

# Eletrromagnetismo Avançado

*2 de outubro*  
*Relatividade restrita*

# Transformação de Lorentz

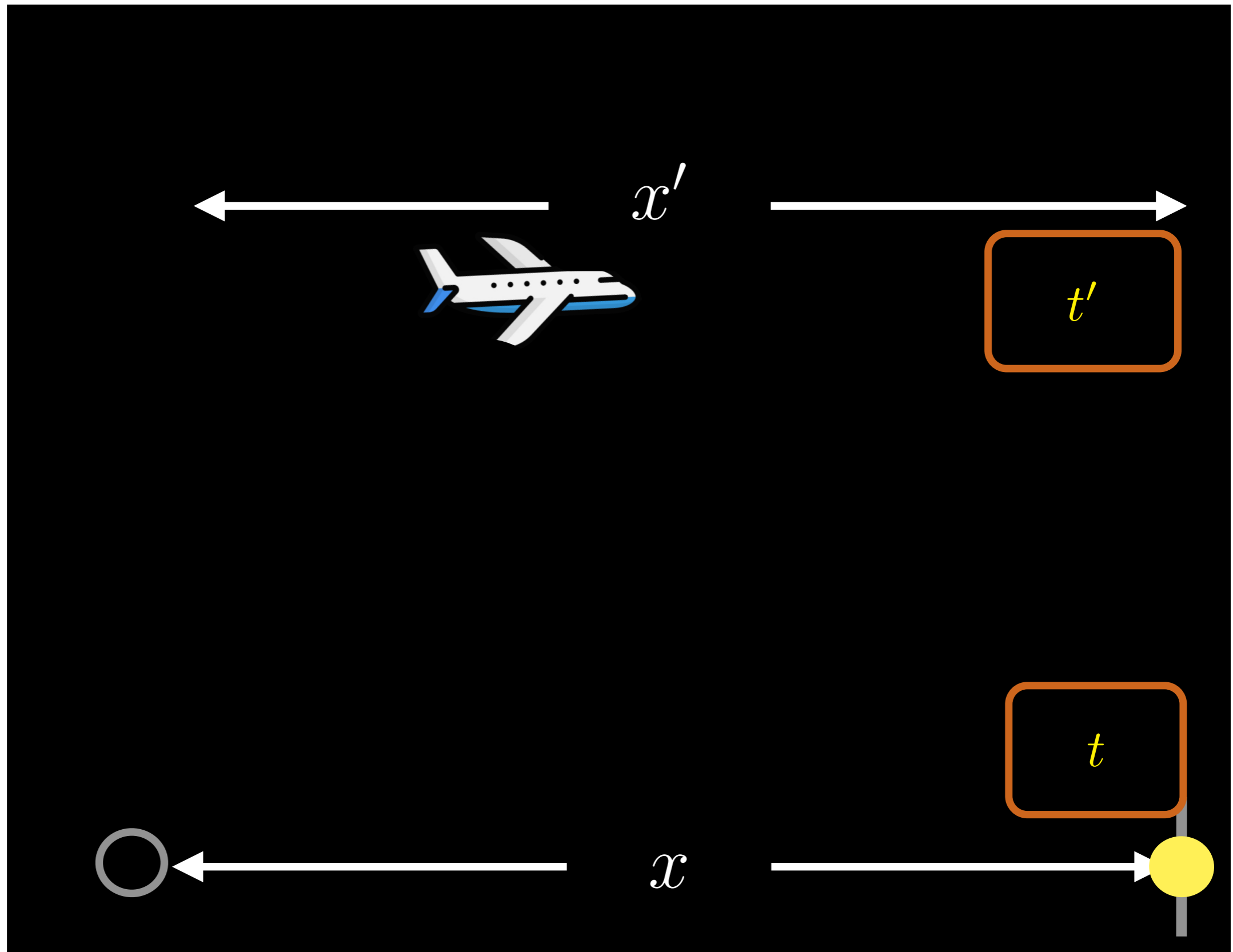
$$t' = 0$$

 $x'$ 

