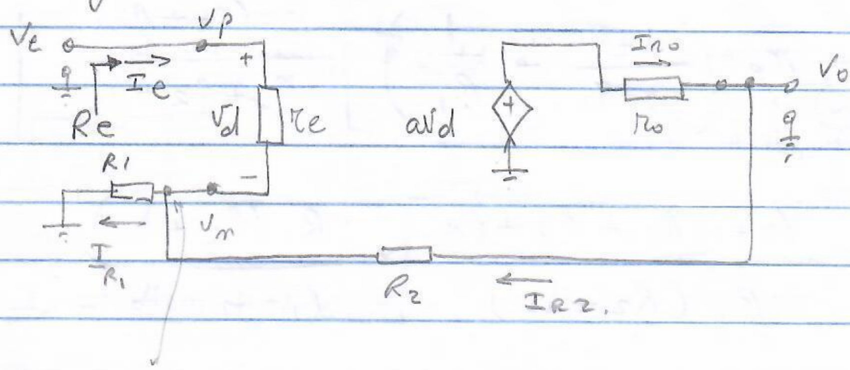
OP-AMP Ñ IDEAL.

a) Configuração amplificador não inversor:



$R_e = ?$

$R_o = ?$

$A = ?$

$V_n = V_e - V_d$

$V_p = V_e$

$V_d = r_e I_e$

$$I_e + I_{R2} = I_{R1}$$

$$I_{R1} = \frac{V_n}{R_1} = \frac{V_e - V_d}{R_1} = \frac{V_e}{R_1} - \frac{r_e I_e}{R_1}$$

$$I_{R_2} = \frac{V_o - V_n}{R_2} = \frac{V_o}{R_2} - \frac{V_e - V_d}{R_2} = \frac{V_o}{R_2} - \frac{V_e}{R_2} + \frac{V_d}{R_2}$$

$$V_o = aV_d - r_o I_{R2}$$

$$I_{R} = \frac{aV_d - r_o I_{R2}}{R_2} - \frac{V_e}{R_2} + \frac{r_e I_e}{R_2} = \frac{a r_e I_e}{R_2} - \underbrace{\frac{r_o I_{R2}}{R_2}} - \frac{V_e}{R_2} + \frac{r_e I_e}{R_2}$$

$$I_{R_2} + \frac{r_o}{R_2} I_{R2} = I_{R2}\left(1 + \frac{r_o}{R_2}\right) = \frac{a r_e I_e}{R_2} - \frac{V_e}{R_2} + \frac{r_e I_e}{R_2}$$

$$I_{R2}\left(1 + \frac{r_o}{R_2}\right) = \left(\frac{a r_e + r_e}{R_2}\right) I_e - \frac{V_e}{R_2}$$

Assim: $$I_e + \left[\frac{a r_e + r_e}{R_2} I_e - \frac{V_e}{R_2}\right] \frac{1}{\left(1 + \frac{r_o}{R_2}\right)} = \frac{V_e}{R_1} - \frac{r_e I_e}{R_1}$$

$$I_e + \frac{(a+1) r_e}{\cancel{R_2}} I_e \frac{\cancel{R_2}}{R_2 + r_o} + \frac{r_e}{R_1} I_e = \frac{V_e}{R_1} + \frac{\cancel{R_2}}{\cancel{R_2}(R_2 + r_o)} V_e$$

$$I_e \left[1 + \frac{(a+1) r_e}{R_2 + r_o} + \frac{r_e}{R_1}\right] = \left(\frac{1}{R_1} + \frac{1}{R_2 + r_o}\right) V_e = \frac{R_2 + r_o + R_1}{R_1 (R_2 + r_o)} V_e$$

$$\frac{V_e}{I_e} = R_e = \frac{1}{\dfrac{R_2+R_1+r_0}{R_1(R_2+r_0)}} \times \left[1 + \frac{r_e(a+1)}{R_2+r_0} + \frac{r_e}{R_1}\right]$$

