

## A decision on the truckload and less-than-truckload problem: An approach based on MCDA



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### ABSTRACT

The quantity and frequency of merchandise shipped from each distribution centre to each customer by Full Truckload (TL) and Less Than Truckload (LTL) represent important factors for distribution planning. The transport of smaller quantities leads to reduced costs associated with retailers' inventories but requires additional freight costs. In contrast, the transport of greater quantities with less frequency may reduce transport costs but increases inventory costs due to the need to maintain higher inventory levels. This decision is not easy because some criteria related to costs and to transit time may conflict, and many other important criteria that influence the choice of TL or LTL carriers are not easily quantifiable. This article proposes a Multi-Criteria Decision Analysis (MCDA) approach for structuring and appraising the transportation modes of a large and complex Joint Transportation and Inventory Problem (JTIP). Because this study is based on the Multi-Attribute Utility (MAUT), the proposed approach enables a robust analysis of the most suitable decision according to the preferences and risk aversion of the company's decision makers, considering a group of criteria that are evaluated simultaneously. Sensitivity analyses verify that incorporating utility functions is key to proposing a solution that is more personalised and aligned with the preferences and real concerns of decision makers. A case study is conducted for a telecommunications company to test the efficiency of the approach in real situations.

### 1. Introduction

The Joint Transportation and Inventory Problem (JTIP) concerns the distribution of products from one or multiple sources along a defined planning horizon. In general, the distribution consists of two typical freight transportation modes: Full Truckload (TL) or Less Than Truckload (LTL). The goal is to find a distribution plan from warehouses to retailers that minimizes the transportation and inventory costs.

To address this distribution problem, logistics managers consider several criteria when making decisions about transportation service and inventory levels, often focusing on the cost of safety inventory and transit time (Meixell and Norbis, 2008). The quantity and frequency of customer service are important factors in distribution planning. The transport of smaller quantities with greater frequency usually leads to reduced inventory costs for retailers but may require additional freight costs. On the other hand, the transport of greater quantities with less frequency may reduce transport costs but increase inventory costs due to the need to maintain higher inventory levels (Kang and Kim, 2010). However, this decision is not easy because some criteria related to these costs and to transit time may conflict, and other qualitative criteria are not easily

quantifiable (Meixell and Norbis, 2008). Traditional approaches to distribution in supply networks still suffer from shortcomings due to considering these criteria separately and to taking the cost as the most relevant factor in the analyses (Aguezzoul and Ladet, 2007). Due to the conflicting criteria considered in optimised modelling, multi-criteria analysis for JTIP may be a good option for quantifying the trade-offs associated with these criteria (Mendoza and Ventura, 2013). Moreover, in most real decision problems, basing a decision solely on one criterion is insufficient. According to Ho and Emrouznejad (2009), traditional optimisation approaches usually fail because most real problems rarely focus on a single criterion and because the objectives are defined as a function of this single factor (Barfod, 2012), which is usually to minimise the total cost or delivery time (Aguezzoul, 2014). Furthermore, it is impossible to find a feasible solution that is optimal for all decision makers (DMs) under each of the criteria considered (Loken et al., 2009).

In our research, we evaluate 25 criteria to select the TL and LTL freight transport modes from the viewpoint of a logistics manager. A Multi-Attribute Utility (MAUT) approach is developed to assist the DMs in ranking, selecting and/or comparing alternatives within a finite set of criteria such that they feel comfortable with the final decision (Belton

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and Stewart, 2002; Chen et al., 2008). Considering this set of criteria in a mathematical model is not feasible mainly because of the huge computational effort required and due to the impossibility of handling all criteria simultaneously. A wide approach with criteria ranking is more appropriate because it allows DMs to choose the most suitable transport mode to meet the needs of the distributor and their customers. Using MAUT, the DMs can understand and identify the fundamental criteria for selecting TL and can avoid making these decisions based solely on habit (Barfod, 2012). This paper proposes an MCDA approach for structuring and appraising the TL and LTL transportation modes of a large and complex JTIP.

The contribution of this research is addressing JTIP by focusing on the preferences and risk aversion of the DMs using joint criteria analysis, which would be difficult to run in a unique mathematical model. The proposal also contributes to the methodology of Montibeller and Franco (2007) and Goodwin and Wright (2014; chapter 3) by aggregating optimisation solutions derived from the mathematical model in the analysis of the other criteria for choosing alternatives. These optimisation solutions inside MCDA affect the final results, which is a theoretical contribution to the literature.

The remainder of the study is organised as follows: Section 2 presents a short literature review concerning hybrid studies and criteria that influence the selection among common fleet transport modes for JTIP. Section 3 presents a methodological approach to multi-criteria decision making. Section 4 evaluates the proposed approach using a case study, and section 5 presents the final considerations.

## 2. Theoretical background

### 2.1. Criteria considered in JTIP

The literature on integrated inventory and transportation problems includes a range of mathematical models and solution approaches (Mendoza and Ventura, 2008, 2013; Hamedani et al., 2013; Sadjady and Davoudpour, 2012; Toptal and Bingol, 2011; Tancrez et al., 2012) that focus on single-echelon risk-pooling network design problems with a single supplier and with warehouses holding the working inventory and the safety stock and serving as the intermediate facilities between the supplier and the retailers (Shu et al., 2012). However, because of their ability to consider several qualitative and quantitative criteria in decision making, multi-criteria optimisation approaches are generally adequate for JTIP (Kulak and Kahraman, 2005; Mohammaditabar and Teimoury, 2008). Recent studies have demonstrated the increased use of these approaches for solving contemporary logistics problems (Ghoniem and Maddah, 2015; Cintron et al., 2010; Ho and Emrouznejad, 2009; Parveen, 2012; Wallenius et al., 2008; Aguezoul, 2014). In these approaches, transportation and inventory costs are the main investigated criteria (Williams et al., 2013; Mendoza and Ventura, 2008; Jaruphongsa et al., 2005), followed by service level (Coltman et al., 2011), relationship quality (Aguetzoul, 2014) and load and transportation capacity (Anderson et al., 2010). The following criteria are also important: punctuality of delivery, lead time of transportation, low delivery error, cargo safety (Crum and Allen, 1997), quick response to emergencies (Voss et al., 2006), geographic coverage of transport and access, shipment tracking (Coulter et al., 1989), compatibility strategies (Peng, 2012), flexible rates for negotiating freight, and flexible schedules.

Table A1 (Appendix A) summarises all the criteria relevant to this research.

### 2.2. Justifying the MAUT approach for JTIP

The selection of multi-criteria methods involves several factors, such as the characteristics of the problem, the current scenario in which the organisation is embedded, and the preferences of the DMs. Ozernoy (1992) stated that the method selection is already a multi-criteria decision problem and that using an inappropriate method can result in a poor

alternative that cannot be justified later on. Figueira et al. (2005) concluded that there is no best method, although methods based on multi-criteria decision analysis are flexible and offer good results for the same problem (Ozernoy, 1992). The MAUT approach has been widely used to represent the preferences of individuals (Ishizaka and Nemery, 2013) and is a compensatory strategy that is reasonable for modelling the complex characteristics of JTIP. The MAUT approach applies linear functions and the additive difference model for curvilinear preference functions (Billings and Maurcus, 1983) and thus also incorporates non-linear results. According to Ishizaka and Nemery (2013), MAUT is recommended when the utility curve for each criterion (i.e., a representation of the perceived utility given the performance of the option on a specific criterion) is known. Then, robust results related to the DMs' risk tolerance (Keeney and Winterfeldt, 2009) can be obtained. This method provides a set of lotteries that in turn assume the existence of a probability scale (Harker and Vargas, 1990) in a recursive way. Goodwin and Wright (2014) claimed that recursive analyses are necessary in hard decisions. MAUT provides good software support for the MCDA (Belton and Stewart, 2002), allowing mixed information and uncertainty to be managed easily, although it is cognitively demanding for the DMs (Cinelli et al., 2014).

