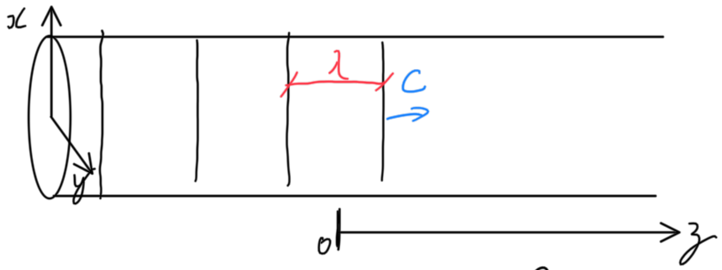
Aula 14 - Ondas em 3D, refração e reflexão



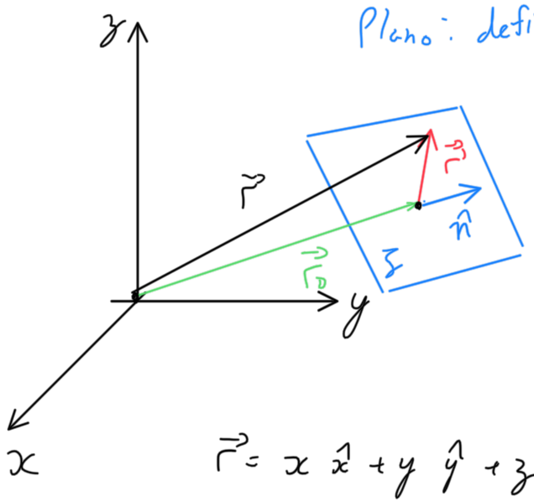
$k = \frac{2\pi}{\lambda}$

Onda senoidal: $p(z,t) = P\cos(kz - \omega t + \varphi)$

x, y? $= P\cos[k\underbrace{(z - vt)}_{z'(z,t) = z - vt} + \varphi]$, $\underline{k \cdot v = \omega}$

Onda plana no espaço?

Plano: definido por um versor normal



$\vec{r}' \cdot \hat{n} = 0 \Rightarrow \vec{r}'$ está no plano Σ

$\vec{r}_0 = r_0 \cdot \hat{n}$ (equivale a dizer que $\vec{r}_0 \times \hat{n} = 0$)

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$; $\vec{k} = k \cdot \hat{n}$

↳ vetor de onda

Produto escalar $\underbrace{\vec{r} \cdot \vec{k}}$ $= x \cdot k(\hat{x} \cdot \hat{n}) + y \cdot k(\hat{y} \cdot \hat{n}) + z \cdot k(\hat{z} \cdot \hat{n})$

Se a onda se propaga na direção $\hat{z} \to \hat{n} = \hat{z}$

$\vec{r} \cdot \vec{k} = k \cdot z$ → como antes!

$\vec{r} = \vec{r}_0 + \vec{r}'$; $\vec{r} \cdot \vec{k} = \vec{r}_0 \cdot \vec{k} + \underbrace{\vec{r}' \cdot \vec{k}}_{=0} = r_0 \cdot \hat{n} \cdot k \cdot \hat{n} = \underline{r_0 k}$

Onda Plana em 3 Dimensões

$$p(x,y,z,t) = p(\vec{r},t) = P_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \varphi)$$

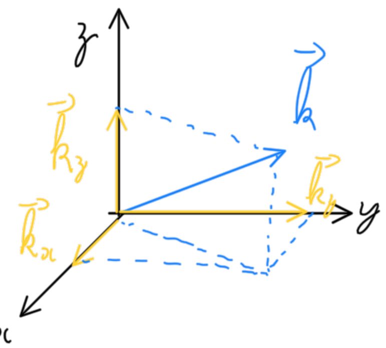
$$\rightarrow \vec{r} = x\cdot\hat{x} + y\cdot\hat{y} + z\cdot\hat{z}$$

$$\vec{r}\cdot\vec{k} = x\cdot\underbrace{k\cdot(\hat{x}\cdot\hat{n})} + y\cdot\underbrace{k\,(\hat{y}\cdot\hat{n})} + z\cdot\underbrace{k\,(\hat{z}\cdot\hat{n})}$$

$$= x\cdot k_x + y\cdot k_y + z\cdot k_z$$

$$\rightarrow \vec{k} = k\cdot\hat{n} =$$

$$= k_x\cdot\hat{x} + k_y\,\hat{y} + k_z\,\hat{z}$$



$$p(\vec{r},t) = P_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \varphi) = P_0 \cos(k_x\cdot x + k_y\cdot y + k_z\cdot z - \omega t + \varphi)$$

