7600026 – Laboratório de Óptica

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Indice de Refração

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CLASSICAL ELETROMAGNETISM

In uniform isotropic linear media, the wave equation is:

$$\nabla^2 \mathbf{E} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\nabla^2 \mathbf{H} - \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$$

They are satisfied by plane wave

$\psi = A\, e^{i(\mathbf{k}\,\mathbf{r} - \omega t)}$

$$k = |\mathbf{k}| = \omega\sqrt{\mu\varepsilon}$$

ψ can be any Cartesian components of **E** and **H**

The phase velocity of plane wave travels in the direction of **k** is

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\nabla\times$

We can define the index of refraction as

$$n = \frac{v}{c} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

Most media are nonmagnetic and have a magnetic permeability $\mu=\mu_0$, in this case

$$n = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

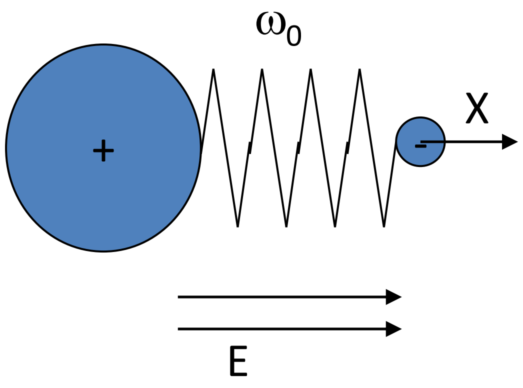
In most media, n is a function of frequency.

Classical Electron Model (Lorentz Model)

Let the electric field of optical wave in an atom be

$$\mathbf{E}=\mathbf{E_0}e^{-i\omega t}$$

the electron obeys the following equation of motion



$$m\frac{d^2}{dt^2}\mathbf{X}+m\gamma\frac{d}{dt}\mathbf{X}+m\omega_0{}^2\mathbf{X}=-e\mathbf{E}$$

X is the position of the electron relative to the atom

m is the mass of the electron

ω_0 is the resonant frequency of the electron motion

γ is the damping coefficient

The solution is

$$\mathbf{X} = \frac{-e\mathbf{E_0}}{m(\omega_0{}^2 - \omega^2 - i\omega\gamma)} e^{-i\omega t}$$

The induced dipole moment is

$$\mathbf{p} = -e\mathbf{X} = \frac{e^2}{m(\omega_0{}^2 - \omega^2 - i\omega\gamma)} \mathbf{E} = \alpha \mathbf{E}$$

α is atomic polarizability

$$\alpha = \frac{e^2}{m(\omega_0{}^2 - \omega^2 - i\omega\gamma)}$$

The dielectric constant of a medium depends on the manner in which the atoms are assembled. Let N be the number of atoms per unit volume. Then the polarization can be written approximately as

$$\mathbf{P} = N\,\mathbf{p} = N\,\alpha\,\mathbf{E} = \varepsilon_0\,\chi\,\mathbf{E}$$

The dielectric constant of the medium is given by

$$\varepsilon = \varepsilon_0 (1+\chi) = \varepsilon_0 (1+N\alpha/ \varepsilon_0)$$

If the medium is nonmagnetic, the index of refraction is

$$n= (\varepsilon /\varepsilon_0)^{1/2} = (1+N\alpha/ \varepsilon_0)^{1/2}$$

$$n^2 = \frac{\varepsilon}{\varepsilon_0} = 1 + \frac{Ne^2}{\varepsilon_0 m({\omega_0}^2 - \omega^2 - i\omega\gamma)}$$

