

1) $P(A) = P(B) = P(A \cap B) = 1/4$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/4 + 1/4 - P(A \cap B) = 2/4 - P(A \cap B)$... (1)

a) A e B indep $\Rightarrow P(A \cup B) = \frac{2}{4} - P(A)P(B) = \frac{2}{4} - \frac{1}{4}(\frac{1}{4}) = 7/16$ S

b) $P(A \cup B) = P(A - B) + P(B - A) + P(A \cap B)$
 $= P(A \Delta B) + P(A \cap B) = 1/4 + P(A \cap B)$... (2)

Usando (1) e (2) $\frac{1}{4} + P(A \cap B) = \frac{2}{4} - P(A \cap B) \Rightarrow P(A \cap B) = 1/8$ S

$\therefore P(A \cup B) = 2/4 - 1/8 = 3/8$

2) X : # de petroleiros por dia

$X \sim \text{Poisson}(\lambda=2)$ $P(X=x) = \frac{e^{-2} 2^x}{x!}, x=0,1,\dots$ Δ

a) $P(X > 3) = 1 - P(X \leq 3) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$
 $= 1 - e^{-2} - 2e^{-2} - \frac{4}{2}e^{-2} - \frac{8}{6}e^{-2} = 1 - (\frac{33}{6})e^{-2} = 0,1428765$
 $\approx 0,14 \rightarrow P(X \leq 3) = 0,86$ 4'

b) $P(X < 6) = 0,95 \rightarrow$ ~~$P(X > 3) = 0,05$~~

$0,95 = P(X \leq 3+c) = P(X \leq 3) + P(X=4) + P(X=5)$

$P(X=4) = \frac{16e^{-2}}{24} = 0,0902$

$P(X=5) = \frac{32e^{-2}}{120} = 0,03608$

$0,86 + 0,009 + 0,03608 \times$

Com $X=4$ atingimos 95%

1pt (um) adicional ✓

3) $X \sim N(0,1), Y \sim N(0,1)$ = 1

a) $E(X+Y) = E(X) + E(Y) = 0 + 0 = 0$ 2

$V(\frac{X-Y}{2}) = V(\frac{1}{2}X) + V(-\frac{1}{2}Y) = \frac{1}{4}V(X) + \frac{1}{4}V(Y) = \frac{1}{4} + \frac{1}{4} = 1/2$ 2

$W = X+Y \sim N(0, 1+1)$ 1

$P(W > 2) = 1 - P(W \leq 2) = 1 - P(Z \leq \frac{2-0}{\sqrt{2}}) = 1 - P(Z < 1,41)$

4

~~$= 1 - 0,9207 = 0,0793$~~

$1 - 0,9207 = 0,0793$

$$\textcircled{4} \quad \frac{1}{k} = \int_0^1 \int_y^2 (x + 3xy^2) dx dy = \int_0^1 \left(\frac{x^2}{2} + \frac{3x^2 y^2}{2} \right) \Big|_y^2 dy$$

$$\textcircled{a} \quad = \int_0^1 \left(2 + 6y^2 - \frac{y^2}{2} - \frac{3}{2} y^4 \right) dy = 2y + \frac{6y^3}{3} - \frac{y^3}{6} - \frac{3}{2} \frac{y^5}{5} \Big|_0^1$$

$$\frac{1}{k} = 2 + 2 - 1/6 - 3/10 = \frac{240 - 40 - 18}{60} \Rightarrow k = \frac{60}{212}$$

$$P(X < 1 / Y > 0,5) = \frac{P(X < 1, Y > 0,5)}{P(Y > 0,5)}$$

$$f(y) = \int_y^2 \frac{60}{212} [x + 3xy^2] dy = \frac{60}{212} \left[\frac{x^2}{2} + \frac{3x^2 y^2}{2} \right] \Big|_y^2$$

$$= \frac{60}{212} \left[2 + 6y^2 - \frac{y^2}{2} - \frac{3}{2} y^4 \right] = \frac{60}{212} \left\{ 2 + \frac{11}{2} y^2 - \frac{3}{2} y^4 \right\} \quad 0 < y < 1$$

$$P(Y > 0,5) = 1 - \int_0^{0,5} \frac{60}{212} \left\{ 2 + \frac{11}{2} y^2 - \frac{3}{2} y^4 \right\} dy$$

$$= 1 - \frac{60}{212} \left\{ 2y + \frac{11}{6} y^3 - \frac{3}{10} y^5 \right\} \Big|_0^{0,5}$$

$$= 1 - \frac{60}{212} \left\{ 1 + \frac{11}{6} (0,5)^3 - \frac{3}{10} (0,5)^5 \right\} = 0,6547 \checkmark$$

$$\textcircled{b} \quad P(X < 1, Y > 0,5) = \int_{0,5}^1 \int_y^1 \frac{60}{212} [x + 3xy^2] dx dy$$

$$= \int_{0,5}^1 \frac{60}{212} \left[\frac{x^2}{2} + \frac{3x^2 y^2}{2} \right] \Big|_y^1 dy$$

$$= \int_{0,5}^1 \frac{60}{212} \left\{ \frac{1}{2} + \frac{3y^2}{2} - \frac{y^2}{2} - \frac{3}{2} y^4 \right\} dy$$

$$= \frac{60}{212} \left\{ \frac{1}{2} y + \frac{y^3}{2} - \frac{y^3}{6} - \frac{3}{10} y^5 \right\} \Big|_{0,5}^1$$

$$= \frac{60}{212} \left\{ \frac{1}{4} + \frac{7}{24} - \frac{3}{10} \left(\frac{31}{32} \right) \right\} = \frac{0,0975}{0,6547} = 0,10$$

$$P(X < 1 / Y > 0,5) = \frac{0,10}{0,6547} = \underline{0,107}$$

5

X	-3	2	4	$P(X=x)$
1	0,1	0,2	0,2	0,5
3	0,3	0,2	0,1	0,5
$P(Y=y)$	0,4	0,3	0,3	1

a

X	$P(X/Y=2)$	$XP(X/Y=2)$
1	$2/3$	$2/3$
3	$1/3$	$3/3$

5 total $E(X/Y=2) = \frac{5}{3} = 1\frac{2}{3}$

b

X	$P(X=x)$	$xP(X=x)$	$x^2P(X=x)$
1	0,5	0,5	0,5
3	0,5	1,5	4,5

$E(X) = 2$ $E(X^2) = 5$

$$V(X) = E(X^2) - E(X)^2$$

$$= 5 - 2^2 = 1$$

Y	$P(Y=y)$	$yP(Y=y)$	$y^2P(Y=y)$
-3	0,4	-1,2	3,6
2	0,3	0,6	1,2
4	0,3	1,2	4,8
$E(Y)$	0,6	9,6	$E(Y^2)$

$$V(Y) = 9,6 - 0,6^2$$

$$= 9,24$$

5 total

$$E(XY) = (1)(-3)(0,1) + (1)(2)(0,2) + (1)(4)(0,2)$$

$$+ (3)(-3)(0,3) + (3)(2)(0,1) + (3)(4)(0,1) = 0$$

$$\text{cov}(X,Y) = E(XY) - E(X)E(Y) = 0 - 2(0,6) = -1,2$$

$$r(X,Y) = \frac{-1,2}{\sqrt{1 \times 9,24}} = -0,3947$$