$$R_e = \left[\left[1 + r_e\left(\frac{a+1}{R_2+r_0} + \frac{1}{R_1}\right)\right]\frac{R_1(R_2+r_0)}{R_1+R_2+r_0}\right] =$$

$$= \left[1 + r_e\frac{(a+1)R_1 + R_2 + r_0}{R_1(R_2+r_0)}\right] \times \frac{R_1(R_2+r_0)}{R_1+R_2+r_0}$$

$$R_e = \frac{R_1(R_2+r_0)}{R_1+R_2+r_0} + \frac{\cancel{R_1(R_2+r_0)}}{R_1+R_2+r_0} \times \frac{(a+1)R_1+R_2+r_0}{\cancel{R_1(R_2+r_0)}} \times r_e$$

$$R_e = r_e\frac{(a+1)R_1+R_2+r_0}{R_1+R_2+r_0} + \frac{R_1(R_2+r_0)}{R_1+R_2+r_0}$$

$$R_e = r_e\frac{aR_1 + (R_1+R_2+r_0)}{R_1+R_2+r_0} + \frac{R_1(R_2+r_0)}{R_1+R_2+r_0} =$$

$$= r_e\left(\frac{aR_1}{R_1+R_2+r_0} + \cancel{\frac{R_1+R_2+r_0}{R_1+R_2+r_0}}\right) + \frac{R_1(R_2+r_0)}{R_1+R_2+r_0} =$$

$$\boxed{= r_e\left(1 + \frac{aR_1}{R_1+R_2+r_0}\right) + \frac{R_1(R_2+r_0)}{R_1+R_2+r_0}}$$

Se $r_0 << R_d$ e $<< R_1+R_2 \Rightarrow$

$$R_e = r_e\left(1 + \frac{aR_1}{R_1+R_2}\right) + \frac{R_1(R_2+r_0)}{R_1+R_2}.$$

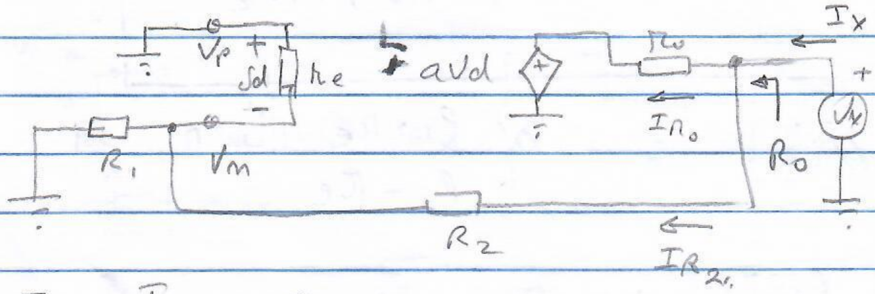
e se $R_1R_2 >> R_1r_0 \Rightarrow R_e = r_e\left(1 + \dfrac{aR_1}{R_1+R_2}\right) + \dfrac{R_1R_2}{R_1+R_2}$

se $R_1 // R_2 << R_d$ e $b = \dfrac{R_1}{R_1+R_2} \Rightarrow \boxed{R_e = r_e(1+ab)}$

$a \to \infty \Rightarrow ab \to \infty \Rightarrow \boxed{R_e \to \infty}$

$R_o = ?$

Para se obter R_o, aterra-se a entrada e aplica-se uma tensão de teste, V_x, aos terminais de saída:



$$I_x = I_{r_o} + I_{R_2}$$

$$I_{r_o} = \frac{V_x - aV_d}{r_o} \quad ; \quad I_{R_2} = \frac{V_x - V_m}{R_2}$$

$$I_x = \frac{V_x}{r_o} - \frac{aV_d}{r_o} + \frac{V_x}{R_2} - \frac{V_m}{R_2} \qquad V_d = -V_m.$$

$$V_m = \frac{(R_1 // r_e)}{(R_1 // r_e) + R_2} V_x \Longrightarrow$$

$$I_x = \frac{V_x}{r_o} + \frac{aV_m}{r_o} + \frac{V_x}{R_2} - \frac{V_m}{R_2} =$$

