

# Eletrromagnetismo Avançado

*6 de novembro*  
*Potenciais e campos*

# Equações de Maxwell

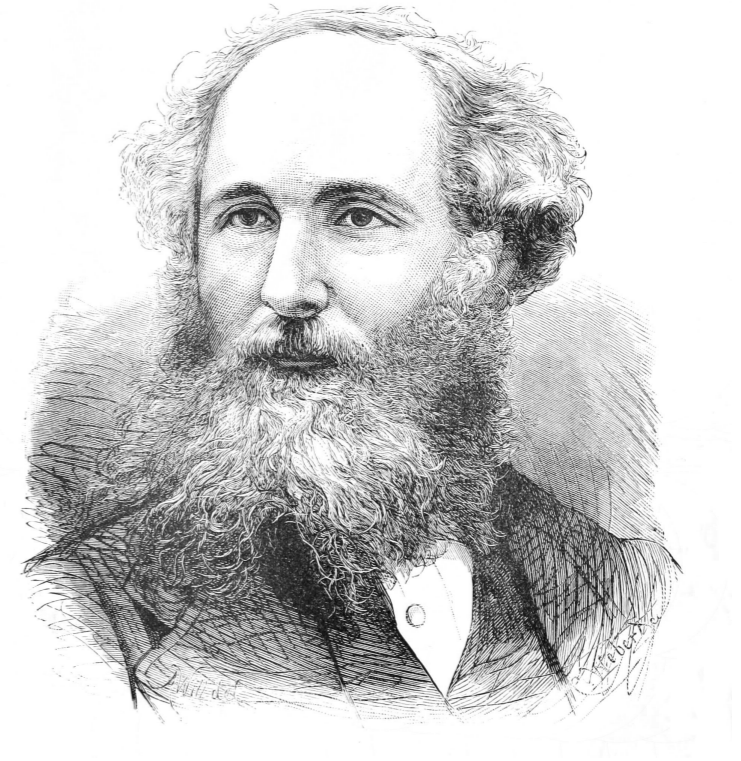
Vácuo

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left( \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$



# Campo elétrico e potenciais

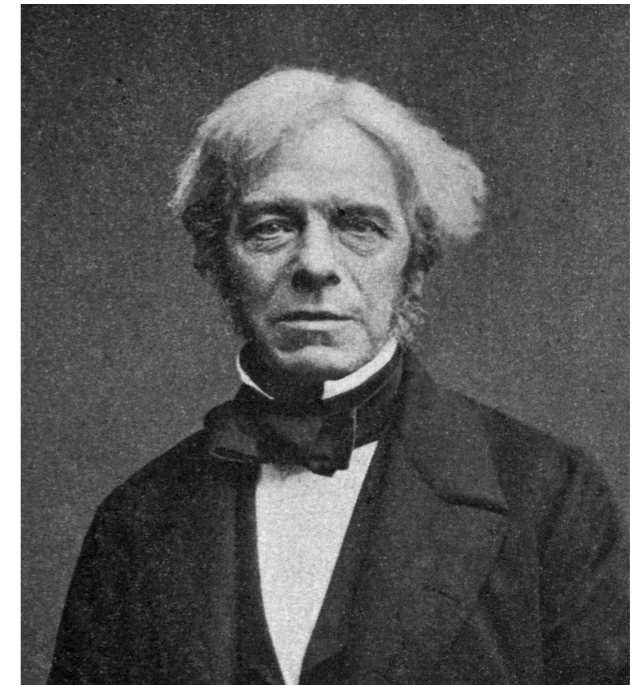
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{\nabla} \times \vec{A}}{\partial t}$$

$$\vec{\nabla} \times \left( \vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$



# Transformações de gauge

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

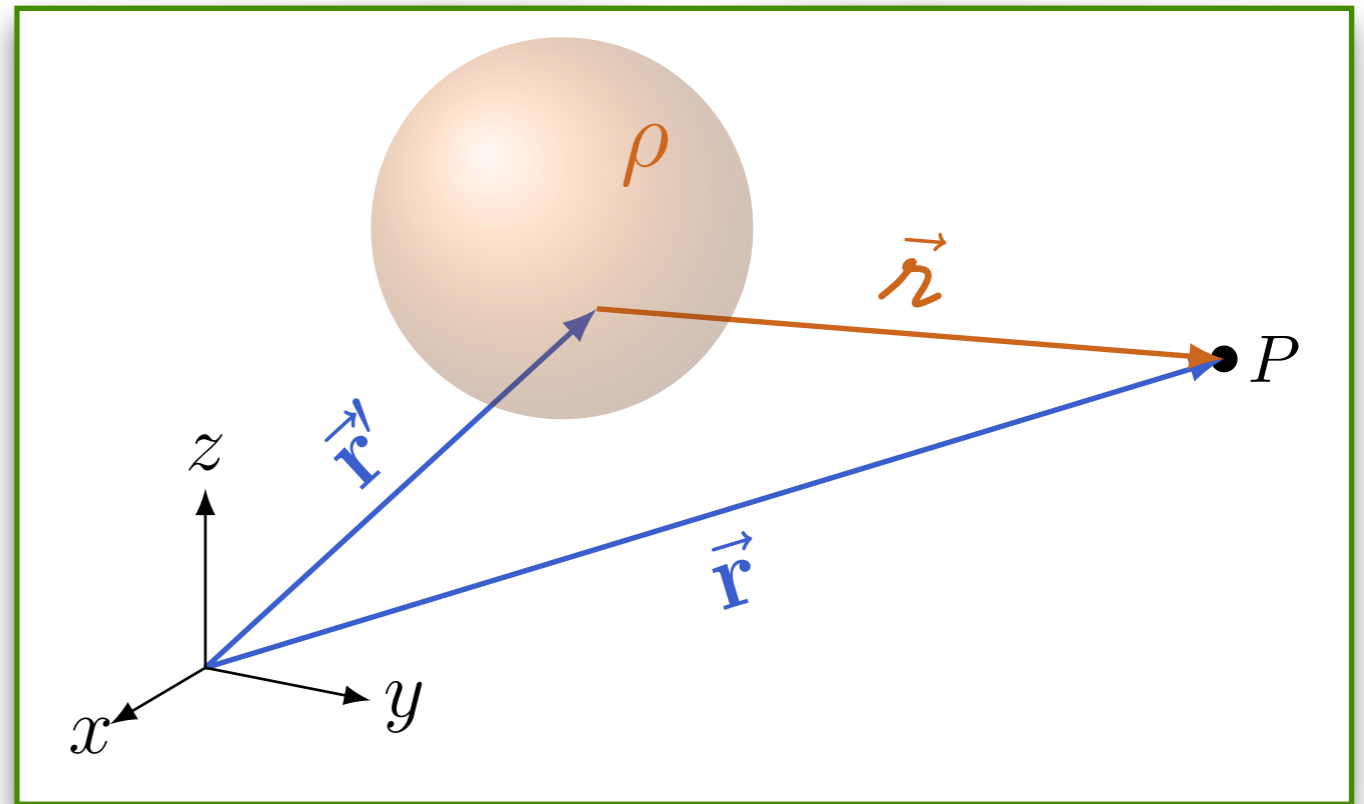
$$\vec{A}' = \vec{A} + \vec{\nabla} \lambda \quad \Rightarrow \quad \begin{cases} \vec{B} = \vec{\nabla} \times \vec{A} \\ \vec{B} = \vec{\nabla} \times \vec{A}' \end{cases}$$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$V' = V - \frac{\partial \lambda}{\partial t} \quad \Rightarrow \quad \vec{E}' = \underbrace{-\vec{\nabla} V' - \frac{\partial \vec{A}'}{\partial t}}_{\vec{E}}$$