If the second term is small enough then

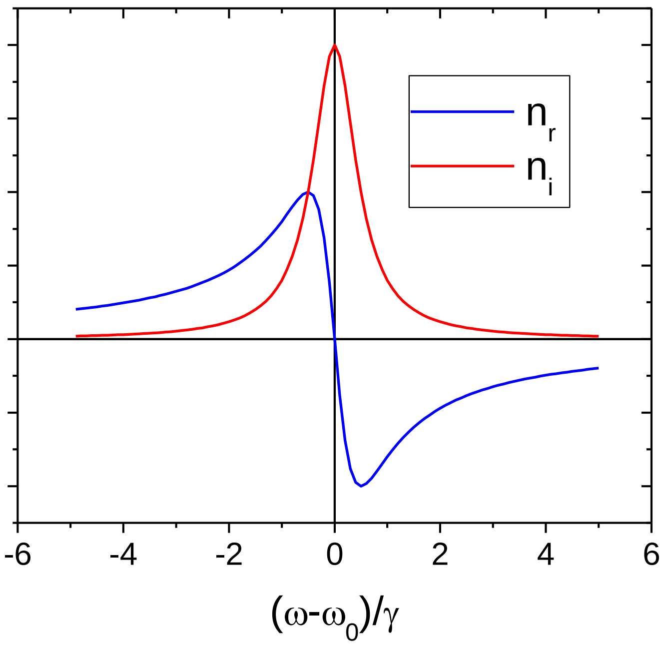
$$n = 1 + \frac{Ne^2}{2\varepsilon_0 m({\omega_0}^2 - \omega^2 - i\omega\gamma)}$$

The complex refractive index is

$$n \rightarrow n_r + in_i = 1 + \frac{Ne^2(\omega_0{}^2 - \omega^2)}{2\varepsilon_0 m[(\omega_0{}^2 - \omega^2)^2 + \omega^2\gamma^2]} + i\frac{Ne^2\gamma\omega}{2\varepsilon_0 m[(\omega_0{}^2 - \omega^2)^2 + \omega^2\gamma^2]}$$

at $\omega \sim \omega_0$,

$$n_r + in_i = 1 + \frac{Ne^2(\omega_0 - \omega)}{4\varepsilon_0 m\omega_0[(\omega_0 - \omega)^2 + (\gamma/2)^2]} + i\frac{Ne^2\gamma}{8\varepsilon_0 m\omega_0[(\omega_0 - \omega)^2 + (\gamma/2)^2]}$$



Normalized plot of n-1 and k versus $\omega - \omega_0$

For more than one resonance frequencies for each atom,

$$n^2 = 1 + \frac{Ne^2}{\varepsilon_0 m} \sum_j \frac{f_j}{(\omega_j{}^2 - \omega^2 - i\omega\gamma_j)} \qquad \sum_j f_j = Z$$

Classical Electron Model (Drude model)

If we set ω_0=0, the Lorentz model become Drude model. This model can be used in free electron metals

