required for different loading conditions.

3. Exemplo de aplicação

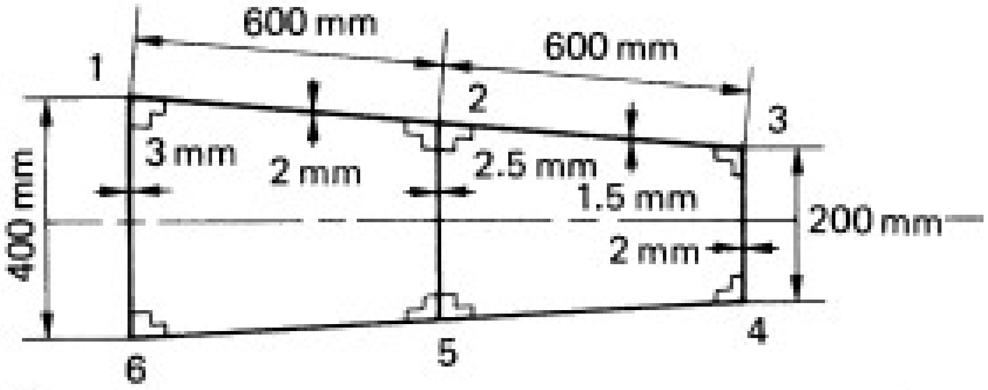
Part of a wing section is in the form of the two-cell box shown in Fig. 20.4(a), in which the vertical spars are connected to the wing skin through angle sections all having a cross-sectional area of 300 mm^2. Idealize the section into an arrangement of direct stress carrying booms and shear stress only carrying panels suitable for resisting bending moments in a vertical plane. Position the booms at the spar/skin junctions.



Solution:

The idealized section is shown in Fig. 20.4(b), in which, from symmetry, $B_1 = B_6$, $B_2 = B_5$, $B_3 = B_4$. Since the section is required to resist bending moments in a vertical plane, the direct stress at any point in the actual wing section is directly proportional to its distance from the horizontal axis of symmetry. Further, the distribution of direct stress in all the panels is linear, so that either Eq. (20.1) or (20.2) may be used. We note that, in addition to contributions from adjacent panels, the boom areas include the existing spar flanges. Hence.

$$B_1 = 300 + \frac{3.0 \times 400}{6}\left(2 + \frac{\sigma_6}{\sigma_1}\right) + \frac{2.0 \times 600}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)$$



(a)

or

$$B_1 = 300 + \frac{3.0 \times 400}{6}(2 - 1) + \frac{2.0 \times 600}{6}\left(2 + \frac{150}{200}\right)$$

which gives

$$B_1 (= B_6) = 1,050 \text{ mm}^2$$

Also,

$$B_2 = 2 \times 300 + \frac{2.0 \times 600}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right) + \frac{2.5 \times 300}{6}\left(2 + \frac{\sigma_5}{\sigma_2}\right) + \frac{1.5 \times 600}{6}\left(2 + \frac{\sigma_3}{\sigma_2}\right)$$

that is,

$$B_2 = 2 \times 300 + \frac{2.0 \times 600}{6}\left(2 + \frac{200}{150}\right) + \frac{2.5 \times 300}{6}(2 - 1) + \frac{1.5 \times 600}{6}\left(2 + \frac{100}{150}\right)$$

from which

$$B_2 (= B_5) = 1,791.7 \text{ mm}^2$$

Finally,

$$B_3 = 300 + \frac{1.5 \times 600}{6}\left(2 + \frac{\sigma_2}{\sigma_3}\right) + \frac{2.0 \times 200}{6}\left(2 + \frac{\sigma_4}{\sigma_3}\right)$$

that is,

$$B_3 = 300 + \frac{1.5 \times 600}{6}\left(2 + \frac{150}{100}\right) + \frac{2.0 \times 200}{6}(2 - 1)$$

so that

$$B_3\ (= B_4) = 891.7\ \mathrm{mm}^2$$

4. Efeito da idealização na análise de vigas sob flexão pura [1-20.3.1]

The analysis presented in Sections 16.1 and 16.2 applies and the direct stress distribution is given by any of Eqs. (16.9), (16.17), or (16.18), depending on the beam section being investigated. In these equations the coordinates (x, y) of points in the cross-section are referred to axes having their origin at the centroid of the direct stress carrying area. Furthermore, the section properties I_{xx}, I_{yy}, and I_{xy} are calculated for the direct stress carrying area only.

