

# Eletrromagnetismo Avançado

*30 de outubro*  
*Relatividade restrita*

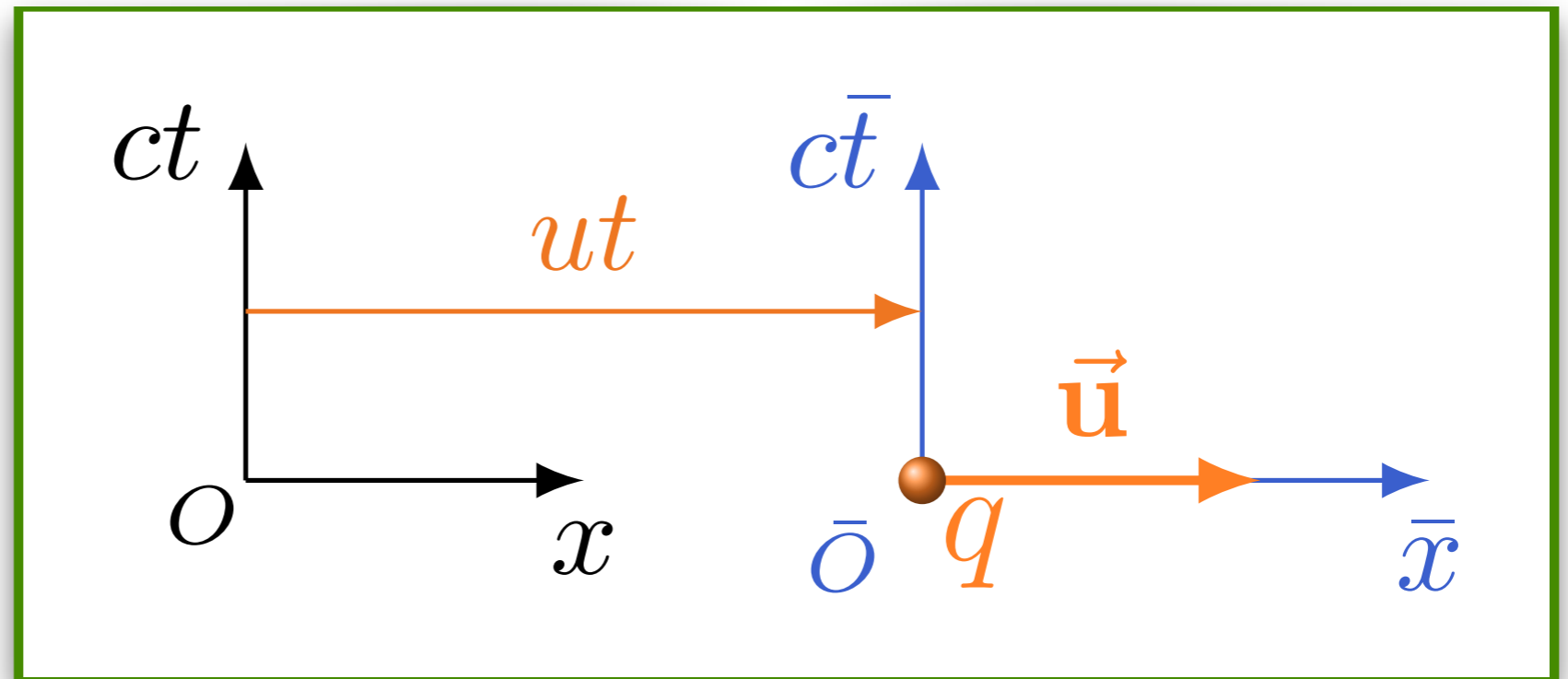
# Potenciais e relatividade

$$A^\mu = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} V/c \\ A^1 \\ A^2 \\ A^3 \end{bmatrix}$$

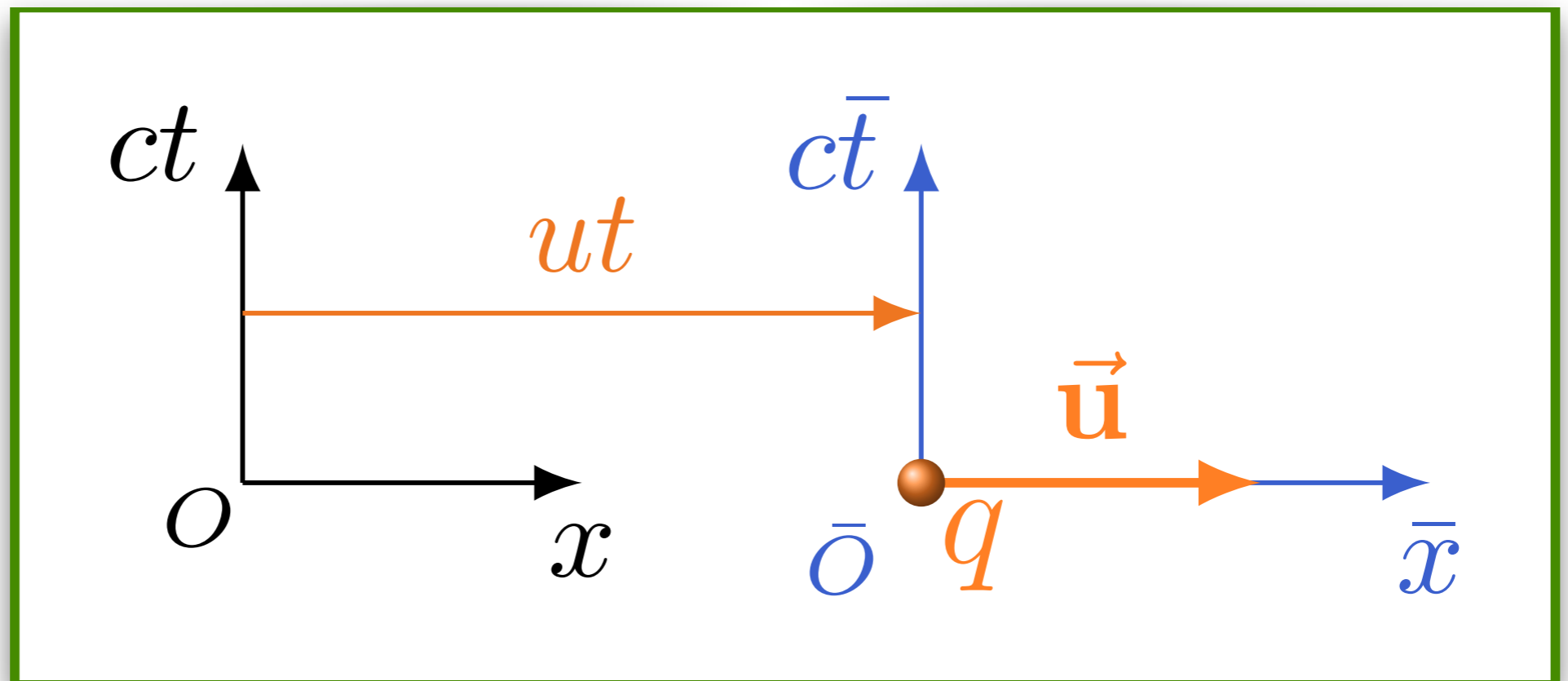
$$A^\mu A_\mu = -V^2/c^2$$

# Potenciais e relatividade

$$A^\mu = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} V/c \\ A^1 \\ A^2 \\ A^3 \end{bmatrix}$$