Equação de onda 3D

$$\frac{\partial^2}{\partial z^2} p(x,y,z,t) = -k_z^2\cdot P_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \varphi) = -k_z^2\, p(\vec{r},t)$$

$$\frac{\partial^2}{\partial x^2} p(\vec{r},t) = -k_x^2\, p(\vec{r},t) \quad ; \quad \frac{\partial^2}{\partial y^2} p(\vec{r},t) = -k_y^2\, p(\vec{r},t)$$

$$\frac{\partial^2}{\partial x^2} p + \frac{\partial^2}{\partial y^2} p + \frac{\partial^2}{\partial z^2} p = -(k_x^2 + k_y^2 + k_z^2)\, p$$

$$= -k^2 p \quad ; \quad |\vec{k}|^2 = \vec{k}\cdot\vec{k} = k^2$$

$$\frac{\partial^2}{\partial t^2} p = -\omega^2 p \qquad k\cdot v = \omega$$

$$= -v^2 k^2 p = v^2\cdot\left(\frac{\partial^2}{\partial x^2} p + \frac{\partial^2}{\partial y^2} p + \frac{\partial^2}{\partial z^2} p\right)$$

Equação de onda: $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)p(\vec{r},t) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}p(\vec{r},t)$

operador Laplaciano

$$\nabla^2 p(\vec{r},t) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}p(\vec{r},t)$$

Som, luz, onda de rádio, etc.

Intensidade: $I = \frac{\text{Potência}}{\text{Área}} \propto P_0^2$

$\therefore$ Potência da onda $\propto P_0^2 \cdot$ Área

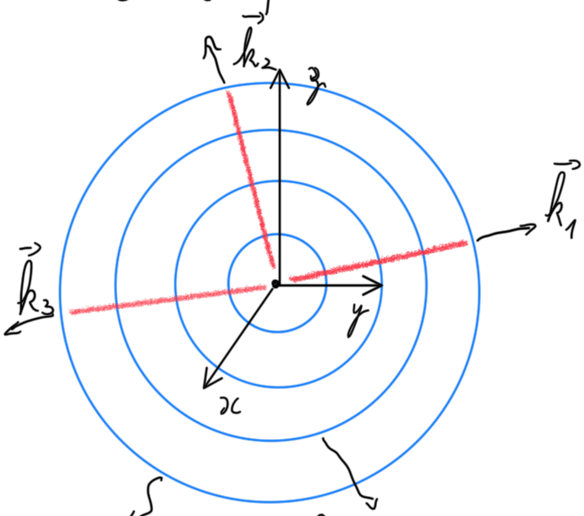
$$p(\vec{r},t) = P_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \varphi)$$

↳ por todo plano transverso

→ Potência diverge!

Potência finita!

Onda partindo de uma fonte simétrica: Onda Esférica



Potência 1 = Potência 2

$$|\vec{k}_n| = k = 2\pi/\lambda = \frac{2\pi}{v}\cdot T = \frac{\omega}{v}$$

$$\vec{k}(\vec{r}) = k\cdot\hat{r}$$

$$p(\vec{r},t) = P_0\cos(\vec{k}\cdot\vec{r} - \omega t + \varphi)$$

$$= P_0 \cos(k\cdot\underbrace{\hat{r}\cdot r\cdot\hat{r}}_{=1} - \omega t + \varphi)$$

$$= P_0(r)\cos(kr - \omega t + \varphi)$$

Conservação de energia

Potência

$$I = P_0^2(r) \cdot \kappa = \frac{P_0^2(r)}{2\rho_0 V} \Rightarrow W = \int_S I \cdot da = I \underbrace{\int_S da}_{\text{área da esfera de raio } r}$$

$$\int da = 4\pi \cdot r^2$$

$$W = \frac{P_0^2(r)}{2\rho_0 V} \cdot 4\pi r^2 = cte \Rightarrow P_0(r) = \sqrt{\frac{W}{2\pi\rho_0 V}} \cdot \frac{1}{r} = \frac{A}{r}$$