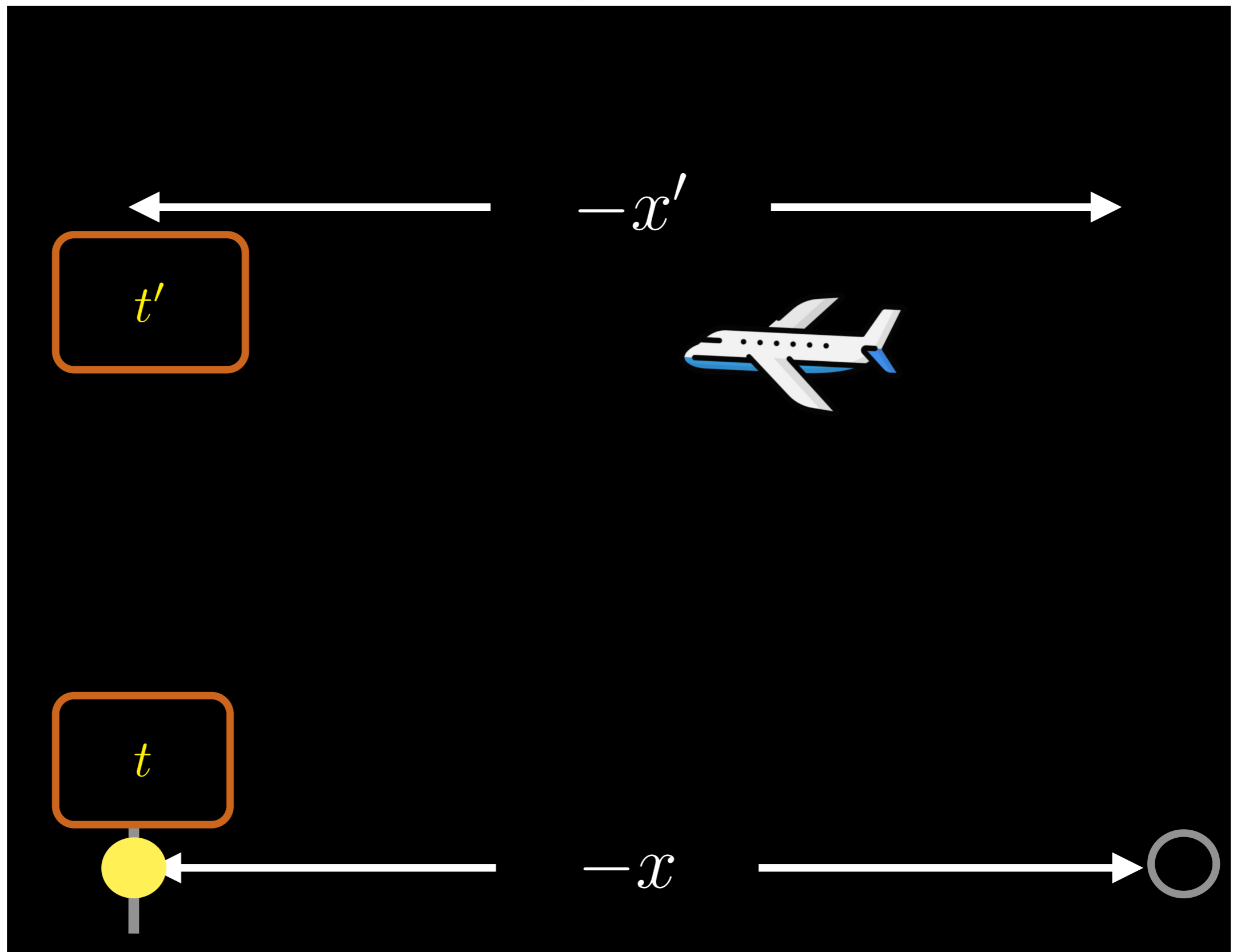
$$t = 0$$

 $x$ 

# Transformação de Lorentz

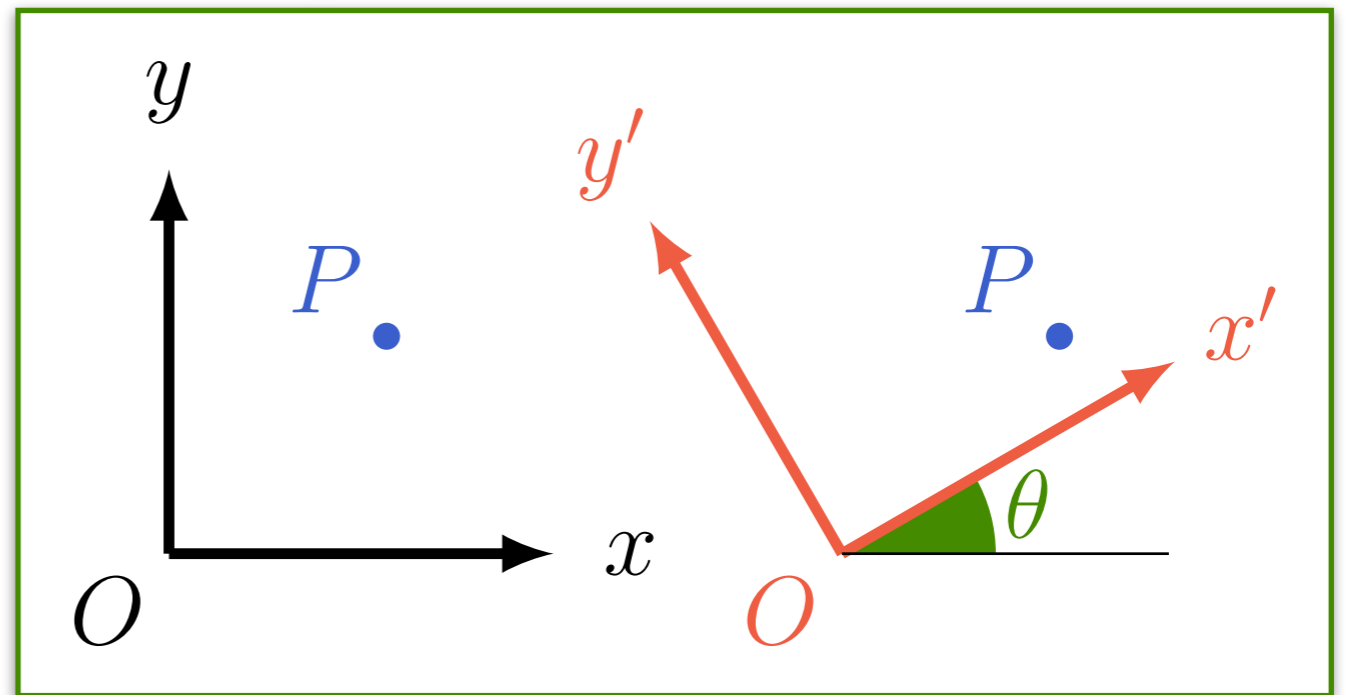


# Transformação de Lorentz



# Transformação de Lorentz