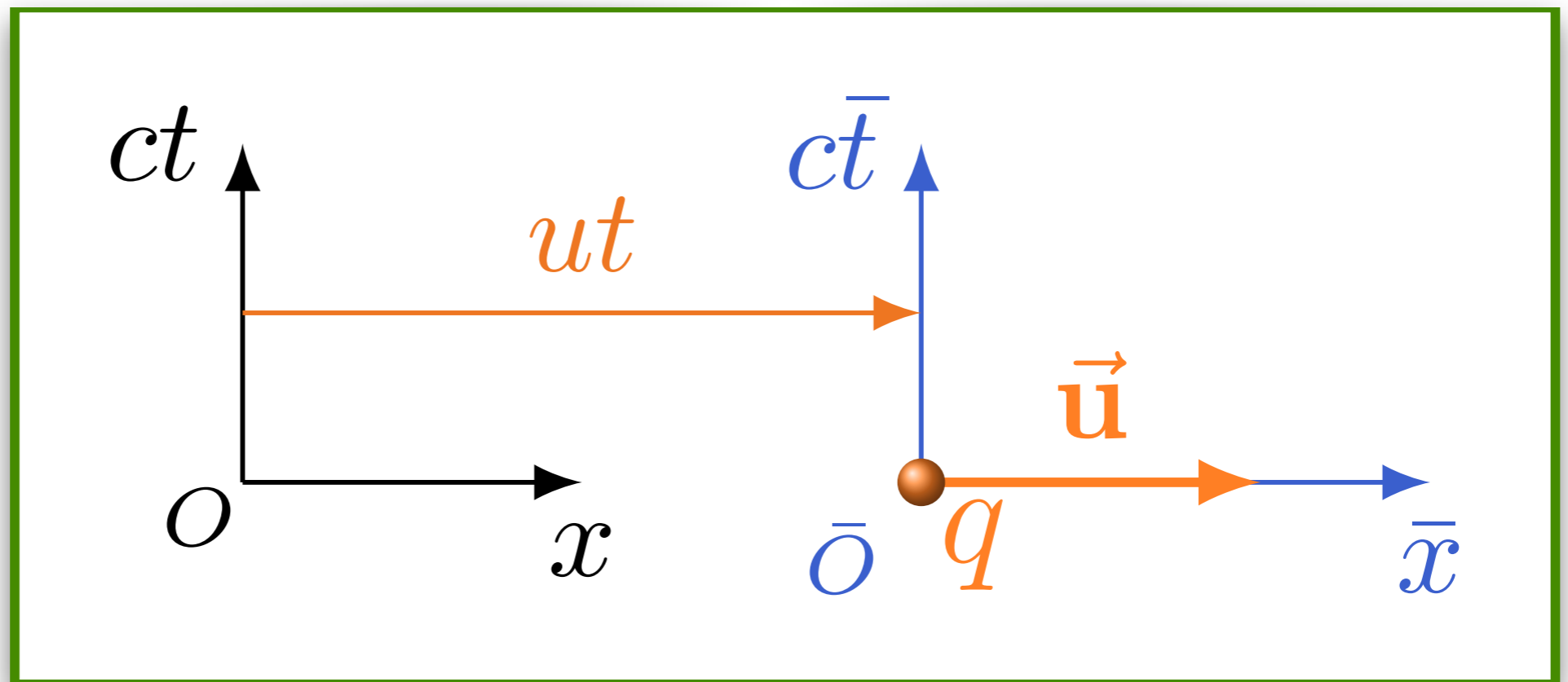
# Potenciais e relatividade



## LABORATÓRIO

Carga parada  $\Rightarrow$  
$$\begin{cases} \bar{V} &= \frac{q}{4\pi\epsilon_0 r} \\ \bar{A} &= 0 \end{cases}$$