# Equações diferenciais para os potenciais

$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$$

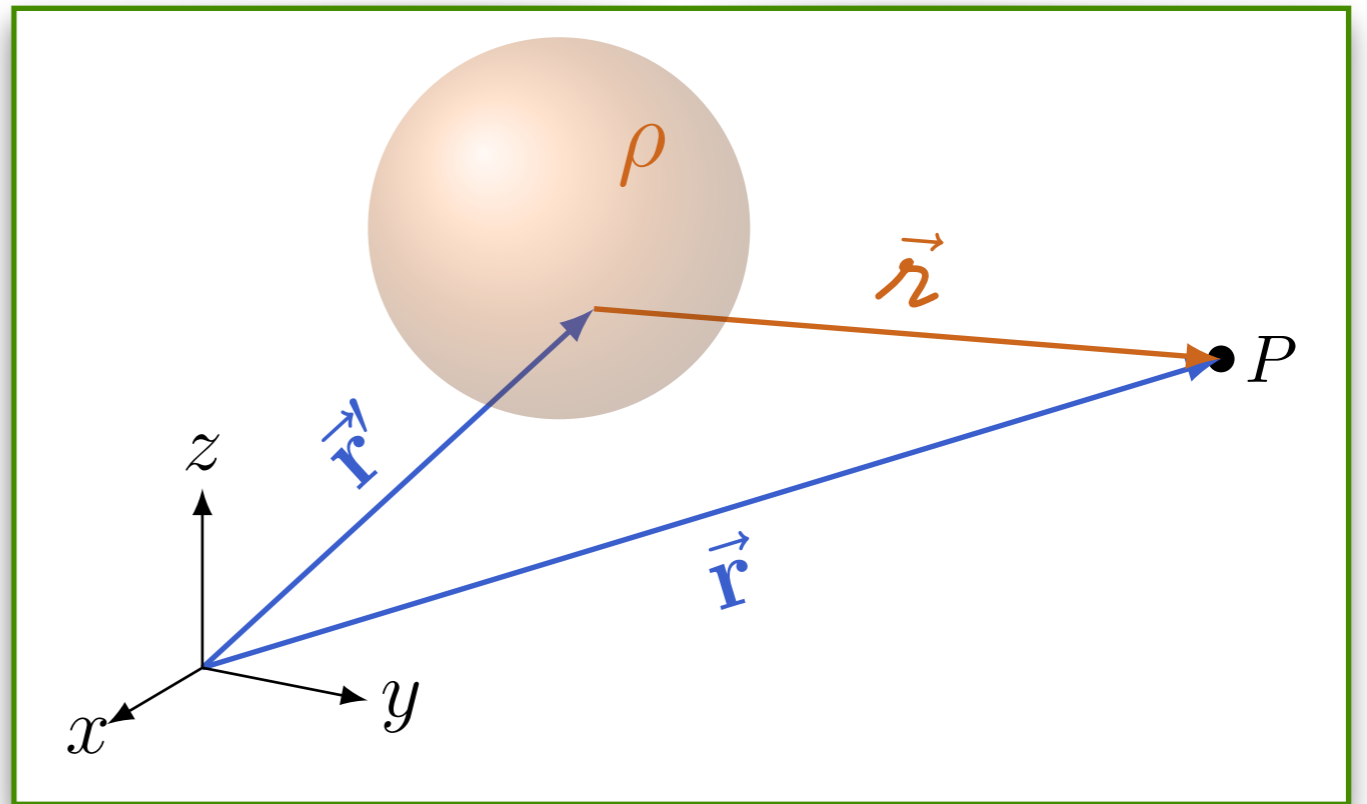


# Equações diferenciais para os potenciais

$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$$

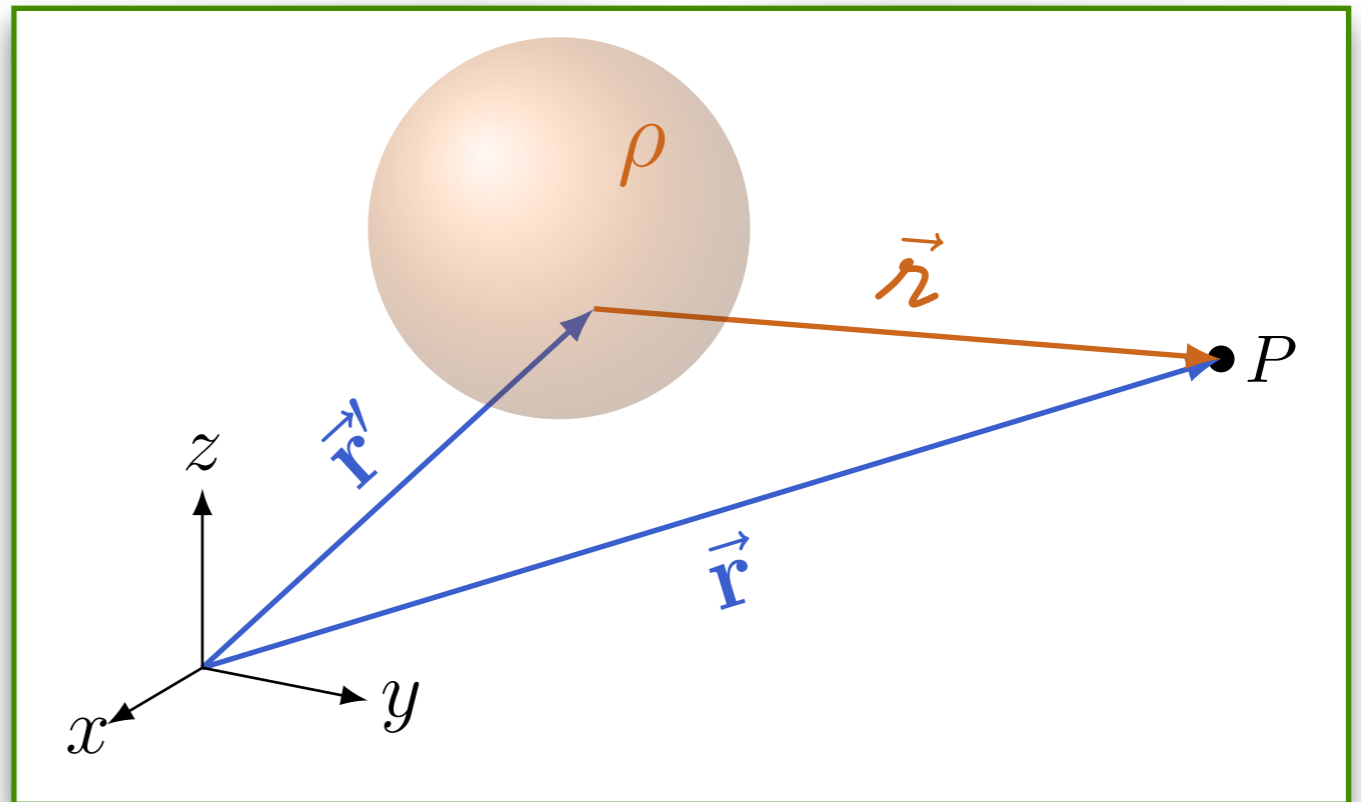
$$\vec{\nabla} \cdot \vec{E} = -\nabla^2 V - \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A}$$