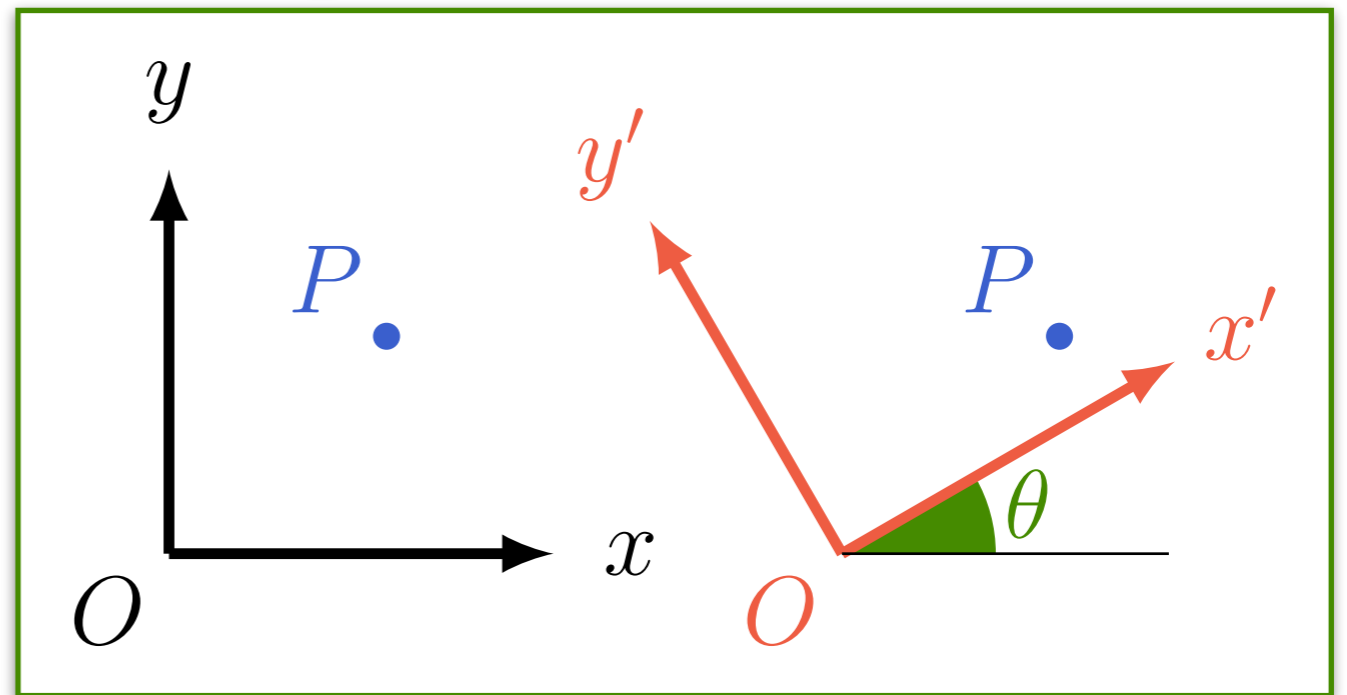
Mudança  
de coordenadas:  
Rotação espacial



$$\begin{bmatrix} y' \\ x' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix}$$