Combining the MCDA approach with optimisation methods is adequate for distribution planning (Cintron et al., 2010). The authors used Analytic Hierarchy Process (AHP) to weight the criteria and used a mathematical method to optimise the profit of the network. However, the combination of MAUT and optimisation is rare in the literature. According to Ho et al. (2010), research from 2000 to 2008 has used the AHP-goal programming combined approach as the most popular method. MAUT was applied as an individual method.

## 3. Methodological approach

The methodological approach is based on MCDA using the MAUT method, i.e. Multi-attribute utility-based theory (Keeney and Winterfeldt, 2009). The methodological approach improves the 8-step model proposed by Montibeller and Franco (2007) and Goodwin and Wright (2014; see chapter 3). Several possible techniques that can be considered in a generic model also are introduced in these steps. Table 1 details the methodological approach in steps and the corresponding objectives and techniques.

Overall, **Steps 1 and 2** are related to constructing the tree according to the main objectives of an organisation (Keeney and Winterfeldt, 2009). First, the DMs define the specific problem that they want to solve (Step 1). Then, the criteria are selected from the literature relevant to a specific problem and are evaluated and validated by the DMs (Step 2). In this Step, other criteria can be selected by the DMs; then, these criteria are added to the tree according to the objective of the organisation. A questionnaire with the criteria from the literature review can be applied, and other criteria can be added according to the interviews with the DMs.

**In Step 3**, the preference or risk aversion of the DMs regarding the criteria is evaluated by plotting the utility curve of each criterion to represent the preferences of the DMs; this information generally is obtained through interviews with the DMs. This approach is used to determine the "certain equivalent" in a lottery or gamble related to the criteria, i.e., given any lottery or gamble for an amount of money that is randomly drawn from a probability distribution, a DMs' certainty equivalent of this gamble is the lowest amount of money-for-certain that the DM would be willing to accept instead of the gamble (Nicholson and Snyder, 2012, p. 216).

**In Step 4**, after determining the tree with all the criteria measured individually, these criteria are prioritised. Then, all of the criteria are weighted using pairwise comparisons according to the preferences of the DMs. The weights can be calculated by several methods, and the most suitable methods for this approach are the following: (1) Simple Multi-Attribute Ranking Technique (SMART) (Edwards and Barron, 1994), which uses swing weighting, is derived by asking the DM to compare a

**Table 1**  
Proposed approach steps. Source: Based on Montibeller and Franco (2007) and Goodwin and Wright (2014).

N	STEP	OBJECTIVE	TECHNIQUES (Authors)
1	Understanding the problem	Description of the problem scope	Interviews of strategic DMs involved in the problem
2	Definition of the main objectives	Construction of the decision tree and definition of the criteria for measuring the objectives	Interviews and questionnaires to determine the relevant criteria from the literature (Keeney and Winterfeldt, 2009)
3	Preferences and evaluation of the value compensations	Development of the utility function	Certainty equivalent to determine the utility curve of each criterion (Belton and Stewart, 2002).
4	Hierarchisation of criteria	Determination of the attribute weights	Aggregation of Individual Judgements (AIJ) of AHP (Saaty, 1980), Swing weight method (Goodwin and Wright, 2014) and ROC method (Edwards and Barron, 1994)
5	Generation of alternatives	Generation of possible alternatives	Multi-objective mathematical model and DMs' empirical solutions (Deshpande et al., 2011).
6	Evaluation of alternatives	Utility value of each alternative and the overall value	Application of the global utility value (Loken et al., 2009)
7	Sensitivity analysis	Verification of the intermediary solution adherence	Variation in weights, techniques and utility functions
8	Recommendation	Indication of the recommended alternative	–

change (or swing) from the least-preferred value to the most-preferred value in one attribute to a similar change in another attribute (Goodwin and Wright, 2014). (2) The Aggregation of Individual Judgements (AIJ) method of AHP (Saaty, 1980) uses geometric averages of the individual judgments to determine the group weights and group of the inconsistency index (Forman and Peniwati, 1998). This method is suitable for avoiding inconsistencies among the responses of the DMs. (3) Rank Order Centroid (ROC) was developed by Edwards and Barron (1994) based on using predefined weights allocated in order of relevance to the criteria. This method eliminates the AHP process of pairwise comparisons between the criteria, which can be tedious and time consuming.

In Step 5, alternatives based on the values of only the criteria directly defined by the DMs are considered. However, this Step also considers alternatives that are attempted separately in a multi-objective mathematical model. Then, based on the best optimisation solutions, we determine the values of utility to all criteria. Thus, all the utility curves are recalculated under optimised solutions.

Step 6 evaluates alternatives. The global utility value is determined using the multi-criteria utility function in its additive form (eq. (1)), as shown by Loken et al. (2009). This is necessary because in Step 5, the utility curves were individually recalculated for the criteria.

**Table 2**  
Profiles of the DMs.

DMs	DM1	DM2	DM3
Position	Logistics supervisor	Senior distribution analyst	Logistics supervisor
Experience	3 years in logistics and distribution	2 years in logistics and distribution	9 years in logistics and supply chain
Supervises	5 people in the area	–	–
Main functions	Freight management and linked costs. Inventory management. Payment and reimbursement to 3PL.	Planning of supply and equipment distribution.	Transportation management. Management of supplies and equipment distribution.

$$U(a) = \sum_{i=1}^m k_i u_i(x_i(a)) \tag{1}$$

where  $a$  is the evaluated alternative,  $U(a)$  is the global value of the utility function of alternative  $a$ ,  $i$  represents the problem's criteria,  $x_i(a)$  is the performance of alternative  $a$  in relation to the attribute of criterion  $i$ ,  $u_i(.)$  is the partial utility function of criterion  $i$ , and  $k_i$  is the weight or preference of criterion  $i$ , which refers to the relative importance of the criterion (Saaty, 2008) is determined according to a point of reference.

In this Step, the solutions presented by mathematical modelling and those empirically used by DMs are evaluated altogether. The objective is to determine the preferred alternative or the alternative that provides better performance considering all of the criteria.

Step 7 conducts the sensitive analysis, i.e., the weights of the criteria, techniques for weighting and preferences of the DMs are varied. If there is no large variation in the results, the robustness of the model is verified. In this Step, we attempt to measure the risk attitude of the DMs.

In Step 8, we select the recommended alternative.

#### 4. Evaluation of the proposed approach through a practical example

In this section, the proposed approach is evaluated using a case study of a telecommunications company operating in Brazil. This company is a large, privately held non-national company that provides equipment and supplies to end customers. The transportation service is completely outsourced, with a large percentage of the transported freight sent to a logistics operator (3PL). The freight cost is defined based on the volume used (cubage), mainly due to the low density of the shipped antennas. All the equipment consists of fragile and dry packaged cargo, with high added value and high risk of damage during transportation.

The company's distribution planning is based on two essential objectives: low freight cost and high service level. The LTL freight option is based on the quantity shipped and the distance travelled, whereas the TL freight option is a quick response alternative, with loads assigned a fixed cost for any shipment. Moreover, the distribution network consists of 70 regional warehouses and two distribution centres. A heterogeneous fleet of TL vehicles was considered for the distribution, with cubic capacities from 4 m<sup>3</sup> to 90 m<sup>3</sup>. The planning horizon was set at 26 weeks, considering the long distances between each O-D arc and the tactical nature of the problem.

Three strategic DMs of the company (see Table 2) participated in defining the problem and characterising the solution variables.

The data for criteria weighting, plotting of utility curves and sensitivity analyses were collected using structured interviews with DMs. The approach in sections 4.1–4.8 follows the previously presented steps in the methodological section.