$$\nabla^2 V + \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} = -\frac{\rho}{\epsilon_0}$$



# Equações diferenciais para os potenciais

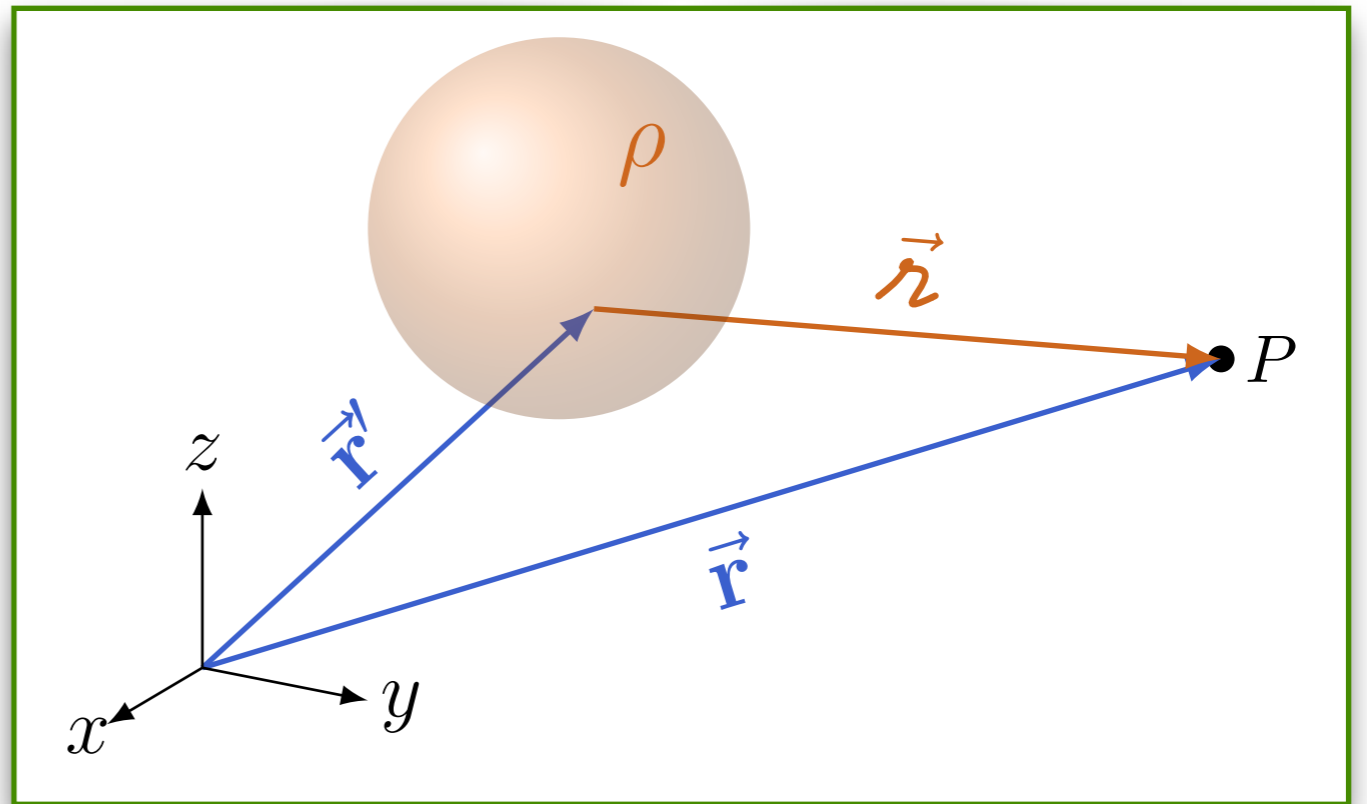
$$\nabla^2 V + \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} = -\frac{\rho}{\epsilon_0}$$