# Transformação de Lorentz

Mudança  
de coordenadas:  
Rotação espacial

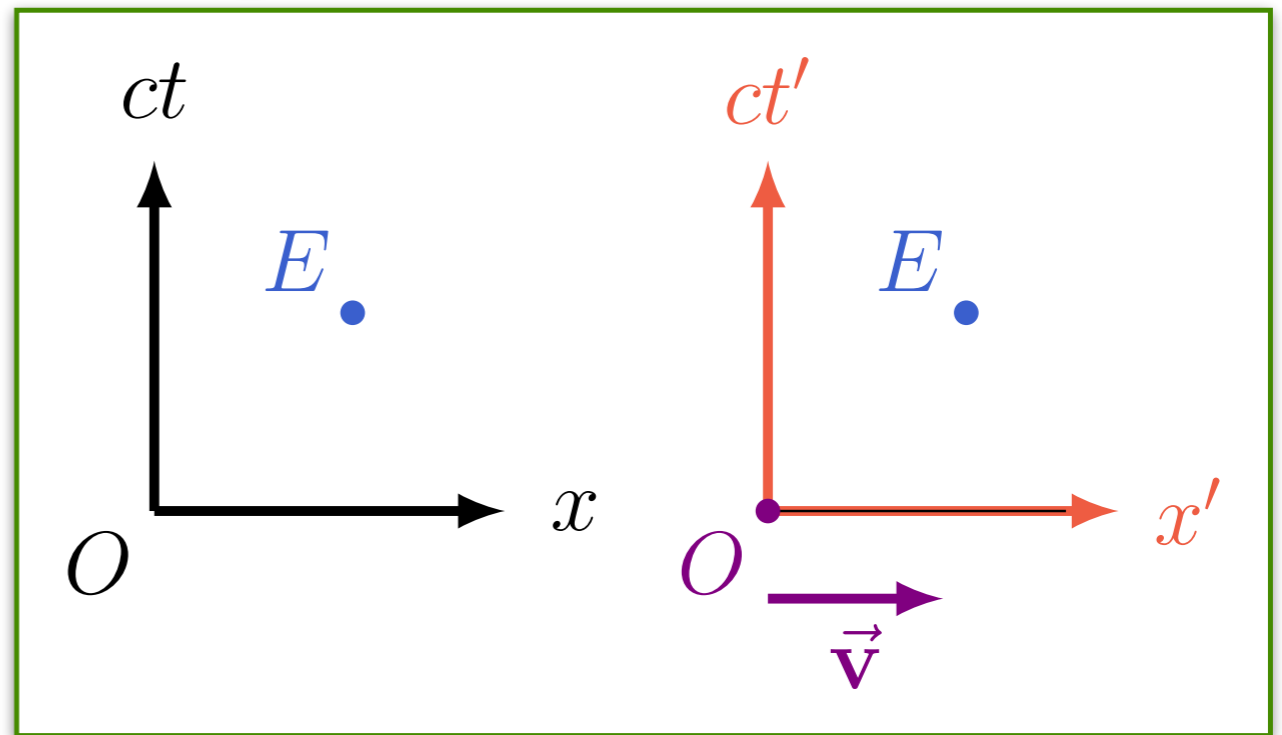


$$\begin{bmatrix} y' \\ x' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix}$$

$$x'^2 + y'^2 = x^2 + y^2$$

# Transformação de Lorentz

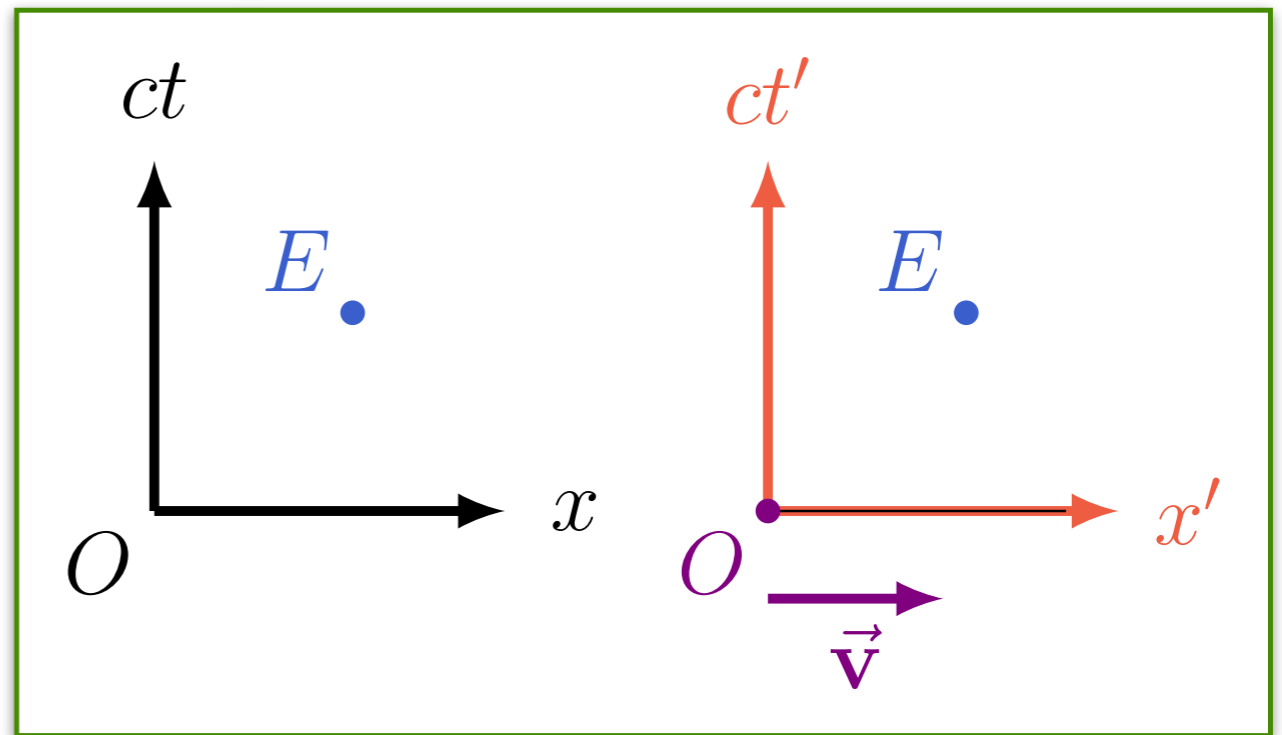
Mudança  
de coordenadas:  
Rotação  
espaço-temporal