$$n^2 = 1 - \frac{Ne^2}{\varepsilon_0 m(\omega^2 + i\omega\gamma)}$$

Relation Between Dielectric Constant and Refractive Index

By definition,

$$n^2 = \frac{\varepsilon}{\varepsilon_0}$$

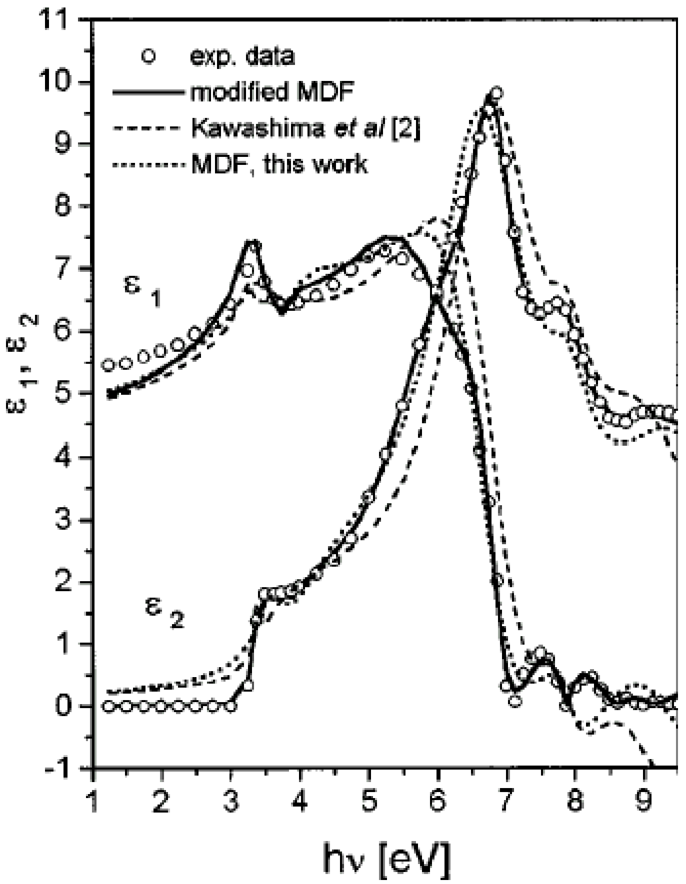
$$n = n_r + i n_i$$

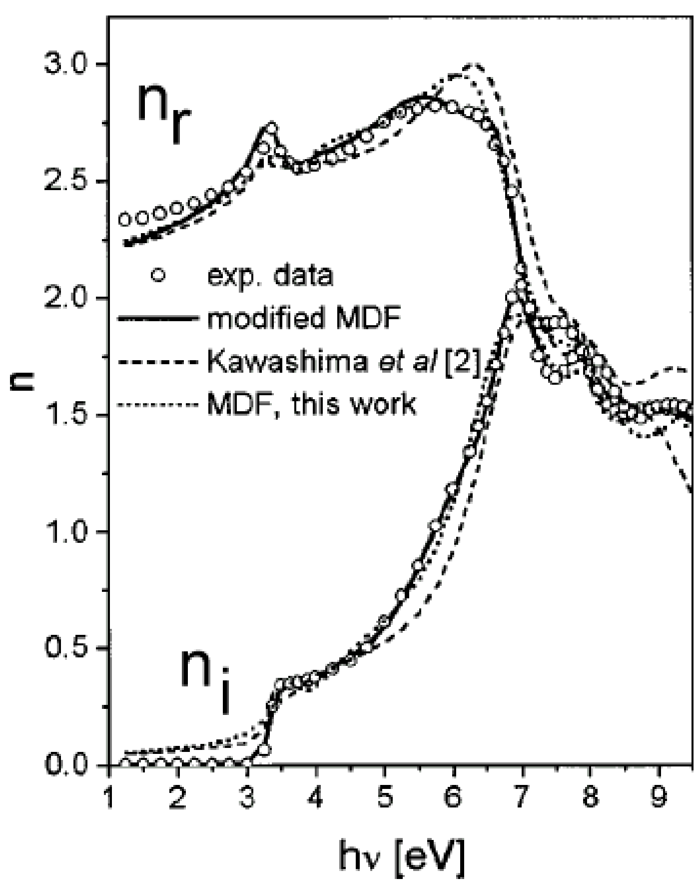
$$\varepsilon = \varepsilon_1 + i\varepsilon_2$$

We can easily get:

$$n_r = \{\frac{1}{2}[(\varepsilon_1{}^2 + \varepsilon_2{}^2)^{1/2} + \varepsilon_1]\}^{1/2} / \varepsilon_0$$

$$n_i = \{\frac{1}{2}[(\varepsilon_1{}^2 + \varepsilon_2{}^2)^{1/2} - \varepsilon_1]\}^{1/2} / \varepsilon_0$$





$$\epsilon(E)=\sum_{\alpha=A,B,C}\left(\sum_{n=1}^{\infty}\frac{A_{0\alpha}^{\mathrm{ex}}}{n^3}\frac{1}{E_{0\alpha}-(G_{0\alpha}^{\mathrm{3D}}/n^2)-E-i\Gamma}\right)$$

Real and imaginary part of the index of refraction of GaN vs. energy;

The real part and imaginary part of the complex dielectric function ε (ω) are not independent. they can connected by Kramers-Kronig relations:

$$\varepsilon_1(\omega) = \varepsilon_0 + \frac{2}{\pi} P \int_0^{\infty} \frac{\varepsilon_2(\omega')\omega'}{\omega'^2 - \omega^2} d\omega'$$

$$\varepsilon_2(\omega) = \frac{2\omega}{\pi} P \int_0^{\infty} \frac{\varepsilon_1(\omega') - \varepsilon_0}{\omega'^2 - \omega^2} d\omega'$$

P indicates that the integral is a principal value integral.