In the case where the beam cross-section has been completely idealized into direct stress carrying booms and shear stress only carrying panels, the direct stress distribution consists of a series of direct stresses concentrated at the centroids of the booms.

$$\sigma_z = \frac{My}{I} \tag{16.9}$$

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y \tag{16.17}$$

$$\sigma_z = \frac{M_x (I_{yy} y - I_{xy} x)}{I_{xx} I_{yy} - I_{xy}^2} + \frac{M_y (I_{xx} x - I_{xy} y)}{I_{xx} I_{yy} - I_{xy}^2} \tag{16.18}$$

Example 20.2

The fuselage section shown in Fig. 20.5 is subjected to a bending moment of 100 kNm applied in the vertical plane of symmetry. If the section has been completely idealized into a combination of direct stress carrying booms and shear stress only carrying panels, determine the direct stress in each boom.

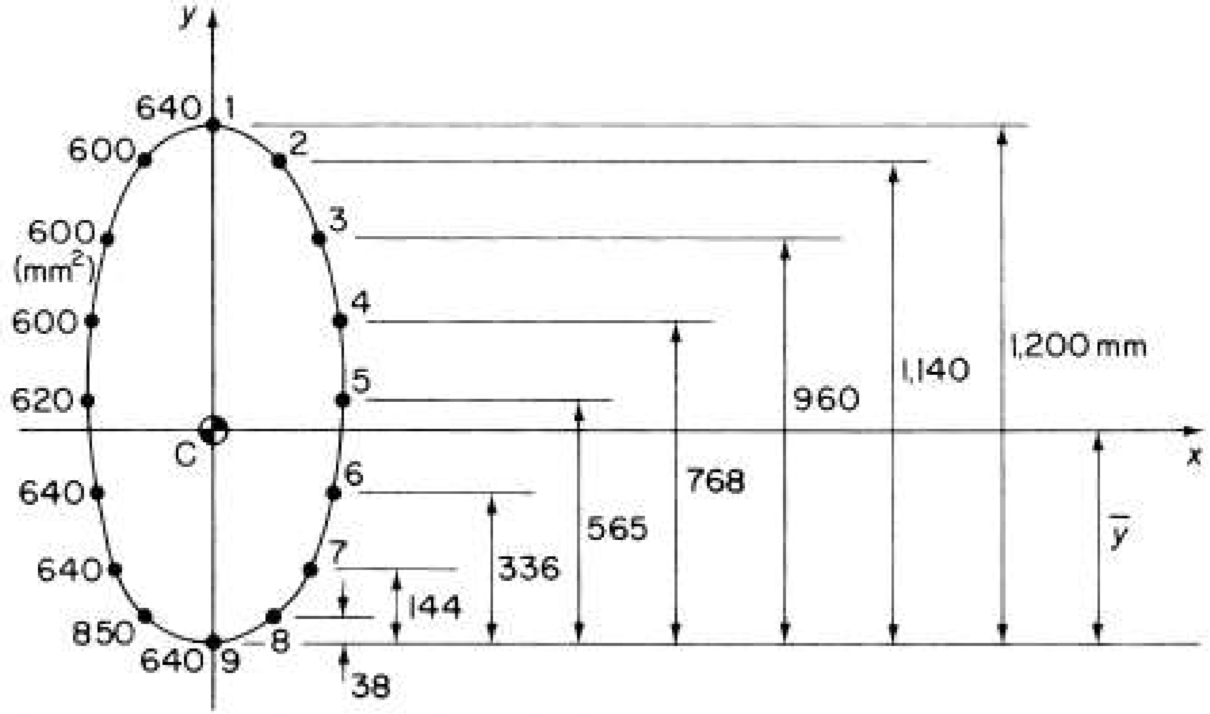


FIGURE 20.5 Idealized fuselage Section of Example 20.2

Solution:

The section has Cy as an axis of symmetry and resists a bending moment $M_x = 100$ kNm. Equation (16.17) therefore reduces to

$$\sigma_z = \frac{M_x}{I_{xx}} y \qquad \text{Eq.(i)}$$

The origin of axes Cxy coincides with the position of the centroid of the direct stress carrying area, which, in this case, is the centroid of the boom areas. Thus, taking moments of area about boom 9,

$$\begin{aligned}&(6 \times 640 + 6 \times 600 + 2 \times 620 + 2 \times 850)\bar{y} \\ &\quad = 640 \times 1{,}200 + 2 \times 600 \times 1140 + 2 \times 600 \times 960 + 2 \times 600 \times 768 \\ &\qquad + 2 \times 620 \times 565 + 2 \times 640 \times 336 + 2 \times 640 \times 144 + 2 \times 850 \times 38\end{aligned}$$

which gives

$$\bar{y} = 540 \text{ mm}$$

The solution is now completed in Table 20.1.

