LGN 5822 - Biometrical Genetics

# L08 - Randomized Complete Block Design 

Michele Jorge Silva Siqueira

## Introduction

"We shall need to judge of the magnitude of the differences introduced by testing our treatments upon the different plots by the discrepancies between the performances of the same treatment in different blocks"

R.A Fisher, 1935<br>The Design of Experiments, Section 26



## Introduction

- If the researcher finds that any factor disturbs the homogeneity of the experimental units or environmental conditions


## Introduction <br> \section*{Iroduction}


.








? $+$

## Introduction

- How can we assure, then, that an observed agronomic difference is the result of a specific treatment, rather than the result of the experimental units to which it was allocated?
- In other words, how do we prevent our treatment results from being confounded with our experimental units?



## Introduction

The heterogeneity of experimental units presents a problem:

- Difficulty recognizing differences between experimental units!
- This could lead us to conclude that the differences in our variables are the result of the treatments applied, when in fact they were caused by the pre existing condition of the experimental units


## Introduction

## Randomized Complete Block Design (RCBD)

- How to control undesirable variation between experimental units?


Minimize Field Differences

## Introduction

## Randomized Complete Block Design (RCBD)

- The first step in using the RCBD is to recognize the source(s) of potential heterogeneity among plots (experimental units)

"Production gradient"


## Introduction

## How to block

- In blocking, we generally place an equal-sized block on every map unit. Each block, in this case, contains four experimental units (plots)
- Each treatment is applied to one experimental unit within the block


Blocking of randomized treatments to account for a known yield potential gradient

## Design Characterization

## Randomized Complete Block Design (RCBD)



## Design Characterization

## Randomized Complete Block Design (RCBD)

- RCBD is the standard design for agricultural experiments where similar experimental units are grouped into blocks or replicates
- A RCBD is the most basic blocking design

RCDB uses the basic principles of repetition, randomization and local control

## Design Characterization

## Randomized Complete Block Design (RCBD)

- It is used to control for variation in an experiment not predicted by the researcher (random error)
- e.g. variation in fertility or drainage differences in a field


## Design Characterization

## Randomized Complete Block Design (RCBD)

- Assume we have $r$ blocks containing $g$ units each


Here, $r=3$ blocks with $g=4$ units
In every of the $r$ blocks we randomly assign the $g$ treatments to the $g$ units, independently of the other blocks

## Design Characterization

## Description of the Design: RCBD

- Probably the most used and useful of the experimental designs
- Takes advantage of grouping similar experimental units into blocks or replicates
- The blocks of experimental units should be as uniform as possible
- The purpose of grouping experimental units is to have the units in a block as uniform as possible so that the observed differences between treatments will be largely due to "true" differences between treatments
- Each block gets its "own" randomization


## Design Characterization

## Description of the Design: RCBD

- We call a blocking design complete if every treatment is used in every block
- Each treatment must appear at least once per replicate
- In general we observe every treatment (only) once in every block, hence we have a total of $\boldsymbol{r}$ (the number of blocks) observations per treatment


## Design Characterization

## Example

- Researchers wanted to evaluate the effect of several different fertilization (nitrogen) timing schedules on leaf tissue of maize
- Treatment: Six different nitrogen application timing and rate schedules (including a control treatment of no nitrogen)
- Response: Leaf tissue nitrogen amount


## Design Characterization

## Example

- Experiment design: irrigated field with a water gradient along one direction, see next slide
- We already know:

Available moisture in the soil (humidity) will have an influence on the response

Design Characterization

## Example

- Layout of Experimental Design



## Design Characterization

## Example

- Layout of Experimental Design
- The differences in plant responses caused by the water gradient will be associated with blocks
- We also say: we control for the water gradient


## Design Characterization

## Advantages of the RCBD

- Generally more precise than the CRD
- No restriction on the number of treatments or replicates
- Some treatments may be replicated more times than others
- Missing plots are easily estimated


## Design Characterization

## Disadvantages of the RCBD

- In experiments with a large number of treatments, a RCDB may become inefficient because each block must contain all treatments. This can significantly increase the size of the experiment, require more resources, and make the analysis more complex
- If the variability between blocks is too large, this can negatively affect the ability of the RCBD to detect differences between treatments


## Design Characterization

## Data table

- Consider an experiment installed at the RCDB with $i$ treatments and $j$ replicates (blocks)

|  | Treatments |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Blocks | 1 | 2 | $\ldots$ | I | Total |
| 1 | $\mathrm{Y}_{11}$ | $\mathrm{Y}_{21}$ | $\ldots$ | $\mathrm{Y}_{11}$ | $\mathrm{~B}_{1}$ |
| 2 | $\mathrm{Y}_{12}$ | $\mathrm{Y}_{22}$ | $\ldots$ | $\mathrm{Y}_{12}$ | $\mathrm{~B}_{2}$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| J | $\mathrm{Y}_{1 \mathrm{~J}}$ | $\mathrm{Y}_{2 \mathrm{~J}}$ | $\ldots$ | $\mathrm{Y}_{1 \mathrm{~J}}$ | $\mathrm{~B}_{\mathrm{J}}$ |
| Total | $\mathrm{T}_{1}$ | $\mathrm{~T}_{2}$ | $\ldots$ | $\mathrm{~T}_{1}$ | G |