# Potenciais e relatividade



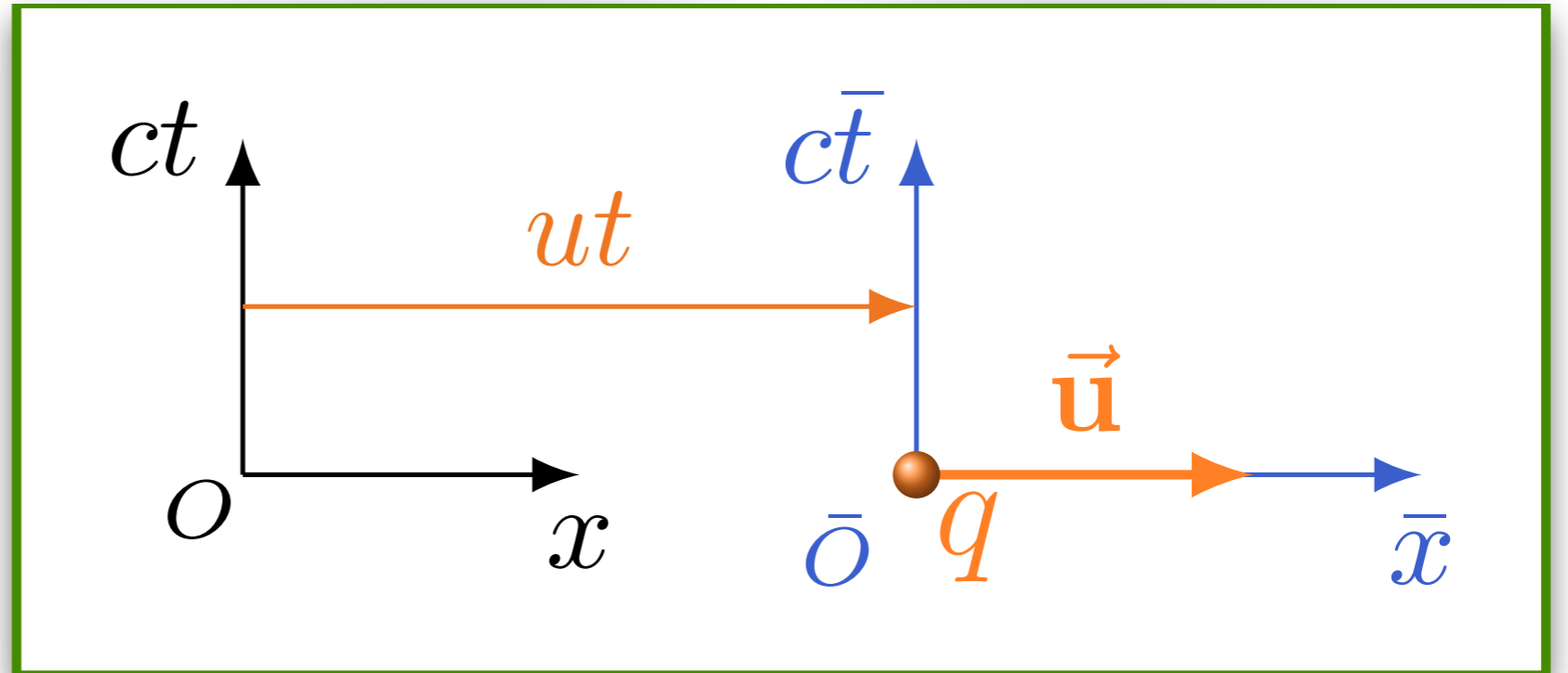
MÓVEL

Carga com velocidade  $\vec{u} \Rightarrow A^\mu = \frac{\mathcal{V}/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$

$\mathcal{V} = ?$

# Potenciais e relatividade

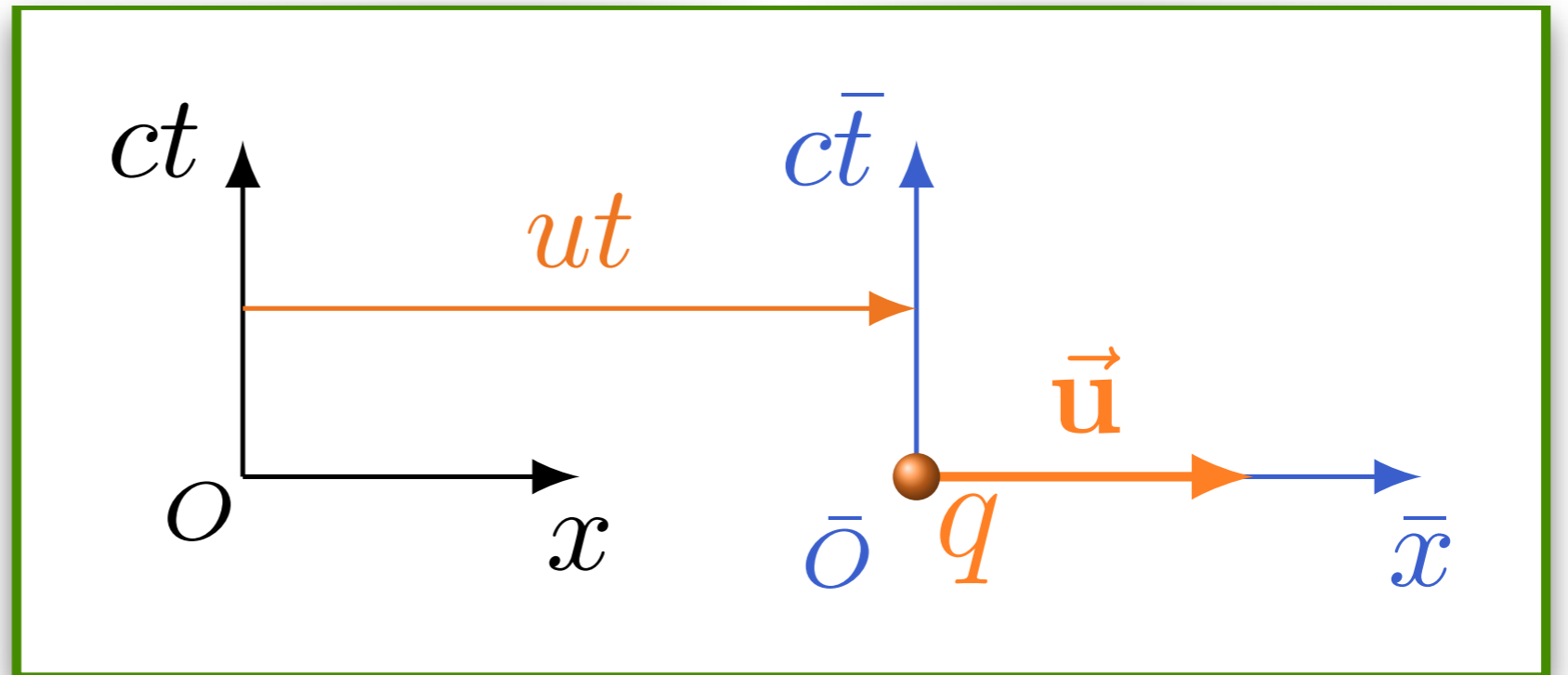
$$A^\mu = \frac{\mathcal{V}/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$$



$$\mathcal{V} = \begin{cases} \text{Invariante} \\ \frac{q}{4\pi\epsilon_0 r} \quad (\text{MÓVEL}) \end{cases}$$

