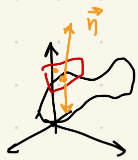
09/10 Parte 3

3.1 $f(x,y) = x\,(x^2+y^2)^{-3/2}\, e^{\operatorname{sen}(x^2 y)}$

$$\frac{\partial f}{\partial x}(1,0) = \dots = -2$$

3.2 Plano Tangente



a) $f(x,y) = y^2 - x^2$, $q = (-4, 5, 9)$

Plano
- Ponto : $(-4, 5, 9)$
- Vetor Normal : $\vec{n} = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1 \rangle = \langle +8, 10, -1 \rangle$

$$\frac{\partial f}{\partial x} = -2x \xrightarrow{q} -2\cdot -4 = +8$$

$$\frac{\partial f}{\partial y} = 2y \xrightarrow{q} 2\cdot 5 = 10$$

Equação do Plano:

$$\overbrace{+8x + 10y - 1z}^{\vec{n}\cdot\langle x,y,z\rangle} = \overbrace{+8\cdot -4 + 10\cdot 5 + -1\cdot 9}^{\vec{n}\cdot P}$$

$$+8x + 10y - z = -32 + 50 - 9$$

$$\boxed{+8x + 10y - z = 9}$$

Plano Tangente: $\frac{\partial f}{\partial x}(p)\,(x - x_p) + \frac{\partial f}{\partial y}(p)\,(y - y_p) = z - z_p$

(2) $f(x,y) = \sqrt{4 - x^2 - 2y^2}$, $q = (\overset{x}{1}, \overset{y}{-1}, \overset{z}{1})$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$

$\frac{\partial f}{\partial x} = \dots \overset{q}{=} -1$

Plano Tangente $\begin{cases} (1,-1,1) \\ \vec{n} = \langle -1, 2, -1 \rangle \end{cases}$

$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{4 - x^2 - 2y^2}} \cdot (-4y) =$

$\boxed{\tilde{\Pi} : x - 2y + z = 4}$

$= \frac{4}{2\sqrt{1}} = 2$]

3.1) $f(x,y) = x \cdot (x^2 + y^2)^{-3/2} \cdot e^{\operatorname{sen}(x^2 y)}$

$\frac{\partial f}{\partial x}(1,0) = \dots = -2$]

P1 2022

$u(x,y) = x^{1/2} y^{1/2}$

$f(x,y) = \sqrt{x^2 + y^2 + 1}$

$g(x,y) = x^2 - 4x - y^2 + 2y$

$h(x,y) = \frac{1}{2}\ln(x) + \frac{1}{2}\ln(y)$

3.4 (df_{Total} : derivada total) ?

(1) $f(t,\theta) = e^t \operatorname{sen}(\theta)$

Ex) $P = (\overset{x}{0}, \overset{y}{\frac{\pi}{3}})$ ↳ (60°)

(θ constante) $\dfrac{\partial f}{\partial t} = e^t \cdot \operatorname{sen}(\theta) \xrightarrow{P} \dfrac{\partial f}{\partial t}(P) = 1 \cdot \dfrac{\sqrt{3}}{2}$

(Se eu aumentar 1 unidade do t, o meu f aumenta aprox. $\sqrt{3}/2$)

Se $\Delta t = 1$: $\Delta f \approx \dfrac{\sqrt{3}}{2}$

$\Delta t = 2$: $\Delta f \approx \sqrt{3}$

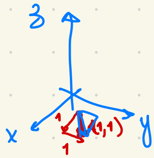
$\Delta t = 0{,}5$: $\Delta f \approx \dfrac{\sqrt{3}}{4}$

Δt : $\boxed{\Delta f \approx \dfrac{\partial f}{\partial t} \cdot \Delta t}$

(t constante) $\dfrac{\partial f}{\partial \theta} = e^t \cdot \cos(\theta)$

$$\boxed{\Delta f \approx \frac{\partial f}{\partial \theta} \cdot \Delta\theta}$$

$$\left(\frac{\Delta f}{\Delta \theta} \approx \frac{\partial f}{\partial \theta}\right)$$



$$\boxed{\Delta f \approx \frac{\partial f}{\partial \theta} \Delta\theta + \frac{\partial f}{\partial t} \cdot \Delta t}$$

$$\boxed{df = \frac{\partial f}{\partial \theta} \cdot d\theta + \frac{\partial f}{\partial t} \cdot dt}$$

Variação Total

df: variação muito pequena do f ($df = \Delta f$, $\Delta f \to 0$)

$d\theta$: $\Delta\theta \to 0$

dt: $\Delta t \to 0$

Resp) $\boxed{df = e^t \cos(\theta)\, d\theta + e^t \operatorname{sen}(\theta)\, dt}$

$$\boxed{(z - z_0) = \frac{\partial f(P)}{\partial x}(x - x_0) + \frac{\partial f(P)}{\partial y}(y - y_0)}$$

Plano Tangente

(x_0, y_0, z_0): Ponto de Tangência (P)