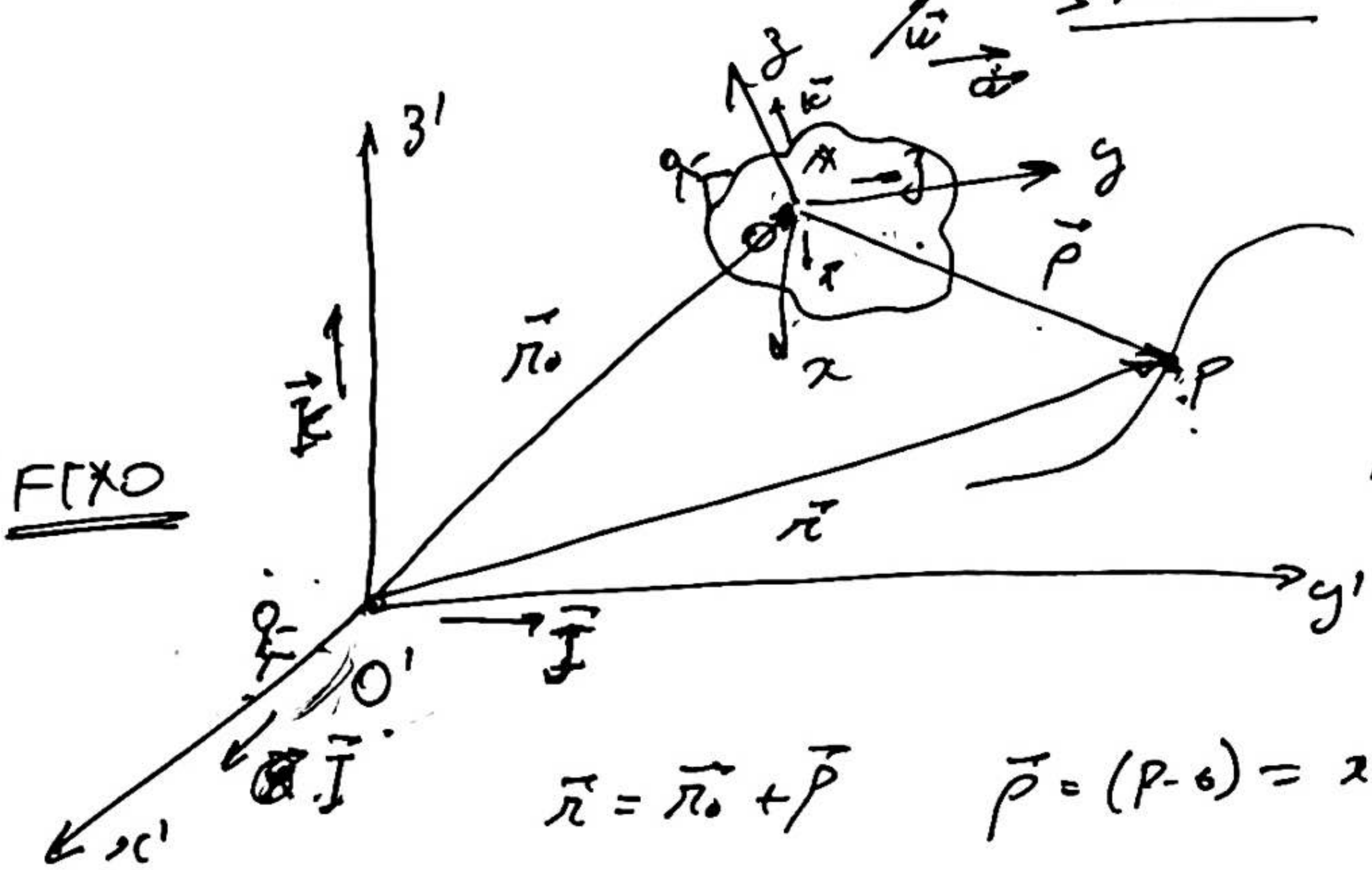
PME 3100 10.10.23

1

COMPOSIÇÃO DE MOVIMENTOS

S MÓVEL $\vec{v}_0, \vec{\omega}, \dot{\vec{\omega}}$



S' FIXO

OBS:

$\dot{\vec{i}} = \vec{\omega} \wedge \vec{i}$

$\dot{\vec{j}} = \vec{\omega} \wedge \vec{j}$

$\dot{\vec{k}} = \vec{\omega} \wedge \vec{k}$

$\vec{r} = \vec{r}_0 + \vec{\rho} \qquad \vec{\rho} = (P-O) = x\vec{i} + y\vec{j} + z\vec{k}$