# Potenciais e relatividade

$$A^\mu = \frac{v/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$$



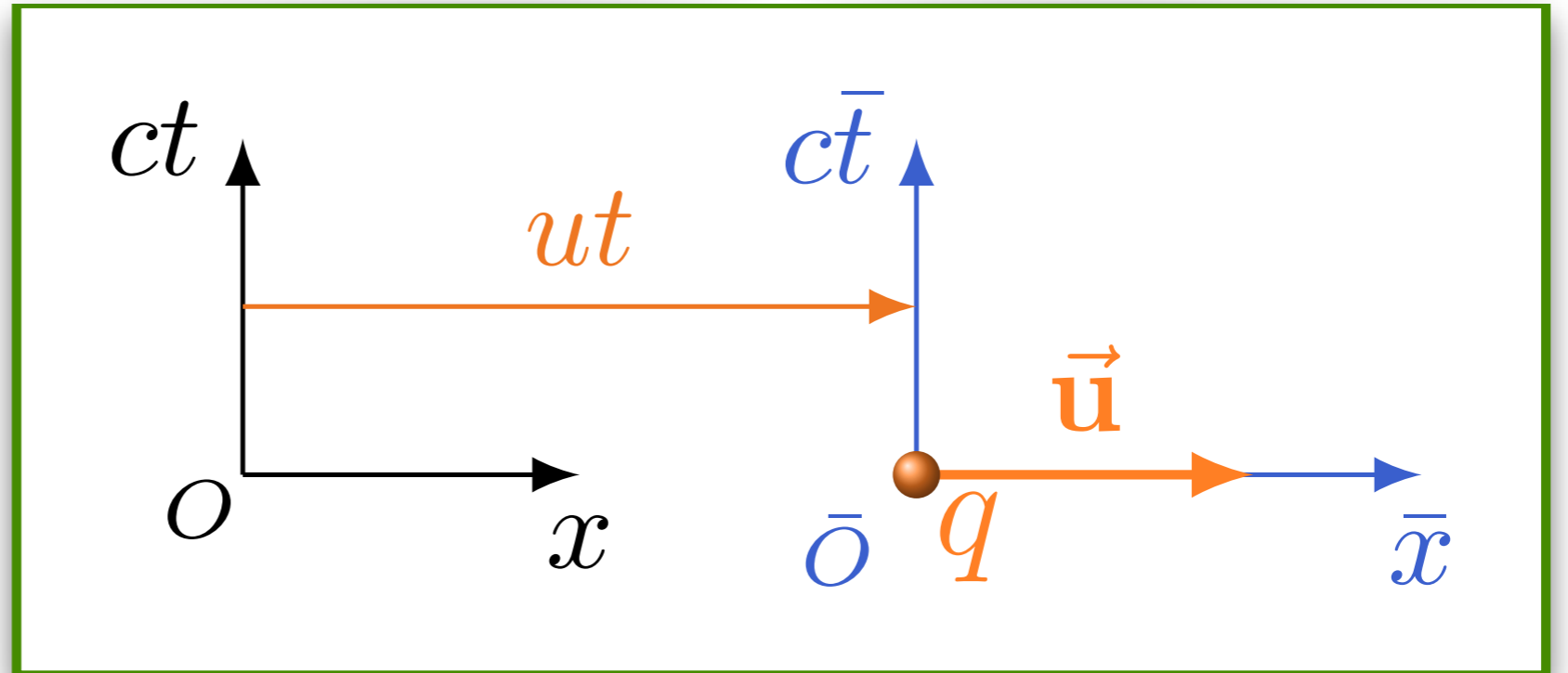
$$v = \begin{cases} \text{Invariante} \\ \frac{q}{4\pi\epsilon_0 r} \quad (\text{MÓVEL}) \end{cases}$$

$r \rightarrow$  Invariante

?

# Potenciais e relatividade

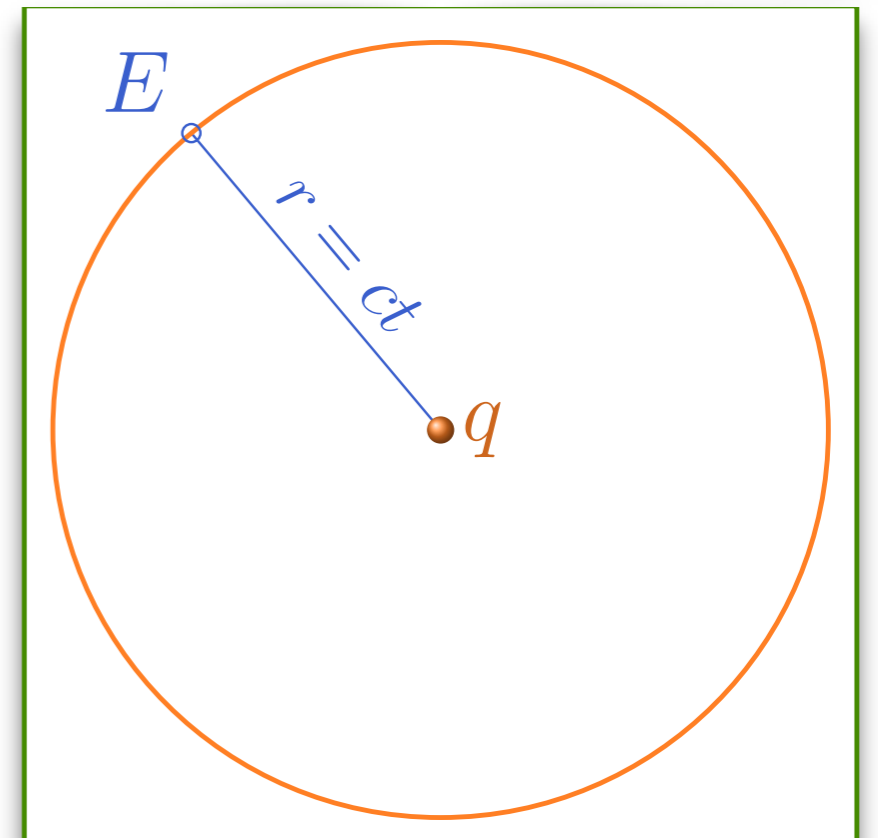
$$A^\mu = \frac{v/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$$



$$v = \begin{cases} \text{Invariante} \\ \frac{q}{4\pi\epsilon_0 r} \quad (\text{MÓVEL}) \end{cases}$$

$r \rightarrow$  Invariante

?