$$= \frac{V_x}{r_o} + \frac{a(R_1 // r_e)}{[(R_1 // r_e) + R_2]} \frac{V_x}{r_o} + \frac{V_x}{R_2} - \frac{(R_1 // r_e)}{(R_1 // r_e) + R_2} \frac{V_x}{R_2}.$$

$$\frac{1}{R_2} - \frac{(R_1 // r_e)}{(R_1 // r_e) + R_2} \times \frac{1}{R_2} = \frac{\cancel{(R_1 // r_e)} + R_2 - \cancel{(R_1 // r_e)}}{[(R_1 // r_e) + R_2] R_2} =$$

$$= \frac{\cancel{R_2}}{[(R_1 // r_e) + R_2] \cancel{R_2}} = \frac{1}{(R_1 // r_e) + R_2}.$$

$$\therefore \quad I_x = V_x \left[\frac{1}{r_o} + \frac{a(R_1 // r_e)}{[(R_1 // r_e) + R_2] r_o} + \frac{1}{(R_1 // r_e) + R_2} \right]$$

$$I_x = V_x \frac{1}{r_o} \left[1 + \frac{a(R_1 // r_e)}{(R_1 // r_e) + R_2} + \frac{r_o}{(R_1 // r_e) + R_2} \right].$$

$$I_x = V_x \frac{1}{r_o} \left[1 + \frac{a(R_1 // r_e) + r_o}{(R_1 // r_e) + R_2} \right]$$

$$\boxed{\frac{V_X}{I_X} = R_o = \frac{r_o}{1 + \dfrac{a(R_1 // r_e) + r_o}{(R_1 // r_e) + R_2}}}$$

$$a(R_1 // r_e) + r_o = a\frac{R_1 \times r_e}{R_1 + r_e} + r_o.$$

Se $a\dfrac{R_1 \times r_e}{R_1 + r_e} >> r_o \Rightarrow$

$$R_o = \frac{r_o}{1 + \dfrac{a(R_1 // r_e)}{(R_1 // r_e) + R_2}}$$

Se $R_1 << r_e \Rightarrow R_o = \dfrac{r_o}{1 + \dfrac{aR_1}{R_1 + R_2}}$

$$\frac{R_1}{R_1 + R_2} = b$$

$$\Rightarrow \boxed{R_o = \frac{r_o}{1 + ab}}$$

$ab \to \infty \Rightarrow R_o \to 0 \quad (R_o = 0)$

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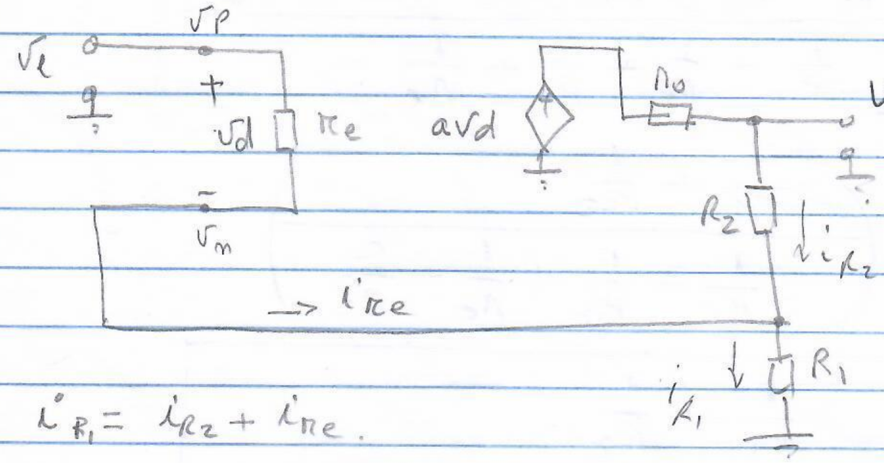
A ?