$\dot{\vec{r}} = \dot{\vec{r}}_0 + \dot{\vec{\rho}} \Rightarrow \dot{\vec{r}}_0 = \vec{v}_0$

$\dot{\vec{\rho}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} + x\dot{\vec{i}} + y\dot{\vec{j}} + z\dot{\vec{k}}$

$\dot{\vec{\rho}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} + x(\vec{\omega} \wedge \vec{i}) + y(\vec{\omega} \wedge \vec{j}) + z(\vec{\omega} \wedge \vec{k})$

$\dot{\vec{\rho}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} + (\vec{\omega} \wedge x\vec{i}) + (\vec{\omega} \wedge y\vec{j}) + (\vec{\omega} \wedge z\vec{k})$

$\dot{\vec{\rho}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} + \vec{\omega} \wedge (x\vec{i} + y\vec{j} + z\vec{k})$

$\dot{\vec{\rho}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} + \vec{\omega} \wedge (P-O)$

$$\dot{\vec{r}} = \vec{v}_P = \underbrace{\vec{v}_0 + \vec{\omega} \wedge (P-O)}_{\text{ARRASTAMENTO}} + \underbrace{\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}}_{\text{RELATIVO}}$$

$\vec{v}_{P,a}$ — VELOCIDADE DE P DE ARRASTAMENTO.

$v_{P,r}$ — VELOCIDADE DE P RELATIVA

$$\boxed{\vec{v}_P = \vec{v}_{P,r} + \vec{v}_{P,a}}$$

ABSOLUTA

DERIVANDO $\dot{\vec{r}} = \vec{v}_p$ EM RELAÇÃO AO TEMPO. 2

$$\ddot{\vec{r}} = \dot{\vec{v}}_p = \vec{a}_p = \frac{d}{dt}\Big[\underbrace{\vec{r}_0}_{1} + \underbrace{\omega \wedge (P-O)}_{2} + \underbrace{\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}}_{3}\Big]$$

1) $\ddot{\vec{r}}_0 = \dot{\vec{v}}_0 = \vec{a}_0$

2) $$\frac{d}{dt}\left(\vec{\omega} \wedge (P-O)\right) = \dot{\vec{\omega}} \wedge (P-O) + \vec{\omega} \wedge (\vec{v}_P - \vec{v}_O) =$$
$$= \dot{\vec{\omega}} \wedge (P-O) + \vec{\omega} \wedge \left[\vec{\omega} \wedge (P-O) + \vec{v}_{P,r}\right] =$$
$$= \dot{\vec{\omega}} \wedge (P-O) + \vec{\omega} \wedge \left[\vec{\omega} \wedge (P-O)\right] + \vec{\omega} \wedge \vec{v}_{P,r}$$

3) $$\frac{d}{dt}(\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}) = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} + \dot{x}\dot{\vec{i}} + \dot{y}\dot{\vec{j}} + \dot{z}\dot{\vec{k}} =$$
$$= \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} + \dot{x}(\vec{\omega} \wedge \vec{i}) + \dot{y}(\vec{\omega} \wedge \vec{j}) + \dot{z}(\vec{\omega} \wedge \vec{k}) =$$
$$= \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} + (\vec{\omega} \wedge \dot{x}\vec{i}) + (\vec{\omega} \wedge \dot{y}\vec{j}) + (\vec{\omega} \wedge \dot{z}\vec{k}) =$$
$$= \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} + \vec{\omega} \wedge (\underbrace{\dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}}_{\vec{v}_{P,r}}) =$$
$$= \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} + \vec{\omega} \wedge \vec{v}_{P,r}$$

$$\vec{a}_P = \underbrace{\ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}}_{\text{RELATIVO}} + \underbrace{\vec{a}_O + \dot{\vec{\omega}} \wedge (P-O) + \vec{\omega} \wedge [\vec{\omega} \wedge (P-O)]}_{\text{ARRASTAMENTO}} + \underbrace{2\vec{\omega} \wedge \vec{v}_{P,r}}_{\text{COMPLEMENTAR}}$$