##### 4.1. Understanding the problem

The problem consists of how to distribute products from one or multiple sources to retailers to minimise the transportation and inventory costs while considering several criteria. In our case, it is important to know the demand at the customer locations for the available freight transportation alternatives (TL or LTL). In other words, it is helpful to

define the amount of merchandise shipped from each distribution centre (DC) and to each customer by TL and LTL. The criteria evaluation will vary depending on the costs and the service quality to meet the local demand. For example, per distance and cubic volume, there is a specific price and service level that will determine whether TL or LTL is chosen. Additionally, quantitative operational efficiency criteria, such as transit time reliability, total transit time, and delivery error, and qualitative operational efficiency criteria, such as good communication, compatibility of strategies between partners, and flexible schedules, must be evaluated. Specifically, in the case study, it is also important to know the safety inventory level defined for each customer (regional DC) to meet the desired level of service. For example, in the northern region, where shipments take approximately 30 days, the safety inventory is higher than in other places. Criteria such as long-term relationships between carriers and partners (supplier and retailers), transport coverage and access area, and tracking of trucks are important for understanding the

entire problem.

4.2. Determination of the main objectives

This section introduces the main objectives used to build the decision tree. From the viewpoint of the MCDA, the objectives encompass criteria and sub-criteria obtained from a literature review based on the DMs' pre-assessments of relevance. The general objective of the problem is to understand the demand at the customer locations for the different freight transport modes to improve the overall system performance. Using the classification suggested by Coulter et al. (1989), the criteria and sub-criteria are presented in Fig. 1. The values in the parentheses present the absolute weights of the criteria, while the others are relative weights. Visual Interactive Sensitivity Analysis (VISA) software was used to build the tree and to assist in determining the values and weights of the

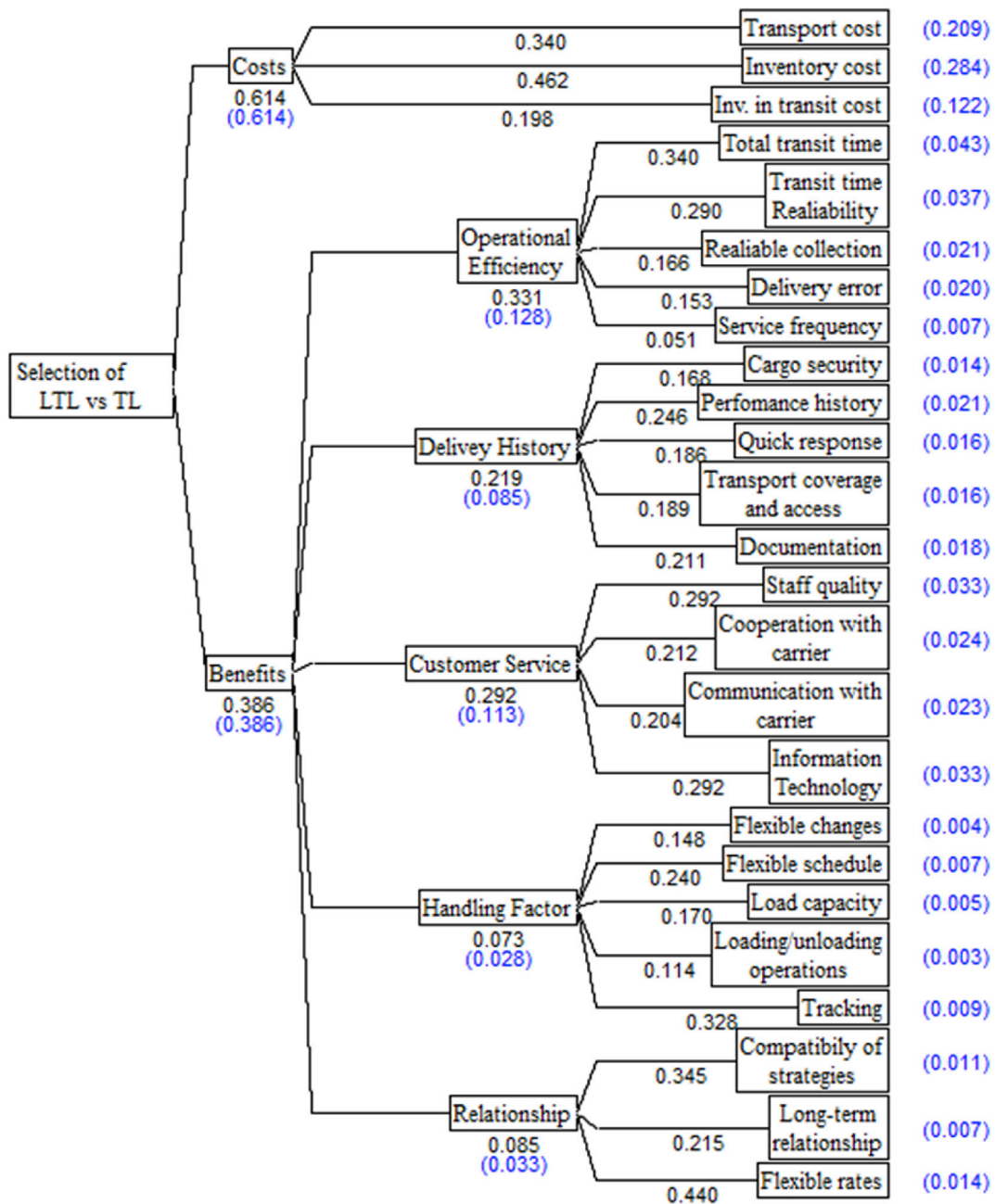


Fig. 1. Decision tree for the case study.



**Table 3**  
Utilities for the transportation cost.

Utility $f(x)$	Costs ( $x$ )
0	$x \geq \$12,600,000$
25	\$12,045,000
50	\$9,825,000
75	\$8,437,500
100	$x \leq R\$ 1,500,000$
Approximated function ( $0 \leq f(x) \leq 100$ )	
$f(x) = \begin{cases} -4 \times 10^{-6}x + 105.41 & x \leq 8,437,500 \\ 2 \times 10^{-12}x^2 - 6 \times 10^{-5}x + 398 & x \leq 12,045,000 \\ -9 \times 10^{-5}x + 1,135 & x > 12,045,500 \end{cases}$	

**4.3. Preferences and evaluation of the value compensations**

This step demands the greatest participation from DMs. It consists of defining the structure of each criterion (indicators and utility functions) according to the preferences of the DMs. As each criterion has different measurement units (\$ for costs, days for time, and so on), the DMs were asked to define the indicators for each quantitative criterion.

The certainty equivalent was applied to plot the utility curves for each criterion. These functions allowed measuring the DMs' preferences between the alternatives in relation to each main objective.

Using a five-point scale for the utility, Table 3 shows an example of the transportation costs and respective utilities and the approximated function according to the costs (see Fig. 2). This curve was drawn using the certainty equivalent.<sup>1</sup> Fig. 2 shows the respective non-linear utility curve plotted from these transportation costs. A variation of annual transportation costs from  $\$1.5 \times 10^6$  to  $\$7 \times 10^6$  generates a loss of utility of 20 units (from 100 to 80), whereas a variation of  $\$7.1 \times 10^6$  to  $\$12.6 \times 10^6$  generates a loss of 80 units. Therefore, this behaviour demonstrates the risk aversion of the DMs.

However, most of the criteria were qualitative, and there were no performance indicators to measure them. Then, the utility curves were plotted based on the perception of the DMs using a five-point linguistic scale (0 - Bad; 25 - Poor; 50 - Normal; 75 - Good; 100 - Excellent) related to three alternatives (100% TL, 50% TL and 100% LTL). As an example of a qualitative criterion, Fig. 3 shows the utility curve for "Low Delivery Error".

According to the DMs, 100% TL has great performance related to low delivery error, whereas 50% TL is considered normal, and 100% LTL has poor performance. Using the TL origin-destination (O-D) points, the vehicles travel directly to regional DCs, where the shipment is checked, and no stopping or loading/unloading occurs along the way. In contrast, using LTL for the same O-D points, loading and unloading occurs for several different clients at some destinations.

**4.4. Hierarchisation of criteria**

The hierarchisation of criteria can be achieved by applying several methods. The most commonly used methods are AHP and the AIJ of AHP (Saaty, 1980). The swing weight method (Goodwin and Wright, 2014) and ROC method (Edwards and Barron, 1994) are also relevant methods.