Table 20.1 Example 20.2

① Boom	② y (mm)	③ B (mm^2)	④ $\Delta I_{xx} = By^2$ (mm^4)	⑤ σ_z (N/mm^2)
1	+660	640	278×10^6	35.6
2	+600	600	216×10^6	32.3
3	+420	600	106×10^6	22.6
4	+228	600	31×10^6	12.3
5	+25	620	0.4×10^6	1.3
6	–204	640	27×10^6	–11.0
7	–396	640	100×10^6	–21.4
8	–502	850	214×10^6	–27.0
9	–540	640	187×10^6	–29.0

From column ④,

$$I_{xx} = 1,854 \times 10^6 \text{ mm}^4$$

and column ⑤ is completed using Eq. (i).

5. Efeito da idealização nos fluxos de cisalhamento de vigas com seções transversais abertas [1-20.3.2]

The derivation of Eq. (17.14) for the shear flow distribution in the cross-section of an open section beam is based on the equilibrium equation (17.2). The thickness t in this equation refers to the direct stress carrying thickness t_D of the skin. Equation (17.14) may therefore be rewritten

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\int_0^s t_D x \, ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\int_0^s t_D y \, ds \qquad (20.3)$$

in which $t_D = t$ if the skin is fully effective in carrying direct stress or $t_D = 0$ if the skin is assumed to carry only shear stresses. Again, the section properties in Eq. (20.3) refer to the direct stress carrying area of the section, since they are those which feature in Eqs. (16.17) and (16.18).

$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0 \qquad (17.2)$$

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\int_0^s t x \, ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\int_0^s t y \, ds \qquad (17.14)$$

Equation (20.3) makes no provision for the effects of booms which cause discontinuities in the skin and therefore interrupt the shear flow. Consider the equilibrium of the rth boom in the elemental length of beam shown in Fig. 20.6(a), which carries shear loads S_x and S_y acting through its shear center S. These shear loads produce direct stresses due to bending in the booms and skin and shear stresses in the skin. Suppose that the shear flows in the skin adjacent to the rth boom of cross-sectional area B_r are q_1 and q_2. Then, from Fig. 20.6(b),

$$\left(\sigma_z + \frac{\partial \sigma_z}{\partial z}\delta z\right)B_r - \sigma_z B_r + q_2 \delta z - q_1 \delta z = 0$$

which simplifies to

$$q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z}B_r \tag{20.4}$$

Substituting for σ_z in Eq. (20.4) from (16.17), we have

$$q_2 - q_1 = -\left[\frac{(\partial M_y/\partial z)I_{xx} - (\partial M_x/\partial z)I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right]B_r x_r - \left[\frac{(\partial M_x/\partial z)I_{yy} - (\partial M_y/\partial z)I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right]B_r y_r$$