Onda esférica: $p(\vec{r},t) = \frac{A}{r}\cos(\vec{k}\cdot\vec{r} - \omega t + \varphi)$

$$r = \sqrt{x^2+y^2+z^2} \qquad = \frac{A}{\sqrt{x^2+y^2+z^2}}\cos(k_x x + k_y y + k_z z - \omega t + \varphi)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)p = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}p$$

$r \to 0$, $\frac{A}{r} \to \infty \to \nexists$ fonte pontual

$p(\vec{r},t) = \frac{A}{r}\cos(\vec{k}\cdot\vec{r} + \omega t + \varphi) \to$ onda convergindo

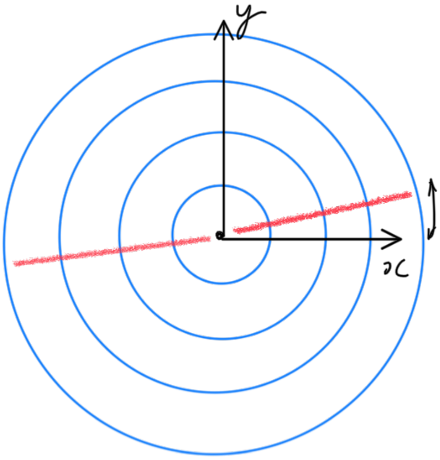
<u>alta intensidade</u> $\leftarrow \frac{A}{r}$ perturbação

<u>Sonoluminescência</u>: bolhas brilhantes excitadas acusticamente

↳ Funciona bem com onda longitudinal

↳ onda transversa é mais complicado

Ondas bidimensionais - ondas circulares



$$W = P^2 \kappa \cdot \underbrace{2\pi r}_{\text{Integral na linha}}$$

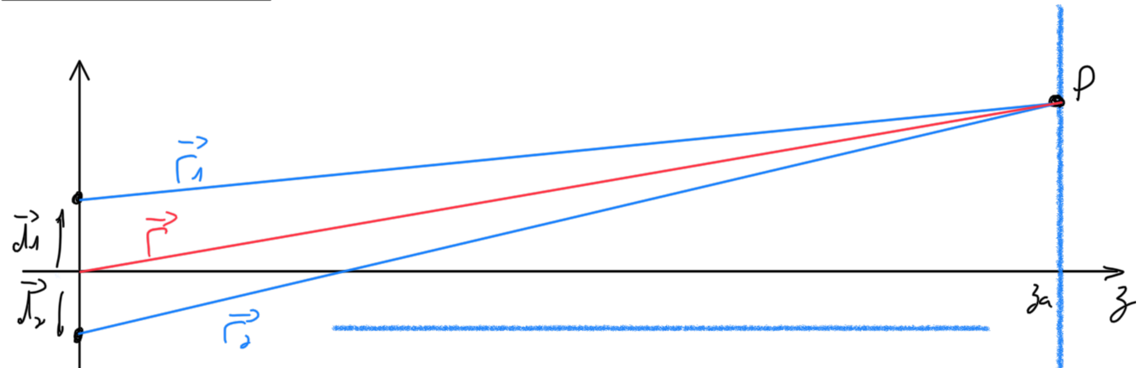
$$P = \frac{A}{\sqrt{r}}$$

$$p(\vec{r},t) = \frac{A}{\sqrt{r}} \cos(\vec{k} \cdot \vec{r} - \omega t + \varphi)$$

$$\vec{k} = k_x \cdot \hat{x} + k_y \cdot \hat{y} \quad ; \quad \vec{r} = x\,\hat{x} + y \cdot \hat{y}$$