In Fig. 1, the tree also presents the grouped weights for the criteria and sub-criteria utilising the AIJ method of the AHP. This figure shows the local weights and cumulative/global weights (in parentheses). The

<sup>1</sup> The current transportation costs are \$7,050,000 annually. According to the DM preference, the best cost would be to spend at most \$1,500,000 annually and the worst alternative would be to spend \$12,600,000 annually. Supposing a lottery, the DM pays \$1,500,000 with a probability of 0.5 or pay \$12,600,000 with a probability of 0.5. Would the DM prefer the current cost of \$7,050,000 to the lottery? If the answer is "No", the next question is if the current costs were \$10,000,000, would the DM prefer this cost to the lottery? If the answer to the first question is "Yes", the next question is if the costs were \$6,000,000, would the DM prefer this cost to the lottery? When the DM is indifferent to both choices, we continue the question closely plotted this curve.

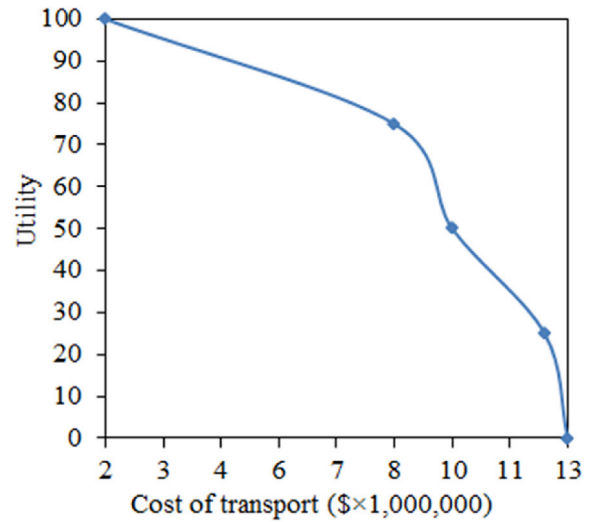


Fig. 2. Utility curve for the transportation cost.

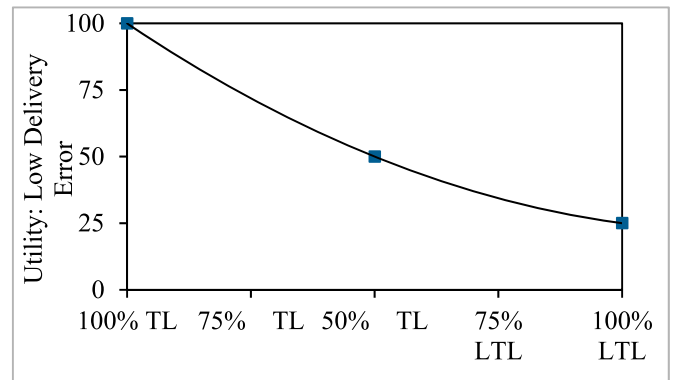


Fig. 3. Utility curve for delivery error.

judgement inconsistency indices vary from 1% to 5% and are below 10% (Saaty, 2008), reflecting the consistency of the respondents' answers and improving the reliability of their results.

In the grouped judgements, a preference for cost over benefit was observed. Briefly, a low cost represents good system performance at least 61.4% of the time, as shown in Fig. 1. The weights in the tree also demonstrate the DMs' priorities in maintaining low inventory levels. Analysing the benefit branch in the tree shows that the 'operational efficiency' criterion had the highest priority (0.331). This indicates that DMs are willing to assign a higher value to solutions that provide better performance in basic transportation operations to ensure effectiveness and efficiency. In particular, the total transit time and reliability of transit time had the highest relevance. The 'customer service' criterion (0.292) also was prioritised. It is understood that the DMs are interested in alternatives that ensure frequent communication and cooperation with the operators in addition to good information technology to ensure faster processes.

**4.5. Generation of alternatives**

In this step, several alternatives, for instance, TL, LTL or a combination of these, are evaluated according to the criteria. Our approach is innovative because it considers alternatives obtained from the multi-objective mathematical modelling that must modify the results. Based on the optimised values of the alternatives, the criteria are re-evaluated to obtain the best solution. This model is classified as the Weighted Sum

Method and is characterised by the weights given to all objectives and by a linear combination of objective functions into a unique function (Ochoa et al., 2006). Oliveira and Saramago (2010) presented other multi-objective optimisation techniques that also can be applied. The goal is to generate a set of efficient alternatives considering a wide solution space.

The objective functions to the JTIP problem minimise the total cost involved, namely, the cost of LTL and TL transportation, the regular inventory costs, the safety inventory costs and the cost of inventory in transit (see Appendix B).

Table 4 shows the solutions presented by the strategies used and those that can be used by companies beyond the ones generated by the multi-objective mathematical model. Initially, the model is solved with each objective function independently to obtain the optimised values:  $F_1^* = 1665.25$ ,  $F_2^* = 1410.89$  and  $F_3^* = 733.52$ , which represent solutions (1), (2) and (3), respectively. Each combination of weights  $\omega$  considers an efficient solution for the problem; no pair of solutions exists such that one of the solutions is the best for all objectives. The weights  $\omega$  of solutions (1)–(7) were generated while exploring all the combinations of the three objectives given a 0 or 1 priority. Solution (4) is the best solution found by the mathematical model when all costs are prioritised. In this case, 82.8% of the demand is fulfilled by LTL, and 17.2% is fulfilled by TL. Solution (8) is obtained using the combined judgements of the DMs through the AHP-AIJ technique. Solutions (4)–(8) were obtained by minimizing  $F_4$ , an integrated objective function based on  $F_1^*$ ,  $F_2^*$ ,  $F_3^*$  and the relative weights  $\omega$  of each objective function. Solution (9) is an approximation of the actual costs of the company: 80% of the demand is fulfilled by the LTL mode. Solutions (10)–(14) were obtained with additional constraints in the model to reach the optimal costs when the cargo distribution is defined a priori. For example, alternative (10) obtains the optimal cost solution when only the LTL is available, whereas alternative (11) obtains the optimal cost solution when 30% of the demand is met with TL shipping.

Based on these optimised costs, the utilities for all criteria were re-calculated. For example, following the previous utility curve of the transport cost (Fig. 4), the utility was re-calculated for all alternatives (i.e., the actual transportation cost of \$2202.90 (9) has an equivalent of 96.6 utility units).

For qualitative criteria, these valuations were obtained through utility curves and by the percentage of demand attribution for each freight transportation mode. For instance, the solution that assigns 100% of the demand to LTL shipping (10) obtained a utility of 25.0 for the ‘delivery error’ criterion (see Fig. 3).

Tables 6 and 7 show the utility valuations of the non-dominated solutions (Table 4) analysed for each criterion. For example, in Table 5, solution (1) has a transportation cost of \$1665.254, which is equivalent

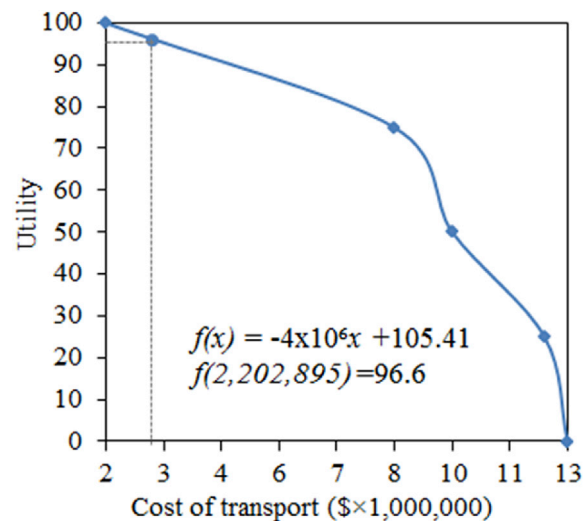


Fig. 4. Curve of transport with optimised cost.

to 98.7 utility units, and a cargo distribution of 38.2% TL – 61.8% LTL, which are equivalent to 45.5 and 41.8 utility units, respectively, with a low collection and delivery error.