K-K relation can also be written in other form, like

$$n(\lambda) = \frac{1}{\pi} P \int_0^{\infty} \frac{\alpha(\lambda')}{1 - (\lambda'/\lambda)^2} d\lambda'$$

$$E(z,t) = E_0 e^{i(k\cdot z - \omega t)}, \qquad k = (n + i\kappa)\omega / c$$
$$= E_0\, e^{-\alpha z} e^{i(\omega n z/c - \omega t)}.$$

Two propagation parameters:
n, α

Model:
Light: electromagnetic wave
Atom and molecule: classical dipole oscillator
$n(\omega)$, $\alpha(\omega)$

Propagation of light in a dense optical medium

Three types of oscillators:
1. bound electron (atomic) oscillator
2. vibrational oscillator;
3. free electron oscillators

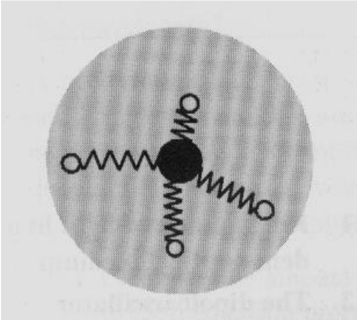
Atomic oscillators

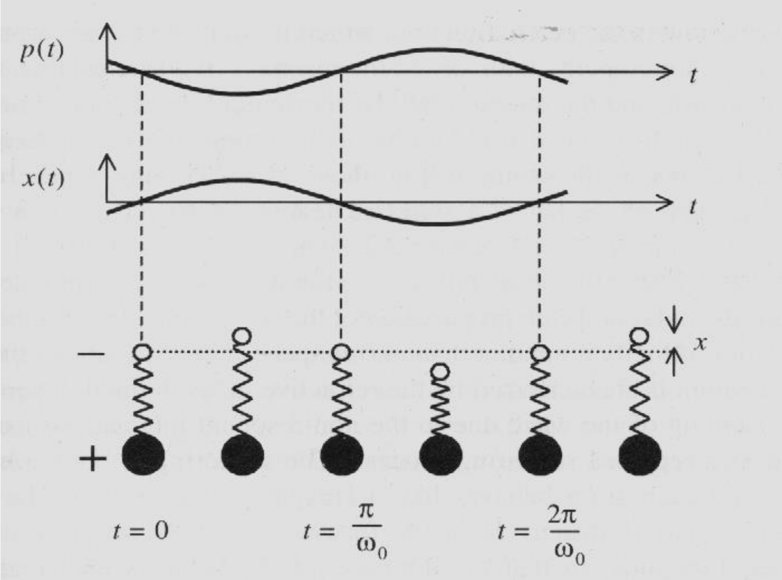
$$\frac{1}{\mu} = \frac{1}{m_0} + \frac{1}{m_N} \approx \frac{1}{m_0},$$

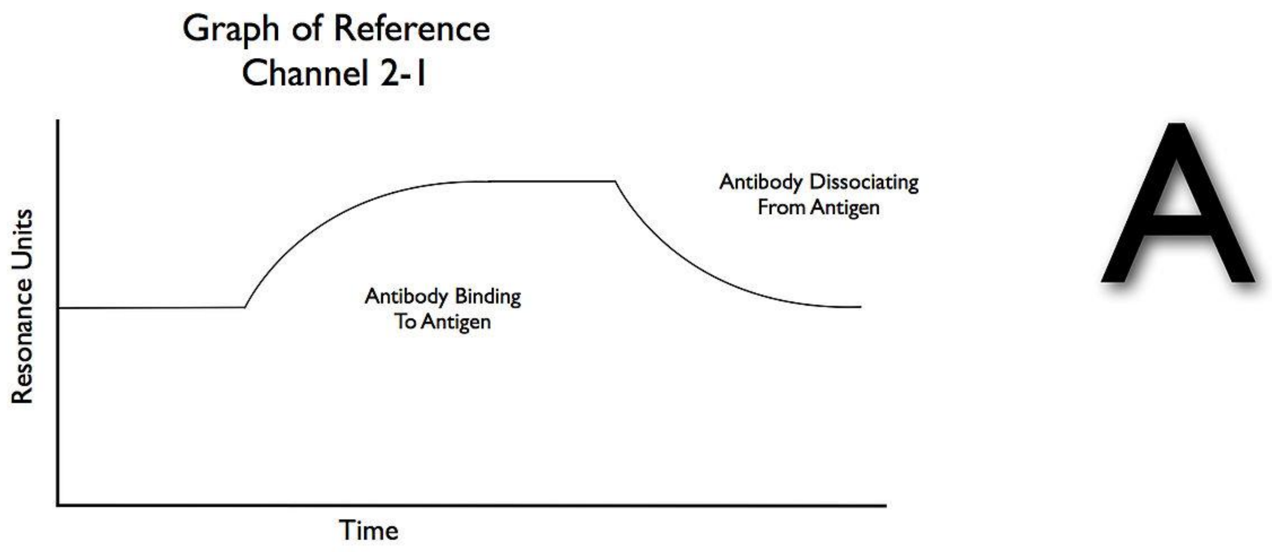
$$\omega_0 = \sqrt{\frac{K_S}{\mu}} = 10^{14} \sim 10^{15},$$