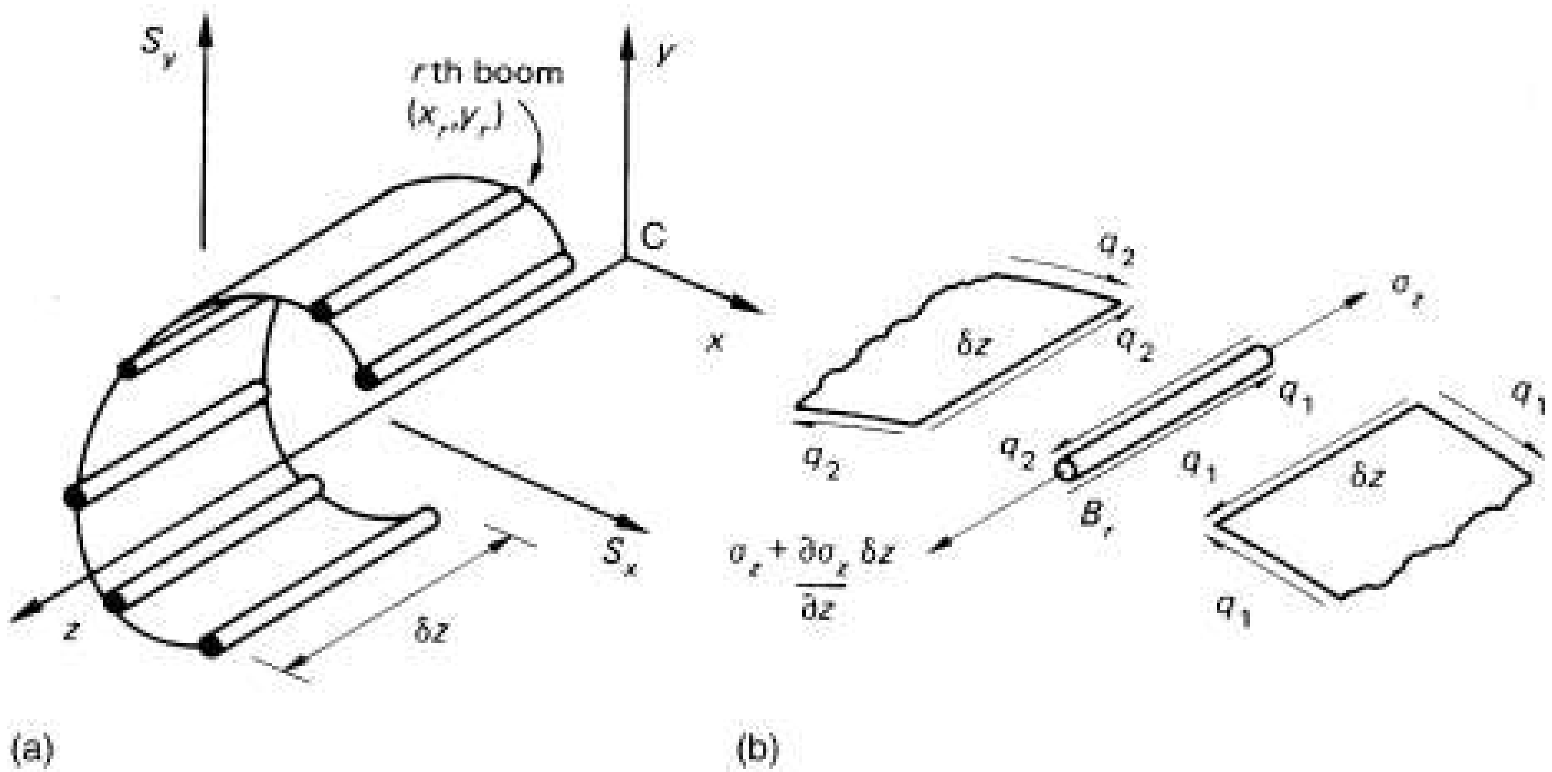


FIGURE 20.6 (a) Elemental Length of Shear Loaded Open Section Beam with Booms; (b) Equilibrium of Boom Element

or, using the relationships of Eqs. (16.22) and (16.23),

$$q_2 - q_1 = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) B_r x_r - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) B_r y_r \qquad (20.5)$$

Equation (20.5) gives the change in shear flow induced by a boom which itself is subjected to a direct load ($\sigma_z B_r$). Each time a boom is encountered, the shear flow is incremented by this amount, so that if, at any distance s around the profile of the section, n booms have been passed, the shear flow at the point is given by

$$\begin{aligned} q_s = &-\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_{\mathrm{D}} x \,\mathrm{d}s + \sum_{r=1}^{n} B_r x_r\right) \\ &-\left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_{\mathrm{D}} y \,\mathrm{d}s + \sum_{r=1}^{n} B_r y_r\right) \end{aligned} \qquad (20.6)$$

6. Exemplo de aplicação

Example 20.3

Calculate the shear flow distribution in the channel section shown in Fig. 20.7 produced by a vertical shear load of 4.8 kN acting through its shear center. Assume that the walls of the section are effective in resisting only shear stresses, while the booms, each of area 300 mm^2, carry all the direct stresses.