# Equações diferenciais para os potenciais

$$\nabla^2 V + \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} = -\frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

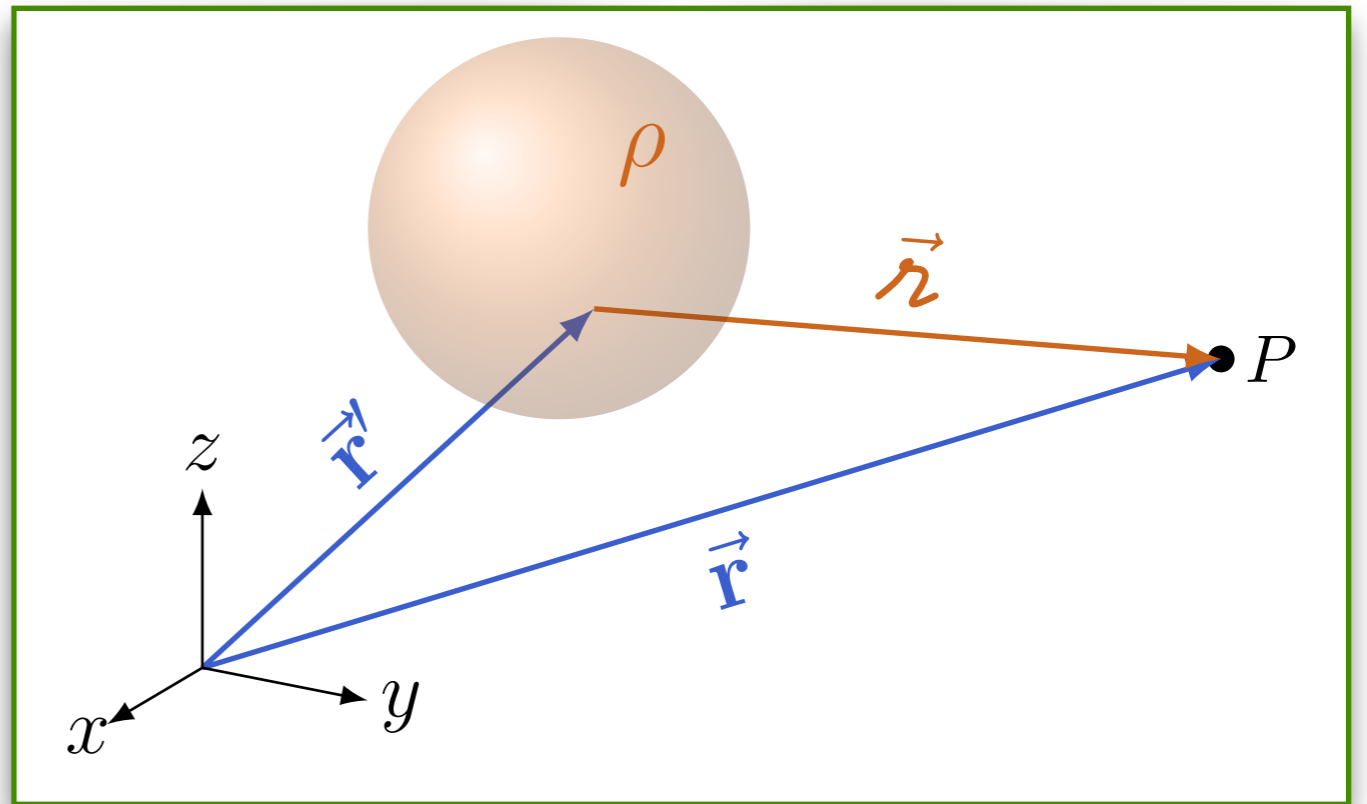




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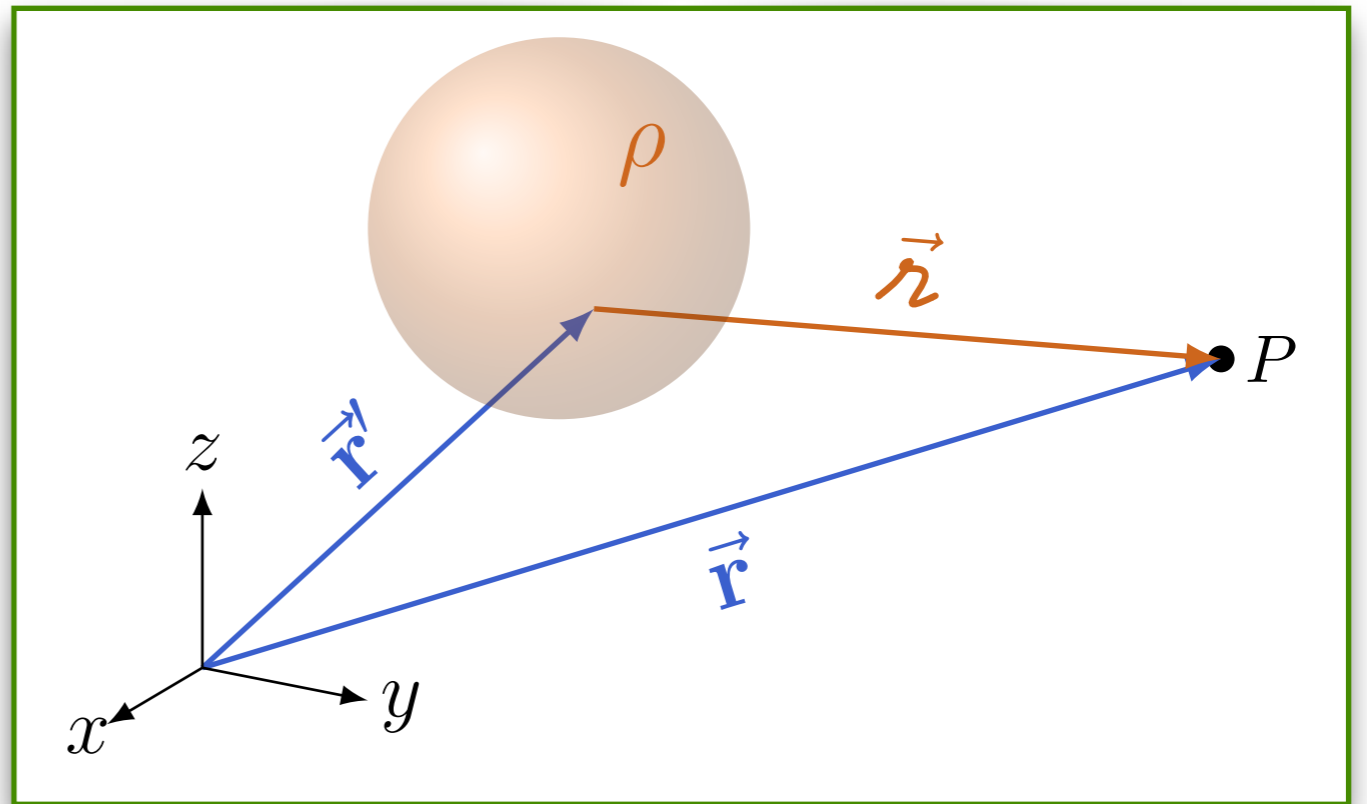


$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J} - \frac{1}{c^2} \frac{\partial}{\partial t} \left( \vec{\nabla} V + \frac{\partial \vec{A}}{\partial t} \right)$$