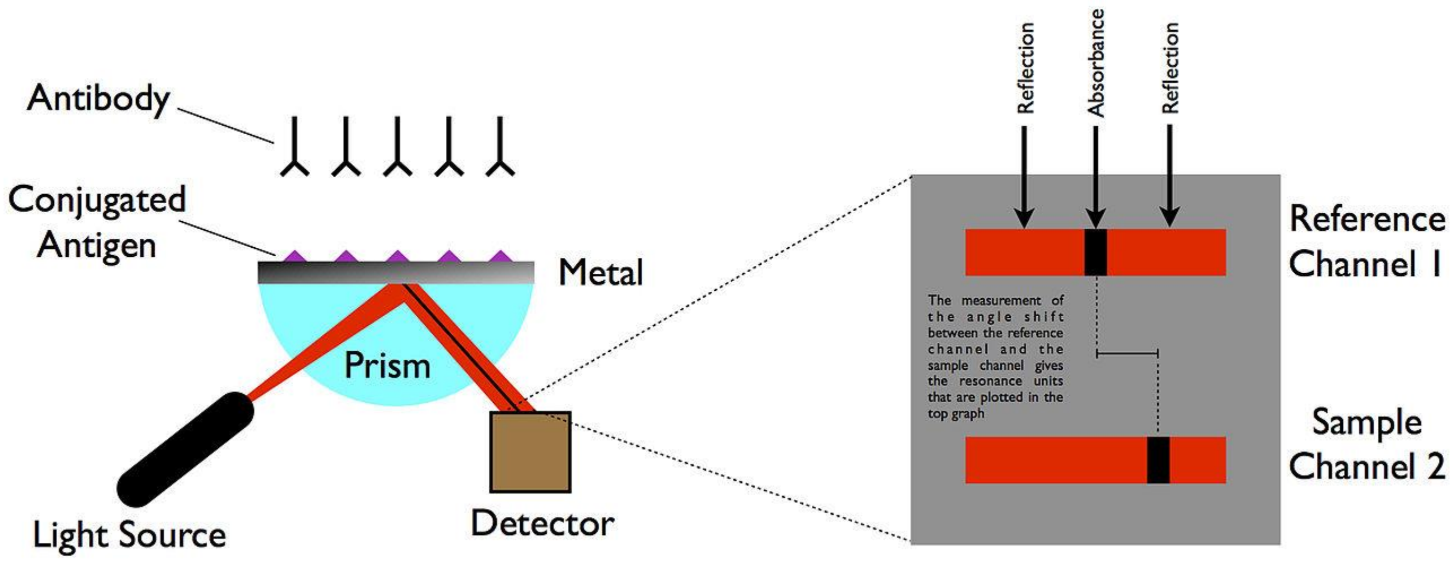
$$\vec{p} = q(\vec{r}_+ - \vec{r}_-)$$

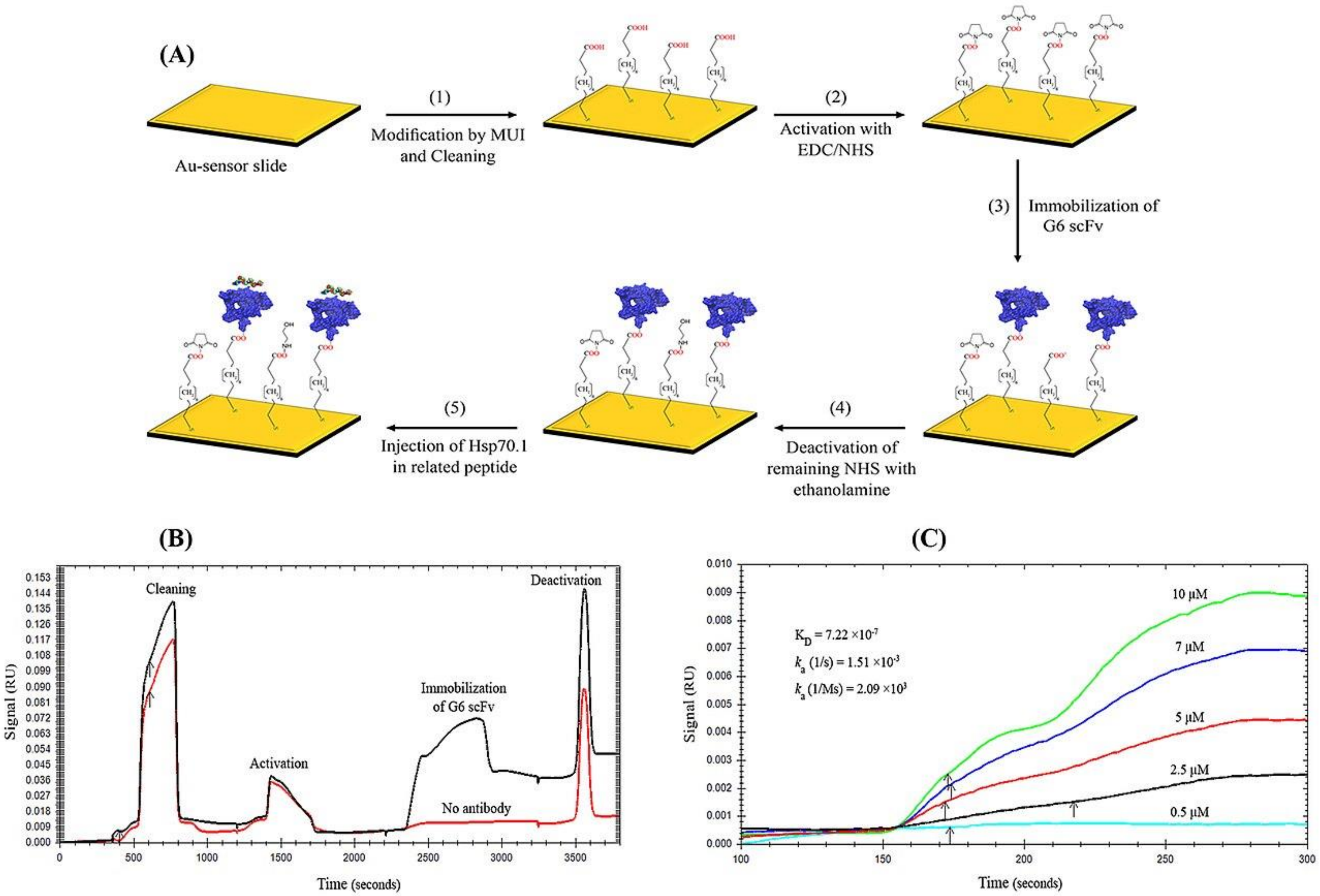
$$p(t) = -ex(t)$$






Graph of Reference
Channel 2-1
Resonance Units
Time
Antibody Binding
To Antigen
Antibody Dissociating
From Antigen
A


Antibody
Conjugated
Antigen
Metal
Prism
Light Source
Detector
Reflection
Absorbance
Reflection
Reference
Channel 1
Sample
Channel 2
The measurement of the angle shift between the reference channel and the sample channel gives the resonance units that are plotted in the top graph


(A)
(1)
Modification by MUI and Cleaning
Au-sensor slide
(2)
Activation with EDC/NHS
(3) Immobilization of G6 scFv
(4)
Deactivation of remaining NHS with ethanolamine
(5)
Injection of Hsp70.1 in related peptide
(B)
Cleaning
Activation
Immobilization of G6 scFv
Deactivation
No antibody
Signal (RU)
Time (seconds)
(C)
$K_D = 7.22 \times 10^{-7}$
k_a (1/s) = 1.51×10^{-3}
k_a (1/Ms) = 2.09×10^{3}
10 µM
7 µM
5 µM
2.5 µM
0.5 µM
Signal (RU)
Time (seconds)