$$? \begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix} ?$$

# Transformação de Lorentz

Mudança  
de coordenadas:  
Rotação  
espaço-temporal



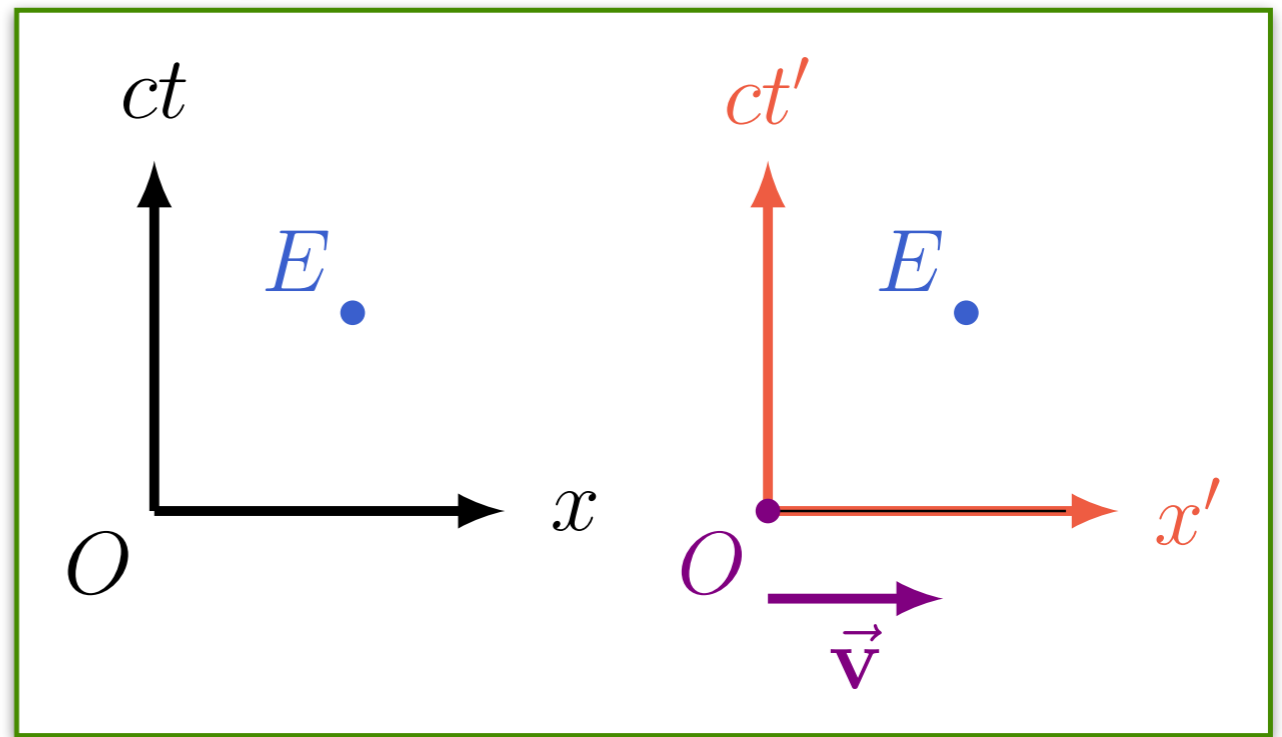
$$? \begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix} ?$$

$$\Rightarrow c^2 t'^2 + x'^2 = c^2 t^2 + x^2 \quad \text{😞}$$



# Transformação de Lorentz

Mudança  
de coordenadas:  
Rotação  
espaço-temporal

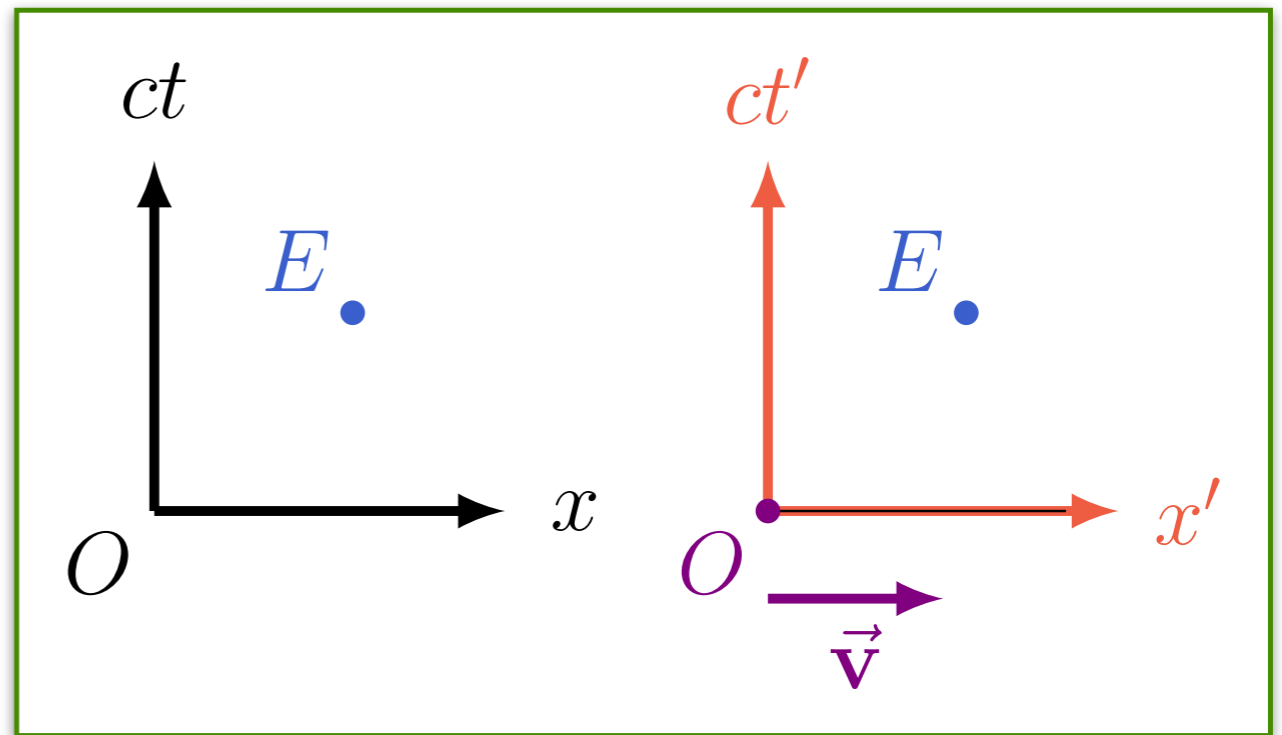


$$\begin{bmatrix} ct \\ x \end{bmatrix} \rightarrow \begin{bmatrix} ict \\ x \end{bmatrix}$$

$$c^2 t'^2 + x'^2 = c^2 t^2 + x^2 \quad \rightarrow \quad -c^2 t'^2 + x'^2 = -c^2 t^2 + x^2$$