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$$\nabla^2 V + \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} = -\frac{\rho}{\epsilon_0}$$

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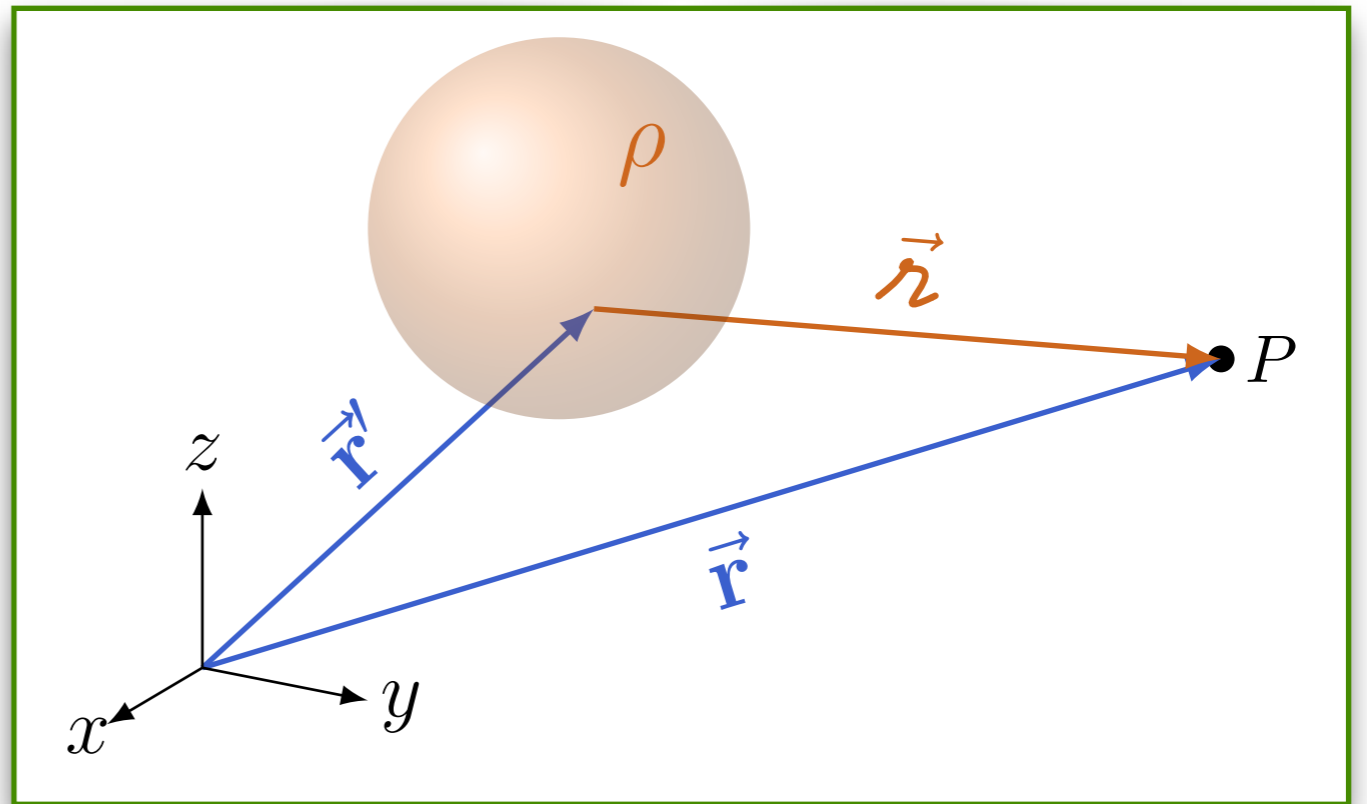
$$\vec{\nabla} (\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J} - \frac{1}{c^2} \vec{\nabla} \frac{\partial V}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2}$$