#### 4.6. Evaluation of alternatives

After the re-calculation of the criteria, the alternatives are available to be evaluated. In this case, the alternatives are evaluated according to the global MAUT equation (Eq. (1)), focusing on a main set of criteria or an individual criterion. Table 7 represents the overall utility evaluations for the considered alternatives. The results are given on a scale from 0% to 100%, indicating the level at which the alternative performs with respect to all criteria.

The results indicate that the solution that assigns 30% of the demand to TL shipping is (11) the most preferred solution, achieving 77.1% of the overall objectives. This is the most adequate solution for improving the performance of the distribution system based on the priorities and risk aversion of the DMs. The solution generated by the combined judgements of the DMs (8) is the second-most preferred alternative, followed by the solution that optimises the costs in a joint manner (4) and the approximate current company solution (9). The solution that minimizes the delivery time (3) achieved the best performance in benefit but the worst in cost and thus the worst overall performance. Therefore, the DM solution presents the best utility based on costs, whereas LEADT\* presents

Table 4  
Non-dominated solutions generated as a function of the weights  $\omega$ .

Sol.	Desc. - %TL/LTL	$\omega_1$	$\omega_2$	$\omega_3$	Transportation costs (\$)	Inventory costs (\$)	Inventory in transit costs (\$)	Total Costs (\$)
1	TRANSP* - 38/62	1	0	0	1,665.25	9,521.16	1,087.99	12,274.40
2	INV* - 64/36	0	1	0	11,953.53	1,410.89	1,193.15	14,557.57
3	LEADT* - 85/15	0	0	1	9,152.93	7,187.34	733.52	17,073.79
4	COST* - 17/83	1	1	1	2,084.29	1,510.61	1,296.31	4,891.21
5	TRANSP + INV - 3/97	1	1	0	2,061.16	1,445.92	1,499.71	5,006.79
6	TRANSP + LT - 45/55	1	0	1	1,815.85	19,067.88	1,000.88	21,884.61
7	INV + LT - 91/9	0	1	1	12,570.19	1,335.92	743.91	14,650.02
8	DM - 11/89	0,34	0,46	0,2	2,106.47	1,493.89	1,147.16	4,747.52
9	ACTUAL - 20/80	1	1	1	2,202.90	1,891.42	1,280.70	5,375.02
10	LTL - 0/100	1	1	1	2,077.54	1,458.10	1,453.92	4,989.56
11	30/70	1	1	1	2,134.80	1,656.80	1,192.78	4,984.38
12	50/50	1	1	1	2,317.33	2,806.42	1,044.03	6,167.78
13	70/30	1	1	1	2,972.35	2,689.24	1,035.55	6,697.14
14	TL - 100/0	1	1	1	5,315.71	3,036.41	834.67	9,186.79

Note: TRANSP = Transportation cost; INV = Inventory cost; LEADT = Lead time; COST = Global costs; DM = DMs' solutions; ACTUAL = Current values used by the company. All costs are stated in MM. The asterisk represent the optimal solutions.

**Table 5**  
Values of alternatives related to the relevant criteria - part I.

SOL.	DESCR.	TL FREIGHT	RELEVANT COSTS			BENEFIT									
			Transport cost	Inventory cost	Inventory in transit cost	OPERATIONAL EFFICIENCY					DELIVERY HISTORY				
						Total transit time	Transit time Reliability	Reliable collection	Delivery error	Service frequency	Cargo security	Performance history	Quick response	Transport coverage and access	Documentation
20.8%	28.4%	12.2%	4.3%	3.7%	2.1%	2.0%	0.7%	1.4%	2.1%	1.6%	1.6%	1.8%			
1	TRANSP*	38.2%	98.7	–	83.5	97.1	87.9	45.5	41.8	78.6	71.4	44.1	16.8	61.8	14.6
2	INV*	64.3%	–	100.0	78.3	4.4	55.1	59.2	61.7	72.4	77.6	57.1	36.7	35.7	–
3	LEADT*	85.2%	16.3	–	100.0	100.0	100.0	80.0	82.6	72.4	77.6	67.6	57.6	14.8	–
4	COST*	17.2%	97.1	100.0	55.0	62.3	31.0	44.4	30.8	88.6	61.4	33.6	5.8	82.8	54.3
5	TR + INV	3.2%	97.2	100.0	–	54.4	6.5	48.5	25.8	97.7	52.3	26.6	0.8	96.8	90.6
6	TR + LT	45.1%	98.1	–	87.9	96.3	88.1	47.8	46.4	76.3	73.7	47.6	21.4	54.9	5.4
7	INV + LT	91.3%	3.8	100.0	100.0	71.9	82.9	87.8	89.5	73.2	76.8	70.7	64.5	8.7	–
8	DM	11.4%	97.0	100.0	80.6	61.6	29.6	45.6	28.5	92.1	57.9	30.7	3.5	88.6	68.3
9	ACTUAL	20.0%	96.6	93.8	59.2	63.5	34.5	44.0	32.0	87.0	63.0	35.0	7.0	80.0	47.9
10	LTL	0.0%	97.1	100.0	23.5	57.1	12.4	50.0	25.0	100.0	50.0	25.0	–	100.0	100.0
11	30/70	30.0%	96.9	98.4	78.3	68.7	49.8	44.0	37.0	82.0	68.0	40.0	12.0	70.0	28.0
12	50/50	50.0%	96.1	–	85.7	79.7	75.0	50.0	50.0	75.0	75.0	50.0	25.0	50.0	–
13	70/30	70.0%	93.5	–	86.1	78.9	75.7	64.0	67.0	72.0	78.0	60.0	42.0	30.0	–
14	TL	100%	84.1	–	96.2	90.6	99.8	100.0	100.0	75.0	75.0	75.0	75.0	–	–

The values in cells are represented in utility units, with values between 0 (no preference) and 100 (most preferred).  
The asterisk represent the optimal solutions.

**Table 6**  
Values of alternatives related to the relevant criteria - part II.

SOL.	DESCR.	TL FREIGHT	BENEFIT											
			CUSTOMER SERVICE				HANDLING FACTOR				RELATIONSHIP FACTOR			
			Staff quality	Cooperation with carrier	Communication with carrier	Information technology	Flexible changes	Flexible schedule	Load capacity	Loading/unloading operations	Tracking	Compatibility of strategies	Long-term relationship	Flexible rates
3.3%	2.4%	2.3%	3.3%	0.4%	0.6%	0.5%	0.3%	1.0%	1.1%	0.7%	1.4%			
1	TRANSP*	38.2%	75.0	85.4	42.7	30.9	27.3	38.2	57.3	45.5	50.0	21.4	40.5	40.5
2	INV*	64.3%	75.0	58.7	55.1	17.9	20.4	64.3	44.9	59.2	50.0	27.6	59.7	59.7
3	LEADT*	85.2%	75.0	27.3	55.2	7.4	10.0	85.2	44.8	80.0	50.0	27.6	70.2	70.2
4	COST*	17.2%	75.0	97.0	22.8	41.4	27.8	17.2	77.2	44.4	50.0	11.4	20.0	20.0
5	TR + INV	3.2%	75.0	99.9	4.7	48.4	25.7	3.2	95.3	48.5	50.0	2.3	3.9	3.9
6	TR + LT	45.1%	75.0	79.7	47.3	27.5	26.1	45.1	52.7	47.8	50.0	23.7	46.2	46.2
7	INV + LT	91.3%	75.0	16.6	53.6	4.3	6.1	91.3	46.4	87.8	50.0	26.8	72.5	72.5
8	DM	11.4%	75.0	98.7	15.9	44.3	27.2	11.4	84.1	45.6	50.0	7.9	13.6	13.6
9	ACTUAL	20.0%	75.0	96.0	26.0	40.0	28.0	20.0	74.0	44.0	50.0	13.0	23.0	23.0
10	LTL	0.0%	75.0	100.0	–	50.0	25.0	–	100.0	50.0	50.0	–	–	–
11	30/70	30.0%	75.0	91.0	36.0	35.0	28.0	30.0	64.0	44.0	50.0	18.0	33.0	33.0
12	50/50	50.0%	75.0	75.0	50.0	25.0	25.0	50.0	50.0	50.0	50.0	25.0	50.0	50.0
13	70/30	70.0%	75.0	51.0	56.0	15.0	18.0	70.0	44.0	64.0	50.0	28.0	63.0	63.0
14	TL	100.0%	75.0	–	50.0	–	–	100.0	50.0	100.0	50.0	25.0	75.0	75.0

The values in cells are represented in utility units, with values between 0 (no preference) and 100 (most preferred).  
The asterisk represent the optimal solutions.