CORIOLIS

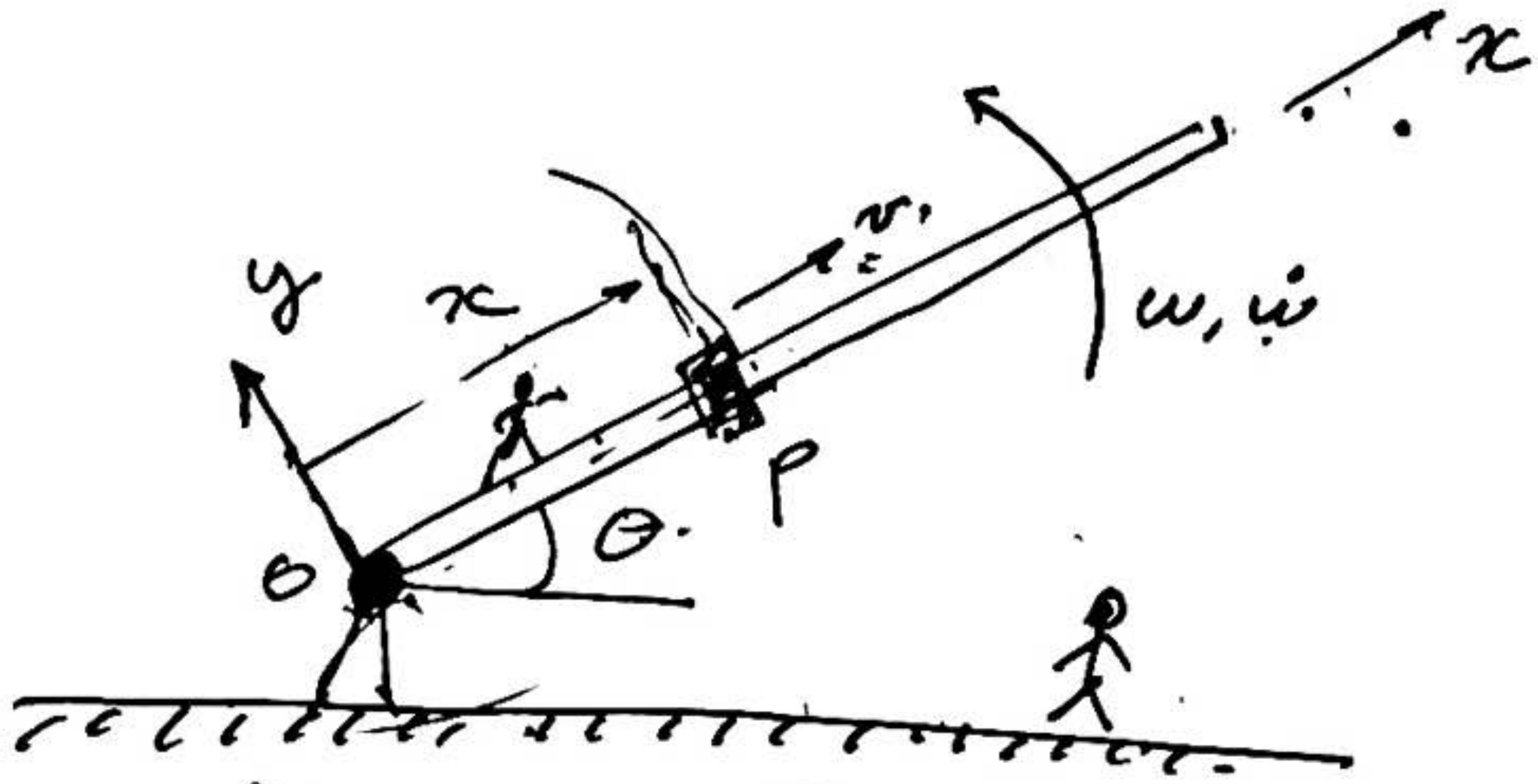
$$\boxed{\vec{a}_P = \vec{a}_{P,r} + \vec{a}_{P,a} + \vec{a}_{P,c}}$$

COMPLEMENTAR
CORIOLIS

ABSOLUTA

RELATIVA

ARRASTAMENTO



$\dot{x} = v = cte$

$\vec{\omega} = \dot{\theta}\vec{k}$

$\dot{\vec{\omega}} = \ddot{\theta}\vec{k}$

$\rightarrow \vec{v}_p = \vec{v}_{p,r} + \vec{v}_{p,a}$ $\qquad \vec{v}_{p,r} = \dot{x}\vec{i} = \underline{v\vec{i}}$