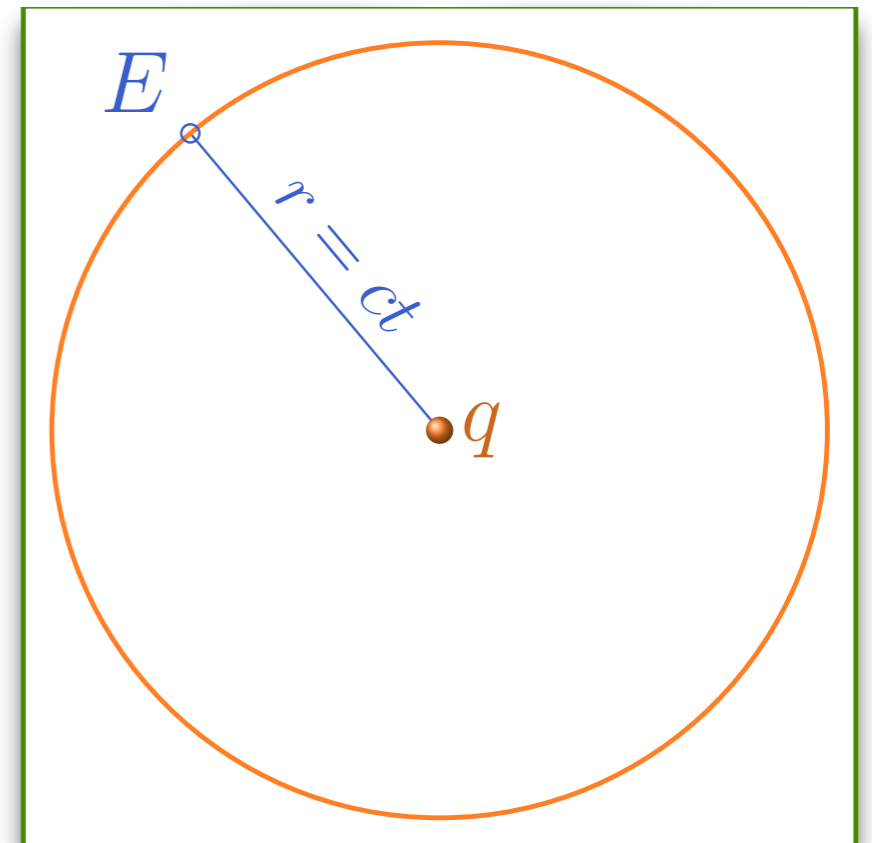




# Potenciais e relatividade

$$A^\mu = \frac{v/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$$

$r \rightarrow$  Invariante

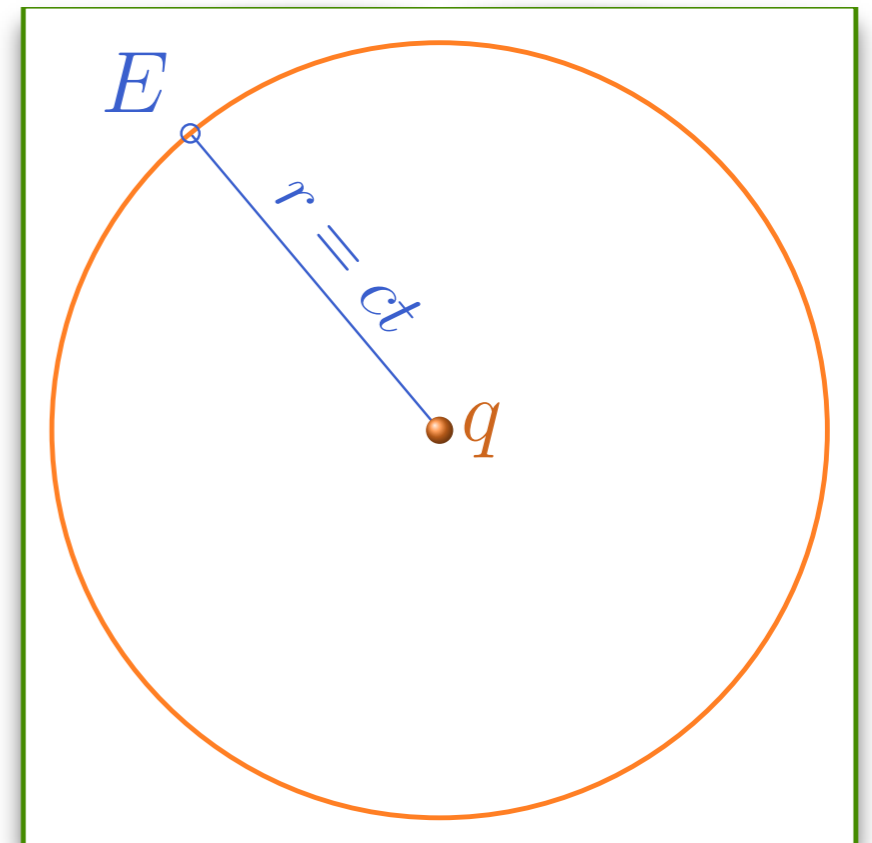


$$x^\mu = \begin{bmatrix} ct \\ x^1 \\ x^2 \\ x^3 \end{bmatrix}$$

# Potenciais e relatividade

$$A^\mu = \frac{v/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$$

$r \rightarrow$  Invariante

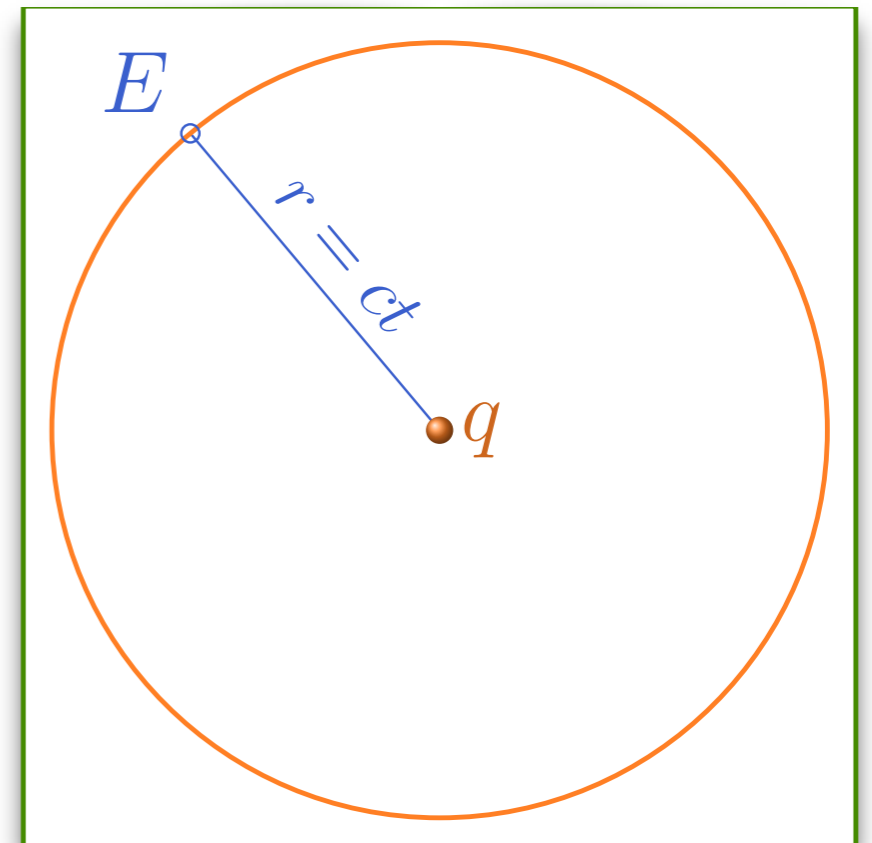


$$x^\mu = \begin{bmatrix} r \\ x^1 \\ x^2 \\ x^3 \end{bmatrix}$$

# Potenciais e relatividade

$$A^\mu = \frac{v/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$$