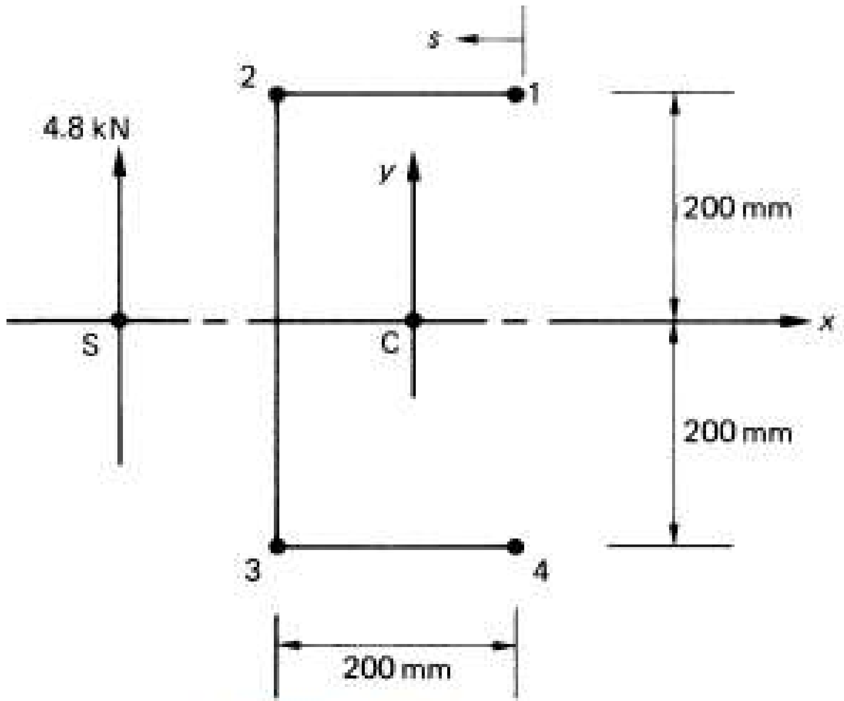


FIGURE 20.7 Idealized Channel Section of Example 20.3

Solution:

The effective direct stress carrying thickness t_D of the walls of the section is zero so that the centroid of area and the section properties refer to the boom areas only. Since Cx (and Cy as far as the boom areas are concerned) is an axis of symmetry, $I_{xy} = 0$; also $S_x = 0$ and Eq. (20.6) thereby reduces to

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r \tag{i}$$

in which $I_{xx} = 4 \times 300 \times 200^2 = 48 \times 10^6\ \text{mm}^4$. Substituting the values of S_y and I_{xx} in Eq. (i) gives

$$q_s = -\frac{4.8 \times 10^3}{48 \times 10^6} \sum_{r=1}^{n} B_r y_r = -10^{-4} \sum_{r=1}^{n} B_r y_r \tag{ii}$$

At the outside of boom 1, $q_s = 0$. As boom 1 is crossed, the shear flow changes by an amount given by

$$\Delta q_1 = -10^{-4} \times 300 \times 200 = -6\ \text{N/mm}$$

Hence, $q_{12} = -6$ N/mm, since, from Eq. (i), it can be seen that no further changes in shear flow occur until the next boom (2) is crossed. Hence,

$$q_{23} = -6 - 10^{-4} \times 300 \times 200 = -12 \text{ N/mm}$$

Similarly,

$$q_{34} = -12 - 10^{-4} \times 300 \times (-200) = -6 \text{ N/mm}$$

while, finally, at the outside of boom 4, the shear flow is

$$-6 - 10^{-4} \times 300 \times (-200) = 0$$

as expected. The complete shear flow distribution is shown in Fig. 20.8.

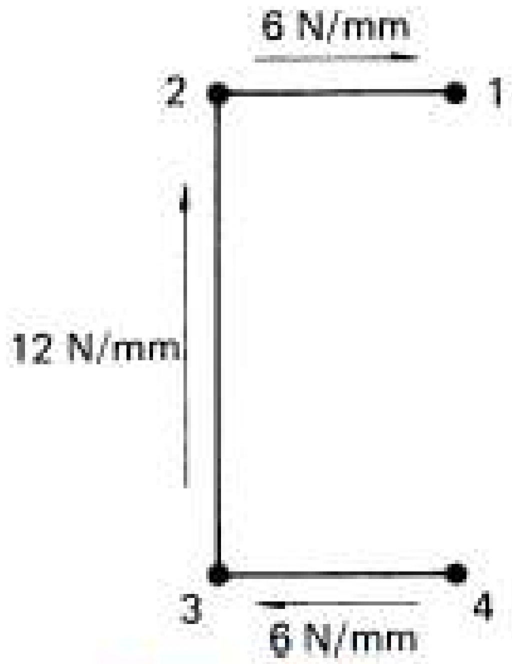


FIGURE 20.8 Shear Flow in Channel Section of Example 20.3

Referências:

[1] Megson, T. H. G. *Aircraft Structures for Engineering Students*. 6th ed., Butterworth-Heinemann, 2016. Cap. 20: Structural Idealization

20.1 – Principles (p.605-606)

20.2 – Idealization of a panel (p.606-608)

20.3 – Effect of idealization on the analysis of open and closed section beams (p.608-620)

20.3.1 – Bending of open and closed section beams (p.609-610)

20.3.2 – Shear of open section beams (p.610-616)

Problems (p.623-627)