16/10

Problema 3.21. Sejam $f : \mathbb{R}^2 \to \mathbb{R}$ definda como $f(x_1, x_2) = \cos(\sqrt{x_1^2 + x_2^2})$ e $p = (\frac{\pi}{6}, \frac{\sqrt{3}\pi}{6})$. Determine o plano tangente do gráfico de f em $(\frac{\pi}{6}, \frac{\sqrt{3}\pi}{6}, f(p))$

$\hookrightarrow z_0 : f\left(\frac{\pi}{6}, \frac{\sqrt{3}\pi}{6}\right) = \frac{1}{2}$

Problema 3.22. Sejam $f : \mathbb{R}^2 \to \mathbb{R}$ definda como $f(x_1, x_2) = \cos(\sqrt{x_1^2 + x_2^2})$ e $p = (\frac{\pi}{6}, \frac{\sqrt{3}\pi}{6})$. Determine a derivada $df(p)$ onde $p = (\frac{\pi}{6}, \frac{\sqrt{3}\pi}{6})$.

3.21 Problema ??: $\underset{(z-\frac{1}{2})}{z} = \frac{\sqrt{3}}{4}(x_1 - \frac{\pi}{6}) + \frac{3}{4}(x_2 - \frac{\sqrt{3}\pi}{6}) \cancel{+ (x_3 - \frac{1}{2})} = 0$

3.22 Problema ??: $df(p) = \begin{bmatrix} -\frac{\sqrt{3}}{4} & \frac{-3}{4} \end{bmatrix}$ ou $\boxed{df = -\frac{\sqrt{3}}{4}dx - \frac{3}{4}dy}$

3.21

$$\frac{\partial f}{\partial x_1} = -\operatorname{sen}\left(\sqrt{x_1^2+x_2^2}\right) \cdot \frac{1}{2\sqrt{x_1^2+x_2^2}} \cdot 2x_1 \xrightarrow{p} -\frac{\sqrt{3}}{4}$$

$$\frac{\partial f}{\partial x_2} = \quad \cdots \quad \cdot 2x_2 \xrightarrow{p} -\frac{3}{4}$$

$$\vec{n} = \left\langle -\frac{\sqrt{3}}{4}; -\frac{3}{4}; -1 \right\rangle$$

$$\vec{n} = \left\langle \frac{\sqrt{3}}{4}, \frac{3}{4}, 1 \right\rangle$$

$$P\left(\frac{\pi}{6}; \frac{\sqrt{3}\pi}{6}; \frac{1}{2}\right)$$

PLANO TANGENTE

Ponto: $\left(\frac{\pi}{6}; \frac{\sqrt{3}\pi}{6}, \frac{1}{2}\right)$

Vetor normal: $\vec{n} = \left\langle \frac{\partial f}{\partial x}(p), \frac{\partial f}{\partial y}(p), -1 \right\rangle$

ou

$$= \left\langle -\frac{\partial f}{\partial x}(p); -\frac{\partial f}{\partial y}(p); +1 \right\rangle$$

$$\boxed{\widetilde{\Pi} : \frac{\sqrt{3}}{4}x + \frac{3}{4}y + 1 \cdot z = \frac{\sqrt{3}}{4} \cdot \frac{\pi}{6} + \frac{3}{4} \cdot \frac{\sqrt{3}\pi}{6} + 1 \cdot \frac{1}{2}}$$

P1 2022

Questão 2.2 (2,5 pt)**.** Considere a função $f(x_1, x_2) = 2\sqrt{x_1^2 + x_2^2 + 1}$.

(a) Esboce o gráfico de f.

(b) Determine a derivada $df(\frac{\sqrt{3}}{2}, \frac{3}{2})$.

(c) Determine a equação Cartesiana do plano tangente ao gráfico de f no ponto $(\frac{\sqrt{3}}{2}, \frac{3}{2}, 4)$.

Respostas:

(b) $df(\frac{\sqrt{3}}{2}, \frac{3}{2}) = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{3}{2} \end{bmatrix}$.

(c) $-\frac{\sqrt{3}}{2}(x_1 - \frac{\sqrt{3}}{2}) - \frac{3}{2}(x_2 - \frac{3}{2})) + (\underbrace{x_3}_{z} - 4) = 0$

Mudança de Nome de Variável

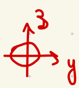
$x_1 = x$

$x_2 = y$

$\vec{n} = \langle -\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}; 1 \rangle$ ou $\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1 \rangle$

Cilindros: $f(x,y) = c$ (ñ tinha z)

Superfícies de Revolução: $f(\underset{x^2+y^2}{r^2}, z) = c$ Supunha $x = 0$



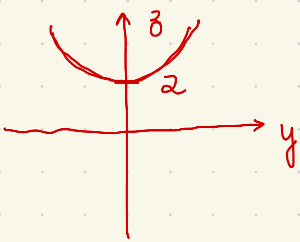
Rotacionava pelo z

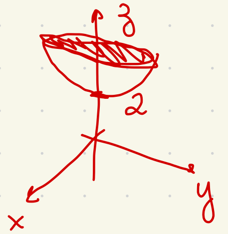
$\overbrace{f(x,y)}^{z} = 2\sqrt{x^2+y^2+1}$

$z = 2\sqrt{x^2+y^2+1} \xrightarrow{x=0} z = 2\sqrt{y^2+1} \longrightarrow \frac{z}{2} = \sqrt{y^2+1}$

$\downarrow {}^2$ ⚠ $(z>0)$

$\boxed{\frac{z^2}{4} - y^2 = 1} \longleftarrow \frac{z^2}{4} = y^2 + 1$





2.9 (3) $S = \{(x,y,z) \in \mathbb{R}^3 / \underbrace{x^2 - y^2 + 4y + z = 4}\}$

$$x^2 - (y^2 - 4y + 4) + z = 4$$

$$x^2 - (y - 2)^2 + z = 4 - 4$$

$$\boxed{x^2 - (y-2)^2 + z = 0}$$

$\hookrightarrow z = (y-2)^2 - x^2$: SELA de CAVALO