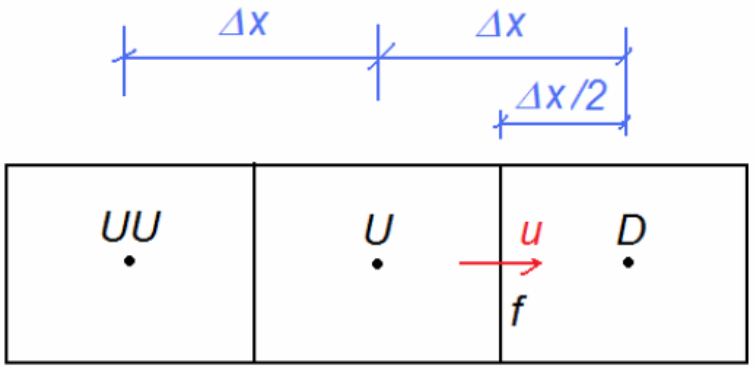
# PME 2556 – Dinâmica dos Fluidos Computacional
# Aula 14 – Esquemas TVD




*D:Downwind cell*

*U:Upwind cell*

*UU: 2nd Upwind cell*

*f:face*

Upwind: $\phi_f = \phi_U$

Linear Upwind: $\phi_f = \frac{3}{2}\phi_U - \frac{1}{2}\phi_{UU}$

QUICK: $\phi_f = \frac{6}{8}\phi_U + \frac{3}{8}\phi_D - \frac{1}{8}\phi_{UU}$

Central Differences (CD): $\phi_f = \frac{1}{2}\phi_U + \frac{1}{2}\phi_D$

Generalized form: $\boxed{\phi_f = \phi_U + \frac{1}{2}\psi(r)(\phi_D - \phi_U)}$

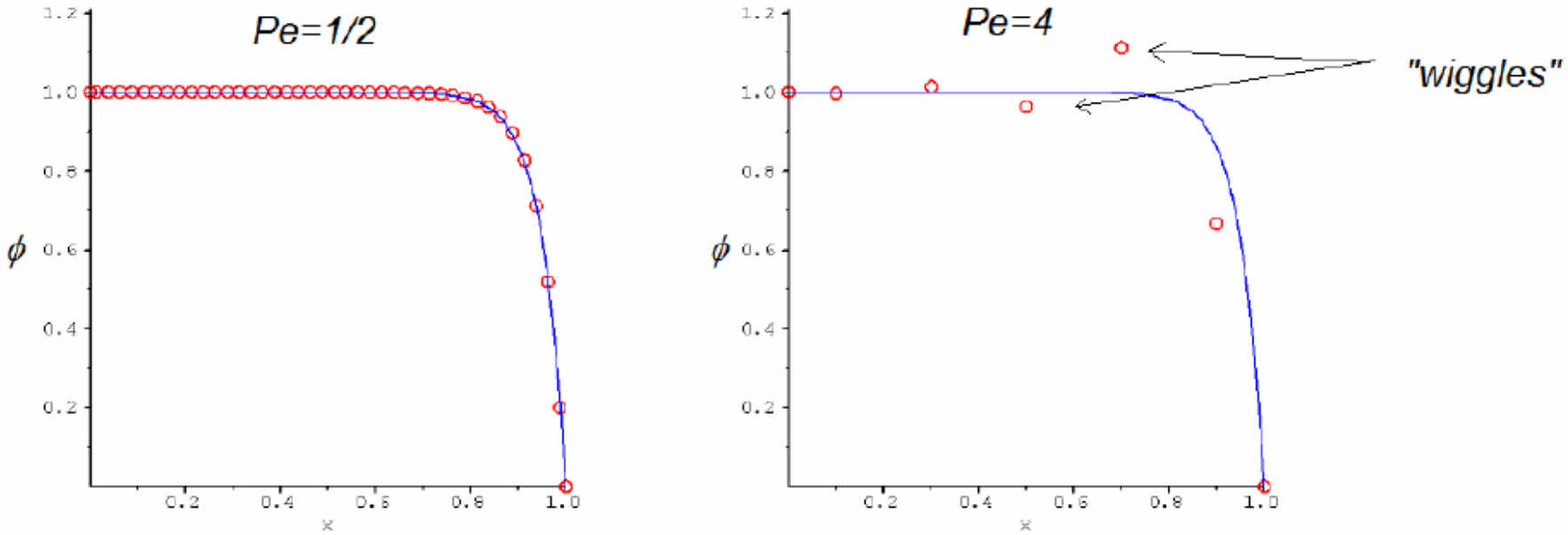
Where $r = \frac{\phi_U - \phi_{UU}}{\phi_D - \phi_U}$

Upwind: $\phi_f = \phi_U \quad \Rightarrow \psi(r) = 0$

Linear Upwind: $\phi_f = \frac{3}{2}\phi_U - \frac{1}{2}\phi_{UU} \Rightarrow \psi(r) = r$

QUICK: $\phi_f = \frac{6}{8}\phi_U + \frac{3}{8}\phi_D - \frac{1}{8}\phi_{UU} \Rightarrow \psi(r) = (3 + r)/4$