# Equações diferenciais para os potenciais

$$\nabla^2 V + \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} = -\frac{\rho}{\epsilon_0}$$

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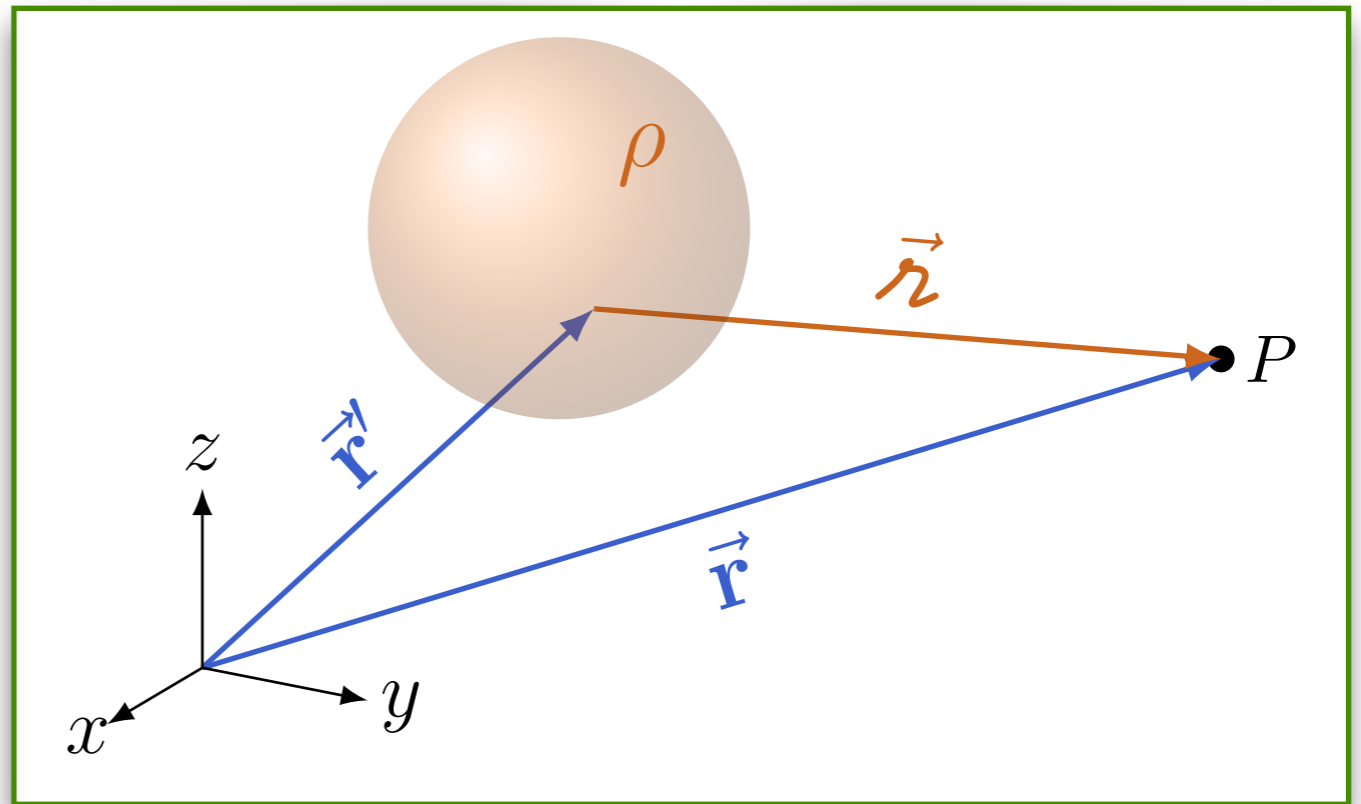
$$\vec{\nabla} (\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J} - \frac{1}{c^2} \vec{\nabla} \frac{\partial V}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2}$$



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$$\nabla^2 V + \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} = -\frac{\rho}{\epsilon_0}$$

