Problema 2.7. eja $\alpha : \mathbb{R} \to \mathbb{R}^2$, $\alpha(t) = (x(t), y(t))$ a curva parametrizada que atende a seguinte E.D.O $\frac{d}{dt}x(t) = \frac{1}{2}y(t)$, $\frac{d}{dt}y(t) = 2x(t)$, $\boxed{x(0) = 0}$ e $y(0) = 2$. Determine em qual curva a imagem de α está contida.

Exemplo 9: Hipérbole

$C_1 = \{(x_1, x_2) \in \mathbb{R}^2 | \frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1, x_1 > 0\}$

$\alpha(t) = (a\cosh(t), b\sinh(t))$ $t \in \mathbb{R}$.

OU

$$\boxed{\alpha(t) = (a\sinh(t), b\cosh(t))} \ *$$

Se

$$x(t) = a\cosh(t) = a \cdot \frac{e^x + e^{-x}}{2}$$

$$x(0) = a \cdot \frac{1+1}{2} = a$$

$$x(t) = a\,\text{senh}(t) = a \cdot \frac{e^x - e^{-x}}{2}$$

$$x(0) = a \cdot \frac{1-1}{2} = 0$$

$x(t) = a\sinh(t)$ $\quad$ $y(t) = b\cosh(t)$

$x'(t) = a\cosh(t)$ $\quad$ $y'(t) = b\sinh(t)$

$a\cosh(t) = \frac{1}{2} b\cosh(t)$ $\quad \to \quad$ $a = \frac{1}{2}b$

$b\sinh(t) = 2a\sinh(t)$ $\quad$ $b = 2a$

$$y(0) = b\cosh(0) = b = 2$$

$$\boxed{\begin{array}{l} b = 2 \\ a = 1 \end{array}}$$

$$\frac{x_2^2}{a^2} - \frac{x_1^2}{b^2} = 1$$

$$x_2^2 - \frac{x_1^2}{4} = 1$$

"trocar sinal" $(\cdot(-1))$

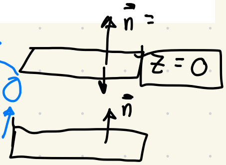
$$\boxed{-y^2 + \frac{x^2}{4} = -1}$$

Problema 3.24. Seja $f : \mathbb{R}^2 \to \mathbb{R}$ definida como $f(x,y) = 2(2x - x^2)(2y - y^2)$. Determine os pontos $p = (p_1, p_2)$ tais que os planos tangentes ao gráfico de f nos pontos $q = (p_1, p_2, f(p))$ sejam paralelos a $\{z = 0\}$.

$$\vec{n}_{GERAL} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1 \right\rangle$$

$$\vec{n}_{z=0} = \langle 0, 0, -1 \rangle$$

$0x + 0y + 1z = 0$



$$\begin{cases} \partial f/\partial x = 0 \\ \partial f/\partial y = 0 \end{cases} \qquad \begin{cases} 2(2y-y^2)\cdot(2-2x) = 0 \\ 2(2x-x^2)(2-2y) = 0 \end{cases}$$

$2y - y^2 = 0 \to \boxed{y=0}$ ou $\boxed{y=2}$

$2 - 2x = 0 \to \boxed{x=1}$

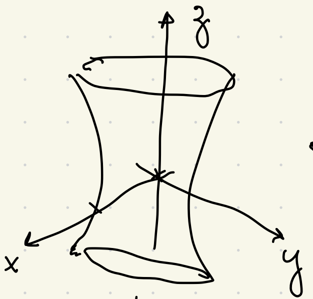
$y=0$	$y=2$	$x=1$
$2(2x-x^2)\cdot 2 = 0$	$2\cdot(2x-x^2)(-2)=0$	$2(1)(2-2y)=0$
$x=0$	$x=0$	$y=1$
$x=2$	$x=2$	

$$\boxed{\begin{matrix} (0;0) & (0,2) & (1;1) \\ (2;0) & (2;2) & \end{matrix}}$$

2.9 (1) $S = \{(x,y,z) \in \mathbb{R}^3 \mid z^2 = 3x^2 + 4y^2 - 12\}$

$$\frac{z^2}{12} = \frac{x^2}{4} + \frac{y^2}{3} - 1$$

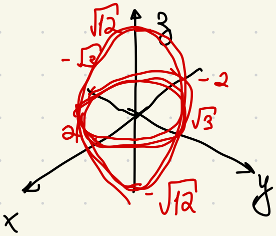
$$\boxed{1 = \frac{x^2}{4} + \frac{y^2}{3} - \frac{z^2}{12}}$$



↳ hiperbolóide (rotação de hipérbole)

PS |

$$+\frac{x^2}{4}+\frac{y^2}{3}+\frac{z^2}{12}=1$$



$\boxed{\text{Elipsoide}}$