$r \rightarrow$  Invariante

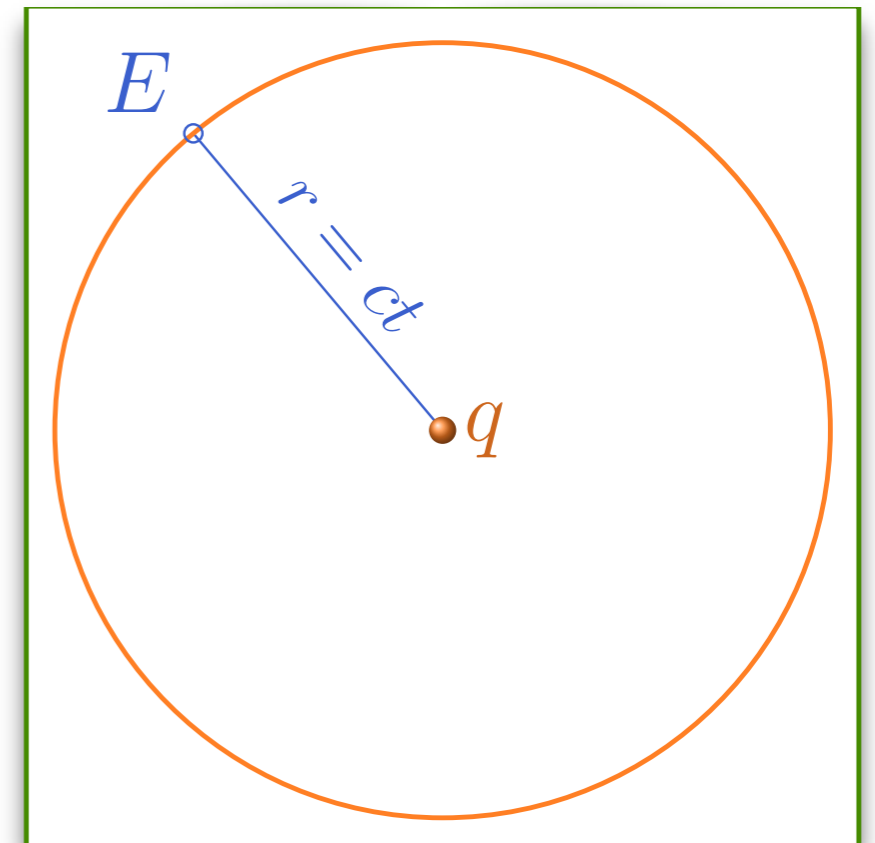


$$x^\mu = \begin{bmatrix} r \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} \Rightarrow x^\mu x_\mu = 0$$



# Potenciais e relatividade

$$A^\mu = \frac{\mathcal{V}/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$$



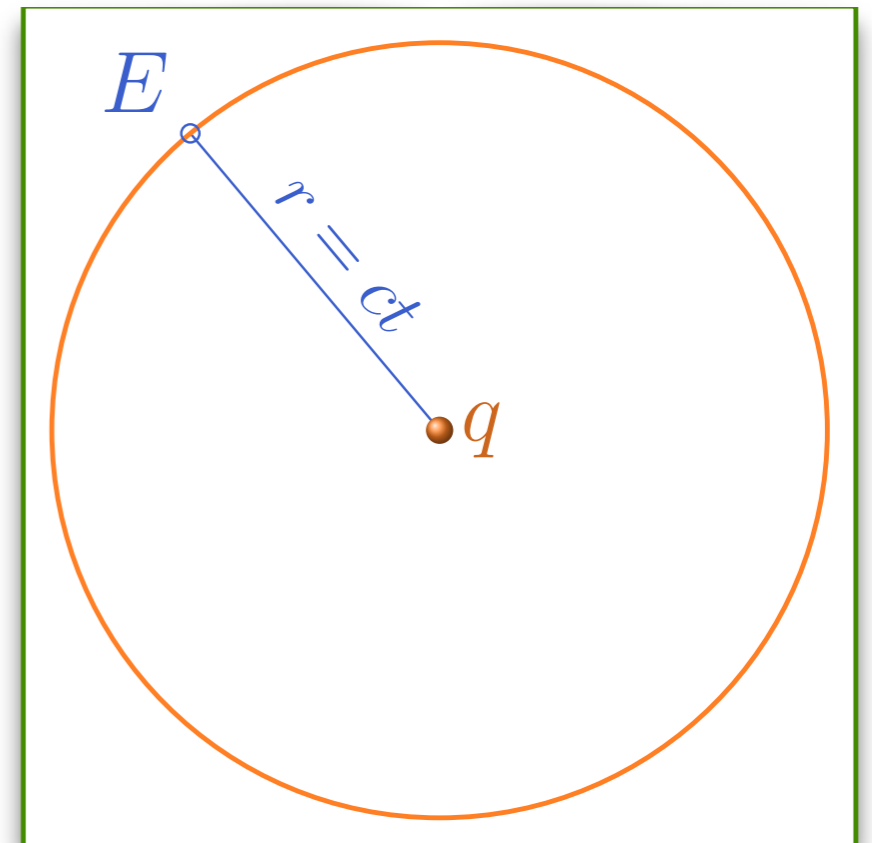
$r \rightarrow$  Invariante

$$x^\mu = \begin{bmatrix} r \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} \quad \eta_\mu = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} -c & u_1 & u_2 & u_3 \end{bmatrix}$$