**Table 7**  
Cost and benefit utility of the alternatives.

SOL.	DESCR.	TL FREIGHT	UTILITY		
			COST	BENEFIT	OVERALL
1	TRANSP*	38.2%	30.7	22.1	52.8
2	INV*	64.3%	37.9	17.3	55.3
3	LEADT*	85.2%	15.6	23.4	39.0
4	COST*	17.2%	55.3	18.4	73.7
5	TR + INV	3.2%	48.6	17.2	65.8
6	TR + LT	45.1%	31.1	22.1	53.2
7	INV + LT	91.3%	41.4	21.7	63.1
8	DM	11.4%	58.4	18.4	76.8
9	ACTUAL	20.0%	53.9	18.6	72.5
10	LTL	0.0%	51.5	17.6	69.0
11	30/70	30.0%	57.7	19.4	77.1
12	50/50	50.0%	30.5	21.0	51.4
13	70/30	70.0%	30.0	21.4	51.4
14	TL	100.0%	29.2	23.2	52.4

The asterisk represent the optimal solutions.

the best utility based on benefit. The DMs value solutions based on low costs, whereas optimised solutions may neglect some criteria focused on benefit. On the other hand, LEADT\* produces solutions based on quick response and great benefits; however, the costs are higher. The results show that alternatives with high percentages of LTL loads have high utilities of costs, and alternatives based on TL loads achieve greater utilities for the benefits.

Comparing the global utility and the demand fulfilled by the TL mode in solutions (8) and (9), it is interesting to note that solution (8) presents higher utility, although this solution fulfils a smaller percentage of the demand than the TL mode. This suggests that global utility is not a linear function of the transport mode; in other words, there is no conclusion as to whether a lesser or greater use of TL mode positively or negatively affects the overall performance of the transport system.

Note that for an approach using only the optimisation model without the DM's information, for instance, using the same weights (see solution 4) would not result in the best choice for the company. Although the global utility of this solution (see Table 7) has presented good results, other solutions appear better for improving the overall performance of the distribution system.

4.7. Sensitivity analyses

Sensitivity analyses are conducted to determine the robustness of the model.

Fig. 5 shows the behaviour of the solutions in response to changes in the preference for the “benefit” criterion. It is observed that when the benefit weight ranges from 0% to 16%, the suggested solution is (8). When the benefit weight ranges from 16% to 82%, the suggested solution alternative (11), and for weights greater than 82%, alternatives (7), (14) and (3) are preferred. Therefore, alternative (11) has a wide range of preference for which it remains the preferred alternative.

For the remaining criteria at the first, second and third levels, the previous solution continues to achieve great performance.

In addition, variations of the utilities of the criteria can be evaluated to check the robustness of the approach. We applied the following strategies, as described in Table 8.

- Varied the weights of the criteria;
- Used ROC to weight the criteria;
- Prioritised DMs, in which DM3 is the best expert in this case; and
- Based the test on a risk-neutral DM.

In the first case, ‘variation of weights of the criteria’, huge changes in the preference of the criteria were necessary to move to another solution; alternative (11) remained the preferred solution.

When ROC was applied, solutions (11) and (8) were equally preferred. This indicates the robustness of the solution considering changes in the weighting technique. This technique represents a valid alternative for obtaining weights when there is relatively little time available.

In the third case, the judgments of the DM3 were considered to have highest priority, and the weighted AIJ technique of the AHP was used to obtain the overall weights. The priorities of DM1, DM2 and DM3 were defined as 20%, 20% and 60%, respectively. In this case, solution (11) remained the preferred solution. In addition, when prioritising the judgments of a DM, the result is a tendency to follow their particular objectives. Due to the preference of DM3 for solutions with high performance in terms of safety in service delivery, the general utility of alternatives (3), (7), (13) and (14) increased; these alternatives mainly use TL freight, contributing to the selection of these alternatives.

The fourth case relates to risk-neutral decision making. DMs are expected to attribute utility values to the alternatives without

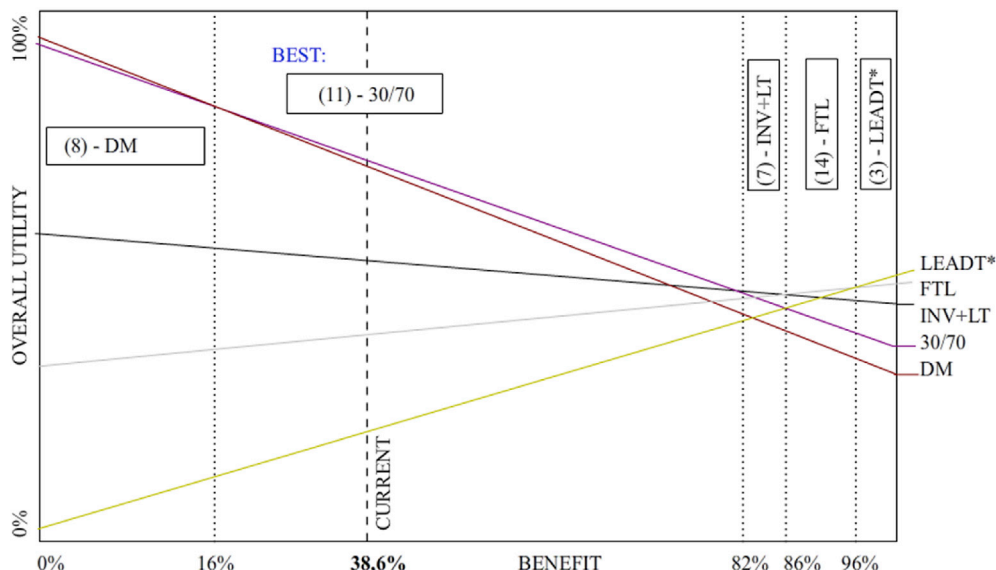


Fig. 5. Sensitivity analyses: variation of benefit preference.



**Table 8**  
Summary of the sensitivity analyses.

SOL.	DESCR.	TL Freight	Normal		Using ROC		Prioritised DM3		Neutrality	
			Utility	Ranking	Utility	Ranking	Utility	Ranking	Utility	Ranking
1	TRANSP*	38.2%	52.8	10	43.5	9	54.9	10	41.5	13
2	INV*	64.3%	55.3	8	63.0	7	55.5	9	63.1	8
3	LEADT*	85.2%	39.0	13	29.1	13	41.4	13	28.4	14
4	COST*	17.2%	73.7	3	83.1	2	73.0	3	82.1	2
5	TR + INV	3.2%	65.8	6	77.5	5	63.3	7	80.4	5
6	TR + LT	45.1%	53.2	9	43.8	8	55.6	8	41.8	12
7	INV + LT	91.3%	63.1	7	70.5	6	63.5	6	65.7	7
8	DM	11.4%	76.8	2	85.2	1	76.9	2	83.4	1
9	ACTUAL	20.0%	72.5	4	80.6	3	72.1	4	74.9	6
10	LTL	0.0%	69.0	5	79.8	4	67.2	5	80.9	4
11	30/70	30.0%	77.1	1	85.2	1	77.2	1	81.3	3
12	50/50	50.0%	51.4	12	42.0	11	54.0	12	56.7	10
13	70/30	70.0%	51.4	12	41.9	12	54.2	11	57.9	9
14	TL	100.0%	52.4	11	42.8	10	55.6	8	47.5	11

The asterisk represent the optimal solutions.

overestimating or underestimating their current performance. When using MAUT, the risk-neutral utility curves are linear functions. An important change in the solution order is observed for this case as solutions (8) and (4) were preferred over the previous solution. This indicates that the DMs' preferences for solution (11) may be motivated by their risk aversion.