Interferência e difração

Interferência



$$p_1(\vec{r},t) = \frac{A_1}{\sqrt{r_1}} \cos(\vec{k}_1 \cdot \vec{r}_1 - \omega t + \varphi_1)$$

$$p_2(\vec{r},t) = \frac{A_2}{\sqrt{r_2}} \cos(\vec{k}_2 \cdot \vec{r}_2 - \omega t + \varphi_2)$$

$$\vec{r}_1 = \vec{r} - \vec{d}_1 \; , \; \vec{r}_2 = \vec{r} - \vec{d}_2 \qquad z_a >> |\vec{d}_1|, |\vec{d}_2|$$

$$; \; \vec{r}_2 = \vec{r} + \vec{d}_1 \qquad \vec{d}_1 = -\vec{d}_2$$

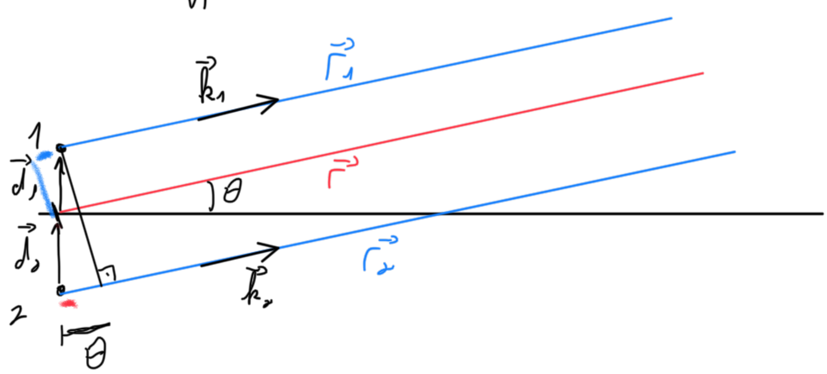
$$\sqrt{r_1} \simeq \sqrt{r}$$

$$\Rightarrow \frac{A_1}{\sqrt{r_1}} \sim \frac{A_1}{\sqrt{r}} \; ; \; \frac{A_2}{\sqrt{r_2}} \sim \frac{A_2}{\sqrt{r}}$$

$$A_1 = A_2 = A$$

$$p(\vec{r},t) = p_1(\vec{r},t) + p_2(\vec{r},t) = \frac{A}{\sqrt{r}} \left[\cos(\vec{k}_1 \cdot \vec{r}_1 - \omega t + \varphi_1) + \cos(\vec{k}_2 \cdot \vec{r}_2 - \omega t + \varphi_2)\right]$$

$\vec{k}_1 \cdot \vec{r}_1$?



$$r_1 \sim r - d \,\text{sen}\theta \qquad r_2 \sim r + d\,\text{sen}\theta$$

$$\left[\cos(\vec{k}_1\cdot\vec{r}_1 - \omega t + \varphi_1) + \cos(\vec{k}_2\cdot\vec{r}_2 - \omega t + \varphi_2)\right] = \mathrm{Re}\left[e^{i k_1 \cdot r_1}\cdot e^{-i\omega t} e^{i\varphi_1} + e^{i k_2 r_2} e^{-i\omega t} e^{i\varphi_2}\right]$$

$$= \mathrm{Re}\left[e^{-i\omega t} e^{i\varphi_2}\left[e^{i k_1 r_1} e^{i(\varphi_1-\varphi_2)} + e^{i k_2 r_2}\right]\right]$$