# Potenciais e relatividade

$$A^\mu = \frac{V/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$$

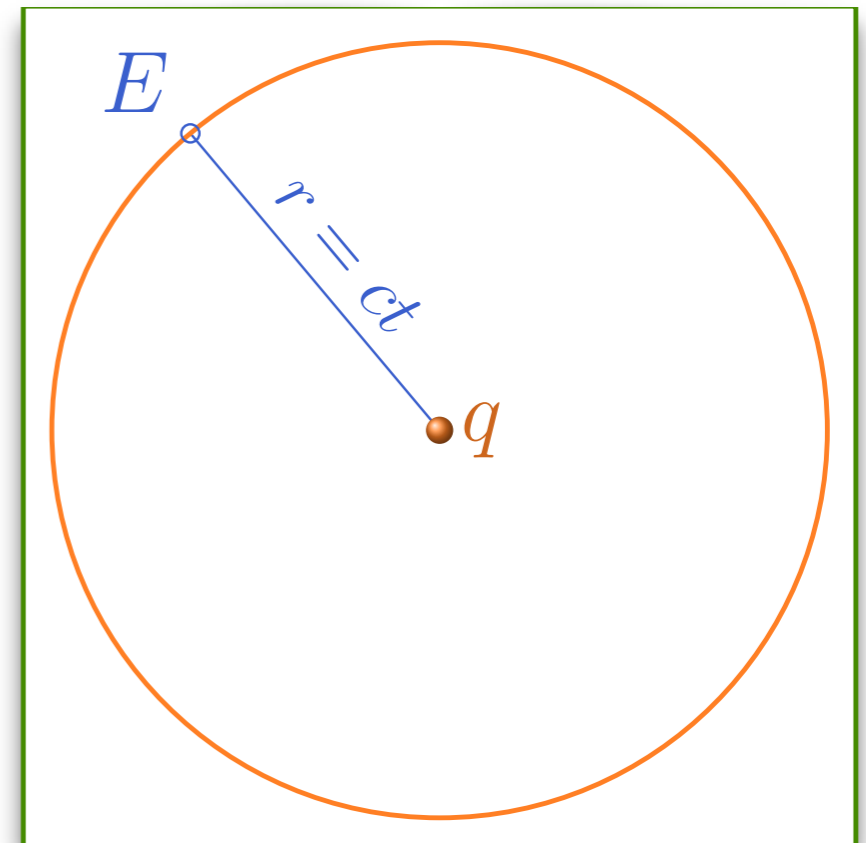
$r \rightarrow$  Invariante



$$\eta_\mu x^\mu = \frac{-cr + \vec{\mathbf{u}} \cdot \vec{\mathbf{r}}}{\sqrt{1 - \frac{u^2}{c^2}}} \Rightarrow -\frac{\eta_\mu x^\mu}{c} = \frac{r - \frac{\vec{\mathbf{u}}}{c} \cdot \vec{\mathbf{r}}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

# Potenciais e relatividade

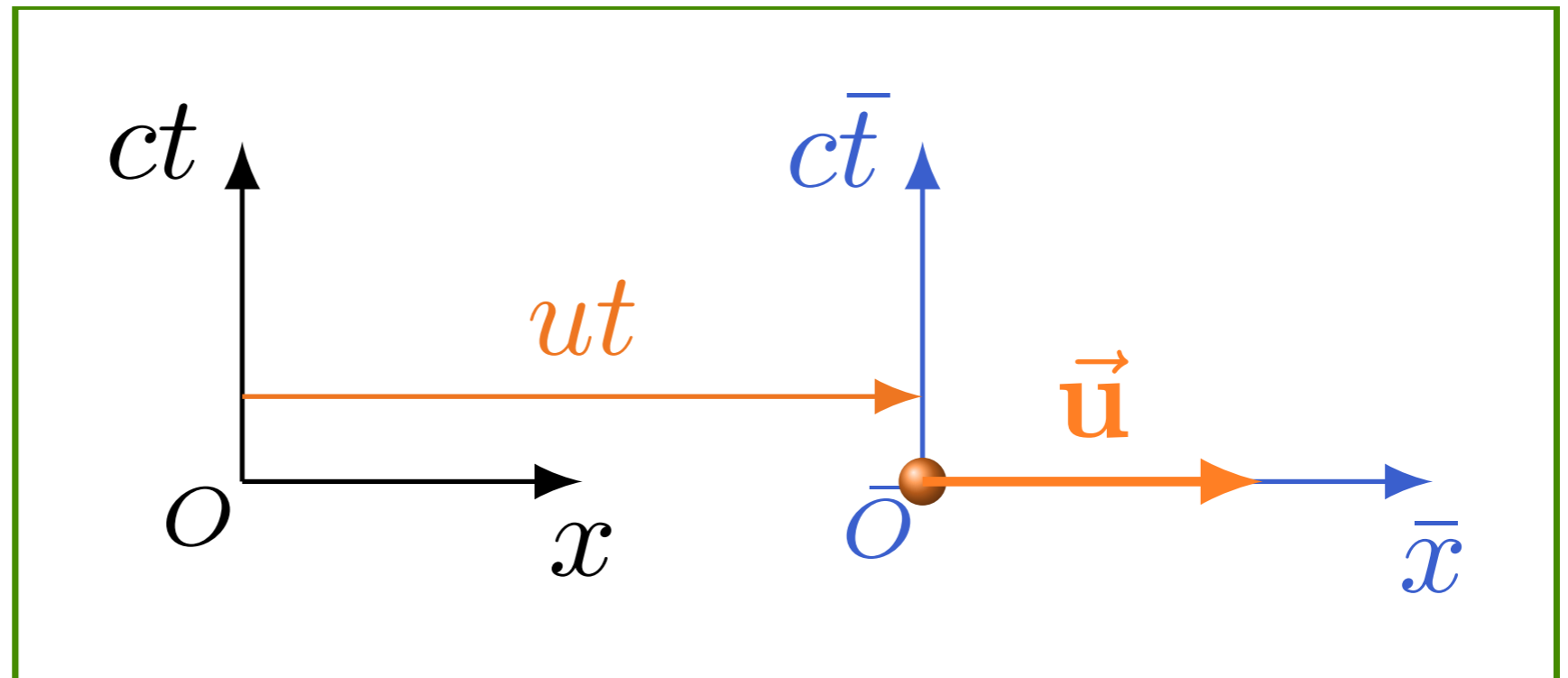
$$A^\mu = \frac{V/c^2}{\sqrt{1 - \frac{u^2}{c^2}}} \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$$



$r \rightarrow$  Invariante

$$r \rightarrow \frac{r - \frac{\vec{u}}{c} \cdot \vec{r}}{\sqrt{1 - \frac{u^2}{c^2}}} \Rightarrow \frac{q}{4\pi\epsilon_0 r} \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \rightarrow \frac{q}{4\pi\epsilon_0 \left( r - \frac{\vec{u}}{c} \cdot \vec{r} \right)}$$

