Eletromagnetismo Avançado

16 de outubro
Relatividade restrita

Geometria do espaço-tempo

Notação: evento E $\equiv \vec{r}, t \equiv$ quadrivetor

$$(ct, x, y, z) \quad \Rightarrow \quad (x_0, x_1, x_2, x_3)$$

Geometria do espaço-tempo

Notação

$(ct, x, y, z) \quad \Rightarrow \quad (x_0, x_1, x_2, x_3) \equiv x_\mu$ Vetor covariante

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Vetor contravariante

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Vetor contravariante

Dualidade

$$(x_0 \quad x_1 \quad x_2 \quad x_3) \Leftrightarrow \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x_3 \end{pmatrix}$$

$$x_0 = -x^0$$

$$x_1 = x^1$$

$$x_2 = x^2$$

$$x_3 = x^3$$

Transformação de Lorentz

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

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$$\bar{x}_{\mu} = \tilde{\Lambda}^{\eta}_{\mu} x_{\eta}$$

Produto escalar

$$\bar{x}^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} \qquad \bar{x}_{\mu} = \tilde{\Lambda}^{\eta}_{\mu} x_{\eta}$$

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Pratique o que aprendeu

$$\bar{a}^{\mu}\bar{b}_{\mu} = a^{\nu}b_{\nu}$$

$$\bar{\Delta}x^{\mu}\bar{\Delta}x_{\mu} = \Delta x^{\nu}\Delta x_{\nu}$$

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$$-c^2\Delta\bar{t}^2 + \Delta\bar{x}^2 + \Delta\bar{y}^2 + \Delta\bar{z}^2 = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 \quad \Rightarrow \quad \bar{I} = I$$