It is noteworthy that although the neutrality characteristic can suggest solutions that seem more logical because it neither underestimates nor overestimates performance, the real risk attitude of the DMs should be accepted. In reality, no risk attitude is particularly preferred over the real attitude. However, for cases where there are many criteria, collecting utility curves can be tedious and time consuming. Accordingly, a linear approximation using two end points can save significant time for the analyst and DMs.

For the four most preferred solutions, {(11), (8), (4), (9)}, the demand percentage transported using TL ranges from 10% to 30%, which reflects the DMs' preference for using LTL shipping.

#### 4.8. Recommendation: indication of the recommended alternative

In this step, the final results are determined based on the sensitivity analysis, and the recommended alternative for the decision problem is selected. Our study shows that large variations in the criteria weights are necessary to generate a change in the suggested solution, which demonstrates the robustness of the solution and the proposed approach. Additionally, for the other techniques, the previous solution remained within the top three solutions in terms of the best overall performance. Therefore, it is recommended that the DMs adopt alternative (11), which is characterised by the demand at the customer locations as 30% TL – 70% LTL. This solution presents a cost difference of only \$93,177 annually, which is equivalent to a 2% increase compared to the best cost solution found by the mathematical model (alternative 4). This solution provides better delivery conditions and is less prone to mistakes and variability and therefore less prone to costs that are not directly visible on the cost worksheets compared to the optimal cost solution.

## 5. Conclusions

This study proposed a methodology to address distribution problems

considering multiple transportation modes and different levels of inventory in the distribution channels. The TL and LTL freight transport modes for a large telecommunications company were used to test the methodology. To our knowledge, studies on JTIP considering multiple criteria analysis are rare, and they all study the problem within the context of fleet management; other studies similar to ours focus on cost and time using optimised approaches. Unlike these studies, our proposal focuses on integrating optimisation with considerations of relevant criteria that are difficult to quantify in a unique mathematical model. This would require huge computational effort, and there are no measurements of most criteria. Therefore, integrating MCDA techniques with mathematical programming allowed for a solution that could not be obtained from implementing the methods independently. Moreover, by considering other criteria external to modelling, this methodology can provide appropriate solutions in a practical context and also can be generalized to similar operations using different mathematical models.

The grouped judgements showed the DMs' preference for cost over benefit. As a result of the case study analysis, the approach suggested a “30% TL - 70% LTL” cargo distribution instead of the “20% TL - 80% LTL” planning used by the company. This suggested solution was closer to the optimal cost and showed better performance in terms of the benefit criteria. The sensitivity analyses verified that the risk aversion of the DMs significantly conditioned the final choice, which suggests that incorporating the utility functions was key to proposing a solution that is more personalised and aligned to the preferences and real concerns of DMs.

The approach requires a strong involvement with the company's professionals because data are not the only resources needed. The most important resource is the participation of key DMs. These DMs must be directly related to the problem because they are the ones who choose measures and prioritise the criteria that influence the final choice. Although this approach is innovative, the criteria should be validated by conducting a survey with companies of different sectors and then applying this generic analysis of the solution.

## Acknowledgements

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## Appendix A

**Table A1**  
Summary of the criteria in the literature.

SELECTION CRITERIA	REFERENCE																	
	Jerman et al. (1978)	Coulter et al. (1989)	Bardi et al. (1999)	McGinnis (1990)	Abshire and Premeaux (1991)	Aguezoul (2014)	Murphy et al. (1997)	Anderson et al. (2010)	Crum and Allen (1997)	Min (1998)	Lu (2003)	Danielis et al. (2005)	Kulak and Kahraman (2005)	Voss et al. (2006)	Mohamm aditabar and Teimoury (2008)	Coltman et al. (2011)	Peng (2012)	Williams et al. (2013)
Associated costs (transportation, inventory)	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Transit time reliability	X	X	X	X				X	X			X	X	X	X	X	X	X
Total transit time	X	X	X	X	X		X	X			X	X			X	X	X	X
Flexible rates	X	X	X	X	X	X	X		X		X				X	X		X
Documentation and billing services		X		X			X		X	X			X	X	X	X		X
Quality of transport personnel (staff quality)	X	X	X	X		X	X		X						X			
Tracking capacity	X		X		X		X	X			X				X	X		
Cargo security	X						X		X					X		X		X
Cooperation between transport staff and carrier	X		X		X			X							X	X		X
Ease of loading and unloading		X	X	X	X		X											
Long-term relationship possibility					X	X	X		X						X			
History of performance experience	X	X			X		X		X						X			
Infrastructure of transport and access; delivery geographic coverage				X	X				X			X			X	X		
Reliable collection service		X			X	X	X		X		X							
Flexible schedule	X		X							X			X		X			
Quick response to emergency deliveries											X			X	X	X		
Load capacity constraints								X	X		X				X	X		
	X			X	X		X	X							X			

(continued on next page)

Table A1 (continued)

SELECTION CRITERIA	REFERENCE																	
	Jerman et al. (1978)	Coulter et al. (1989)	Bardi et al. (1999)	McGinnis (1990)	Abshire and Premeaux (1991)	Aguezzoul (2014)	Murphy et al. (1997)	Anderson et al. (2010)	Crum and Allen (1997)	Min (1998)	Lu (2003)	Danielis et al. (2005)	Kulak and Kahraman (2005)	Voss et al. (2006)	Mohamm aditabar and Teimoury (2008)	Coltman et al. (2011)	Peng (2012)	Williams et al. (2013)
Easy to make changes to dispatched cargo (flexible changes)																		
Service frequency (availability of service)	X		X								X			X				X
Frequent communication with carrier	X										X					X		X
Information technology for storage and transport							X	X			X						X	X
Low delivery error (reliable delivery)					X		X		X									X
Compatibility of strategies and company culture															X	X	X	

**Appendix B. Mathematical modelling**

To introduce the model, the following mathematical notations are used:

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<b>Index and sets</b>	
$i$	Index of distribution centres ( $i = 1, 2, \dots, CD$ )
$j$	Index of clients or small distribution centres ( $j = 1, 2, \dots, CL$ )
$k$	Types of vehicles TL ( $k = 1, \dots, K$ )
$s$	Range costs for LTL ( $s = 1, \dots, S$ )
$t$	Periods ( $t = 1, \dots, T$ )
$CD$	Number of distribution centres
$CL$	Number of clients
$K$	Number of types of vehicles for TL shipment
$S$	Number of price ranges for LTL shipment
$T$	Total of periods
<b>Parameters</b>	
$D_{jt}$	Average demand from client $j$ in period $t$
$d_j$	Average daily demand for client $j$
$\sigma_j^D$	Standard deviation of average demand for $j$
$c_{ijk}$	Transport cost from the distribution centre $i$ to client $j$ with type of dedicated vehicle $k$
$B_k$	Capacity of vehicle $k$
$L_{ij}^{FTL}$	Average lead time of transport from DC $i$ to client $j$ with TL shipment
$\sigma_{ij}^{FTL}$	Standard deviation of <i>lead time</i> from DC $i$ to client $j$ with TL shipment
$a_{ijs}$	Costs of single transport in cost range $s$ with LTL shipment to serve client $j$ from DC $i$
$p_s$	Breakpoint in cost range $s$ for LTL shipment
$L_{ij}^{LTL}$	Average lead time of transport from DC $i$ to client $j$ with LTL shipment
$\sigma_{ij}^{LTL}$	Standard deviation of lead time of transport from DC $i$ to client $j$ with LTL shipment
$h_j$	Single cost to maintain inventory in client $j$
$e_j$	Single cost to maintain inventory in transit for client $j$
$\alpha$	Desired level of service, that is, no mistakes during lead time
$\phi_\alpha$	Value of standard normal distribution so that $P(Z \leq \phi_\alpha) = \alpha$
$Q_i^{max}$	Maximum quantity allowed for transport from DC $i$ in week $t$
$G_j$	Maximum storage capacity for client $j$
$RM_{ijk}$	Indicates whether vehicle type $k$ may go from DC $i$ to serve client $j$ , that is, determines access restrictions
$R_{ij}^{FTL}$	Indicates whether client $j$ is a period away from being served with TL shipment from DC $i$
$R_{ij}^{LTL}$	Indicates whether client $j$ is a period away from being served with LTL shipment from DC $i$
$M$	Sufficiently large number
<b>Decision Variables</b>	
$X_{ijkt}$	Number of vehicle type $k$ sent in week $t$ from DC $i$ to serve client $j$
$Y_{ijst}$	Quantity sent in week $t$ in LTL shipment within cost range $s$ from DC $i$ to serve client $j$
$\overline{SS}_j$	Safety stock for client $j$
<b>Auxiliary Decision Variables</b>	
$Q_{jt}$	Quantity of goods received by client $j$ in week $t$
$I_{jt}$	Quantity of inventory in client $j$ at the end of period $t$
$W_{ijkt}$	Indicates whether or not vehicle type $k$ is sent in week $t$ from DC $i$ to serve client $j$ ( $W_{ijkt} = 1$ )
$SS_{jt}$	Determines the safety stock level for client $j$ in period $t$
$Z_{ijst}$	Indicates whether or not an LTL shipment within cost range $s$ is used in week $t$ from DC $i$ to serve client $j$ ( $Z_{ijst} = 1$ )