$\vec{v}_{p,a} = \vec{v}_{O,a} + \vec{\omega} \wedge (P-O) = \vec{0} + \dot{\theta}\vec{k} \wedge x\vec{i} = \underline{\dot{\theta}x\vec{j}}$

$\boxed{\vec{v}_p = v\vec{i} + \dot{\theta}x\vec{j}}$

$\rightarrow \vec{a}_p = \vec{a}_{p,r} + \vec{a}_{p,a} + \vec{a}_{p,c}$

$\vec{a}_{p,r} = \underline{\vec{0}}$

$\vec{a}_{p,a} = \vec{a}_{O,a} + \dot{\vec{\omega}} \wedge (P-O) + \vec{\omega} \wedge [\vec{\omega} \wedge (P-O)]$

$\vec{a}_{p,a} = \vec{0} + \ddot{\theta}\vec{k} \wedge x\vec{i} + \dot{\theta}\vec{k} \wedge [\underbrace{\dot{\theta}\vec{k} \wedge x\vec{i}}_{\vec{j}}]$

$\vec{a}_{p,a} = \underline{\ddot{\theta}x\vec{j} - \dot{\theta}^2 x\vec{i}}$

$\vec{a}_{p,c} = 2\vec{\omega} \wedge \vec{v}_{p,r} = 2\dot{\theta}\vec{k} \wedge v\vec{i} = \underline{2\dot{\theta}v\vec{j}}$

$\boxed{\vec{a}_p = -\dot{\theta}^2 x\vec{i} + (\ddot{\theta}x + 2\dot{\theta}v)\vec{j}}$

OUTRA POSSIBILIDADE

$$\vec{a}_{P.r} = \dot{\vec{v}}_{P.r} = \frac{d}{dt}(v\vec{i}) \Rightarrow$$ (NO MOVIMENTO RELATIVO)

$$\vec{a}_{P.r} = \vec{0}$$

$$\vec{a}_{P.a} = \dot{\vec{v}}_{P.a} = \frac{d}{dt}(\dot{\theta} x \vec{j}) \Rightarrow$$ (NO MOVIMENTO DE ARRASTAMENTO).

$$\frac{d}{dt}(\dot{\theta} x \vec{j}) = \ddot{\theta} x \vec{j} + \dot{\theta} x \dot{\vec{j}} = *$$

$$\dot{\vec{j}} = \vec{\omega} \wedge \vec{j} = \dot{\theta} \vec{k} \wedge \vec{j} = \underline{-\dot{\theta} \vec{i}}$$

$$* = \boxed{\ddot{\theta} x \vec{j} - \dot{\theta}^2 x \vec{i}}$$

$$\vec{a}_P = \dot{\vec{v}}_P = \frac{d}{dt}(v\vec{i} + \dot{\theta} x \vec{j}) \Rightarrow$$ (NO MOVIMENTO ABSOLUTO).

$$\frac{d}{dt}(v\vec{i} + \dot{\theta} x \vec{j}) = v\dot{\vec{i}} + \ddot{\theta} x \vec{j} + \dot{\theta} \dot{x} \vec{j} + \dot{\theta} x \dot{\vec{j}} = *$$