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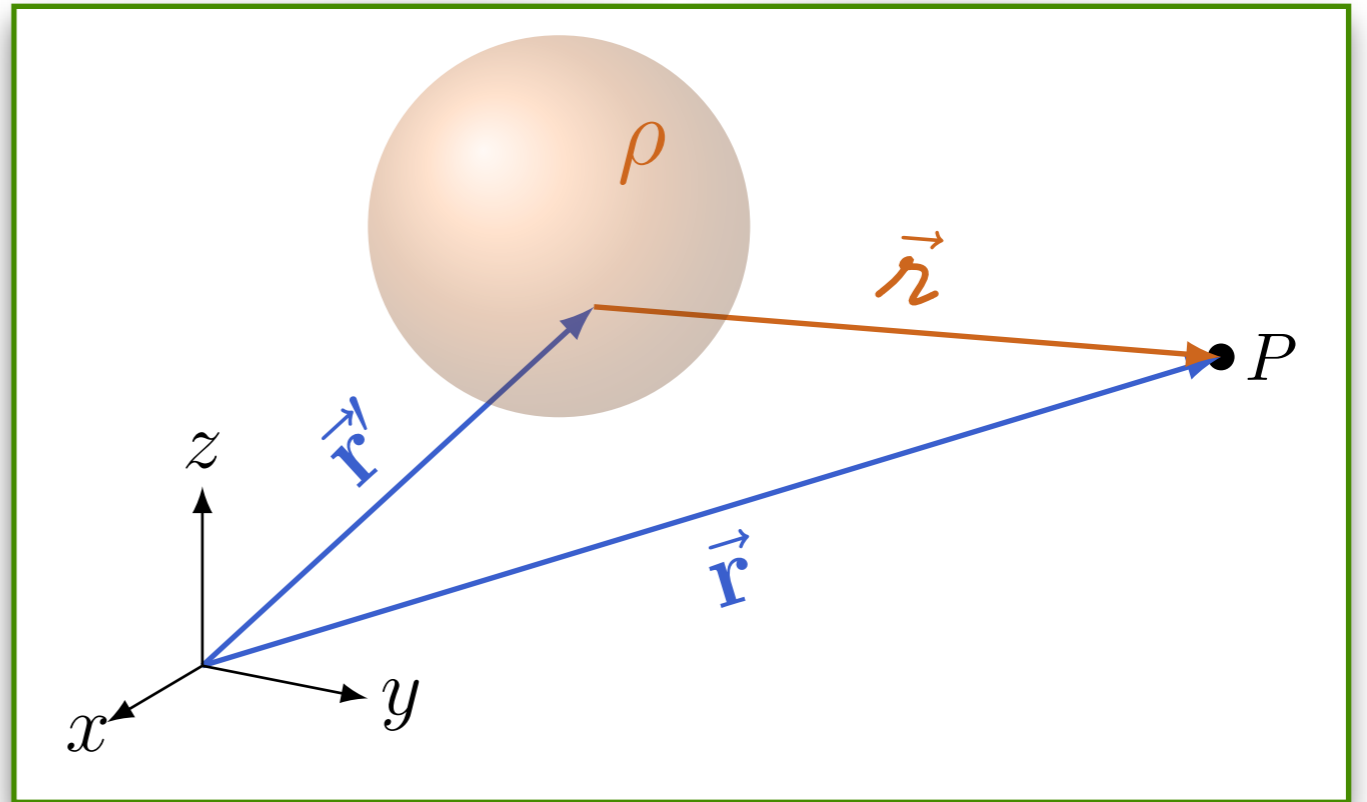
$$\vec{\nabla} \left( \vec{\nabla} \cdot \vec{A} \right) - \nabla^2 \vec{A} = \mu_0 \vec{J} - \frac{1}{c^2} \vec{\nabla} \frac{\partial V}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} + \vec{\nabla} \left( \vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} \right)$$

# Equações diferenciais para os potenciais

$$\nabla^2 V + \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} = -\frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



$$\vec{\nabla} \left( \vec{\nabla} \cdot \vec{A} \right) - \nabla^2 \vec{A} = \mu_0 \vec{J} - \frac{1}{c^2} \vec{\nabla} \frac{\partial V}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2}$$

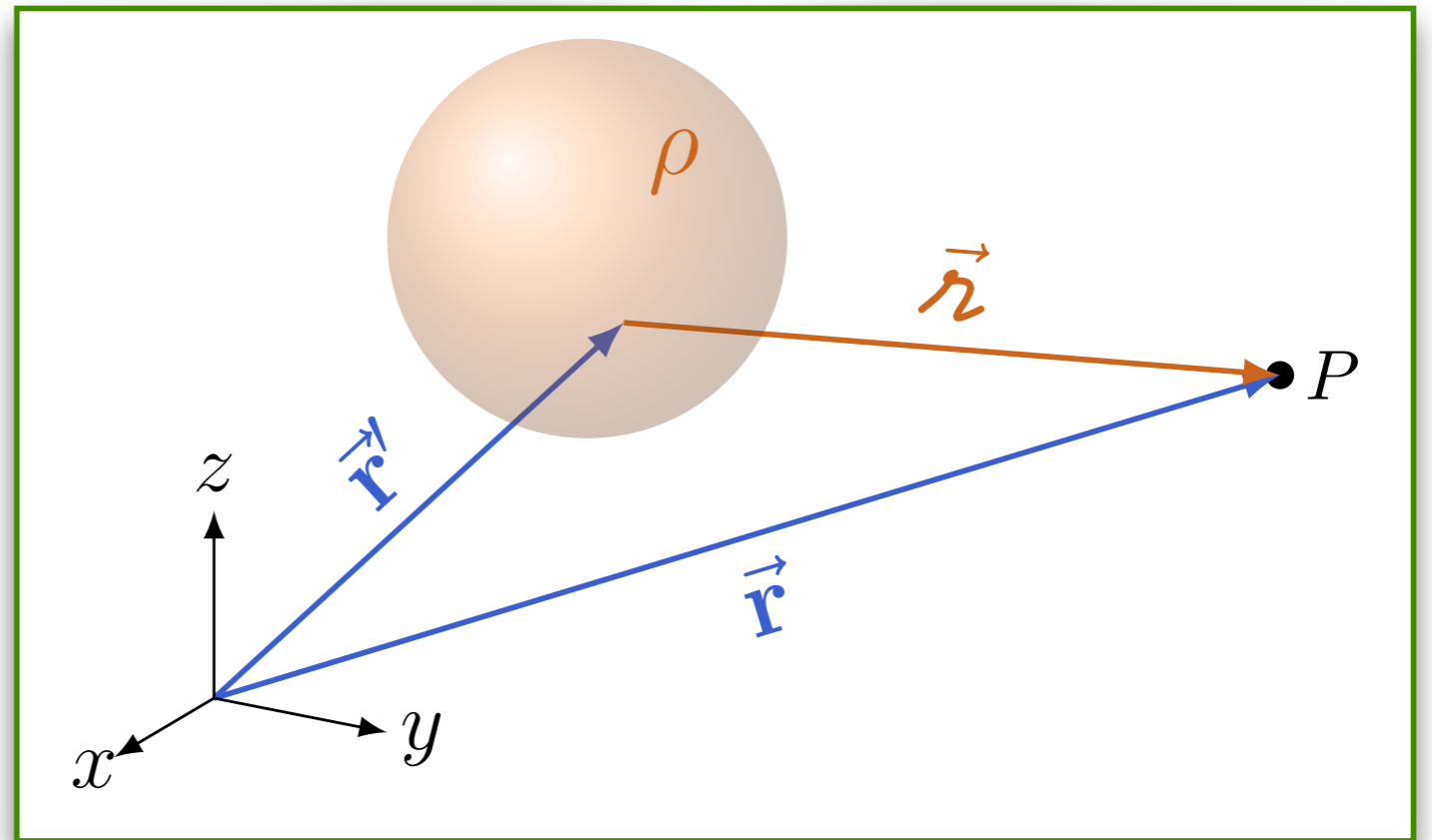
$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} + \underbrace{\vec{\nabla} \left( \vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} \right)}$$