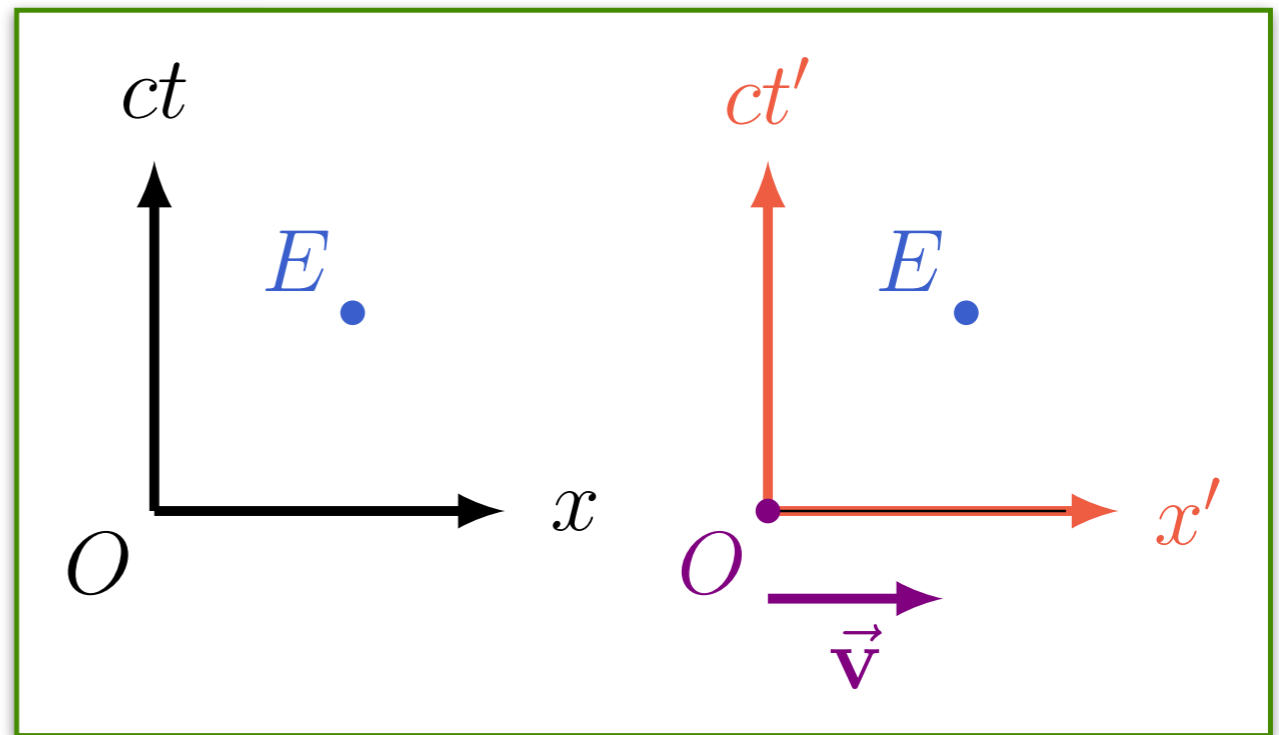


# Transformação de Lorentz



$$\begin{bmatrix} ict' \\ x' \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} ict \\ x \end{bmatrix}$$