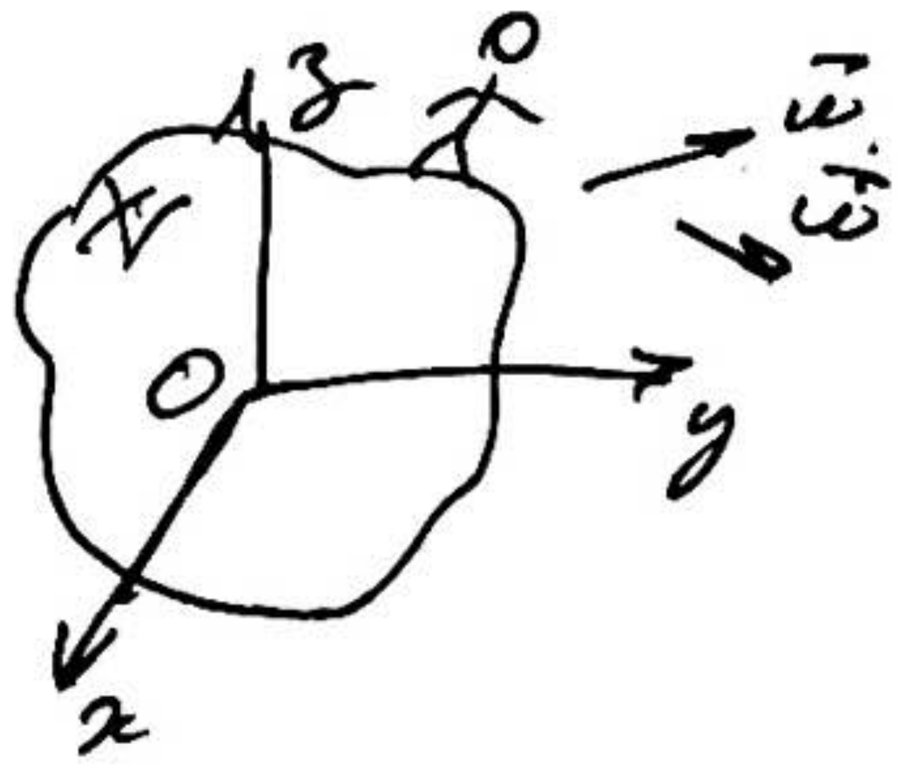
$$\dot{\vec{i}} = \vec{\omega} \wedge \vec{i} = \dot{\theta} \vec{k} \wedge \vec{i} = \underline{\dot{\theta} \vec{j}}$$

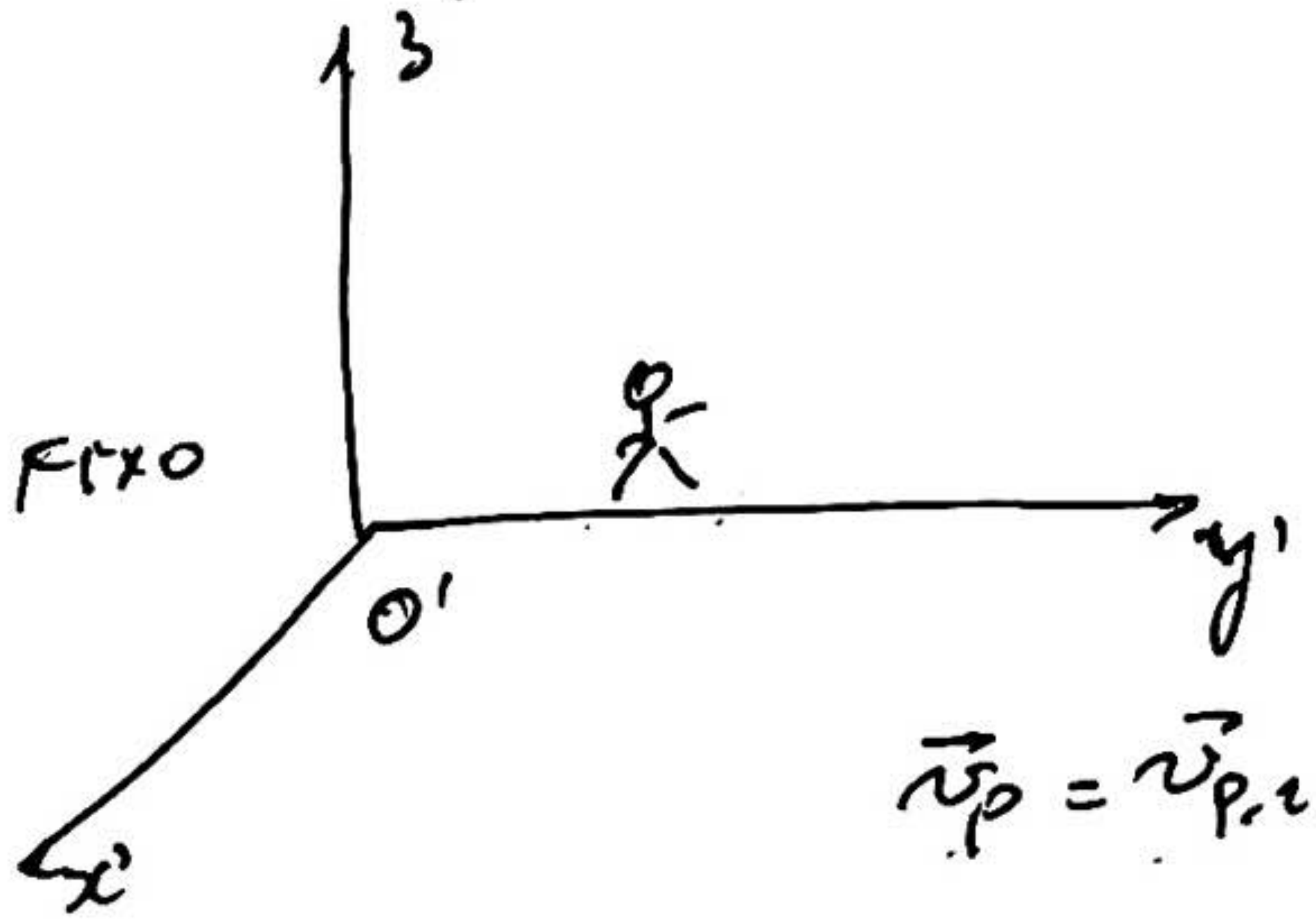
$$* = \dot{\theta} v \vec{j} + \ddot{\theta} x \vec{j} + \dot{\theta} \underbrace{\dot{x}}_{v} \vec{j} + \dot{\theta} x (-\dot{\theta} \vec{i}) =$$