# Potenciais e relatividade



$$V = \frac{q}{4\pi\epsilon_0 \left( r - \frac{\vec{u}}{c} \cdot \vec{r} \right)}$$

$$\bar{V} = \frac{q}{4\pi\epsilon_0 r}$$

$$\vec{A} = \mu_0 \frac{q\vec{u}}{4\pi \left( r - \frac{\vec{u}}{c} \cdot \vec{r} \right)}$$

$$\vec{\bar{A}} = 0$$

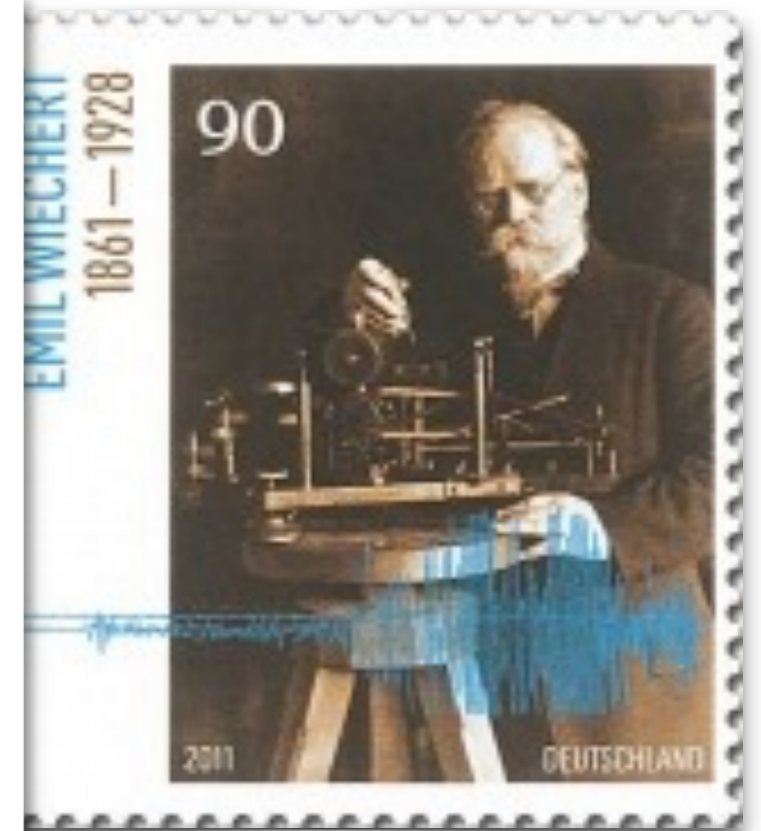
# Potenciais de Liénard e Wiechert

$$V = \frac{q}{4\pi\epsilon_0 \left( r - \frac{\vec{u}}{c} \cdot \vec{r} \right)}$$

$$\vec{A} = \mu_0 \frac{q\vec{u}}{4\pi \left( r - \frac{\vec{u}}{c} \cdot \vec{r} \right)}$$



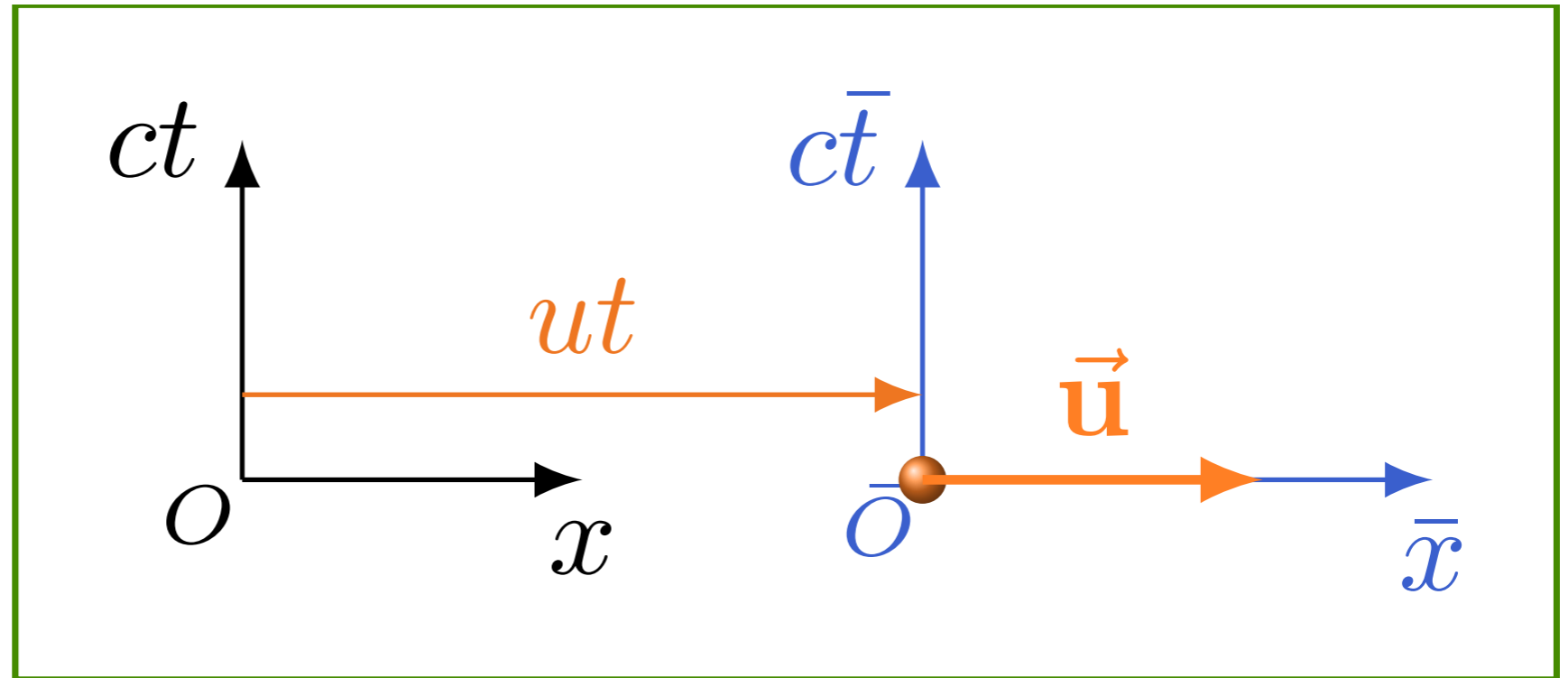
1898



1900



# Potenciais e relatividade



$$\bar{V} = \frac{q}{4\pi\epsilon_0 r}$$

$$\vec{\bar{A}} = 0$$