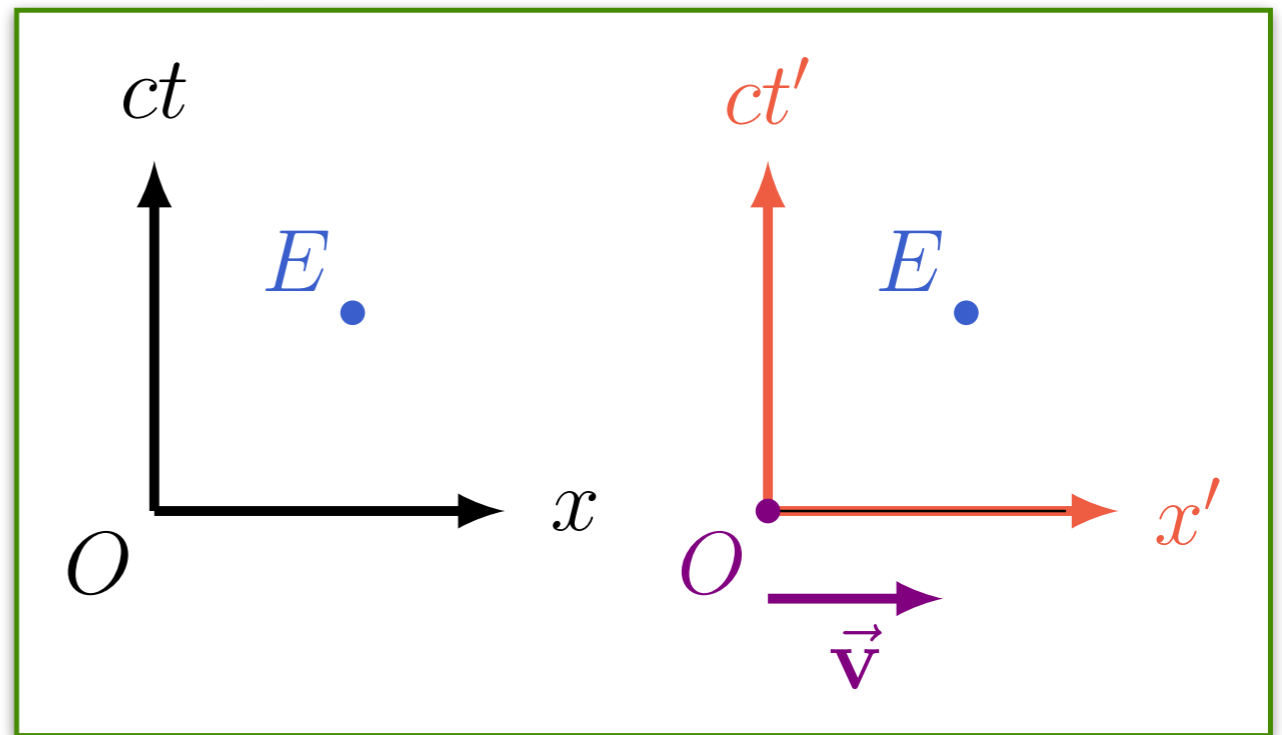
# Transformação de Lorentz



$$\begin{bmatrix} ict' \\ x' \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} ict \\ x \end{bmatrix}$$

$$x = vt \Rightarrow x' = 0$$

# Transformação de Lorentz



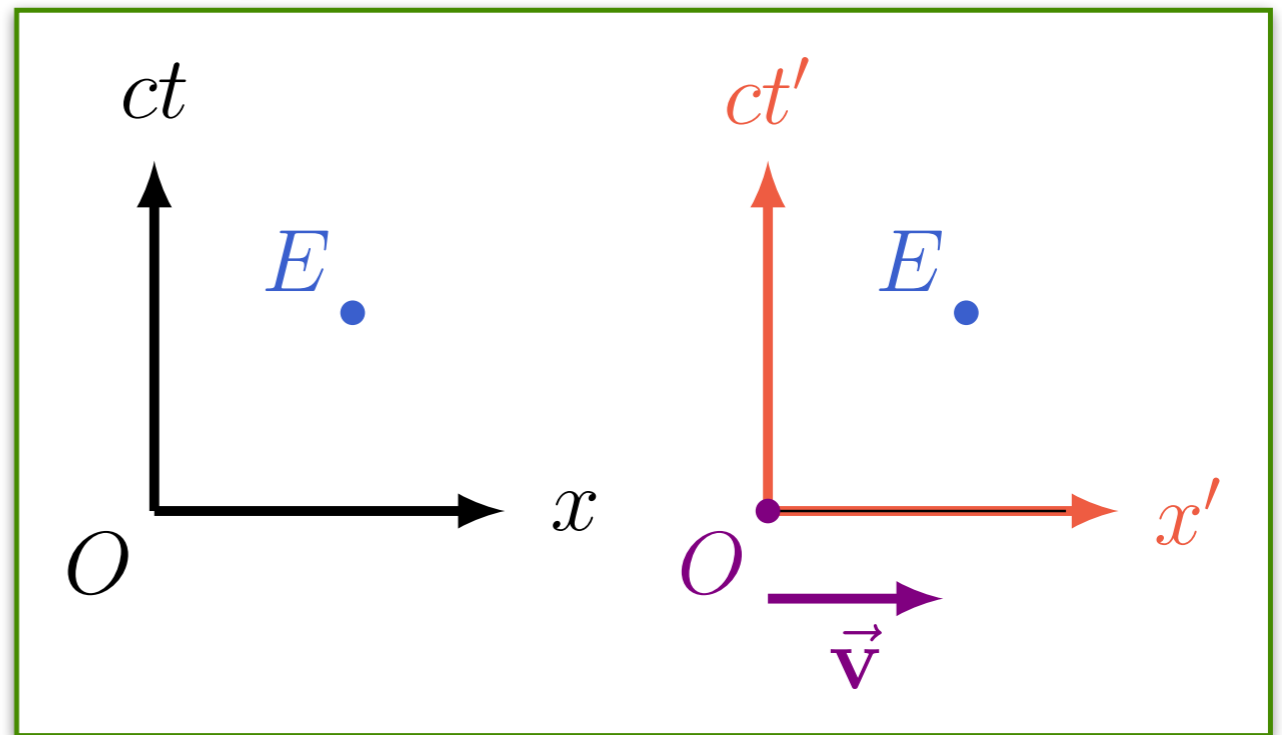
$$\begin{bmatrix} ict' \\ x' \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} ict \\ x \end{bmatrix}$$