Central Differences (CD): $\phi_f = \frac{1}{2}\phi_U + \frac{1}{2}\phi_D \Rightarrow \psi(r) = 1$

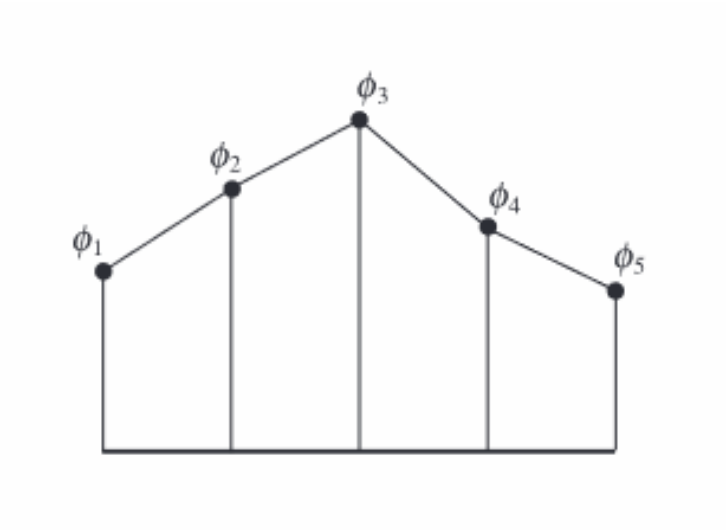



Advection-Diffusion solution using Central Differences: exact solution (blue line) compared to simulation (red circles).

To avoid wiggles, a desirable scheme:
a)Must not create local extrema;
b)The value of an existing local minimum must be non-decreasing and that of a local maximum must be non-increasing

Such a scheme is called a **monotonicity preserving** scheme.

Monotonicity preserving schemes do not create new undershoots and overshoots in the solution or accentuate existing extremes.



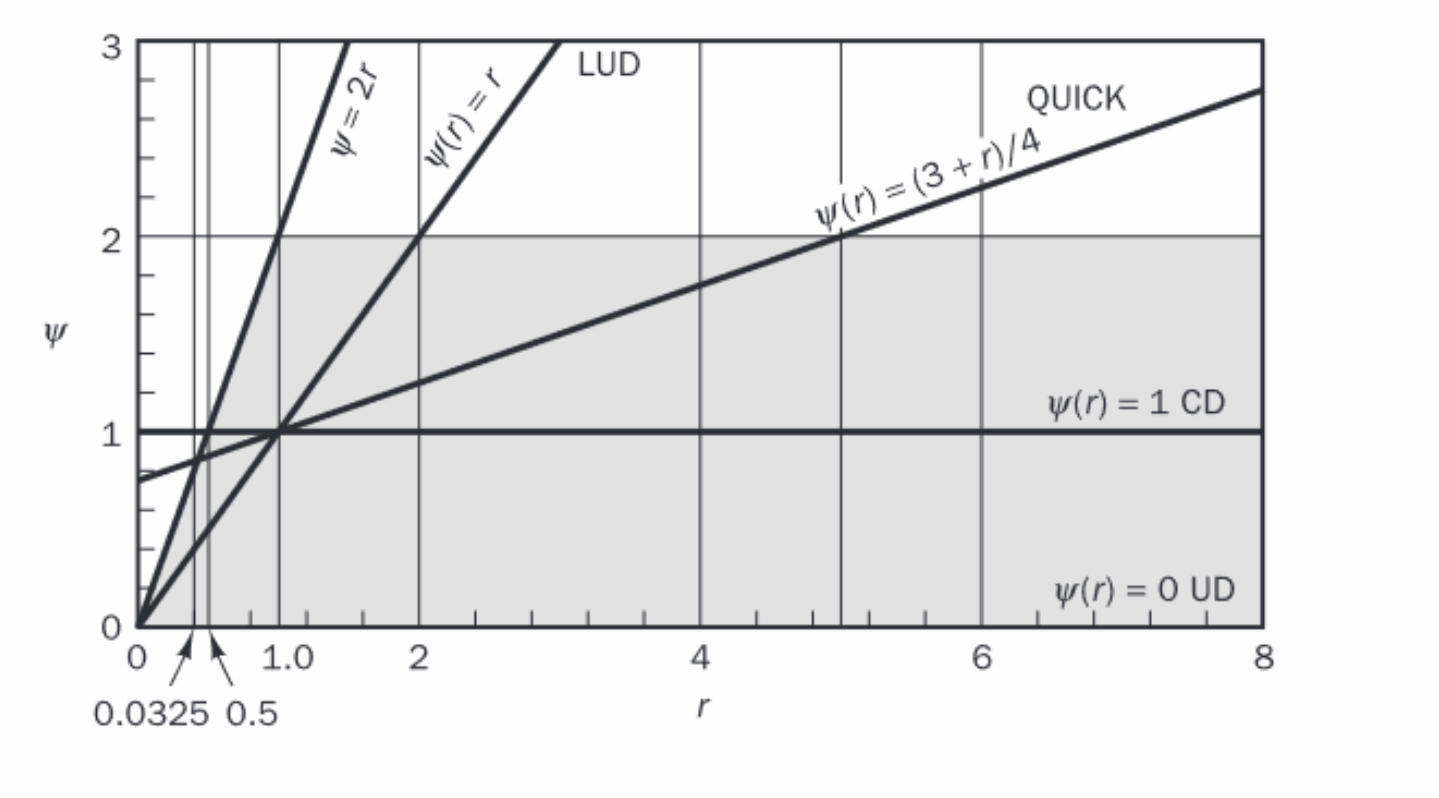

Total Variation of the data set:

$$TV(\phi) = |\phi_2 - \phi_1| + |\phi_3 - \phi_2| + |\phi_4 - \phi_3| + |\phi_5 - \phi_4|$$
$$= |\phi_3 - \phi_1| + |\phi_5 - \phi_3|$$

Monotonicity-preserving schemes have the property that the total variation of the solution should diminish with time. So, they are called Total Variation Diminishing or TVD schemes.

According to Sweby (1984), the necessary and sufficient conditions for a scheme to be TVD are:

$$r \le 0 \Rightarrow \psi(r) = 0$$
$$0 < r < 1 \Rightarrow \psi(r) \le 2r$$
$$r \ge 1 \Rightarrow \psi(r) \le 2$$

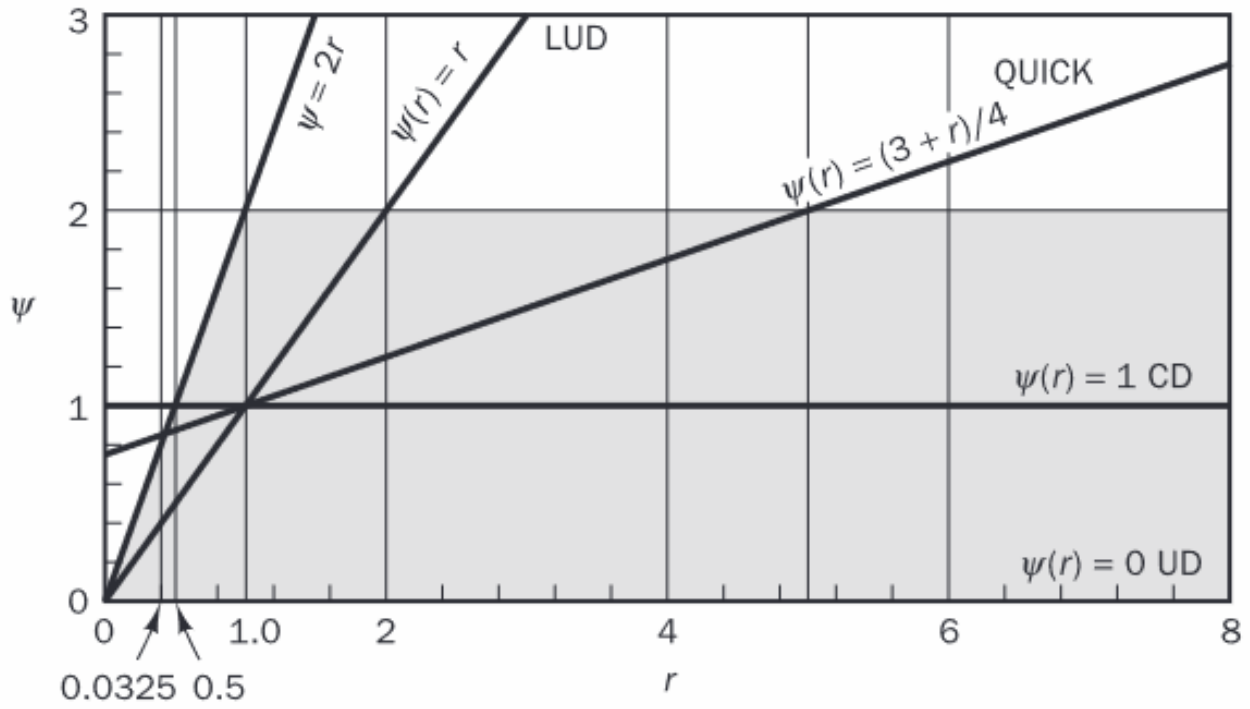


# TVD-compliant version of some schemes:

Linear Upwind: $\psi(r) = MAX(MIN(r,2),0)$

QUICK: $\psi(r) = MAX(MIN(MIN((3+r)/4, 2r)\,2),0)$

Central Differences (CD): $\psi(r) = MAX(MIN(2r,1),0)$



# Other TVD schemes:

MIN-MOD: $\psi(r) = MAX\,(\,MIN(r,1),0)$

SUPERBEE: $\psi(r) = MAX\,(0\,, MAX\,(MIN(2r,1), MIN(r,2)))$

Van Leer: $\psi(r) = \dfrac{r + |r|}{1 + r}$

MUSCL: $\psi(r) = MAX(\,MIN(\,MIN(\,2r\,,\,0.5\,r + 0.5\,),\,2),\,0)$

Reference: An Introduction to Computational Fluid Dynamics – The Finite Volume Method, second edition. H K Versteeg, W Malalasekera.