$i_{R_1} = i_{R2} + i_{re}$.

$i_{R1} = \frac{v_m}{R_1}$; $i_{R2} = \frac{v_o - v_m}{R_2}$; $i_e = \frac{v_p - v_m}{r_e}$

$\frac{v_m}{R_1} = \frac{v_o - v_m}{R_2} + \frac{v_p - v_m}{r_e}$. $\quad v_p = v_e$.

$\frac{v_m}{R_1} = \frac{v_o}{R_2} - \frac{v_m}{R_2} + \frac{v_e}{r_e} - \frac{v_m}{r_e}$.

$\hookrightarrow \frac{a(v_p - v_m) - i_2 r_o}{R_2} - \frac{v_m}{R_2}$

Para $r_o << R_2 \Rightarrow \frac{v_m}{R_1} = \frac{av_e}{R_2} - \frac{av_m}{R_2} + \frac{v_e}{r_e} - \frac{v_m}{r_e}$

$v_m\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{r_e} + \frac{a}{R_2}\right) = v_e\left(\frac{1}{r_e} + \frac{a}{R_2}\right)$

$v_m = v_e\left(\frac{1}{r_e} + \frac{a}{R_2}\right) \times \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{a}{R_2} + \frac{1}{r_e}}$

$V_o = a(v_p - v_m) = av_e - a\,v_e\left(\frac{1}{r_e} + \frac{a}{R_2}\right) \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{a}{R_2} + \frac{1}{r_e}}$

$A_V = \frac{V_o}{V_e} = a\left[1 - \left(\frac{1}{r_e} + \frac{a}{R_2}\right) \times \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{a}{R_2} + \frac{1}{r_e}}\right]$

$$A_v = a\left[\frac{\frac{1}{R_1}+\frac{1}{R_2}+\cancel{\frac{a}{R_2}}+\cancel{\frac{1}{Re}}-\cancel{\frac{1}{Re}}-\cancel{\frac{a}{R_2}}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{a}{R_2}+\frac{1}{Re}}\right]$$

$$A_v = a\left(\frac{\frac{1}{R_1}+\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{Re}+\frac{a}{R_2}}\right)$$

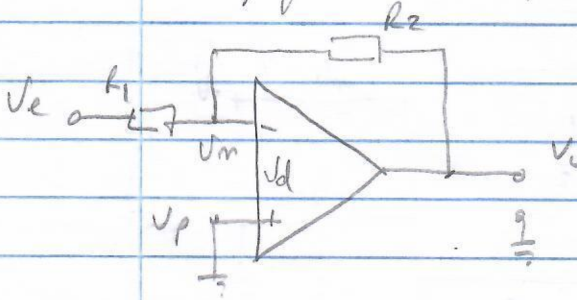
$$\boxed{A_v = \frac{\frac{1}{R_1}+\frac{1}{R_2}}{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{Re}\right)\frac{1}{a}+\frac{1}{R_2}}}$$

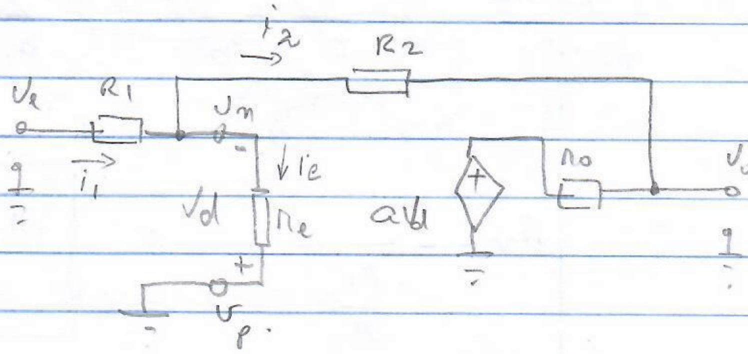
Se $a \to \infty \Rightarrow A_v = \dfrac{\frac{1}{R_1}+\frac{1}{R_2}}{\frac{1}{R_2}} = \dfrac{\frac{R_1+R_2}{R_1R_2}}{\frac{1}{R_2}}$

$$A_v = \frac{R_1+R_2}{R_1\cancel{R_2}}\times\cancel{R_2} = \frac{R_1+R_2}{R_1}$$

$$\boxed{A_v = \left(1+\frac{R_2}{R_1}\right)}$$

b) Configuração amplificador inversor:





$V_d = v_p - v_n = -v_n$.