$$x = vt \Rightarrow x' = 0$$

$$ict' = ict \cos \alpha - vt \sin \alpha$$

$$0 = ict \sin \alpha + vt \cos \alpha$$

# Transformação de Lorentz



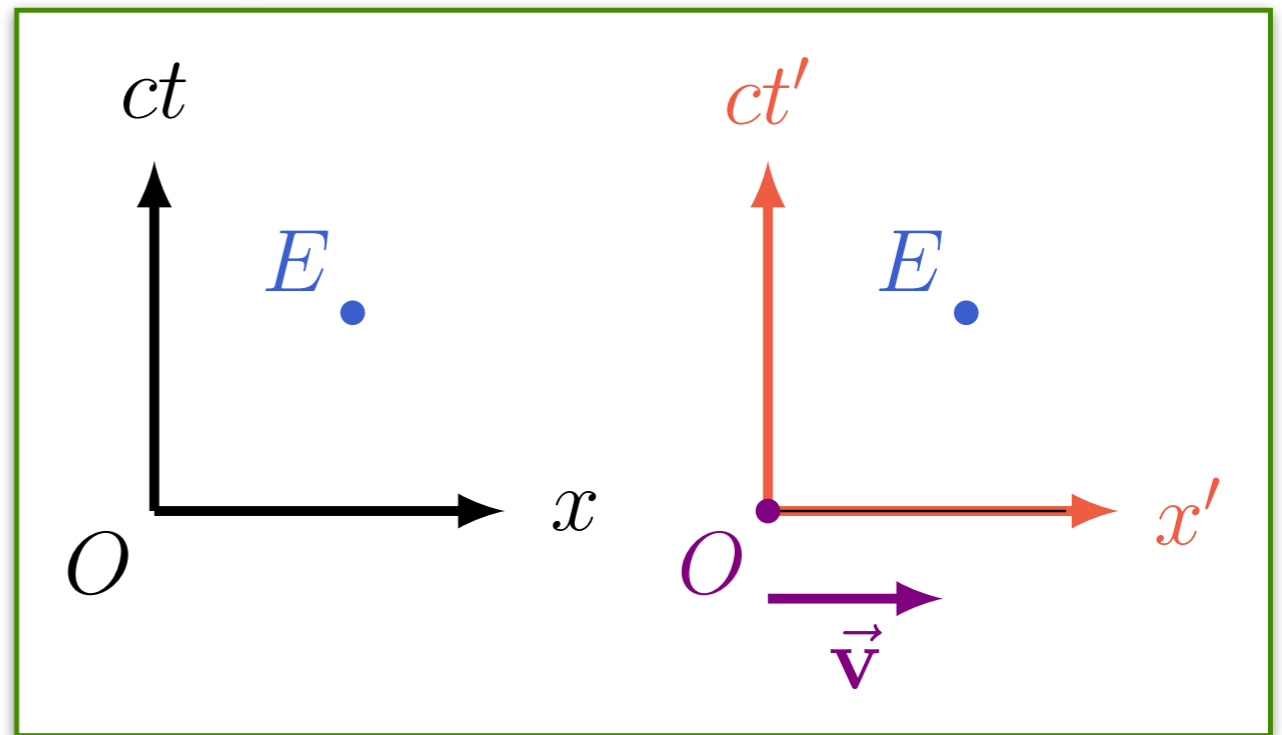
$$\begin{bmatrix} ict' \\ x' \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} ict \\ x \end{bmatrix}$$

$$x = 0 \Rightarrow x' = vt'$$

$$\sin \alpha = i \frac{v}{c} \cos \alpha$$

$$\tan \alpha = i\beta$$

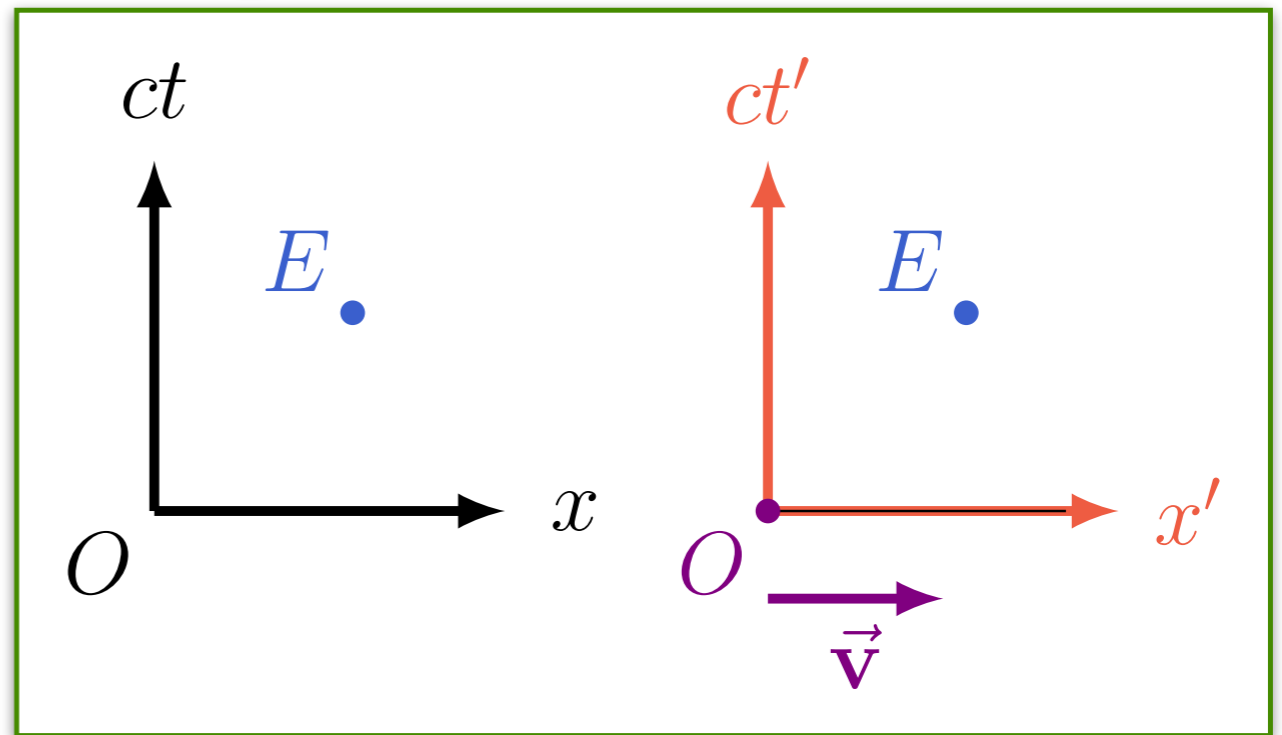
# Transformação de Lorentz



$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$\beta \equiv \frac{v}{c} \quad \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$$

# Transformação de Lorentz



3D

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$