$$= 2\dot{\theta} v \vec{j} + \ddot{\theta} x \vec{j} - \dot{\theta}^2 x \vec{i} =$$

$$= \boxed{-\dot{\theta}^2 x \vec{i} + (\ddot{\theta} x + 2v\dot{\theta})\vec{j}}$$

PARA CORPOS RÍGIDOS:





$$\vec{v}_P = \vec{v}_{P,r} + \vec{v}_{P,a}$$

$$\vec{v}_P = \vec{v}_A + \vec{\Omega} \wedge (P-A) \qquad \text{(ABSOLUTO)}$$

$$v_{P,r} = \vec{v}_{A,r} + \vec{\Omega}_r \wedge (P-A) \qquad \text{(RELATIVO)}$$

$$v_{P,a} = v_{A,a} + \vec{\omega} \wedge (P-A)$$

$$\vec{v}_A + \vec{\Omega} \wedge (P-A) = \vec{v}_{A,r} + \vec{\Omega}_r \wedge (P-A) + \vec{v}_{A,a} + \vec{\omega} \wedge (P-A)$$

$$\cancel{\vec{v}_A = \vec{v}_{A,r} + \vec{v}_{A,a}}$$

$$\underline{\vec{v}_A} + \vec{\Omega} \wedge (P-A) = \underline{\vec{v}_{A,r} + \vec{v}_{A,a}} + (\vec{\Omega}_r + \vec{\omega}) \wedge (P-A)$$

$$\Rightarrow \quad \vec{\Omega} \wedge (P-A) = (\vec{\Omega}_r + \vec{\omega}) \wedge (P-A) \rightarrow \boxed{\vec{\Omega} = \vec{\Omega}_r + \vec{\omega}}$$

ABSOL RELATIVO ARRAST.

$$\vec{\Omega}_r = \underline{\Omega_{rx}\vec{i} + \Omega_{ry}\vec{j} + \Omega_{rz}\vec{k}}$$

$$\dot{\vec{\Omega}} = \frac{d}{dt}(\vec{\Omega}_r + \vec{\omega}) = \dot{\vec{\omega}} + (\dot{\Omega}_{rx}\vec{i} + \dot{\Omega}_{ry}\vec{j} + \dot{\Omega}_{rz}\vec{k}) +$$

$$+ \Omega_{rx}\dot{\vec{i}} + \Omega_{ry}\dot{\vec{j}} + \Omega_{rz}\dot{\vec{k}} =$$

$$= \dot{\vec{\omega}} + \dot{\vec{\Omega}}_r + (\Omega_{rx}(\vec{\omega} \wedge \vec{i}) + \Omega_{ry}(\vec{\omega} \wedge \vec{j}) + \Omega_{rz}(\vec{\omega} \wedge \vec{k})) =$$

$$= \dot{\vec{\omega}} + \dot{\vec{\Omega}}_r + \vec{\omega} \wedge \underline{(\Omega_{rx}\vec{i} + \Omega_{ry}\vec{j} + \Omega_{rz}\vec{k})}$$

$$\rightarrow \boxed{\dot{\vec{\Omega}} = \dot{\vec{\omega}} + \dot{\vec{\Omega}}_r + \vec{\omega} \wedge \vec{\Omega}_r}$$

ARRAST RELATIVO COMPLEMENTAR

<u>REAL</u>.