Lorenz  $\Rightarrow$  = 0

# Equações diferenciais para os potenciais

$$\nabla^2 V - \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

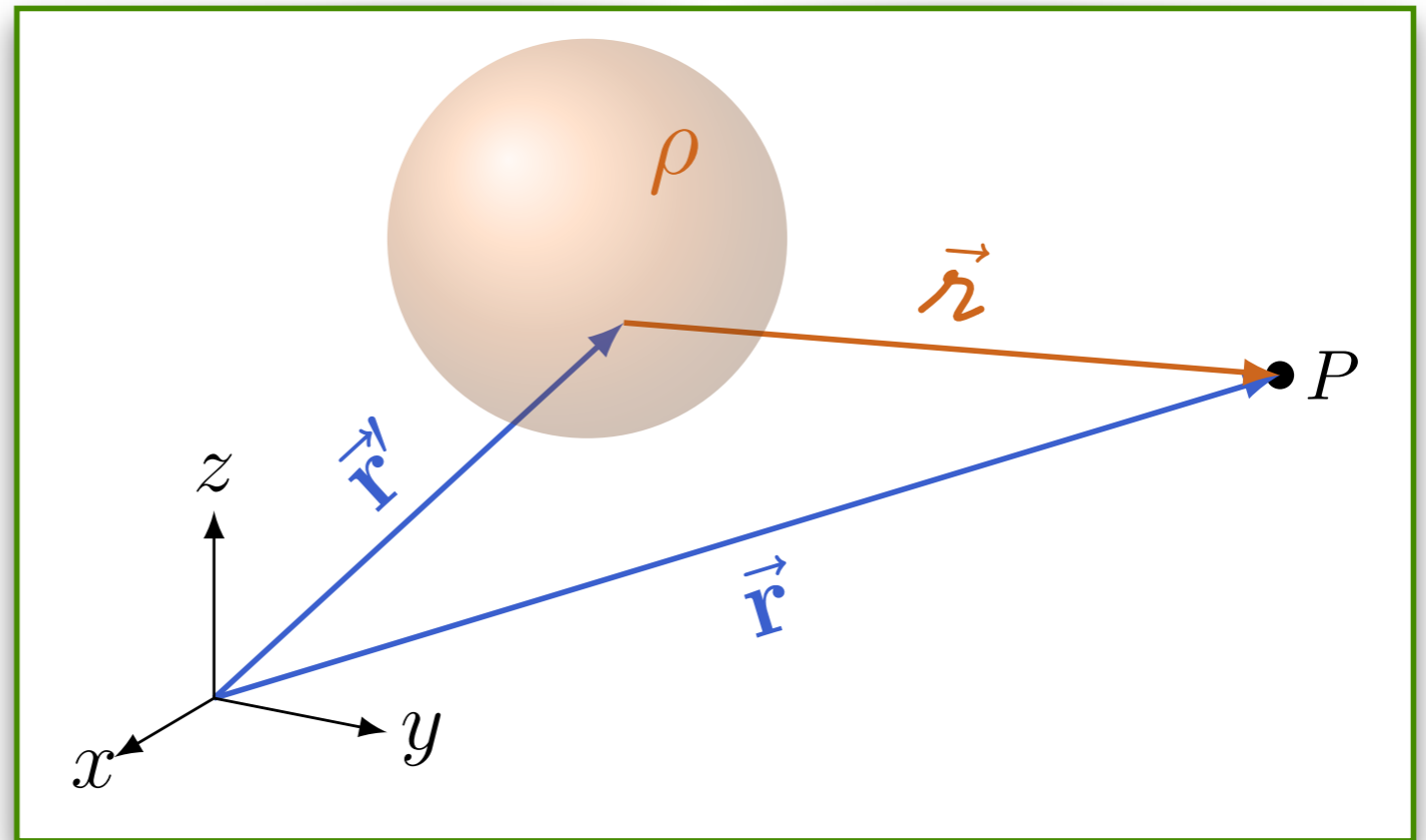
$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$



# Equações diferenciais para os potenciais

$$\nabla^2 V - \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$



$$\square^2 A^\mu = -\mu_0 J^\mu$$