$A_v = ?$

$i_1 = i_2 + i_e \qquad i_1 = \dfrac{V_e - V_n}{R_1}\ ;\ i_e = \dfrac{v_n}{r_e}\ ;$

$$i_2 = \frac{V_n - V_o}{R_2}.$$

Para $r_o << R_2 \Rightarrow V_o = aV_d = a(v_p - v_n) = -a v_n$.

$$\text{e} \quad i_2 = \frac{v_n - (-a v_n)}{R_2} = \frac{v_n(1+a)}{R_2}.$$

$$\frac{V_e - v_n}{R_1} = \frac{v_n(1+a)}{R_2} + \frac{v_n}{r_e} = \frac{V_e}{R_1} - \frac{v_n}{R_1}$$

$$v_n\left(\frac{1}{R_1} + \frac{1}{r_e} + \frac{(1+a)}{R_2}\right) = \frac{V_e}{R_1}$$

$$V_n = \frac{V_e}{R_1} \times \frac{1}{\frac{1}{R_1} + \frac{1}{r_e} + \frac{1+a}{R_2}} = V_e \frac{1}{1 + \frac{R_1}{r_e} + \frac{(1+a)R_1}{R_2}}$$

$$V_o = -aV_n = -\frac{aV_e}{1 + \frac{R_1}{r_e} + \frac{(1+a)R_1}{R_2}}$$

$$\boxed{A_v = \frac{V_o}{V_e} = -\frac{a}{1 + \frac{R_1}{r_e} + \frac{(1+a)R_1}{R_2}}}$$

Para $a \to \infty \Rightarrow$

$$A_V = -\frac{1}{\cancel{\frac{1}{a}} + \cancel{\frac{R_1}{a r_e}} + \cancel{\frac{(1+a)}{a}}\frac{R_1}{R_2}}$$

$$\underset{0}{} \quad \underset{0}{} \quad \underset{1}{}$$

$$A_V = -\frac{1}{\frac{R_1}{R_2}} \quad \therefore \quad \boxed{A_V = -\frac{R_2}{R_1}}$$

$$R_e = ? \qquad R_e = \frac{V_e}{i_e} = \frac{V_e}{i_1}$$

$$i_1 = \frac{V_e - V_m}{R_1} = i_e$$

$$i_e = \frac{V_e - \frac{V_e}{1 + \frac{R_1}{r_e} + (1+a)\frac{R_1}{R_2}}}{R_1} = \frac{V_e}{R_1}\left[1 - \frac{1}{1 + \frac{R_1}{r_e} + (1+a)\frac{R_1}{R_2}}\right]$$

$$i_e = \frac{V_e}{R_1}\left[\frac{\cancel{1} + \frac{R_1}{r_e} + (1+a)\frac{R_1}{R_2} - \cancel{1}}{1 + \frac{R_1}{r_e} + (1+a)\frac{R_1}{R_2}}\right] = \frac{V_e}{R_1}\,\frac{\frac{R_1}{r_e} + \frac{(1+a)R_1}{R_2}}{1 + \frac{R_1}{r_e} + (1+a)\frac{R_1}{R_2}}$$