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The multi-objective mixed integer linear (MIP) mathematical formulation of the problem is given by:

$$\text{Min F1} = \sum_{i \in CD} \sum_{j \in CL} \sum_{k \in K} \sum_{t \in T} c_{ijk} X_{ijkt} + \sum_{i \in CD} \sum_{j \in CL} \sum_{s \in S} \sum_{t \in T} a_{ijs} Y_{ijst} \tag{1.a}$$

$$\text{Min F2} = \sum_{j \in CL} \sum_{t \in T} h_j \overline{SS}_j + \sum_{j \in CL} \sum_{t \in T} h_j I_{jt} \tag{1.b}$$

$$\text{Min F3} = \sum_{i \in CL} \sum_{j \in CL} \sum_{t \in T} e_j \left( L_{ij}^{FTL} \sum_{k \in K} (X_{ijkt} B_k) + L_{ij}^{LTL} \sum_{s \in S} Y_{ijst} \right) \tag{1.c}$$

The objective function F1 minimizes the transport cost involved (1.a), namely LTL and TL transportation. The objective function F2 minimizes the cost of regular inventory and the safety inventory costs (1.b), and the objective function F3 minimizes the cost of inventory in transit (1.c).

Subject to:

$$I_{jt-1} + Q_{jt} - D_{jt} = I_{jt} \quad \forall j \in CL, t \in T \tag{2}$$

$$Q_{jt} = \sum_i \left( \left( 1 - R_{ij}^{FTL} \right) \sum_{k \in K} X_{ijkt-1} B_k RM_{ijk} + \left( 1 - T_{ij}^{LTL} \right) \sum_{s \in S} Y_{ijst-1} \right) + \sum_i \left( R_{ij}^{FTL} \sum_{k \in K} X_{ijkt} B_k RM_{ijk} + R_{ij}^{LTL} \sum_{s \in S} Y_{ijst} \right) \quad \forall j \in CL, t \in T \tag{3}$$

$$\sum_{j \in CL} \left( \sum_{k \in K} (X_{ijkt} B_k RM_{ijk}) + \sum_{s \in S} Y_{ijst} \right) \leq Q_i^{max} \quad \forall i \in CD, t \in T \tag{4}$$

$$\overline{SS}_j + I_{jt-1} + Q_{jt} \leq G_j \quad \forall j \in CL, t \in T \tag{5}$$

$$ss_{jt} \geq \phi_\alpha W_{ijkt} \sqrt{L_{ij}^{FTL} (\sigma_j^D)^2 + (d_j \sigma_{ij}^{FTL})^2} \quad \forall i \in CD, j \in CL, k \in K \tag{6}$$

$$ss_{jt} \geq \phi_\alpha Z_{ijst} \sqrt{L_{ij}^{LTL} (\sigma_j^D)^2 + (d_j \sigma_{ij}^{LTL})^2} \quad \forall i \in CD, j \in CL, s \in S, t \in T \tag{7}$$

$$\overline{SS}_j = \frac{1}{T} \sum_{t \in T} ss_{jt} \quad \forall j \in CL \tag{8}$$

$$X_{ijkt} \leq W_{ijkt} M \quad \forall i \in CD, j \in CL, k \in K, t \in T \tag{9}$$

$$Y_{ijst} \leq Z_{ijst} M \quad \forall i \in CD, j \in CL, s \in S, t \in T \tag{10}$$

$$p_s Z_{ijst} \leq Y_{ijst} \quad \forall i \in CD, j \in CL, s \in S, t \in T \tag{11}$$

$$X_{ijkt} \in \mathbb{Z}^+ \quad \forall i \in CD, j \in CL, k \in K, t \in T \tag{12}$$

$$W_{ijkt}, Z_{ijst} \in \{0, 1\} \quad \forall i \in CD, j \in CL, k \in K, s \in S, t \in T \tag{13}$$

$$Y_{ijst}, Q_{jt}, I_{jt}, ss_{jt}, \overline{SS}_j \in \mathbb{R}^+ \quad \forall i \in CD, j \in CL, s \in S, t \in T \tag{14}$$

Constraints (2) corresponds to inventory balance and ensures that the demand is met by delivered or stored goods. Constraints (3) determines the quantities received for each distribution centre considering all shipments available. The first term represents the quantities shipped during the period prior to TL and LTL shipments since the transportation lead time is greater than one period, i.e. these quantities will only be available in the current period. The second term represents the quantities shipped during the current period for both types of shipments and that will be available to be used within the same period. These constraints also consider access limitations to TL vehicles from distribution centres to clients located in cities with access restrictions to large vehicles, that is, they include the  $RM_{ijk}$  parameter.

Constraints (4) ensure that the capacity of the distribution centres is considered. On the other hand, constraints (5) limit the quantities kept by the clients given their storage capacity.

Constraints (6) and (7) together determine the maximum safety stock levels to be kept within the period in case different types of shipments may be used or if the client is served by different distribution centres. In these constraints, although the parameters are included in the root, the decision variables are located outside as a type of activator to avoid non-linear representations. The safety stock levels for each client are defined as an average for each period, as shown in equation (8).

Constraints (6) and (7) are based an advanced push inventory control model, specifically the method of order point with uncertainty in demand and delivery time. Adding demand variance to the delivery variance results in a revised formula for the safety stock, as presented by Ballou (1999, p. 334).

Constraints (9) and (10) prevent TL and LTL loading, respectively, are used in case no quantity is attributed to them. These restrictions make  $W_{ijkt} = 1$  and  $Z_{ijst} = 1$  when the defined loads are shipped from the specified distribution centre to the stated client within the specified week.

Constraints (11) guarantee that the LTL shipped quantities are at least in one of the cost ranges and that a determined quantity must be greater than the range breakpoint as defined in the all-units discount structure. Finally, constraints (12)–(14) define the domain of the decision variables.

An integrated objective function (F4) was also used to generate multiple solutions (eq. (15)). Note that these solutions are presented by the alternatives in the approach:

$$\text{Min F4} = \frac{\omega_1 (F_1 - F_1^*)}{F_1^*} + \frac{\omega_2 (F_2 - F_2^*)}{F_2^*} + \frac{\omega_3 (F_3 - F_3^*)}{F_3^*} \tag{15}$$

where  $\omega_1, \omega_2$  and  $\omega_3$  are the relative weights of each objective function, which can take values between 1 (maximum priority) and 0 (no priority) at the discretion of the DMs for the generation of alternatives. Here,  $F_1^*, F_2^*$  and  $F_3^*$  are the optimal values of each objective function. Thus, equation (15) minimizes the weighted sum of the percentage deviations of the proposed objectives.

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