## Design Characterization

## Data table

- Number of experimental units: $\mathrm{N}=\mathrm{I} \mathrm{x}$ J
- Total for treatment $i: \mathrm{T}_{\mathrm{i}}=\sum_{\mathrm{j}=1}^{\mathrm{j}} \mathrm{Y}_{\mathrm{ij}}=\mathrm{Y}_{\mathrm{i}}$
- Total for block $j: \mathrm{B}_{\mathrm{i}}=\sum_{\mathrm{i}=1}^{i} \mathrm{Y}_{\mathrm{ij}}=\mathrm{Y}_{\mathrm{j}}$
- Mean for treatment $i: \widehat{m_{i}}=\frac{T_{i}}{J}$
- Mean for block j: $\quad \widehat{m}_{j}=\frac{B_{j}}{I}$
- General mean of the experiment: $\hat{m}=\frac{G}{I J}$


## Design Characterization

## Statistical model

Model

- Data from the RCBD can be described with the following model:

$$
y_{i j}=\mu+\alpha_{i}+\beta_{j}+\varepsilon_{i j}
$$

where $\mu$ is the intercept (an overall mean), $\alpha_{i}$ is the effect of treatment $i, \beta_{j}$ is the effect of block $j$ and $\varepsilon_{i j}$ is the associated random error term

## Design Characterization

## Anova table

| Source | Sum of <br> Squares | Degrees of <br> Freedom | Mean <br> Square | F-stat |
| :--- | :---: | :---: | :---: | :---: |
| Treatment | SST | $t-1$ | $M S T$ | $F^{*}=M S T / M S E$ |
| Block | $S S B$ | $b-1$ | $M S B$ |  |
| Error | $S S E$ | $(t-1)(b-1)$ | $M S E$ |  |
| Total | $T S S$ | $t b-1$ |  |  |

- Remember about the sum of squares!
- Typically, we are not making inference about blocks (we already know that blocks are different!)
- It is interesting to evaluate whether there is a difference between the treatments, which can be verified using the $F$ test for treatments


## Design Characterization

## Hypotheses

- Treatments
$H_{0}: t_{i}=0, i=1,2, \ldots I$
$H_{1}$ : at least one value of $t_{k} \neq 0$
- Blocks
$H_{0}: b_{j}=0, j=1,2, \ldots J$
$H_{1}$ : at least one value of $b_{k} \neq 0$

In cases where the variation between blocks is doubtful, the researcher can perform the $F$ test for blocks, to serve as guidance for setting up future experiments!

## Let's Practice 01!

\#Load the "agridat" package
\#Choose the dataset "federer.tobacco" from the agridat package

- Data: RCB of tobacco, height plants exposed to radiation: 56 observations
- Evaluation of plant growth in different doses of radiation
\#Perform an analysis of variance (ANOVA) for the RCDB
\#Tukey test for multiple comparisons of treatments

Let's Practice!


## TODO:

$>$ head(federer.tobacco)
an

$$
\begin{array}{lllll}
1 & 1 & 1 & 2500 & 1299.2
\end{array}
$$

$$
\begin{array}{lllll}
2 & 1 & 2 & 250 & 1369.2
\end{array}
$$

$$
\begin{array}{lllll}
3 & 1 & 3 & 0 & 1169.5
\end{array}
$$

$$
\begin{array}{lllll}
4 & 1 & 4 & 2500 & 1219.1
\end{array}
$$

$$
\begin{array}{lllll}
5 & 1 & & 5 & 2500 \\
6 & 1120.0
\end{array}
$$

$$
>\mid
$$

## Let's Practice!

```
> summary(model_anova)
Error: block
    Df Sum Sq Mean Sq F value Pr(>F)
Residuals 1 278088 278088
Error: Within
    Df Sum Sq Mean Sq F value Pr(>F)
dose 1 185289 185289 6.688 0.0125 *
Residuals 53 1468400 27706
signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```


## References

- Chapter 9 - Analysis of Variance II: The Randomized Complete Block Design¹ (for a more classical view)
- Chapter 3 - Complete Block Designs²

1. Steel, R. G. \& Torrie, J. H. Principles and Procedures of Statistics: A Biometrical Approach. 2nd Edition. (1980).
2. Casella, G. Statistical Design. (2008).