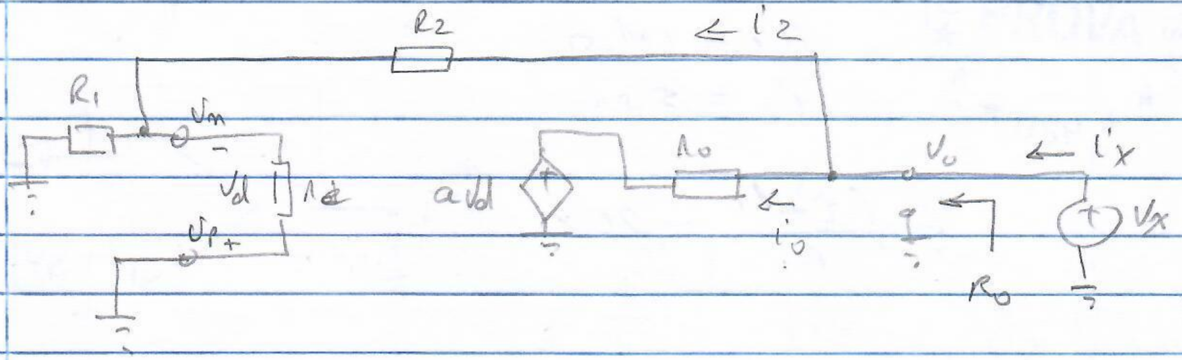
$$R_e = \frac{V_e}{i_e} = \frac{\cancel{V_e}\,R_1\left(1 + \frac{R_1}{r_e} + \frac{(1+a)R_1}{R_2}\right)}{\cancel{V_e}\left(\frac{R_1}{r_e} + \frac{(1+a)R_1}{R_2}\right)} = R_1\frac{1 + \frac{R_1}{r_e} + \frac{(1+a)R_1}{R_2}}{\frac{R_1}{r_e} + \frac{(1+a)R_1}{R_2}}$$

$$R_e = \frac{\frac{1}{a} + \frac{R_1}{a r_e} + \frac{(1+a)}{a}\frac{R_1}{R_2}}{\frac{R_1}{a r_e} + \frac{(1+a)}{a}\frac{R_1}{R_2}}\,R_1$$

Para $a \to \infty \Rightarrow R_e = \frac{\cancel{\frac{R_1}{R_2}}}{\cancel{\frac{R_1}{R_2}}}\,R_1$

e $\boxed{R_e = R_1}$

R_o ?



$V_e = 0$

$$V_n = \frac{R_1}{R_1+R_2} V_x \quad \text{e} \quad i_x = i_2 + i_o$$

$$i_2 = \frac{V_x - V_n}{R_2} \quad \text{e} \quad i_o = \frac{V_x - aV_d}{r_o} = \frac{V_x - a(0 - V_n)}{r_o}$$

$$i_2 = \frac{V_x}{R_2} - \frac{1}{R_2}\frac{R_1}{R_1+R_2} V_x$$

$r_d >> R_1$

$$i_x = \frac{V_x}{R_2} - \frac{1}{R_2}\frac{R_1}{R_1+R_2}V_x + \frac{V_x}{r_o} - \frac{a}{r_o}(0 - V_n)$$

$$i_x = \frac{V_x}{R_2} - \frac{1}{R_2}\frac{R_1}{R_1+R_2}V_x + \frac{V_x}{r_o} + \frac{a}{r_o}\frac{R_1}{R_1+R_2}V_x$$

$$i_x = V_x\left(\frac{1}{R_2} - \frac{R_1}{R_2(R_1+R_2)} + \frac{1}{r_o} + \frac{a}{r_o}\frac{R_1}{R_1+R_2}\right)$$

$$R_o = \frac{V_x}{i_x} = \frac{\cancel{V_x}}{\cancel{V_x}\left(\frac{1}{R_2} - \frac{R_1}{R_2(R_1+R_2)} + \frac{1}{r_o} - \frac{a}{r_o}\frac{R_1}{R_1+R_2}\right)} =$$

$$= \frac{1}{\frac{R_1}{R_1+R_2}\left(\frac{1}{R_2} - \frac{a}{r_o}\right) + \frac{1}{r_o}(1-a) + \frac{1}{R_2}}$$

Para $a \to \infty \Rightarrow R_o \to 0$.