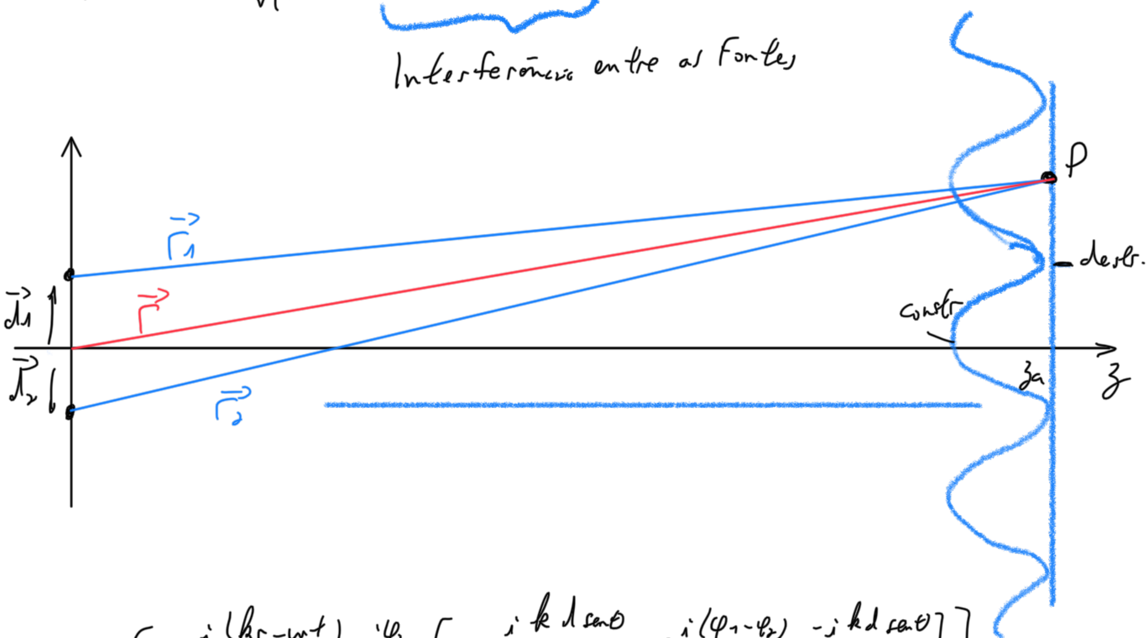
$$= \mathrm{Re}\left[e^{-i\omega t} e^{i\varphi_2}\left[e^{i kr} e^{-i k d \operatorname{sen}\theta} e^{i(\varphi_1-\varphi_2)} + e^{i kr} e^{i k d \operatorname{sen}\theta}\right]\right]$$

$$= \mathrm{Re}\left[e^{i(kr-\omega t)} e^{i\varphi_2}\left[e^{i k d \operatorname{sen}\theta} + e^{i(\varphi_1-\varphi_2)} e^{-i k d \operatorname{sen}\theta}\right]\right]$$

Fontes en fase: $\varphi_1 = \varphi_2 \qquad e^{i(\varphi_1-\varphi_2)} = 1$

$$e^{i k d \operatorname{sen}\theta} + e^{-i k d \operatorname{sen}\theta} = 2\cos(k d \operatorname{sen}\theta)$$

$$p(\vec{r}, t) = \frac{A}{\sqrt{2}}\cdot 2\underbrace{\cos(k d \operatorname{sen}\theta)}_{\text{Interferência entre as Fontes}}\cos(kr - \omega t + \varphi_2)$$



$$= \mathrm{Re}\left[e^{i(kr-\omega t)} e^{i\varphi_2}\left[e^{i k d \operatorname{sen}\theta} + e^{i(\varphi_1-\varphi_2)} e^{-i k d \operatorname{sen}\theta}\right]\right]$$

Fontes em oposição de fase: $\varphi_1 = \varphi_2 + \pi \qquad e^{i(\varphi_1-\varphi_2)} = -1$

$$e^{i k d \operatorname{sen}\theta} - e^{-i k d \operatorname{sen}\theta} = 2i\operatorname{sen}(k d \operatorname{sen}\theta)$$

$$p(\vec{r}, t) = \frac{A}{\sqrt{2}}\cdot 2\underbrace{\operatorname{sen}(k d \operatorname{sen}\theta)}_{\text{Interferência destrutiva entre as Fontes para } \theta=0}\operatorname{sen}(kr - \omega t + \varphi_2)$$